Rock Failure

Topics

- Compressive Strength
- Rock Strength from Logs
- Polyaxial Strength Criteria
- Anisotropic Rock Strength
- Tensile Strength
Key Points

1. When rock fails in compression, the compressive stress exceeds the compressive strength.
2. Even poorly-consolidated sands are characterized by a high coefficient of internal friction. Weak rocks are weak because they lack cohesion.
3. Intermediate principal stress ($S_2$) probably influences compressive strength only at very great depth.
4. Complex failure criterion are unnecessarily complicated and difficult to use. Strong rocks are strong. Weak rocks are not.
5. Are log-based strength estimates valid? Sometimes, especially if calibrated!
6. Under special circumstances, anisotropic rock strength (finely laminated shales) can be important, especially for wellbore stability studies.
7. Tensile strength of rock is extremely low and can generally be assumed to be zero.
8. John Jaeger, the god of rock mechanics (if one exists) said there is only two things you need to know about friction – it is always 0.6, and it will always make a monkey out of you.
9. The frictional strength of faults in Earth’s crust limits the maximum difference between stress magnitudes.
Stress-Strain Curves for Rand Quartzite
Strength Depends on Confining Pressure
Mohr Circles in Two Dimensions

Describes the stress on a plane at failure. Assumes $\sigma_1$, only depends on the magnitude of $\sigma_3$. Applicable to "triaxial" rock mechanics tests ($\sigma_2 = \sigma_3$).

$$\tau = -0.5 (\sigma_1 - \sigma_3) \sin 2\beta$$

$$\sigma_n = 0.5 (\sigma_1 + \sigma_3) + 0.5 (\sigma_1 - \sigma_3) \cos 2\beta$$
Mohr Envelope in Two Dimensions

Assumes $\sigma_1$, only depends on the magnitude of $\sigma_3$

Applicable to "triaxial" rock mechanics tests

$(\sigma_2 = \sigma_3)$

\[
\tau = S_0 + \mu_i \sigma_n
\]

\[
C_0 = 2S_0 \left[ \left( \frac{\mu_i^2 + 1}{2} \right)^{1/2} + \mu_i \right]
\]
Rock Strength Measurements

3086 meters

- $S_0 = 13$ MPa
- $T_0 = 7.3$ MPa
- $C_0 = 65$ MPa
- $P_{\text{conf}} = 10$ MPa
- $P_{\text{conf}} = 15$ MPa
- Linear Mohr Envelope $\mu = 0.83$
- Mohr Failure Envelope

3065 meters

- $S_0 = 21$ MPa
- $T_0 = 6.3$ MPa
- $C_0 = 105$ MPa
- $P_{\text{conf}} = 10$ MPa
- $P_{\text{conf}} = 15$ MPa
- Mohr Failure Envelope
- Linear Mohr Envelope $\mu = 0.9$
Strong Rocks/Weak Rocks

Weak rocks have low cohesion
Practical Guide to Determination of $C_o$ and $\mu_i$

From a series of rock strength tests at different confining pressures

\[ m = \frac{\sqrt{\mu_i^2 + 1} + \mu_i}{\sqrt{\mu_i^2 + 1} - \mu_i} \]

\[ \mu_i = \frac{m - 1}{2\sqrt{m}} \]
Compressive Strength

Sandstone

Limestone

Shale
Static Young’s Modulus

Sandstone

Limestone

Shale
Porosity
Limestone

Static Young's Modulus vs Porosity

Compressive Strength vs Porosity

Compressive Strength vs Static Young's Modulus
Mechpro (Schlumberger)

\[ C'_o = E \left( 0.008 V_{clay} + 0.0045 \left( 1 - V_{clay} \right) \right) \text{ but not used directly (lab based)} \]

\[ S_o = 3.626 \times 10^{-6} K C_o, \text{ MPa} \]

\[ C'_o = 3.464 S_o \text{ (assumes } \phi = 30^\circ) \]

**which yields:**

\[ C'_o = 5.7 \times 10^{-8} KE(1 + 0.78 V_{clay}) \]

or

\[ C'_o = 1.9 \times 10^{-20} \rho^2 V_p^4 \left( \frac{1+v}{1-\nu} \right) (1-2\nu)(1 + 0.28 V_{clay}) \]

\( C'_o \) in MPa, \( \rho \) in kg/m\(^3\) and \( V_p \) in m/s
Mechanical Properties From Field Data

![Graph showing mechanical properties from field data.](image-url)
Sand Strength Log

Sand Strength Log (Uncalibrated)

Sand Strength Log (Calibrated)

Cohesive Strength (MPa)

Relative Depth (m)

Mechpro

Measured

Calibrated Mechpro

Measured

(Holt et al., 1989)
Clean Sandstones

\[ C_o = C_o' \ (1 - \gamma \phi)^2 \]

\( C_o' \) UCS of zero porosity equivalent (granite)
\[ C_o' = 254 \text{ MPa (Westerly granite)} \]
\( \gamma = 0.027 \) (empirical)

Leads to

\[ \tau = \frac{508(1 - 0.027 \phi^2)}{12 - 0.01 \phi} + (7.63 \ e^{-0.046 \phi})\sigma^{0.68} \]
Vernik et al. (1993) Data

\[ C_0 = C_0' \left(1 - \psi \right)^2 \]
Clean Coarse Sandstone and Conglomerates
Moos, Zoback and Bailey (1999)

\[ C_o = -3043 + 253M \]
\[ C_o \text{ (psi)} \]
\[ M \text{ (GPa)} - P \text{ wave modulus } (\rho V_p^2) \]
Log-Derived Strengths for Sands and Shales
Failure Criterion that Describe Rock Strength in Compression

Over the years, comprehensive laboratory studies have yielded a variety of failure criterion to describe rock strength in compression which are summarized below. However, to quote Mark Twain,

The efforts of many researchers have already cast much darkness on the subject: and it is likely that, if they continue, we will soon know nothing about it at all.

This statement, reflective of Twain’s inherent cynicism, is unfortunately applicable of the degree to which concepts about rock failure based on laboratory rock mechanics has made the subject of rock strength sufficiently complex that it can almost never be practically applied in case studies. Thus, the most important thing to keep in mind is that

Strong rock is strong, weak rock isn't

Our first goal is to capture the essential rock strength. Using advanced failure criterion to describe rock strength is a worthy, but secondary, objective.
Failure Criterion

The *Hoek and Brown (HB)* criterion is like the *Mohr Coulomb (MC)* criterion in that it is two-dimensional and depends only on knowledge of $s_1$ and $s_3$. However 3 parameters are used to describe a curved failure surface and thus to can better fit Mohr envelopes than the linear approximation discussed previously. The *Tresca* criterion (TR), is a simplified form of the linearized Mohr Coulomb criterion as it utilizes $m_i = 0$, as commonly found in the description of the strength of metals which have a yield strength but do not strengthen with confining pressure. Other failure criterion like *Drucker Prager (DP)*, (inscribed and circumscribed, both extensions of the von Mises criterion) and *Weibols and Cook (WC)* incorporate the dependence of rock strength on the intermediate principal stress, $s_2$, but require true polyaxial rock strength measurements. The *Modified Lade Criterion (ML)* is a three dimensional strength criterion but requires only two empirical constants, equivalent to $C_0$ and $m_i$, to be determined. Note that all failure criteria are based on effective stresses which are defined as total stress minus the product of Biot’s coefficient and pore pressure ($\sigma_i = S_i - \alpha Pp$).
Strength Criteria in Which the Stress at Failure, $\sigma_1$, Depends Only on $\sigma_3$

**Linearized Mohr-Coulomb criterion (Jaeger and Cook, 1979)**

$$\sigma_1 = q\sigma_3 + C_0 \quad q = (\sqrt{\mu^2 + 1 + \mu})^2$$

**Tresca criterion (Jaeger and Cook, 1979)**

$$\sigma_1 - \sigma_3 = 2C_0$$

**Empirical criterion of Hoek and Brown (1980)**

$$\sigma_1 = \sigma_3 + C_0 \sqrt{m \frac{\sigma_3}{C_0}} + s$$

where $m$ and $s$ are constants that depend on the properties of the rock and on the extent to which it was broken before being subjected to the failure.
Polyaxial Strength Criteria
(The Stress at Failure, $\sigma_1$, Depends on $\sigma_2$ and $\sigma_3$)

Circumscribed Drucker-Prager criterion (Zhou, 1994)

\[
J_2^{1/2} = a + bJ_1 \\
J_1 = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \\
J_2^{1/2} = \sqrt{\frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]} \\
a = \frac{\sqrt{3}C_0}{q + 2} \\
b = \frac{\sqrt{3}(q - 1)}{q + 2}
\]

Inscribed Drucker-Prager criterion (Veeken et al., 1989)

\[
J_2^{1/2} = c + dJ_1 \\
c = \frac{3C_0 \cos \Phi}{2\sqrt{q}\sqrt{3\sin^2 \Phi + 9}} \\
d = \frac{3\sin \Phi}{\sqrt{3\sin^2 \Phi + 9}} \\
\tan \Phi = \mu
\]
Polyaxial Strength Criteria
(The Stress at Failure, $\sigma_1$, Depends on $\sigma_2$)

Modified Wiebols-Cook criterion (Zhou, 1994)

\[
J_2^{1/2} = e + fJ_1 + gJ_1^2
\]

\[
g = \frac{\sqrt{27}}{2C_1 + (q-1)\sigma_3 - C_0} \left[ \frac{C_1 + (q-1)\sigma_3 - C_0}{2C_1 + (2q + 1)\sigma_3 - C_0} - \frac{q-1}{q+2} \right]
\]

\[
e = \frac{C_0}{\sqrt{3}} - \frac{C_0}{3} f - \frac{C_0^2}{9} g
\]

\[
f = \frac{\sqrt{3}(q-1)}{q+2} - \frac{g}{3} [2C_0 + (q+2)\sigma_3]
\]

Modified Lade criterion (Ewy, 1998)

\[
\frac{(I_1)^3}{I_3} = 27 + \eta
\]

\[
I_1 = (\sigma_1 + S_1) + (\sigma_2 + S_1) + (\sigma_3 + S_1)
\]

\[
I_3 = (\sigma_1 + S_1)(\sigma_2 + S_1)(\sigma_3 + S_1)
\]

\[
S_1 = S_o / \tan \Phi
\]

\[
\eta = 4\mu^2 \frac{9\sqrt{\mu^2 + 1 - 7\mu}}{\sqrt{\mu^2 + 1 - \mu}}
\]
Failure Envelopes in Stress Space
Shirahama Sandstone

Modified Wiebols and Cook Criterion

\[ C_0 = 75 \text{ MPa} \]
\[ \mu = 0.65 \]

Mean Misfit = 14.71 MPa
Shirahama Sandstone

Modified Wiebols and Cook Criterion (C₀ = 75 MPa, m = 0.65, Mean Misfit = 14.71 MPa)
# Shirahama Sandstone

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<th>Criterion</th>
<th>Co [MPa]</th>
<th>$\mu_\ast$</th>
<th>m</th>
<th>s</th>
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Strength Anisotropy

Parallel Planes of Weakness (Bedding/Foliation)

\[ \sigma_1 = \sigma_3 = \frac{2(S_w + \mu_w \sigma_3)}{(1 - \mu_w \cot \beta_w) \sin 2\beta} \]

if \( \tan 2\beta_w = -\frac{1}{\mu_w} \)

\[ \sigma_{1\min} = \sigma_3 + 2(S_w + \mu_w \sigma_3) \left[ \left( \mu_w^2 + 1 \right)^{\frac{1}{2}} + \mu_w \right] \]
Highly Foliated Gneiss

\[
\begin{array}{c|c|c}
\text{Orientation} & 90^\circ & 30^\circ \\
\text{Initial Stiffness} (s_o) & 35 & 24 \\
\text{Mean Principal Stress} (\mu_i) & 0.78 & 0.49 \\
\text{Residual Strength} (c_o) & 143 & 77 \\
\end{array}
\]

\(\beta = 0^\circ, 90^\circ\)

\(\beta = 30^\circ\)

Shear Stress (MPa)

Normal Stress (MPa)

Strength (\(\sigma_1\), MPa)

\(\mu\)

\(0^\circ, 90^\circ\)

\(30^\circ\)

\(5, 20, 35\) MPa
Compressive and Tensile Strength Compared
Propagation of a Mode I Fracture

\[ K_i = (P_p - S_3) \pi L^{1/2} \]

When \( L > 1 \text{ m} \),

\( P_p - S_3 \) is negligible
Fracture Mechanics

Irwin and de Witt (1983) define fracture mechanics as describing: “... the fracture of materials in terms of the laws of applied mechanics and the macroscopic properties of materials. It provides a quantitative treatment, based on stress analysis, which relates fracture strength to the applied load and structural geometry of a component containing defects”.

\[ K_{IC} = \text{Fracture Toughness} \]

\[ K_{IC} \approx 1-5 \text{ MPa m} \text{ - for rock} \]
Tensile Strength of Mode I Cracks in Sedimentary Rocks

![Graph showing tensile strength of different rock types](image)

- Very Strong Sandstone
- Dolomite
- Weak Sandstone

Pressure - $S_3$ (psi) vs. Fracture Length (m)