Modeling response uncertainty
Distances: multi-dimensional scaling

Calculate Euclidean distance

\[ d_{ij} = \sqrt{(x_i - x_j)^T (x_i - x_j)} \]
MDS summary

matrix of all Earth models $X = [x_1, x_2, \ldots, x_L]^T$

$\Downarrow$

Euclidean distance matrix $A$

$\Downarrow$

Dot-product $B = XX^T = V \Lambda V^T$

$\Downarrow$

Reconstruction $X = V \Lambda^{1/2}$
Example: Earth models

Created 405 2D channel Earth models with different proportion, direction, width
Basic questions

What is the uncertainty in Oil production over time in terms of [P10, P90]?

What are the most influencing factors?

True response takes too long to evaluate on 405 models.

We have a proxy response
Multi-dimensional scaling in MATLAB

- Calculating distances
  - `dvector = pdist(response)`
  - `d = squareform(dvector)`

- Basic command for MDS
  - `[Y e] = cmdscale(d)`

- Plotting
  - `scatter` for 2D plots
  - `scatter3` for 3D plots
  - `plot_MDS_response_value` for adding a color that corresponds to some (single) response value
Kernel transformation

Distance

<table>
<thead>
<tr>
<th>D</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>1</td>
<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>$\sqrt{2}$</td>
<td>1</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
<td>$\sqrt{2}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
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</tbody>
</table>

New distance

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<th>3</th>
<th>4</th>
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<td>0.86</td>
<td>0.94</td>
</tr>
<tr>
<td>2</td>
<td>0.86</td>
<td>0</td>
<td>0.94</td>
<td>0.86</td>
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<tr>
<td>3</td>
<td>0.86</td>
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<tr>
<td>4</td>
<td>0.94</td>
<td>0.86</td>
<td>0.86</td>
<td>0</td>
</tr>
</tbody>
</table>

$k = 1 - \exp(-d)$

$\lambda_1 = \lambda_2 = 1$
$\lambda_3 = \lambda_4 = 0$

$\lambda_1 = \lambda_2 = 0.44$
$\lambda_3 = 0.31$
$\lambda_4 = 0$
Kernel transformation

Transformation from one metric space to another does not require knowledge of $\varphi$, only knowledge of the dot-product $\varphi^T \varphi$

$$\varphi^T (x) \varphi(y) = k(x, y)$$

**example**

$$k(x, y) = \exp\left(-\frac{(x - y)^T (x - y)}{\sigma}\right)$$

Euclidean distance obtained with MDS

**Role of the Kernel**
Increase dimension, separability and linearity
Command

- Radial basis kernel
- Input = coordinates after projection with MDS
- Output = Kernel matrix K

K=rbf_kernel(coord)
Clustering Earth models

1. Multiple Earth models
2. Definition of a distance
3. Scatter plot of data points
4. Clustering result
5. Visualization of clusters over time
6. Final clustered data representation
7. Response over time graph
Script

- `kmedoids.m`

- Input (in script)
  - Number of clusters
  - Number of iterations
Response calculation

Response of 7 selected Earth models

Calculated P10, P50 and P90
Command

- Quantiles_Calculation

- Input
  - Responses
  - Vector containing percentages corresponding to the quantiles you want to calculate, e.g. [0.10 0.50 0.90]

- Output: quantiles of the responses
Command

- Sensitivity_Analysis_Classical

Input
- Parameter values (experimental design indicators)
- Responses belong to those parameter values
- Parameter names
- Flag variable to indicate whether you want to study interaction

Output
- Effect estimates for all parameters