Rock Failure

Topics

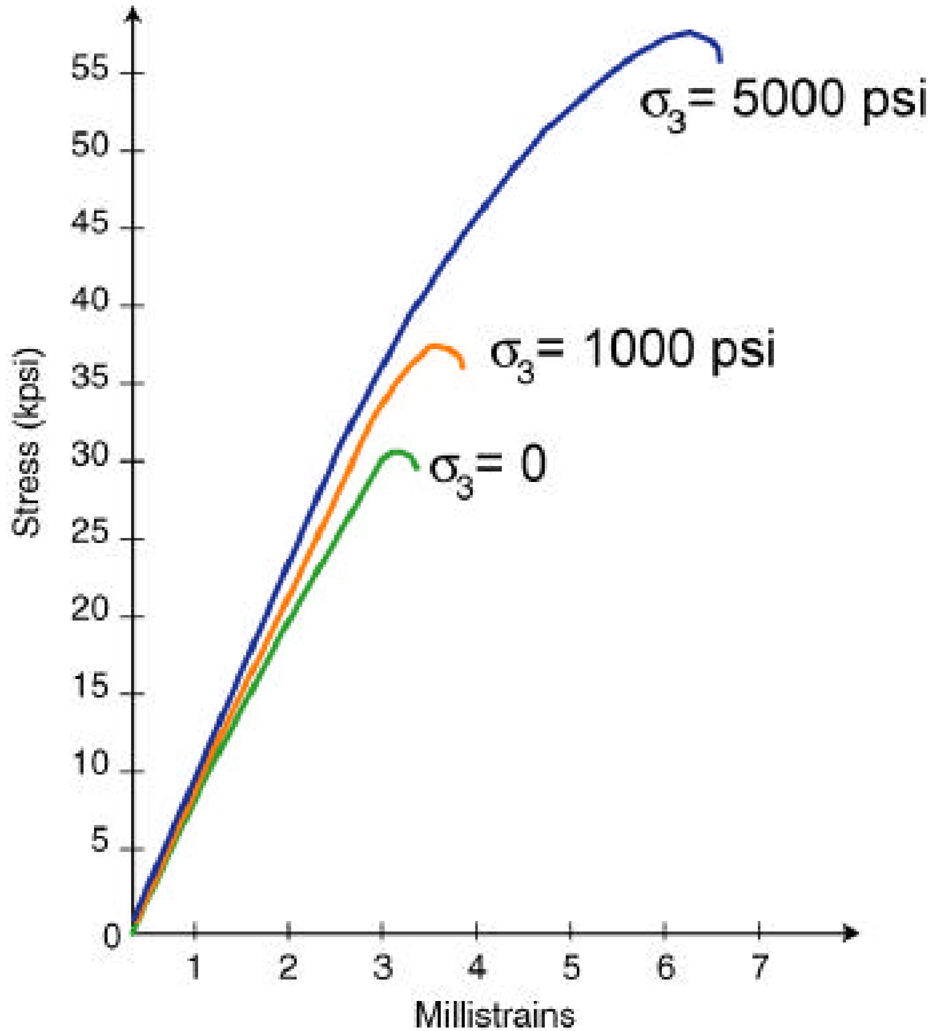
- Compressive Strength
- Rock Strength from Logs
- Polyaxial Strength Criteria
- Anisotropic Rock Strength
- Tensile Strength

Key Points

1. When rock fails in compression, the compressive stress exceeds the compressive strength.
2. Even poorly-consolidated sands are characterized by a high coefficient of internal friction. Weak rocks are weak because they lack cohesion.
3. Intermediate principal stress (S_2) probably influences compressive strength only at very great depth.
4. Complex failure criterion are unnecessarily complicated and difficult to use. Strong rocks are strong. Weak rocks are not.
5. Are log-based strength estimates valid? Sometimes, especially if calibrated!
6. Under special circumstances, anisotropic rock strength (finely laminated shales) can be important, especially for wellbore stability studies
7. Tensile strength of rock is extremely low and can generally be assumed to be zero.
8. John Jaeger, the god of rock mechanics (if one exists) said there is only two things you need to know about friction – it is always 0.6, and it will always make a monkey out of you.
9. The frictional strength of faults in Earth's crust limits the maximum difference between stress magnitudes.

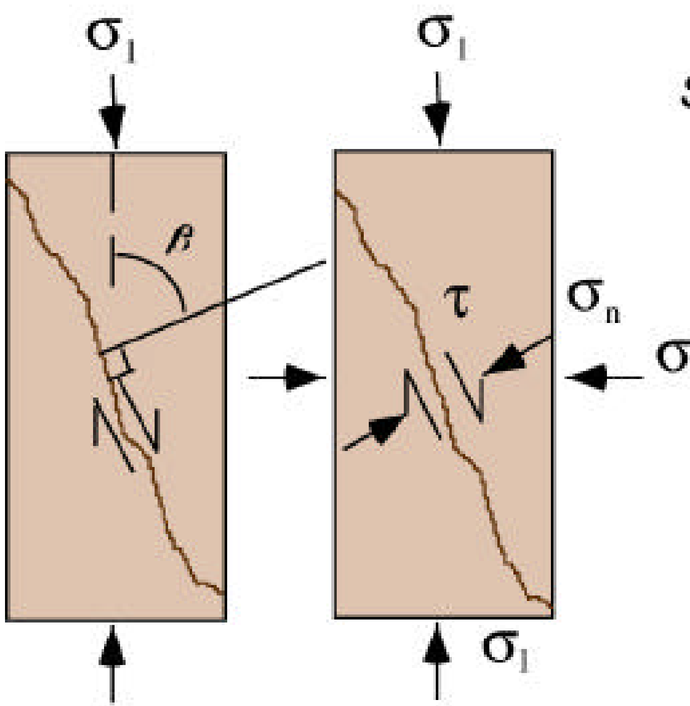
Stress-Strain Curves for Rand Quartzite

Strength Depends on Confining Pressure

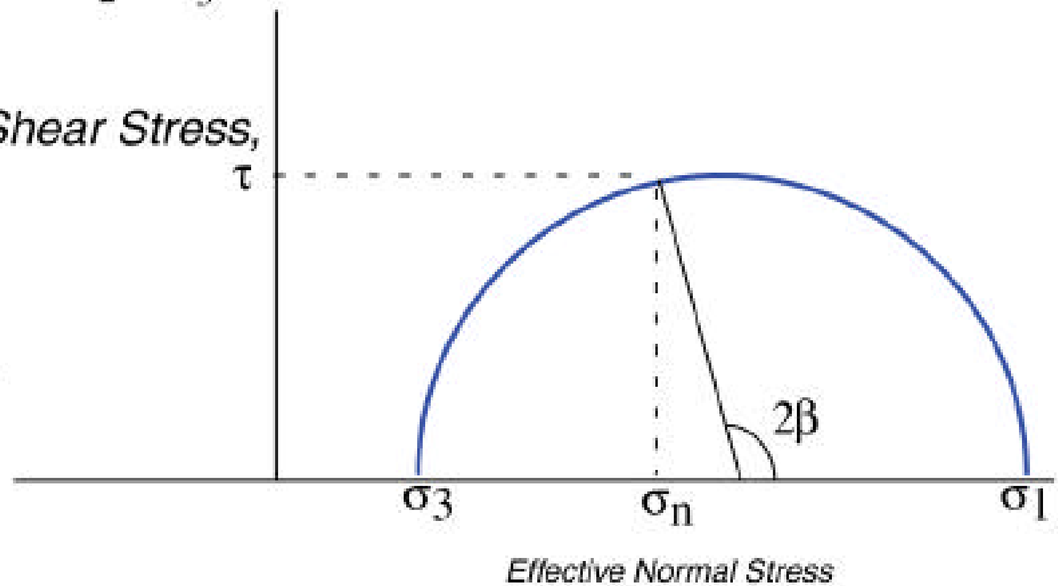


Mohr Circles in Two Dimensions

Describes the stress on a plane at failure. Assumes σ_1, only depends on the magnitude of σ_3. Applicable to "triaxial' rock mechanics tests ($\sigma_2 = \sigma_3$).



Triaxial Compression



$$\tau = -0.5\,(\sigma_1 - \sigma_3)\ \ sin 2\beta$$

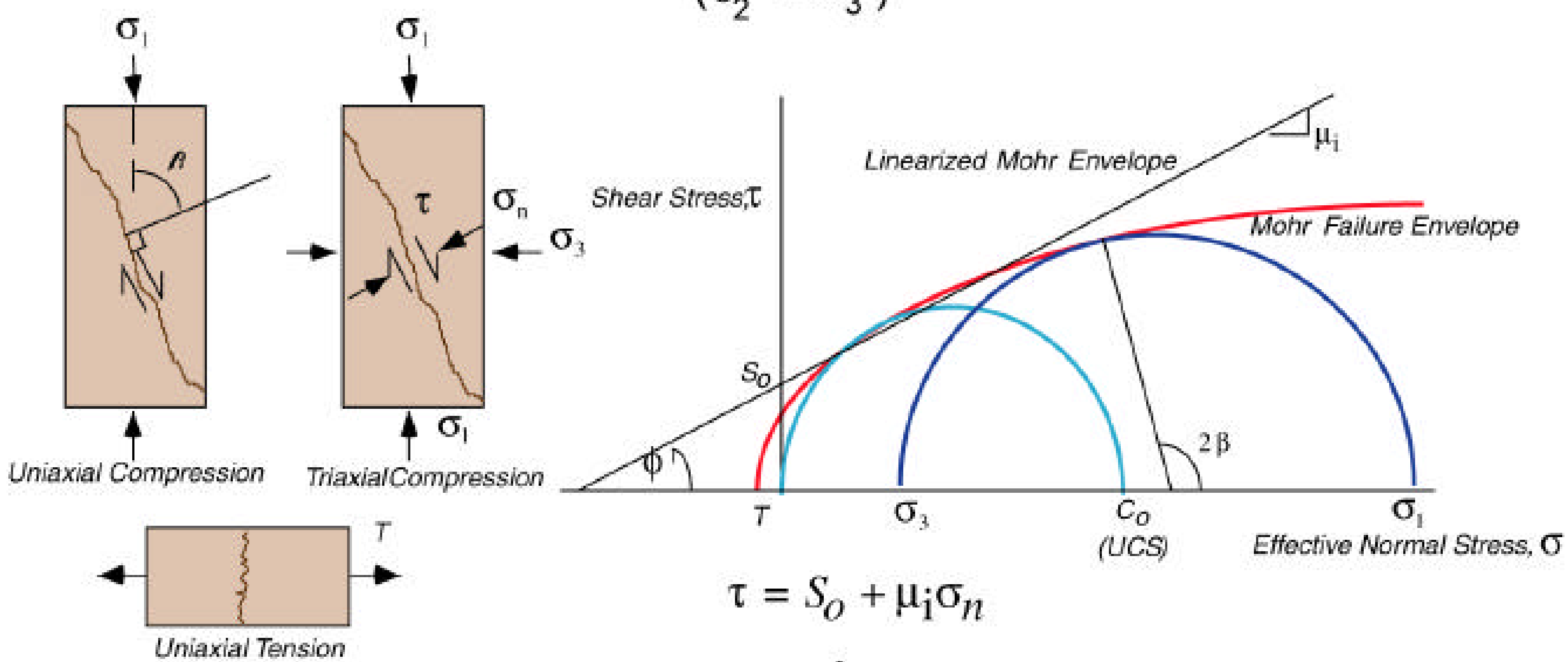
$$\sigma_n = 0.5\,(\sigma_1 + \sigma_3) + 0.5\,(\sigma_1 - \sigma_3)\ cos 2\beta$$

Mohr Envelope in Two Dimensions

Assumes σ_1, only depends on the magnitude of σ_3

Applicable to "triaxial" rock mechanics tests

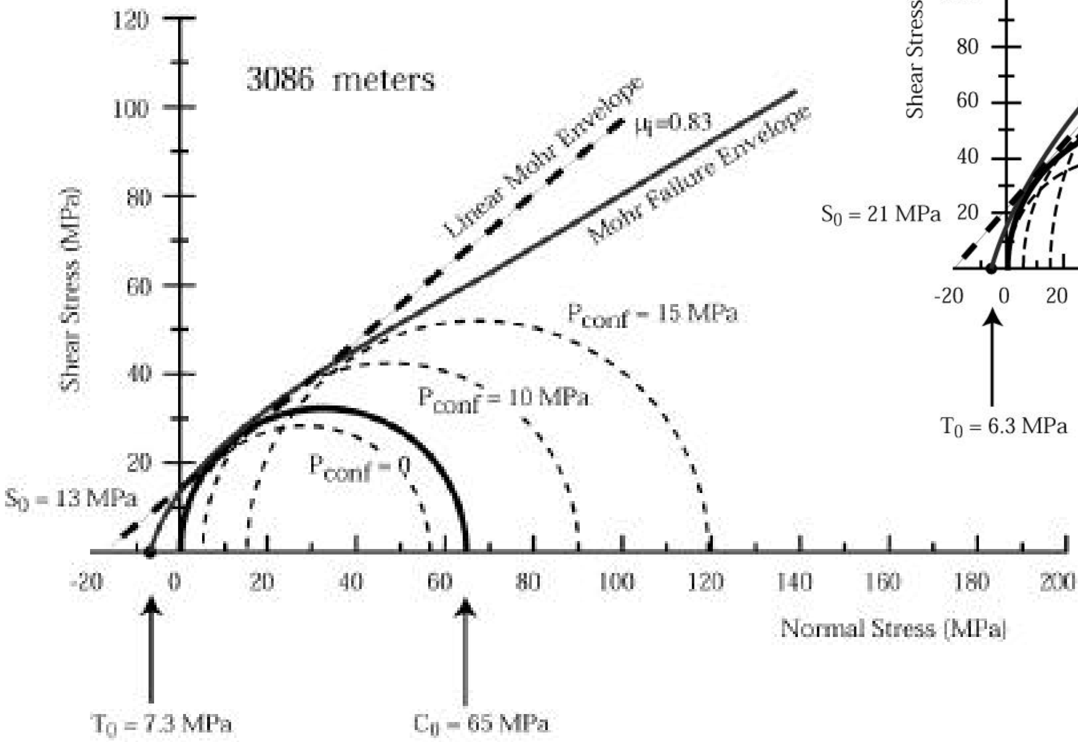
$(\sigma_2 = \sigma_3)$

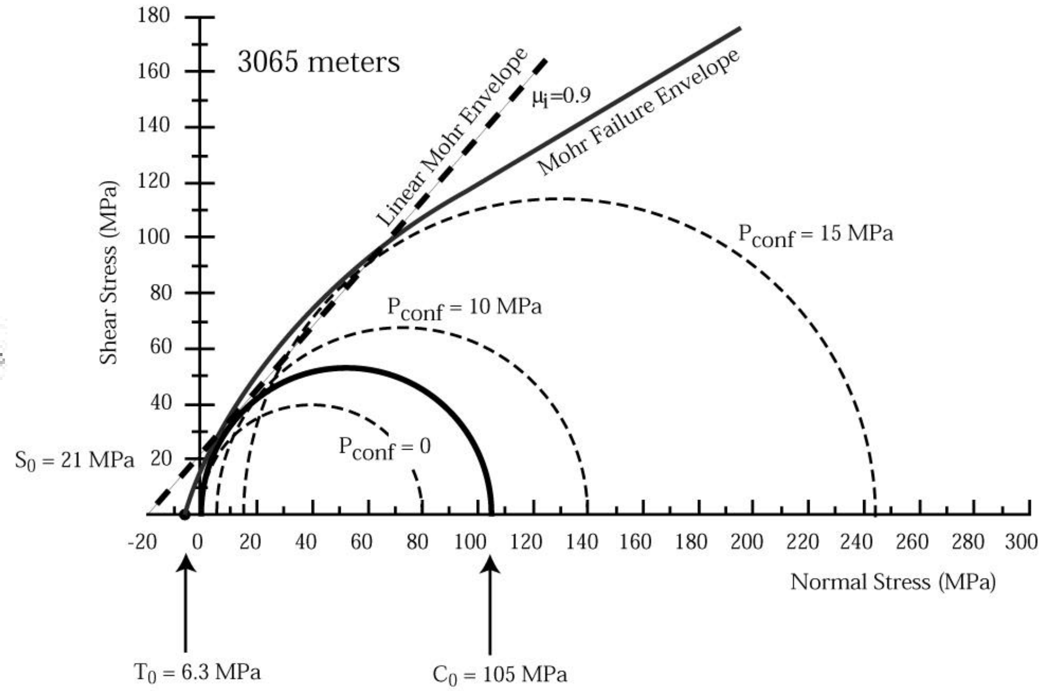


$$\tau = S_o + \mu_i \sigma_n$$

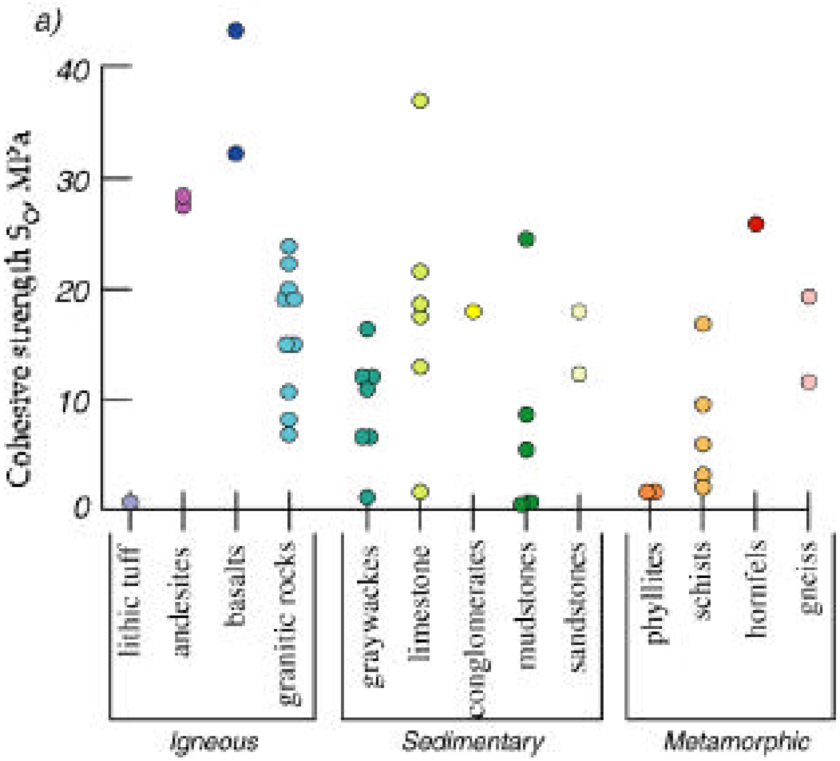
$$C_o = 2\,S_o\,[\,(\mu_i^2 + 1)^{1/2} + \mu_i\,]$$

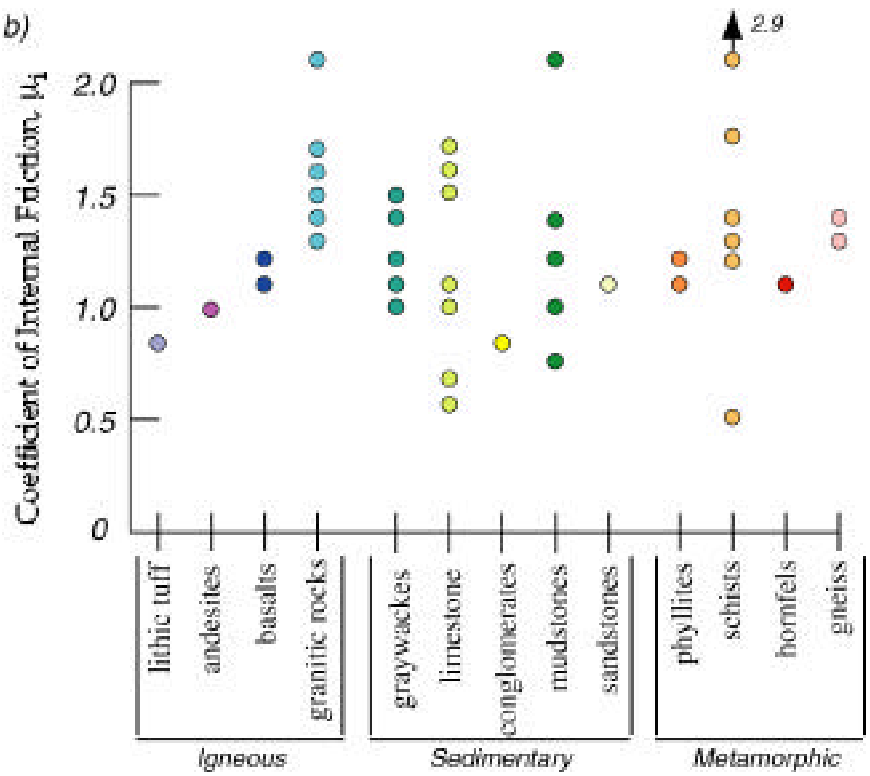
Rock Strength Measurements





Strong Rocks/Weak Rocks

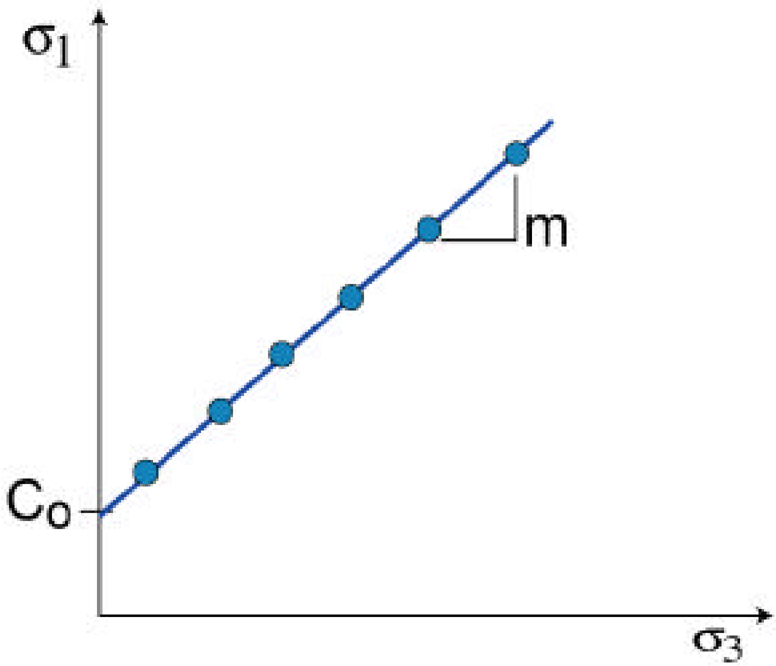




Weak rocks have low cohesion

Practical Guide to Determination of C_o and μ_i

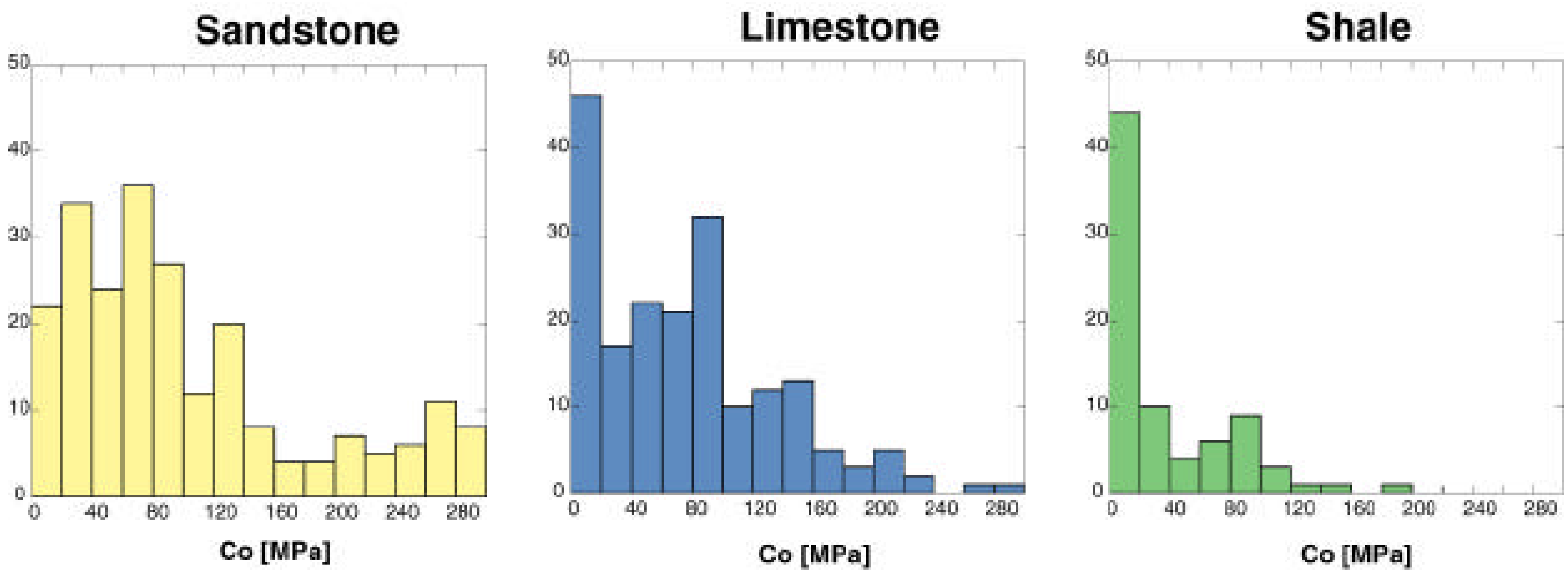
From a series of rock strength tests at different confining pressures



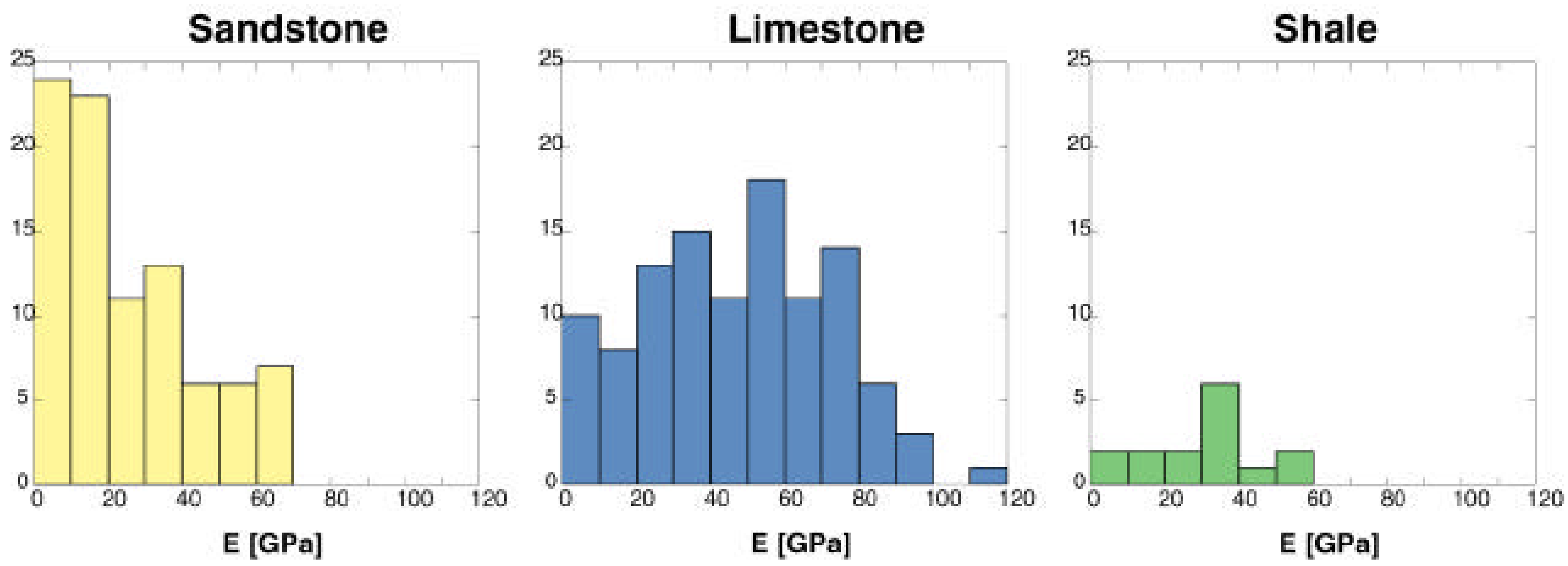
$$m = \frac{\sqrt{\mu_i^2 + 1} + \mu_i}{\sqrt{\mu_i^2 + 1} - \mu_i}$$

$$\mu_i = \frac{m - 1}{2\sqrt{m}}$$

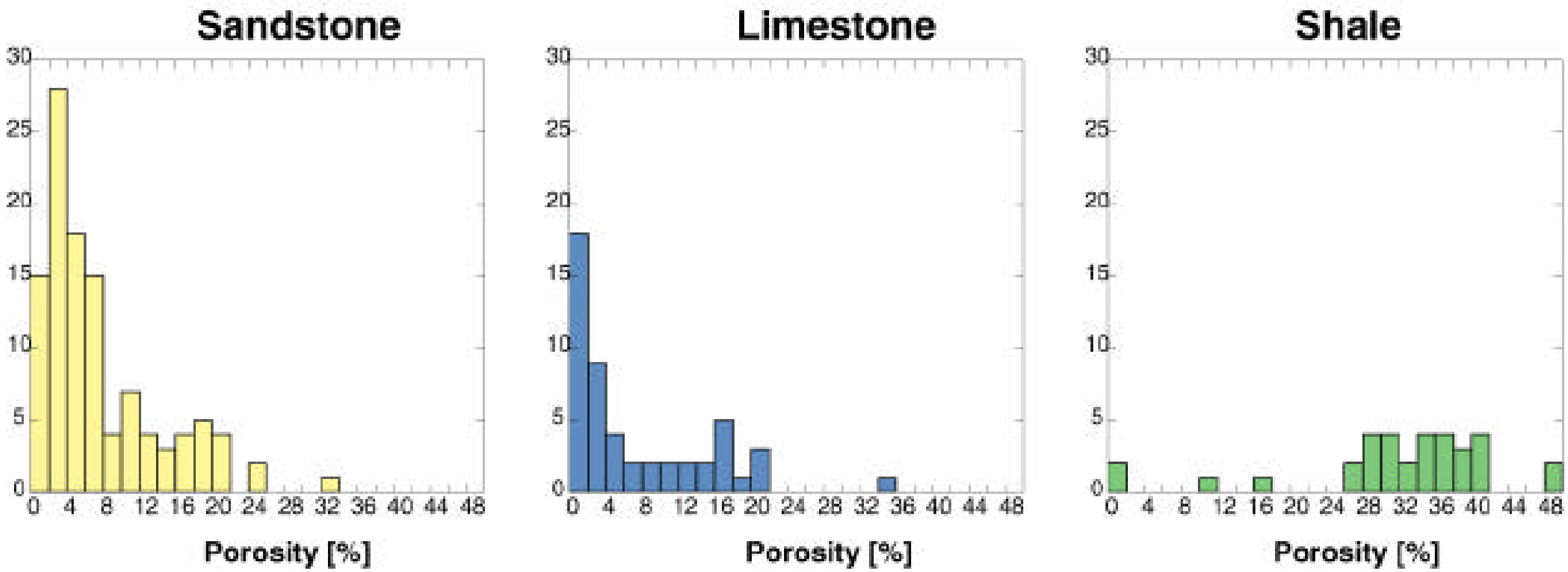
Compressive Strength



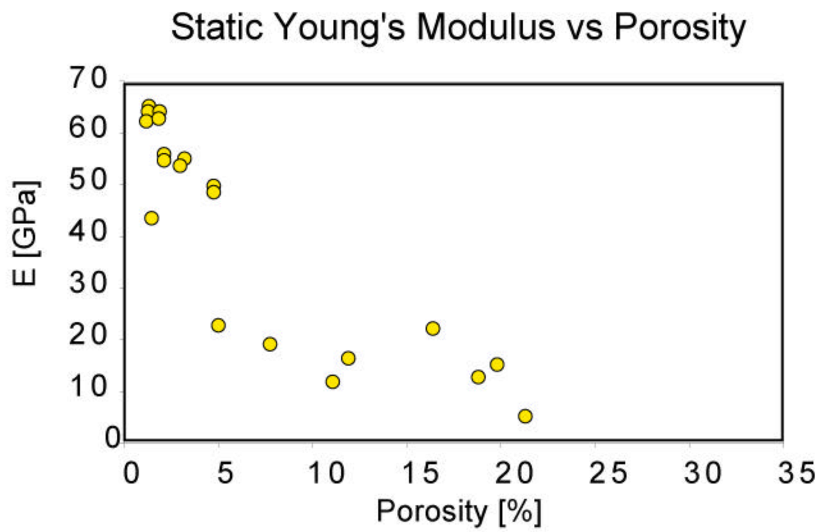
Static Young's Modulus



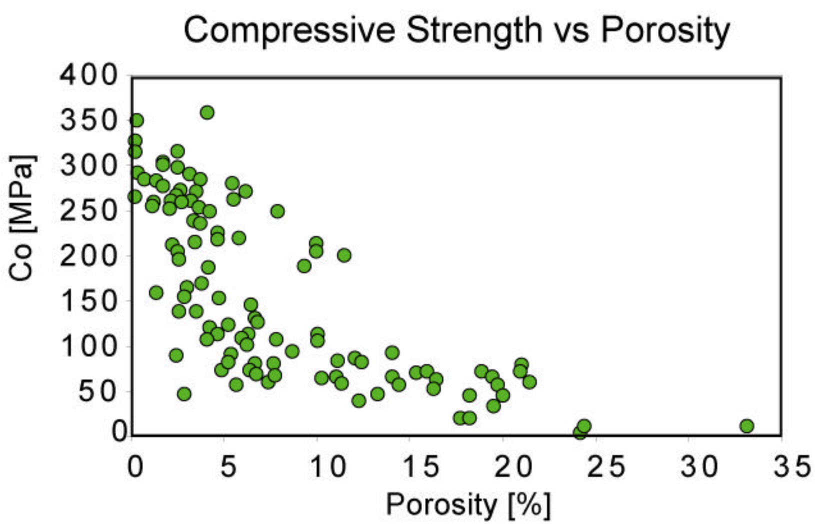
Porosity


Sandstone
Limestone
Shale
30
25
20
15
10
5
0
0 4 8 12 16 20 24 28 32 36 40 44 48
Porosity [%]

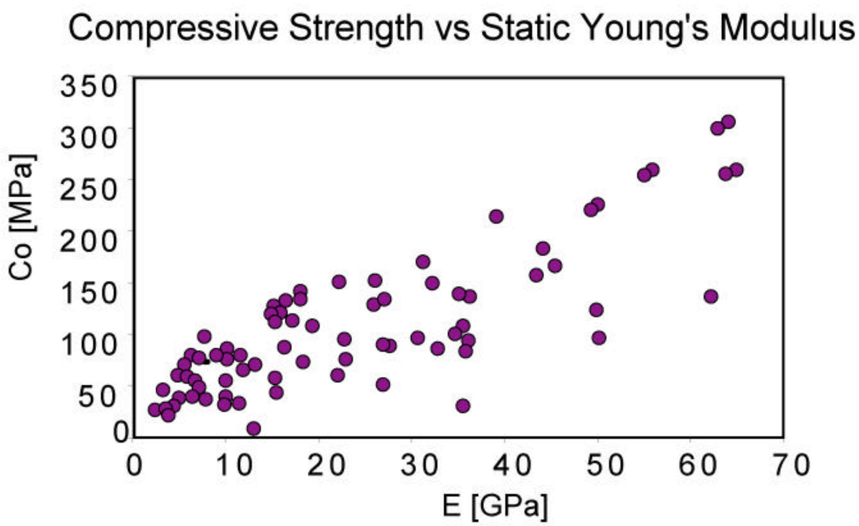
Sandstone



Static Young's Modulus vs Porosity
E [GPa]
0
10
20
30
40
50
60
70
0
5
10
15
20
25
30
35
Porosity [%]

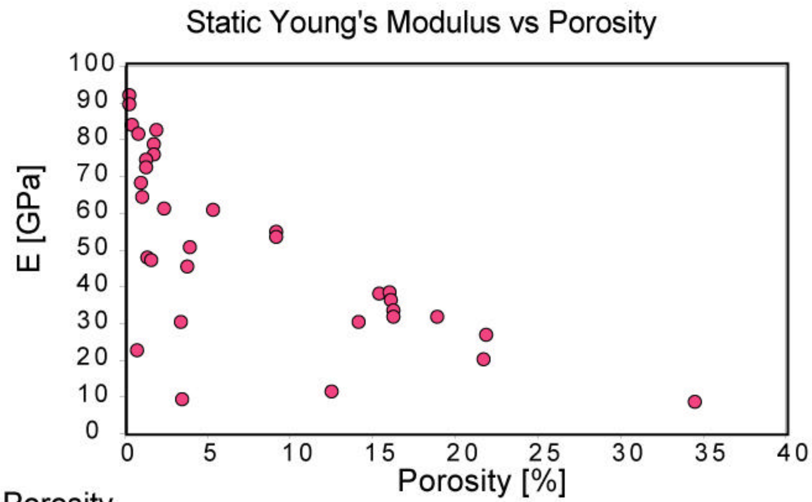


Compressive Strength vs Porosity
Co [MPa]
0
50
100
150
200
250
300
350
400
0
5
10
15
20
25
30
35
Porosity [%]

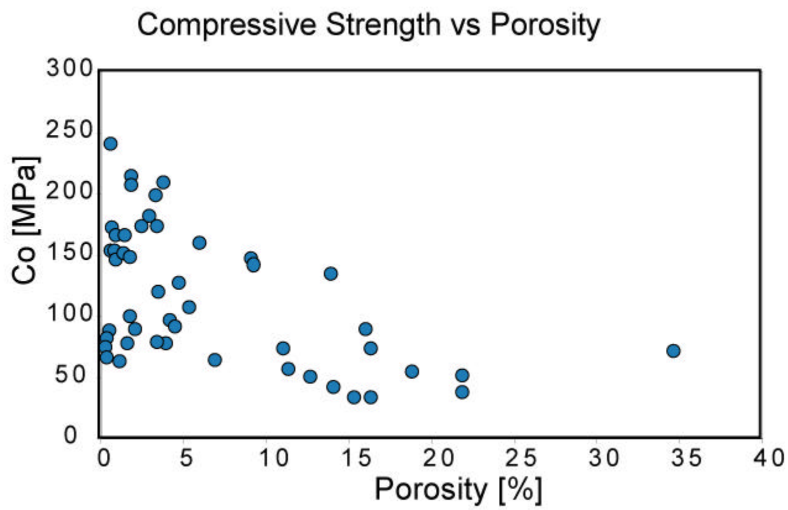


Compressive Strength vs Static Young's Modulus
Co [MPa]
0
50
100
150
200
250
300
350
0
10
20
30
40
50
60
70
E [GPa]

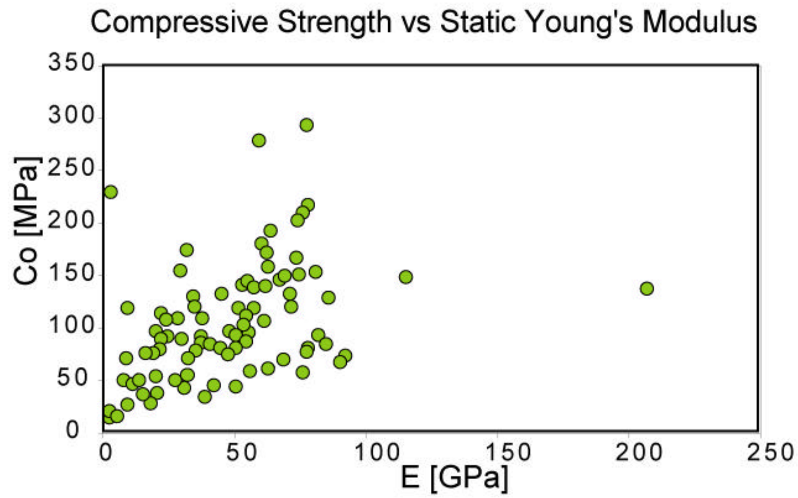
Limestone



Static Young's Modulus vs Porosity
E [GPa]
Porosity [%]
100
90
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40



Compressive Strength vs Porosity
Co [MPa]
Porosity [%]
300
250
200
150
100
50
0
0
5
10
15
20
25
30
35
40



Compressive Strength vs Static Young's Modulus
Co [MPa]
E [GPa]
350
300
250
200
150
100
50
0
0
50
100
150
200
250

Strength from Logs

Mechpro (Schlumberger)

$$C_o = E\left(0.008\ V_{clay} + 0.0045\left(1 - V_{clay}\right)\right)$$ but not used directly (lab based)

$$S_o = 3.626 \times 10^{-6}\ KC_o,\ \text{MPa}$$

$$C_o' = 3.464\ S_o \quad (\text{assumes } \varphi = 30°)$$

which yields:

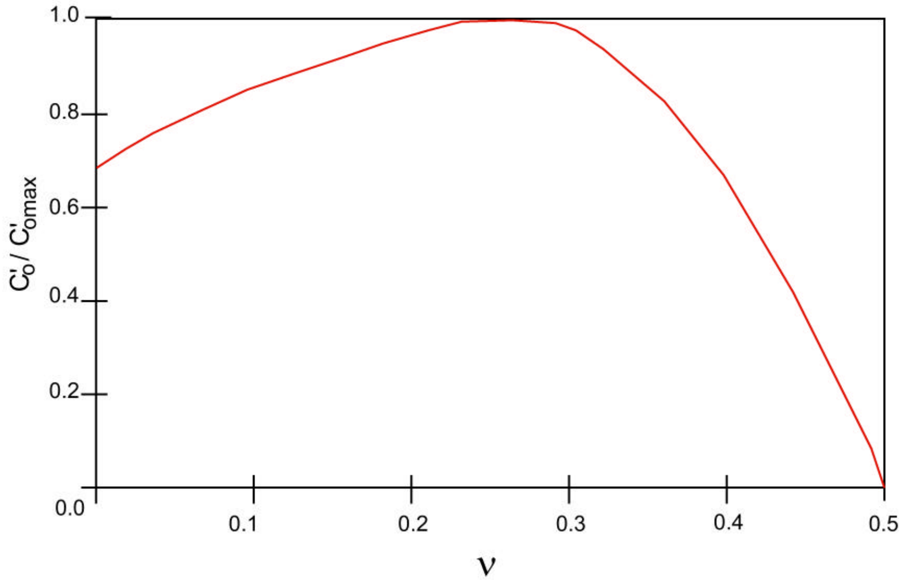
$$C_o' = 5.7 \times 10^{-8}\ KE(1 + 0.78\ V_{clay})$$

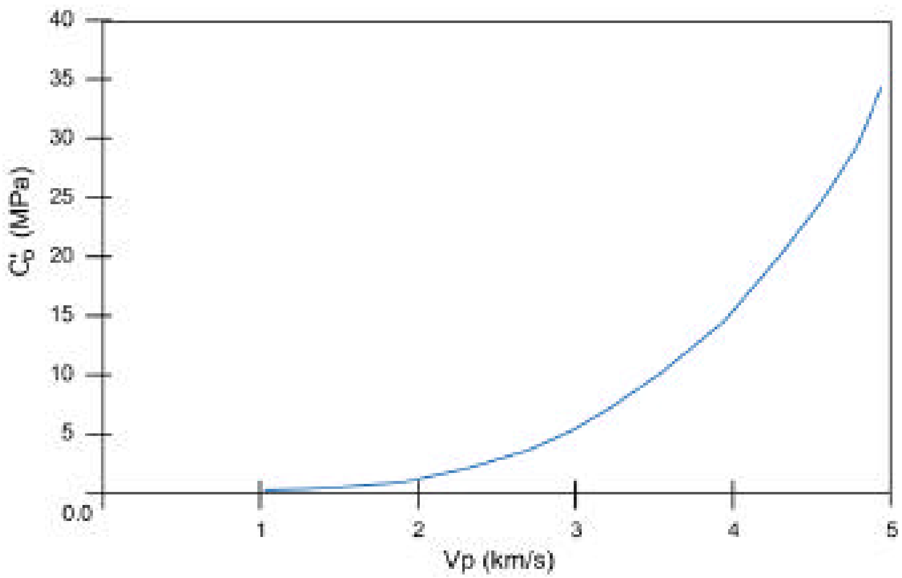
or

$$C_o' = 1.9x10^{-20}\,\rho^2 V_p^4 \left(\frac{1+\nu}{1-\nu}\right)(1-2\nu)\left(1+0.28\ V_{clay}\right)$$

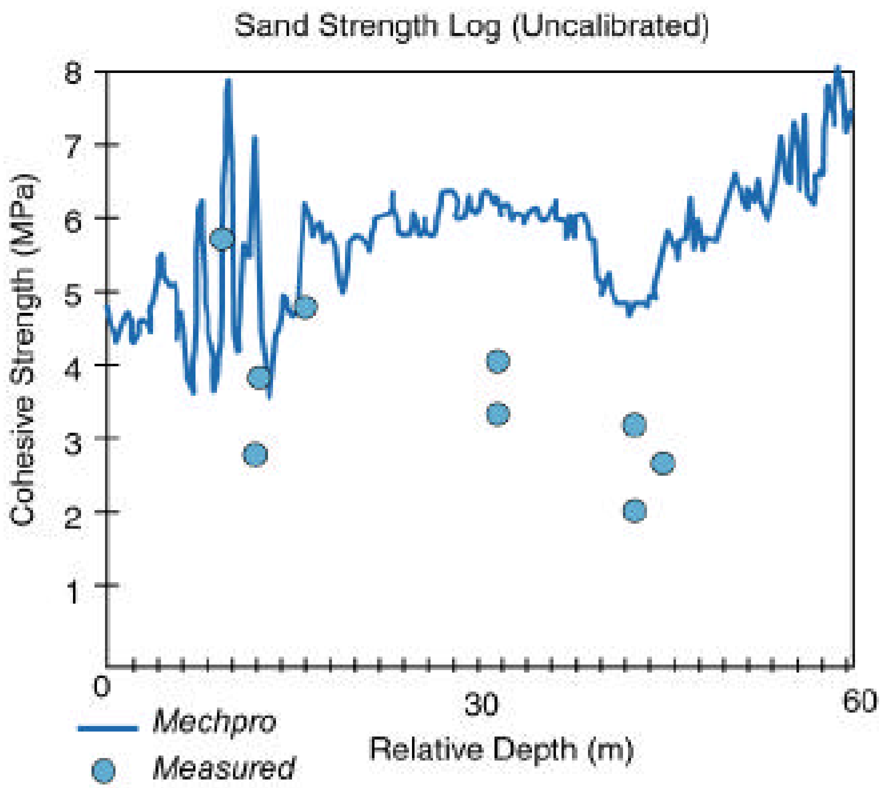
C_o' in MPa, ρ in kg/m^3 and V_p in m/s

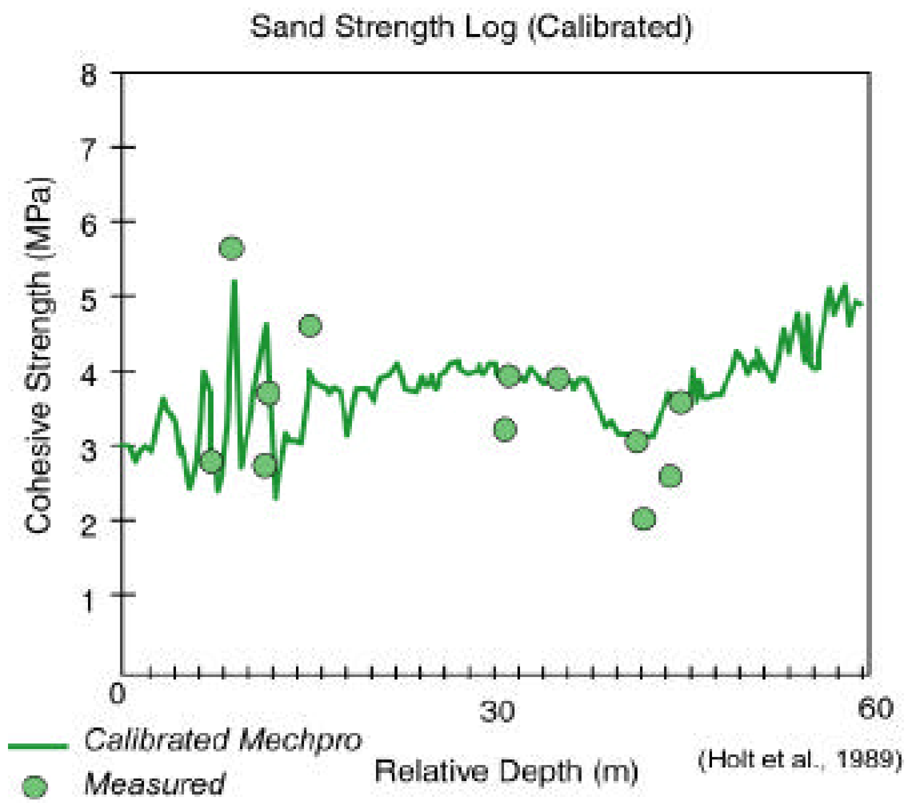
Mechanical Properties From Field Data





Sand Strength Log





Vernik, Bruno and Bovberg (1993)

Clean Sandstones

$C_o = C_o'(1 - \gamma\varphi)^2$

C_o' UCS of zero porosity equivalent (granite)

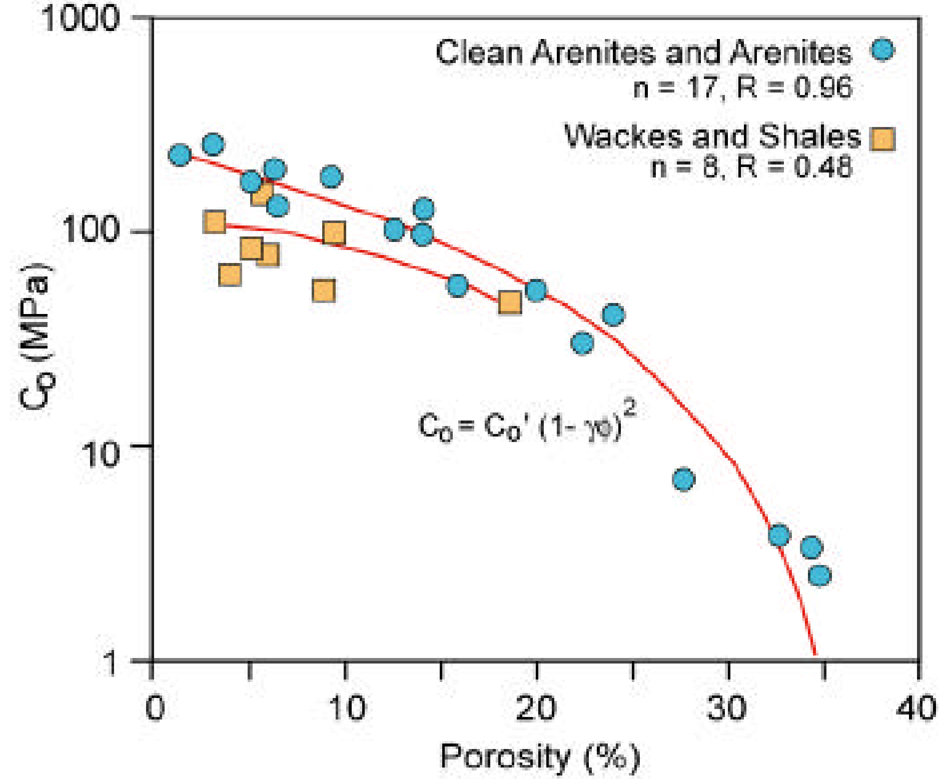
C_o' = 254 MPa (Westerly granite)

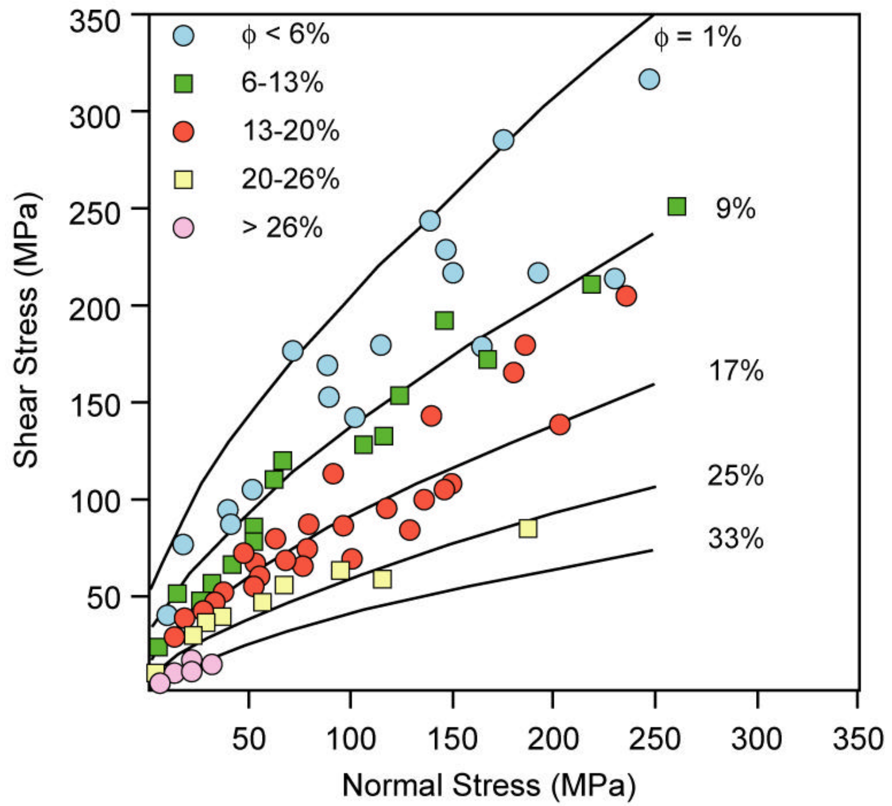
γ= 0.027 (empirical)

Leads to

$$\tau = \frac{508\left(1 - 0.027\ \varphi^2\right)}{12 - 0.1\ \varphi} + \left(7.63\ e^{-0.046\ \varphi}\right)\sigma^{0.68}$$

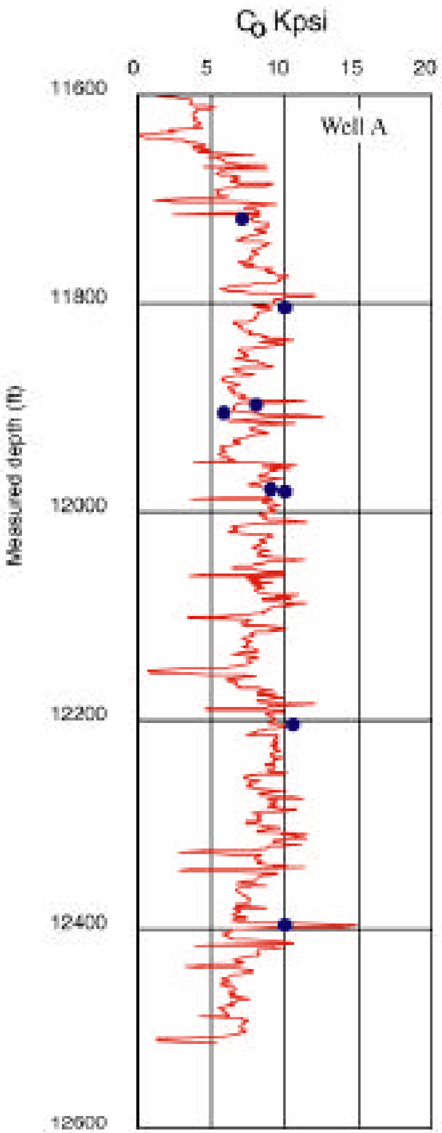
Vernik et al. (1993) Data

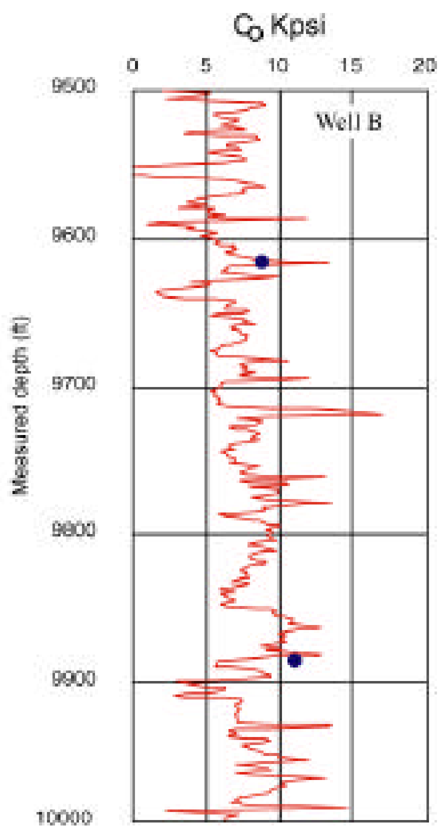




Clean Coarse Sandstone and Conglomerates

Moos, Zoback and Bailey (1999)



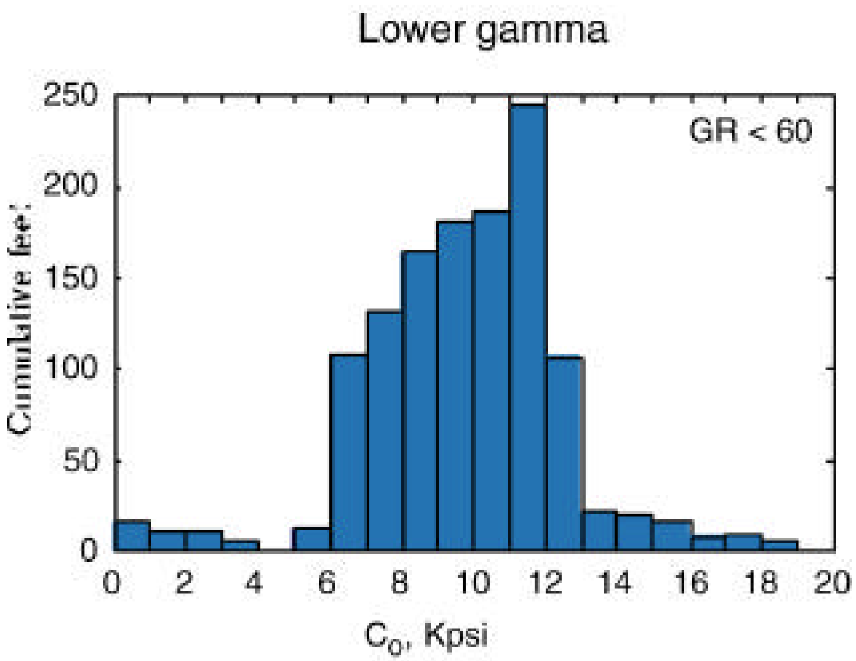


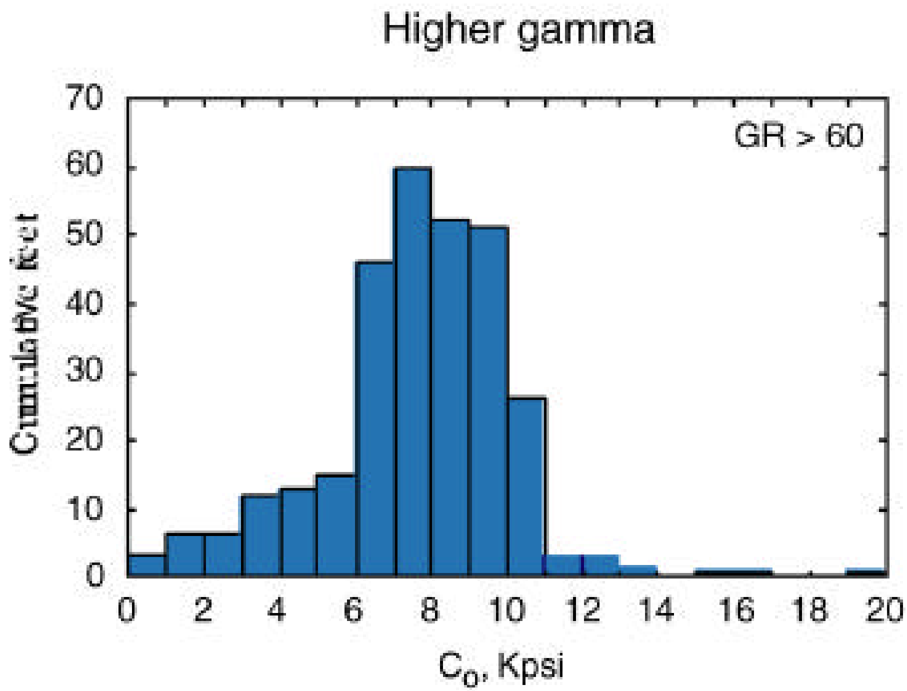
$C_o = -3043 + 253M$

C_o (psi)

M (GPa) – P wave modulus (ρV_p^2)

Log-Derived Strengths for Sands and Shales





Failure Criterion that Describe Rock Strength in Compression

Over the years, comprehensive laboratory studies have yielded a variety of failure criterion to describe rock strength in compression which are summarized below. However, to quote Mark Twain,

> ***The efforts of many researchers have already cast much darkness on the subject: and it is likely that, if they continue, we will soon know nothing about it at all.***

This statement, reflective of Twain's inherent cynicism, is unfortunately applicable of the degree to which concepts about rock failure based on laboratory rock mechanics has made the subject of rock strength sufficiently complex that it can almost never be practically applied in case studies. Thus, the most important thing to keep in mind is that

Strong rock is strong, weak rock isn't

Our first goal is to capture the essential rock strength. Using advanced failure criterion to describe rock strength is a worthy, but secondary, objective.

Failure Criterion

The *Hoek and Brown* (HB) criterion is like the *Mohr Coulomb* (MC) criterion in that it is two-dimensional and depends only on knowledge of s_1 and s_3. However 3 parameters are used to describe a curved failure surface and thus to can better fit Mohr envelopes than the linear approximation discussed previously. The *Tresca* criterion (TR), is a simplified form of the linearized Mohr Coulomb criterion as it utilizes $m_i = 0$, as commonly found in the description of the strength of metals which have a yield strength but do not strengthen with confining pressure. Other failure criterion like *Drucker Prager* (DP), (inscribed and circumscribed, both extensions of the von Mises criterion) and *Weibols and Cook* (WC) incorporate the dependence of rock strength on the intermediate principal stress, s_2, but require true polyaxial rock strength measurements. The *Modified Lade Criterion* (ML) is a three dimensional strength criterion but requires only two empirical constants, equivalent to C_o and m_i, to be determined. Note that <u>all failure criteria are based on effective stresses</u> which are defined as total stress minus the product of Biot's coefficient and pore pressure ($\sigma_i = S_i - \alpha$ Pp).

Strength Criteria in Which the Stress at Failure, σ_1, Depends Only on σ_3

Linearized Mohr-Coulomb criterion (Jaeger and Cook, 1979)

$$\sigma_1 = q\sigma_3 + C_0 \qquad q = \left(\sqrt{\mu^2 + 1} + \mu\right)^2$$

Tresca criterion (Jaeger and Cook, 1979)

$$\sigma_1 - \sigma_3 = 2C_0$$

Empirical criterion of Hoek and Brown (1980)

$$\sigma_1 = \sigma_3 + C_0\sqrt{m\frac{\sigma_3}{C_0} + s}$$

where m and s are constants that depend on the properties of the rock and on the extent to which it was broken before being subjected to the failure.

Polyaxial Strength Criteria
(The Stress at Failure, σ_1, Depends on σ_2 and σ_3)

Circumscribed Drucker-Prager criterion (Zhou, 1994)

$$J_2^{1/2} = a + bJ_1$$

$$J_1 = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$J_2^{1/2} = \sqrt{\frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]}$$

$$a = \frac{\sqrt{3}C_0}{q+2}$$

$$b = \frac{\sqrt{3}(q-1)}{q+2}$$

Inscribed Drucker-Prager criterion (Veeken et al., 1989)

$$J_2^{1/2} = c + dJ_1$$

$$c = \frac{3C_0 \cos\Phi}{2\sqrt{q}\sqrt{3\sin^2\Phi + 9}}$$

$$d = \frac{3\sin\Phi}{\sqrt{3\sin^2\Phi + 9}}$$

$$\tan\Phi = \mu$$

Polyaxial Strength Criteria
(The Stress at Failure, σ_1, Depends on σ_2)

Modified Wiebols-Cook criterion (Zhou, 1994)

$$J_2^{1/2} = e + fJ_1 + gJ_1^2$$

$$g = \frac{\sqrt{27}}{2C_1 + (q-1)\sigma_3 - C_0}\left[\frac{C_1 + (q-1)\sigma_3 - C_0}{2C_1 + (2q+1)\sigma_3 - C_0} - \frac{q-1}{q+2}\right]$$

$$C_1 = (1 + 0.6\,\mu)C_0 \qquad e = \frac{C_0}{\sqrt{3}} - \frac{C_0}{3}f - \frac{C_0^2}{9}g$$

$$f = \frac{\sqrt{3}(q-1)}{q+2} - \frac{g}{3}[2C_0 + (q+2)\sigma_3]$$

Modified Lade criterion (Ewy, 1998)

$$\frac{(I_1)^3}{I_3} = 27 + \eta$$

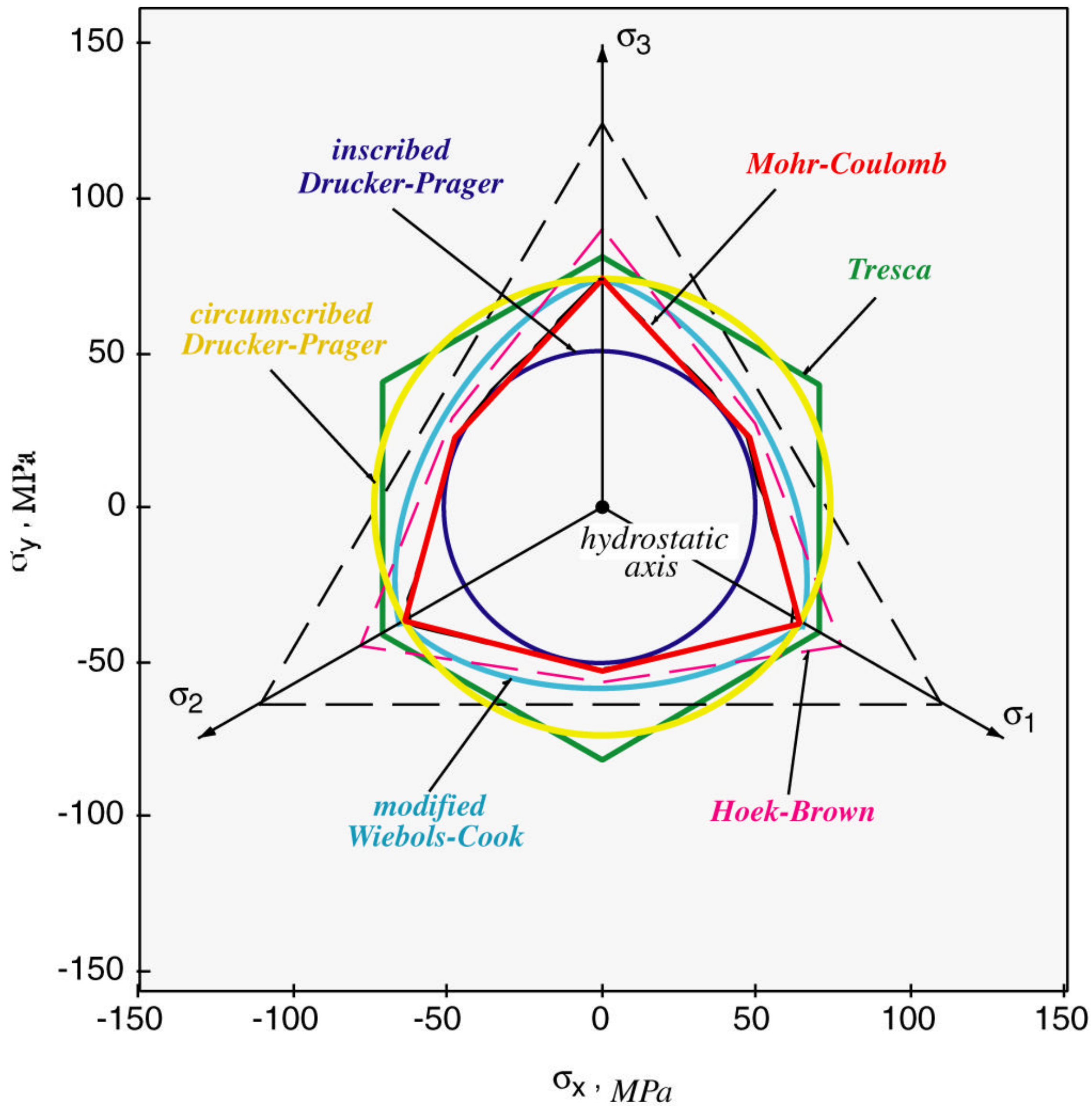
$$I_1 = (\sigma_1 + S_1) + (\sigma_2 + S_1) + (\sigma_3 + S_1)$$

$$I_3 = (\sigma_1 + S_1)(\sigma_2 + S_1)(\sigma_3 + S_1)$$

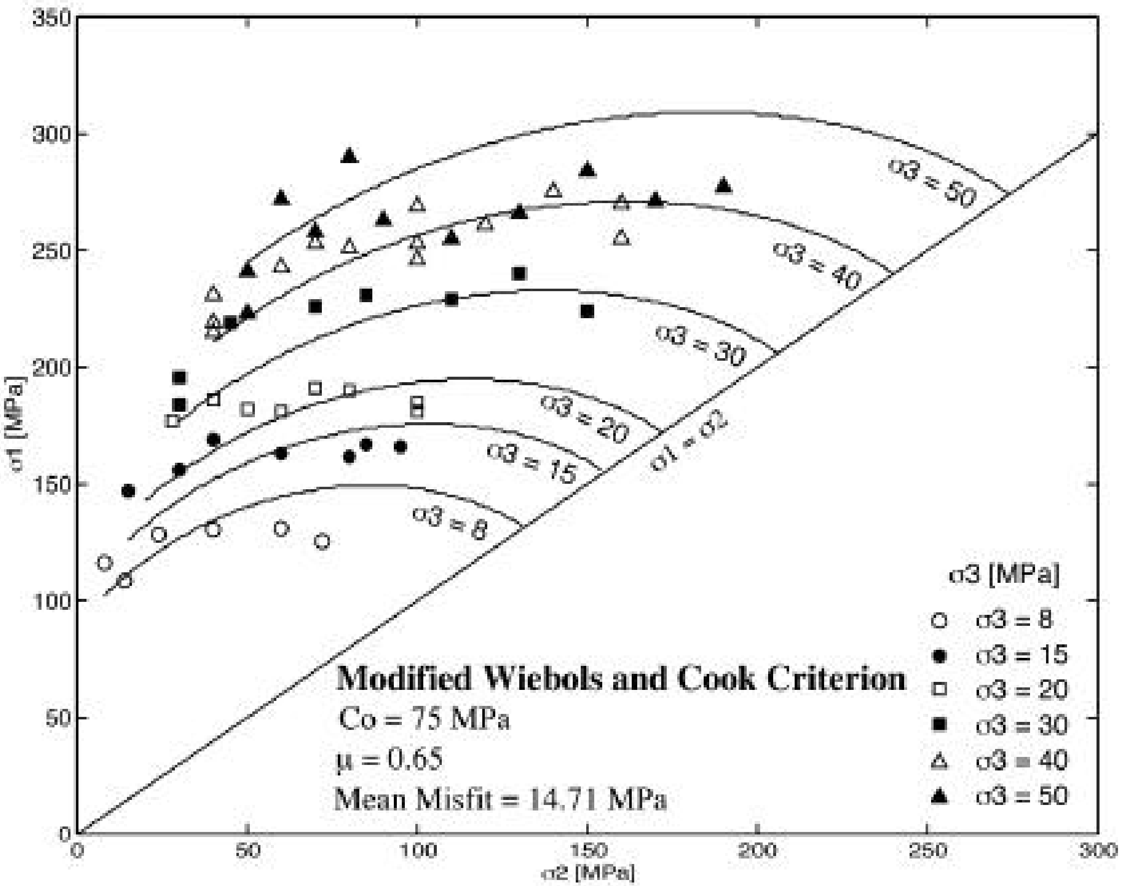
$$S_1 = S_o / \tan\Phi$$

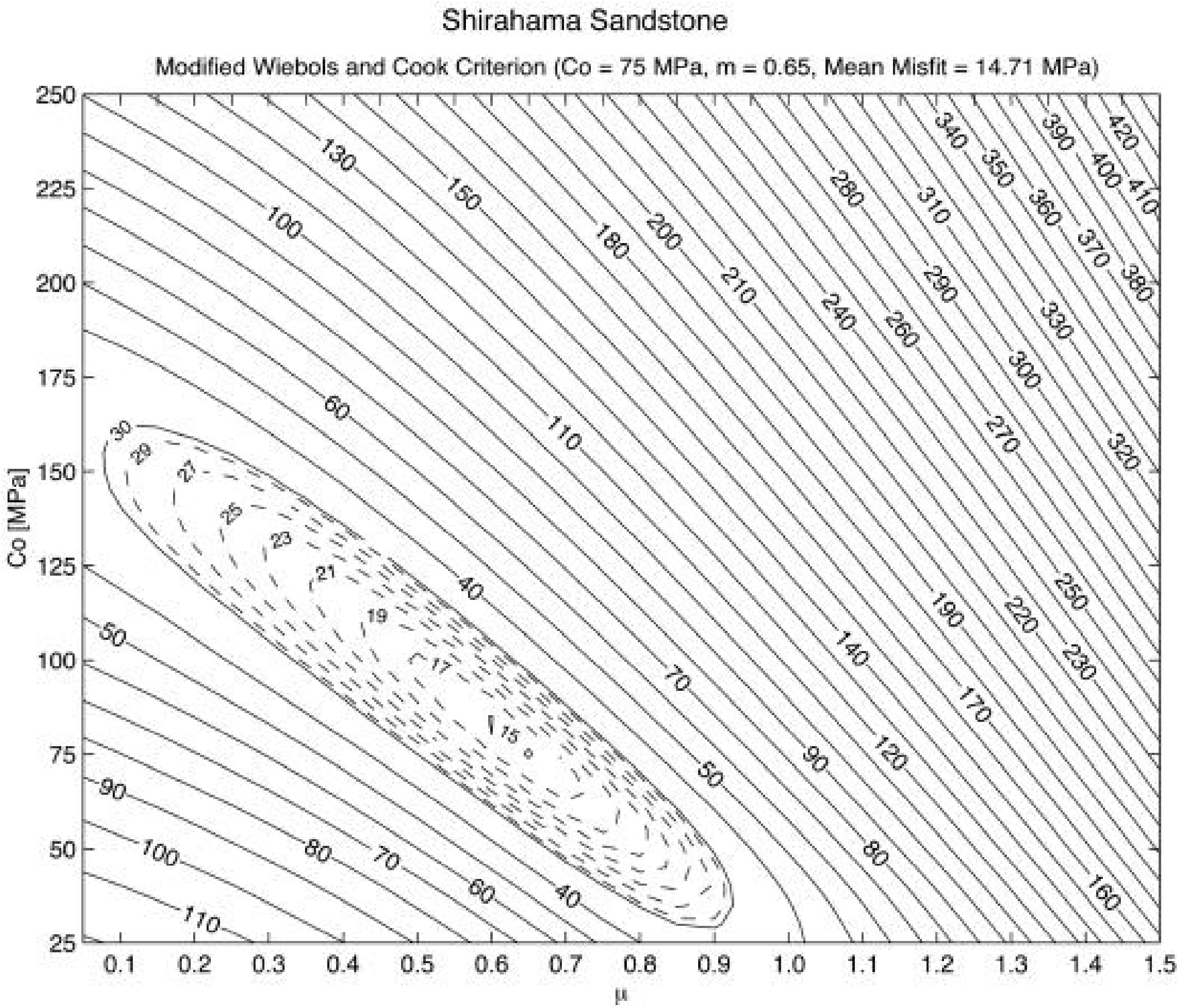
$$\eta = 4\mu^2 \frac{9\sqrt{\mu^2+1} - 7\mu}{\sqrt{\mu^2+1} - \mu}$$

Failure Envelopes in Stress Space



Shirahama Sandstone

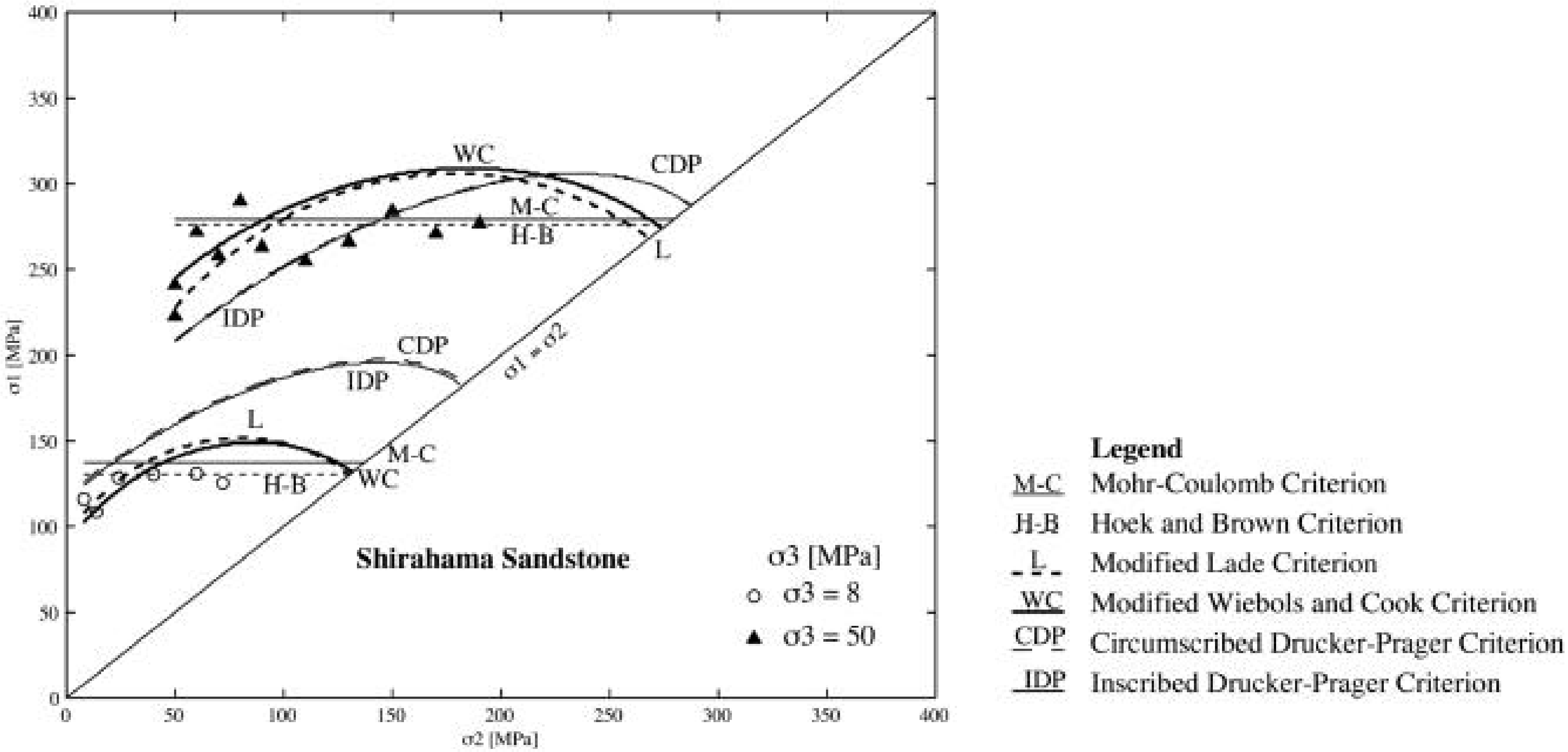




Shirahama Sandstone
Modified Wiebols and Cook Criterion (Co = 75 MPa, m = 0.65, Mean Misfit = 14.71 MPa)
Co [MPa]
μ

Shirahama Sandstone

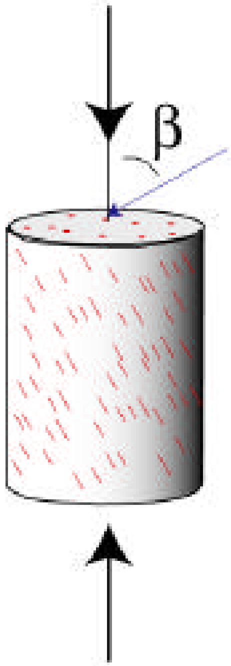
Criterion	Co [MPa]	μ_	m	s	Mean Misfit [MPa]
Triaxial:					
Mohr-Coulomb	110	0.65	-	-	15.42
Hoek-Brown	90	-	9.6	1	13.91
Polyaxial:					
Modified Wiebols-Cook	75	0.65	-	-	14.71
Modified Lade	85	0.55	-	-	15.75
Inscribed Drucker-Prager	175	0.5	-	-	23.38
Circumscribed Drucker-Prager	110	0.35	-	-	23.39

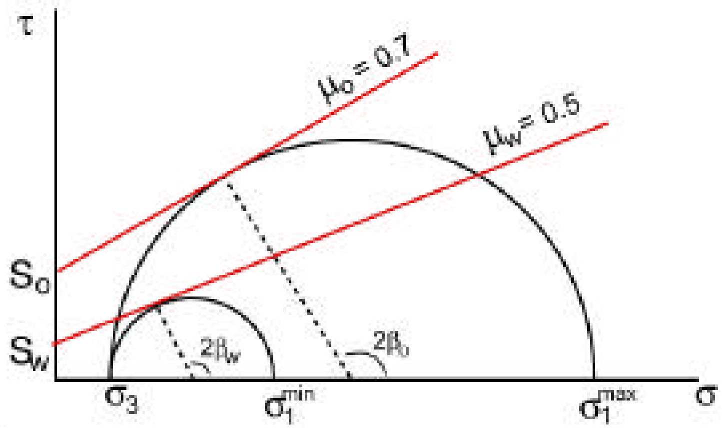


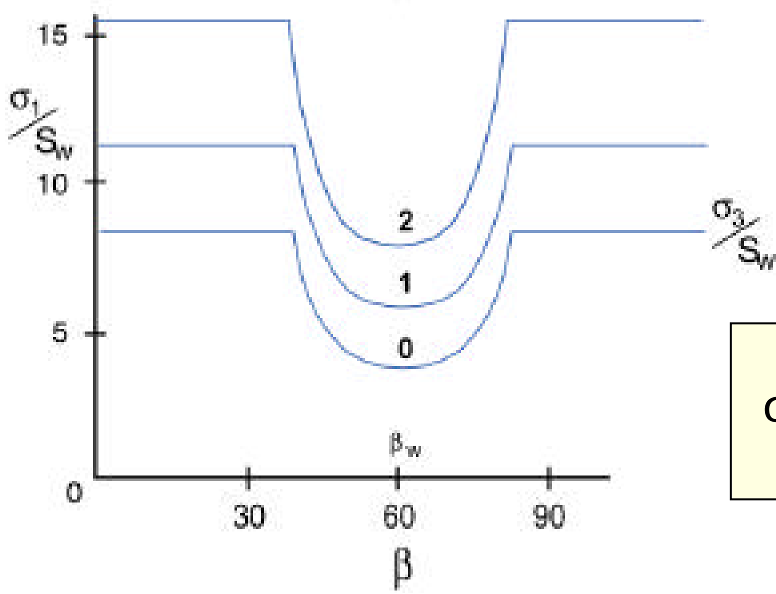
Shirahama Sandstone
σ3 [MPa]
σ3 = 8
σ3 = 50
σ1 = σ2
σ1 [MPa]
σ2 [MPa]
WC
CDP
M-C
H-B
L
IDP
Legend
M-C Mohr-Coulomb Criterion
H-B Hoek and Brown Criterion
L Modified Lade Criterion
WC Modified Wiebols and Cook Criterion
CDP Circumscribed Drucker-Prager Criterion
IDP Inscribed Drucker-Prager Criterion

Strength Anisotropy

Parallel Planes of Weakness (Bedding/Foliation)





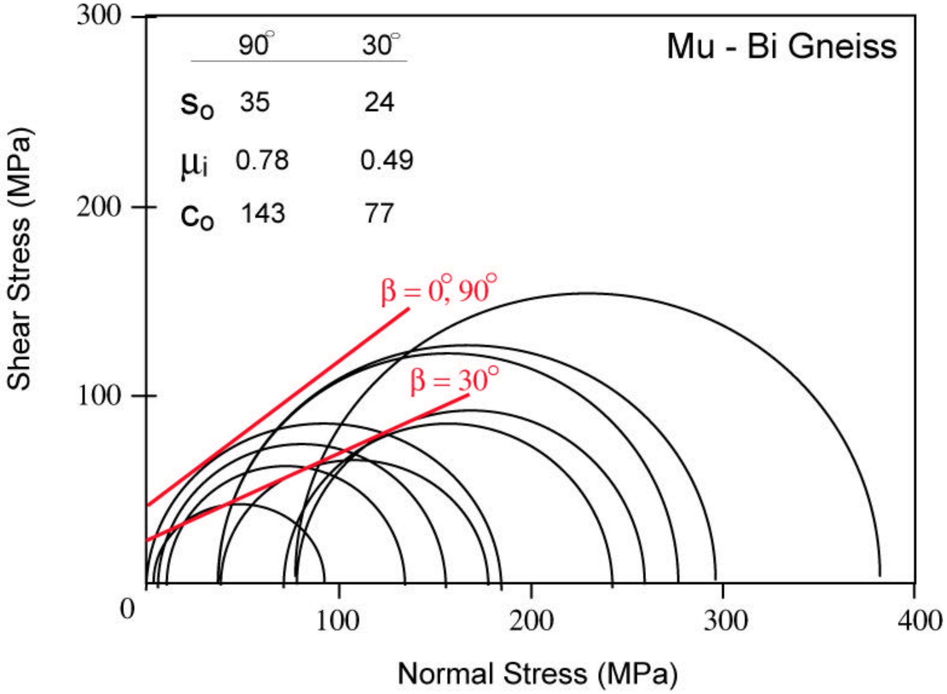


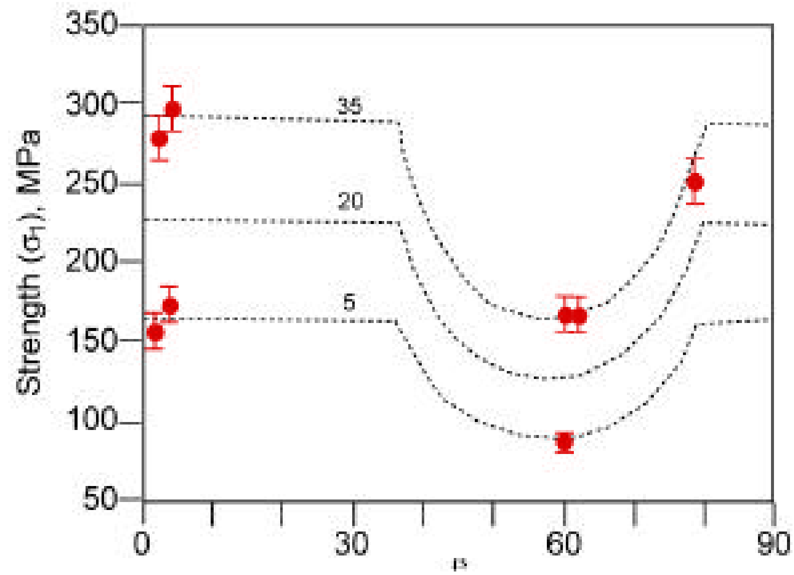
$$\sigma_1 = \sigma_3 = \frac{2(S_w + \mu_w \sigma_3)}{(1 - \mu_w \cot \beta_w)\sin 2\beta}$$

if $\tan 2\beta_w = -\frac{1}{\mu_w}$

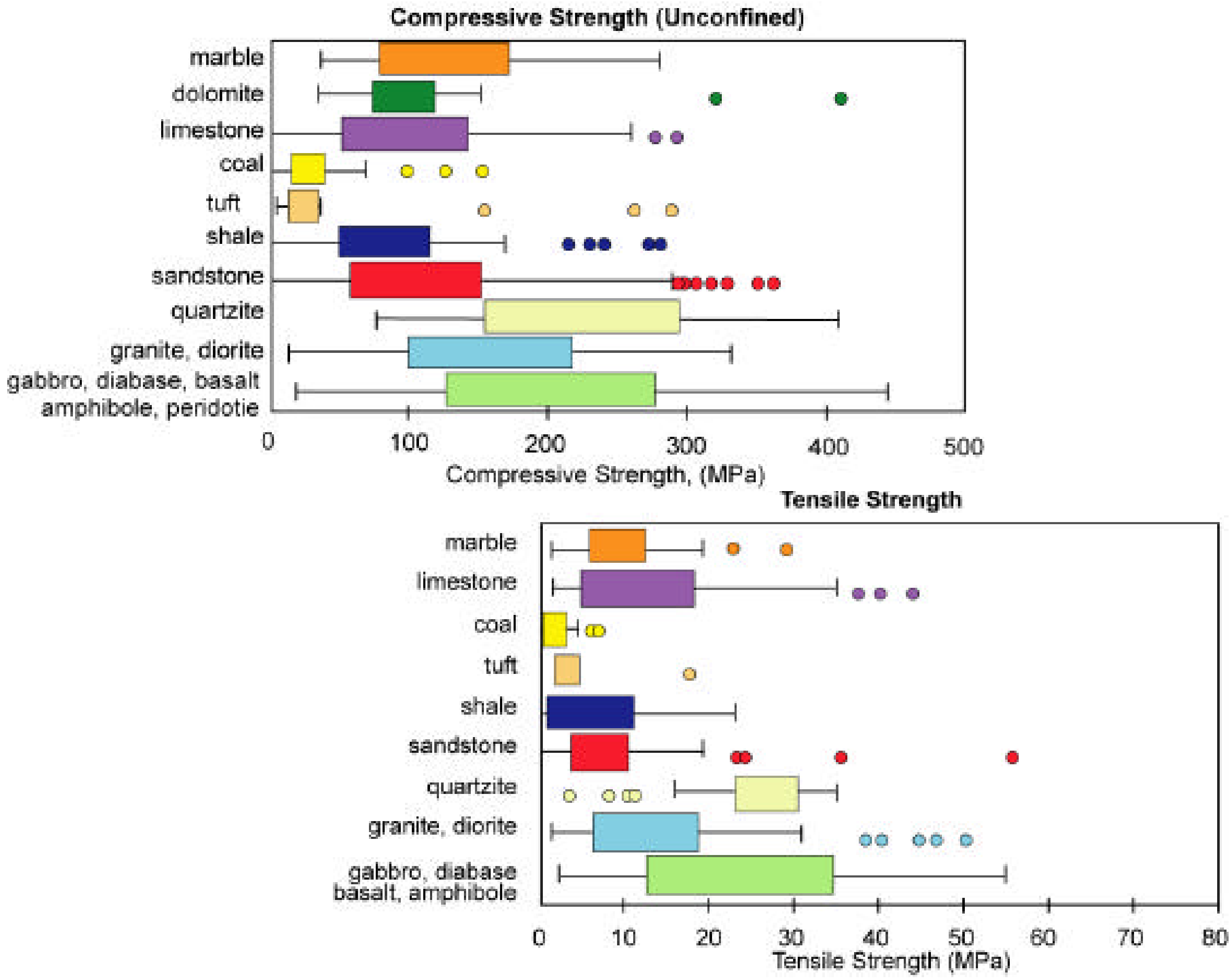
$$\sigma_1^{min} = \sigma_3 + 2(S_w + \mu_w \sigma_3)\left[(\mu_w^2 + 1)^{\frac{1}{2}} + \mu_w\right]$$

Highly Foliated Gneiss

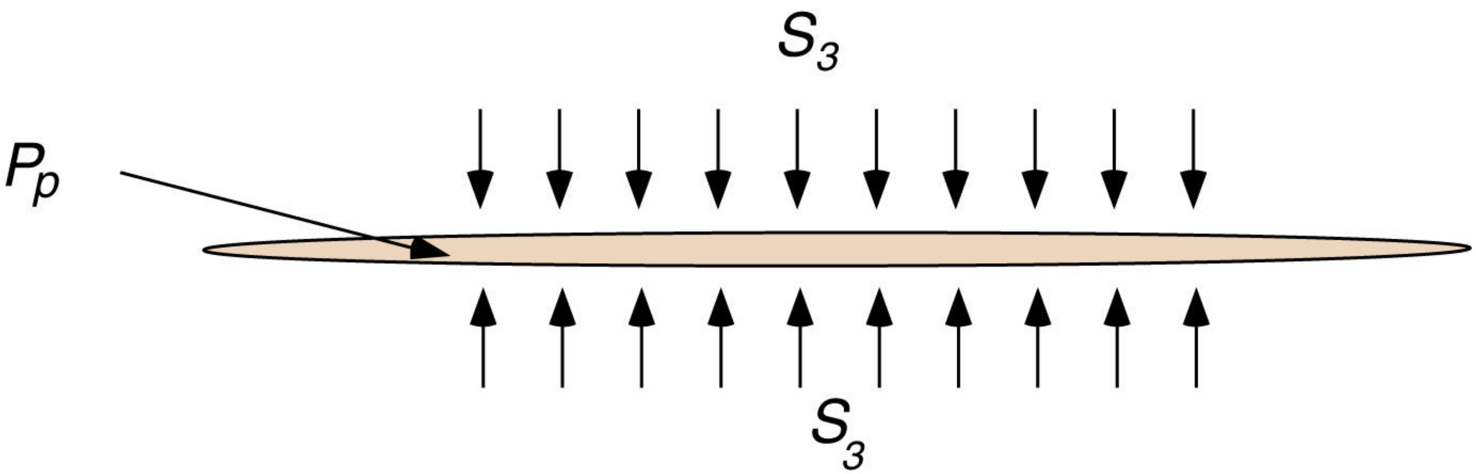




Compressive and Tensile Strength Compared



Propagation of a Mode I Fracture



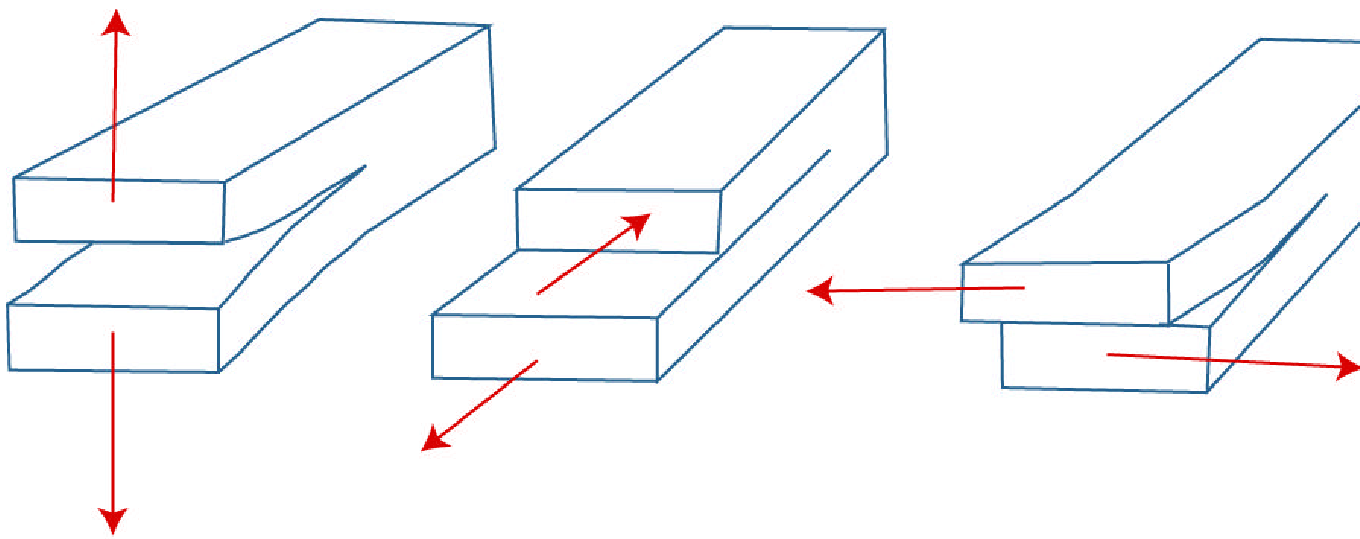
$$K_I = (P_p - S_3)\,\pi\, L^{1/2}$$

When L > 1 m,

$P_p - S_3$ *is negligible*

Fracture Mechanics

Irwin and de Witt (1983) define fracture mechanics as describing: "... the fracture of materials in terms of the laws of applied mechanics and the macroscopic properties of materials. It provides a quantitative treatment, based on stress analysis, which relates fracture strength to the applied load and structural geometry of a component containing defects".



Mode I **Mode II** **Mode III**

K_{IC} = Fracture Toughness
$K_{IC} \approx$ 1-5 MPa m – for rock

Tensile Strength of Mode I Cracks in Sedimentary Rocks