Growth and erosion of fold-and-thrust belts with an application to the Aconcagua fold-and-thrust belt, Argentina

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[1] The development of topography within and erosional removal of material from an orogen exerts a primary control on its structure. We develop a model that describes the temporal development of a frontally accreting, critically growing Coulomb wedge whose topography is largely limited by bedrock fluvial incision. We present general results for arbitrary initial critical wedge geometries and investigate the temporal development of a critical wedge with no initial topography. Increasing rock erodibility and/or precipitation, decreasing mass flux accreting to the wedge front, increasing wedge sole-out depth, decreasing wedge and basal decollement overpressure, and increasing basal decollement friction lead to narrow wedges. Large power law exponent values cause the wedge geometry to quickly reach a condition in which all material accreted to the front of the wedge is removed by erosion. We apply our model to the Aconcagua fold-and-thrust belt in the central Andes of Argentina where wedge development over time is well constrained. We solve for the erosional coefficient that is required to recreate the field-constrained wedge growth history, and these values are within the range of independently determined values in analogous rock types. Using qualitative observations of rock erodibilities within the wedge, we speculate that power law exponents of \( \frac{1}{3} \leq m \leq 0.4 \) and \( 2/3 \leq n \leq 1 \) characterize the erosional growth of the Aconcagua fold-and-thrust belt. This general model may be used to understand the development of mountain belts where orogenic wedges grow as they deform at their Coulomb failure limit.

INDEX TERMS: 8010 Structural Geology: Fractures and faults; 8102 Tectonophysics: Continental contractional orogenic belts; 8020 Structural Geology: Mechanics; 1815 Hydrology: Erosion and sedimentation; 1824 Hydrology: Geomorphology (1625);

KEYWORDS: critical Coulomb wedge, fluvial bedrock incision, Sierra Aconcagua, Argentina


1. Introduction

[2] Field and modeling studies increasingly show that the interaction between topographic construction, erosional processes, and deformation at least partially controls the structure of orogens [e.g., Beaumont et al., 1992, 2001; Horton, 1999; Willett, 1999; Willett and Brandon, 2002]. In particular, studies by Davis et al. [1983], Dahlen et al. [1984], and Dahlen [1984] showed that topography and stable structural geometries are linked in an orogenic wedge. By inference, erosional processes removing material from orogens may alter the mechanical balance between orogenic slopes and deformation, and hence influence the development of deformation within the orogenic wedge [e.g., Dahlen and Suppe, 1988; Dahlen and Barr, 1989].

[3] Numerical models that explicitly link plate tectonic convergence rates, temperature dependent rheologies, and erosional processes have been used to explore the general interaction between deformation and erosion [e.g., Willett, 1999; Willett and Brandon, 2002]. These models demonstrate that erosion may exert an important control on the deformation, thermal structure, and thermochronologic age distributions within orogens [Willett and Brandon, 2002]. In addition, fully coupled thermomechanical erosion models that allow heterogeneous material properties within the crust have been successfully employed to model conditions as diverse as the South Island, New Zealand [Beaumont et al., 1996; Pysklywec et al., 2002], the Pyrenees [Beaumont et al., 2000], the Swiss Alps [Pfiffner et al., 2000], and the general problems of episodic accretion of terranes [Ellis et al., 1999]. The geodynamic models have provided valuable insight into the relative roles of accretion, rheology, and erosion in the development of these orogens. However,
these models suffer from at least two limitations: (1) Their complexity allows exploration of a wide variety of mechanical, thermal, and erosional conditions that may be appropriate for real mountain belts; however, this same complexity limits quantitative comparisons with field data and formal inversions for model parameters; and (2) the erosion laws employed are exceedingly simple (and perhaps unrealistic [Whipple and Tucker, 2002]) relative to the complexity allowed for the rheologies in these models. This latter limitation complicates model-based interpretations of the role of erosion on orogenic structure and may prohibit the comparison of erosional parameters used within these models to values that have been determined elsewhere for specific erosional processes [e.g., Stock and Montgomery, 1999].

[4] Alternatively, simple kinematic models have been used to understand how erosive wedges may change their forms with time [e.g., DeCelles and DeCelles, 2001]. These models lack the complex and realistic rheologies treated in the coupled thermomechanical-erosional models; however, their formulation allows geologic data, such as relative convergence velocities and thickness of accreted foreland material, to be directly used to understand how erosive wedges develop. Using orogenic wedge geometries and frontal wedge propagation rates from the Bolivian Andes and Himalaya, DeCelles and DeCelles [2001] found reasonable correspondence between their model and the field-measured quantities of wedge tip propagation rates and orogenic widths. The simple form of these models opens the possibility for quantitatively determining model parameters based on field data. However, as with the coupled thermomechanical-erosional models, these kinematic models treat erosion as a simple function whose parameters cannot be directly compared to erosional constants determined directly from geomorphic erosional studies. In particular, erosion is required to remain constant in space and time, which neglects the potentially important acceleration of erosional processes as slopes within the orogen steepen in response to prolonged uplift [e.g., Ahnert, 1970].

[5] In this paper, we construct a simple kinematic model of orogenic wedge growth in which realistic erosion laws are used to examine the development of wedge geometry to varying erosional conditions. This simplicity provides several important advantages to previous models. First, geologic information determined in the field may be directly used to model wedge growth. Second, the simplicity of our model allows us to formally invert geologic data to determine the unknown rate constants in the analysis, and specifically, the erosional constants acting to denude the wedge. Because these rate constants apply to specific erosional processes, they can be directly compared to independently derived estimates of these values to provide a direct and quantitative test of the influence of erosion on orogen geometry. In addition, we can use these formulations to explicitly explore effects such as changing climate and variable rock-type erodibility on orogenic structure. [6] We used the bedrock power law incision model [Howard and Kerby, 1983; Whipple et al., 1999; Whipple and Tucker, 1999, 2002] as our primary orogenic-scale erosion process [Whipple et al., 1999], and merged this with the Critical Coulomb Wedge (CCW) [Davis et al., 1983; Dahlen, 1984] and kinematic frontal accretion [DeCelles and DeCelles, 2001] models to consider the development of a critically growing orogenic wedge subjected to erosion and frontal accretion of material. Using a series of forward models of wedge development, we found that this growth is profoundly influenced by the erosional exponents, and when these values are low, rock erodibility and climate may play a strong role in the development of the wedge. Finally, we tested this model using field observations from the Aconcagua fold-and-thrust belt (AFTB) in the central Andes of Argentina, where geologic field studies constrain the growth of the wedge [Ramos et al., 2002; Giambiagi and Ramos, 2002]. Using this history of wedge development, we solve for the poorly known bedrock erosional constant for different values of the power law exponents. We speculate that lower power law exponents apply to this wedge based on comparisons of qualitative observations of the relative erodibilities of different rock types exposed within the wedge with our model results. We found that our model well represents orogenic growth of this wedge, and where judiciously applied, may be used to understand the coupled deformation-erosion response of other orogenic wedges.

2. General Model Description

[7] Many studies have sought to explain the relationship between stable topography and structural geometries in accretionary wedges and fold-and-thrust belts by idealizing an orogen as a simple wedge geometry (Figure 1). In this case, the decollement flattens at the back of the wedge (hereafter referred to as the “decollement sole-out depth”). Likewise, the surface slope of the orogenic wedge maintains a uniform slope along its width. Convergence between the orogenic wedge and the foreland may be accommodated by introduction of material into the back of the wedge (left side in Figure 1), incorporation of foreland material into the front of the wedge, and/or stable sliding of the wedge over the foreland. The mass added to the wedge from the first two of these mechanisms may be removed from the wedge by a variety of surficial processes or may induce a change in the cross-sectional area of the wedge. Therefore the wedge geometry (as gauged by its cross-sectional area) is controlled by the rates of addition and removal of mass by material accretion and erosion, respectively.

[8] We simplify the process of wedge growth by assuming that the wedge deforms at its Coulomb failure limit [e.g., Davis et al., 1983] during its entire development. This assumption allows us to explicitly link the surface slope of the wedge (α in Figure 1) to its basal fault angle (β in Figure 1) at all times during orogenic development. Wedges may undergo phases of subcritical and supercritical growth due to changes in wedge frictional properties or pore pressures over time [e.g., DeCelles and Mitra, 1995]. However, consideration of these types of fluctuations are beyond the scope of this study, and so to a first order, we expect the wedge to remain at its critical taper during most of its history. We fix our reference coordinate system to the backstop of the wedge (left side in Figure 1) and assume that mass enters the front of the wedge by accretion of a thickness of foreland material, T, at the velocity that the foreland converges with respect to the back of the wedge, v. The depth at which the basal wedge decollement flattens at
the back of the wedge is fixed to \( D \), and thus the wedge grows as the average decollement angle decreases rather than self-similar growth [e.g., Davis et al., 1983]. Finally, we assume that the rate-limiting process of removal from the surface of the wedge corresponds to the rate at which rivers can incise bedrock [e.g., Whipple et al., 1999]. This incision rate is idealized as a power function that depends on the upstream contributing area, the local channel slope, two power law exponents, \( m \) and \( n \) that may be related to the processes eroding the channel bed [e.g., Whipple et al., 2000], and an erodibility factor, \( K \). \( K \) includes the effects of downstream allometric changes in the channel geometry, effective precipitation, and the resistance of bedrock to fluvial incision [Whipple and Tucker, 1999]. Therefore \( K \) generally tends to decrease with increasing resistance of bedrock to erosion [Stock and Montgomery, 1999] and decreasing effective precipitation [Whipple and Tucker, 1999]. Finally, the geometry of the watersheds draining the orogen is encapsulated in the parameters \( k_a \) and \( h \) [Hack, 1957].

By combining the tectonic and erosional fluxes of material entering and leaving the wedge [DeCelles and DeCelles, 2001; Whipple et al., 1999; Whipple and Tucker, 1999] with the kinematic equations of critical wedge growth [e.g., Dahlen, 1984; DeCelles and DeCelles, 2001], we can express the rate of change of the average fault decollement angle as a function of the mechanical and erosional properties of the wedge:

\[
\frac{d\beta}{dt} = \left[ \frac{-D^2 \tan \alpha}{\sin \beta \cos \beta} + \frac{D^2 \cot^2 \beta g(\beta)}{2 \cos^2 f(\beta)} - \frac{D^2 \csc^2 \beta}{2} \right]^{-1} \left[ v T - K k_a h^{m-1} \frac{D^{l(m-1)+1}}{\sin \beta S^n} \right]
\]

where \( f(\beta) \) and \( g(\beta) \) are functions that relate the surface slope of the wedge to the decollement angle and the rate of change of the surface slope to the wedge decollement angle, respectively. A detailed description of the derivation of this expression, and the parameters in equation (1) and their physical significance is provided in Appendix A and the notation section of this paper. When an initial wedge geometry is prescribed, equation (1) may be numerically integrated to determine the temporal development of the model wedge.

3. Assumptions and Limitations of the Model

[10] There are several assumptions of our model that warrant discussion. First, we simplify the coupled erosional-tectonic system by assuming that the wedge grows continuously in its critical state. In many orogens, preexisting structures may cause deformation to move in discrete steps toward the foreland [e.g., Strecker et al., 1989; DeCelles and Mitra, 1995]. Also, orogenic wedges may begin their histories in noncritical states and develop toward criticality as material is incorporated into the wedge [Willett, 1992]. We neglect these effects in our model to elucidate the first-order features of developing wedges without introducing additional model parameters that may be difficult or impossible to constrain with field observations. Second, when the depth of flattening is controlled by the brittle-plastic transition in the crust, \( \beta \) may decrease continuously near this depth, rather than abruptly flatten as the geometry of our model requires. Therefore our models only apply to orogenic wedges where this ideal geometry is approximated. Third, our models assume that the material accreted to the wedge does not change its density as it is incorporated into the orogen. Compaction of foreland material may increase the density of this material, decreasing the volume flux entering the wedge. However, the relatively small change in \( v T \) that results from this effect likely does not influence the first-order features observed in our models. Fourth, we assume that the wedge is both mechanically and erosional homogeneous. However, orogens often expose disparate rock types with different mechanical and erosional properties. In addition, orographically controlled precipitation may alter the erosional capacity of different sections of the orogen [e.g., Willett, 1999]. These spatial heterogeneities in mechanical and erosional properties present fruitful paths of future research on the coupled erosional-tectonic development of orogenic wedges; however, assessing complex variations in these properties are beyond the scope of this work. Finally, we consider bedrock incision to be the primary erosional agent that
limits the topography in the orogenic wedge. Likewise, the mean wedge slopes in our models mimic those of the bedrock channels. In many high-altitude portions of an orogen, glaciotectonic processes may be an important erosional process. Also, the spatial distribution of exposed bedrock within channels may change in space and time [Montgomery et al., 1996], leading to “mixed” bedrock-alluvial channels [Whipple and Tucker, 2002]. The mechanics of bedrock channels have been only recently explored [e.g., Howard and Kerby, 1983; Seidl and Dietrich, 1992; Whipple and Tucker, 1999, 2002], and research on these mixed bedrock-alluvial channels is only in its infancy. In addition, quantitative rules for relating incision to basin properties and uplift for glacial transport have been elusive [Whipple et al., 1999]. Therefore we followed currently established transport rules for modeling erosional removal from bedrock channels, rather than include more complex and less well understood processes.

4. Model Results

4.1. Model Parameter Values

[11] We used the model of critical wedge growth to explore the impacts of changing each of the model parameters on the rate of change of the basal decollement angle (dθ/dt), wedge propagation rate (dW/dt), and wedge width (W) through time. A number of the parameters in equation (1) are not varied in our forward models for reasons of simplicity. In this study, we fix \( k_a = 4 \) and \( h = 1.4 \). We assume that within the wedge, the coefficient of friction (\( μ = \tan θ \)) is equal to 0.85, as predicted by Byerlee’s friction law [Byerlee, 1978], although the effective friction may be greater in the wedge due to cohesion [Dahlen et al., 1982]. In addition, we do not allow the Hubbert-Rubey fluid pressure ratio within and outside of the wedge to differ (\( λ_1 = λ_2 \)).

[12] We varied the erosion power exponents (\( m \) and \( n \)), rock erodibility (\( K \)), volume flux per unit width of accreted material (\( vT \)), depth at which the decollement flattens (\( D \)), the wedge and basal Hubbert-Rubey fluid pressure ratio (\( λ \) and \( λ_0 \)), and the coefficient of friction of the basal decollement (\( μ_0 = \tan φ_0 \)). We chose a range for each of these values encountered in the field, and the values used in our forward models are shown in Table 1. On the basis of theoretical and field studies, the ratio of \( m/n \) is close to 0.5; however, the value of \( n \) likely ranges between 2/3 and 7/2 [e.g., Whipple et al., 2000]. For our reference value, we considered \( m \) and \( n \) values of 0.4 and 1, respectively [Stock and Montgomery, 1999]. Stock and Montgomery [1999] investigated a series of bedrock channels and found that \( K \) varies primarily with rock type over possibly three orders of magnitude. Their study provided average \( K \) values of \( 1 \times 10^{-4}, 6 \times 10^{-6}, \) and \( 1 \times 10^{-6} \) for mudstones/volcaniclastics, basalt flows, and granitoid/metasedimentary rocks mainly in humid regions, respectively. In addition, lower precipitation may lead to a decrease in the discharge in channels and higher values of \( K \) [Howard et al., 1994]. We chose volume fluxes (\( vT \)) that represent the accretion of 2, 5, and 15 km of foreland material at convergence velocities similar to the Himalayan orogenic wedge (\( v \sim 17.5 \text{ mm/yr} \) [Bilham et al., 1997; DeCelles et al., 1998; Powers et al., 1998]). We varied the depth to the basal decollement (\( D \)) between 10 and 25 km. The low value of \( D \) may characterize wedges in which the presence of weak evaporite layers result in pressure-independent plastic flow at the base of the wedge [e.g., Stocklin, 1968], whereas large values of \( D \) represent the upper depth bound for the location of the brittle-plastic transition in cool continental crust [Brace and Kohlstedt, 1980]. We vary the Hubbert-Rubey fluid pressure ratio between 0 and 0.9 to represent dry and wet wedges, respectively. Finally, we let the coefficient of friction of the basal decollement (\( μ_0 \)) vary between 0.85 and 0.2, representing those that obey Byerlee’s friction law and weak decollements, respectively. Four of these five parameters were fixed to their reference values, shown in Table 1, while the other was varied. These base conditions represent a basaltic lithology eroded by the influence of moderate (\( \sim 1000 \text{ mm/yr} \)) precipitation [Stock and Montgomery, 1999], the volume flux estimated to be entering the Himalayan orogenic wedge [DeCelles and DeCelles, 2001, and references therein], a moderate to deep brittle-plastic transition, a wedge with little fluid overpressure, and a wedge with a basal decollement that obeys Byerlee friction.

[13] Finally, we calculated the rate of change of the average basal decollement angle (\( dθ/dt \)) and the propagation rate of the wedge toward the foreland (\( dW/dt \)) as a function of \( β \), and constructed example histories of an initially flat-surfaced wedge over time, calculating \( dW/dt \) and \( W \) as a function of time. We examined the relative response times of the wedge as each parameter changes. We qualitatively define the response time as the time between initiation of wedge growth and the time at which the propagation rate of the wedge decreases to a value close to zero. This gauges the length of time required for accretion to be approximately balanced by erosion. We focus on the transient behavior of the growing or shrinking critical wedge in this paper, with the results of a steady state analysis of this model presented by Hilley and Streeker [2003].

4.2. Variation in \( K \)

[14] The effects of changing \( K \) are shown in Figures 2 and 3. Negative and positive \( dθ/dt \) values correspond to wedges that widen and narrow with time, respectively. Low \( K \) values require high topographic slopes to evacuate an equivalent amount of material per time relative to wedges undergoing vigorous erosion (Figure 2). Therefore most of the material accreted to the wedge slowly increases the topographic slope of the wedge and causes

Table 1. Model Parameters Used in Forward Models of Wedge Growth

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base Value</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>1</td>
<td>1/3</td>
<td>0.4</td>
<td>5/4</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>1</td>
<td>2/3</td>
<td>3/2</td>
<td>5/2</td>
<td></td>
</tr>
<tr>
<td>( K ), m^1/2 m yr^{-1} *</td>
<td>( 5 \times 10^{-6} )</td>
<td>( 1 \times 10^{-7} )</td>
<td>( 1 \times 10^{-6} )</td>
<td>( 1 \times 10^{-5} )</td>
<td>( 1 \times 10^{-4} )</td>
</tr>
<tr>
<td>( vT ), m^3 m yr^{-1} *</td>
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<td>35</td>
<td>87.5</td>
<td>175</td>
<td>262.5</td>
</tr>
<tr>
<td>( D ), km</td>
<td>20</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>( λ_1 ) *</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>( μ_0 ) *</td>
<td>0.85</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.85</td>
</tr>
</tbody>
</table>

\* Denotes the parameters used in the forward model calculations.
Figure 2. Effect of changing erodibility constant \((K)\) on (top) \(d\beta/dt\) and (bottom) \(dW/dt\) for various power law exponents, \(m\) and \(n\) (left, middle, and right). See text and Table 1 for unvarying parameter values.

Figure 3. Effect of changing erodibility constant \((K)\) on the history of (top) \(dW/dt\) and (bottom) wedge width \(W\) over 10 Myr of model time for an initially flat wedge. The effects of changing power law exponents, \(m\) and \(n\), are shown in the each of the columns.
the wedge to grow at a relatively constant \( \frac{dW}{dt} \), with little erosional removal of material. However, as the wedge slopes become steep and erosion more effective, successively larger volumes of material accreted to the front are removed as the wedge widens and \( \beta \) decreases, resulting in rapidly decreasing wedge propagation rates.

[15] Figure 2 highlights the importance of the initial critical wedge geometry on the rate and direction of wedge growth. Orogenic wedges that start their history with relatively steep surface slopes (e.g., “A” in Figure 2) result in a larger erosional flux than accreted flux entering there. In this case, the wedge loses volume as the average basal decollement steepens and surface slopes diminish. In addition, the wedge propagation rate is often large when the initial surface geometry of the wedge is steep sloped. Conversely, a wedge that starts its tectonic history with no initial topography (point “B” in Figure 2) will consequently lose little material by erosion until slopes are developed that cause significant erosion. In this case, the initial propagation rate of the wedge tip represents that due solely to frontal accretion. However, as the wedge grows, erosion becomes progressively more efficient until the amount of material accreted to the wedge is balanced by erosional removal of material.

[16] The propagation rate and wedge width history of an initially flat-surfaced wedge is shown in Figure 3. First, the response time of the wedge increases with decreasing \( K \), because propagation rates close to those expected in the absence of erosion (at \( t = 0 \)) are sustained for longer periods of time. The prolonged high propagation rates lead to large wedge widths over time. For example, in the case that \( m = 1/3, n = 2/3, \) and \( K = 1 \times 10^{-3} \), the wedge geometry changes little during its history and quickly attains a geometry that does not change with time. This rapid response time results from the fact that the invariant geometry is close to that of the initial wedge geometry. Therefore only a limited amount of accretion is necessary to provide the material required by this narrow wedge. In contrast, when \( K = 1 \times 10^{-7} \), propagation rates and wedge widths are both large after 10 Myr of model time. The high propagation rates at the end of the model time suggest that the wedge may not reach a geometry in which frontal accretion is balanced by erosion for several tens of millions of years. Under these extremely low erosion conditions, the time at which surficial erosion becomes a significant agent of mass transport from the wedge may be far longer than the life of the orogen itself.

[17] In this and all model parameter variations hereafter, the power law incision exponents \( m \) and \( n \) exert a first-order

![Figure 4](image-url)  
**Figure 4.** Effect of changing accreted volume flux (\( \nu T \)) on (left) \( \frac{d\beta}{dt} \) and (right) \( \frac{dW}{dt} \). See text and Table 1 for unvarying parameter values.

![Figure 5](image-url)  
**Figure 5.** Effect of changing accreted volume flux (\( \nu T \)) on the history of (left) \( \frac{dW}{dt} \) and (right) wedge width \( W \) over 10 Myr of model time.
control on the response of the wedge. Because the effect of the power law exponents on the wedge behavior is similar for a wide range of varied parameters, we do not show the effects of changing $m$ and $n$ on wedge growth in Figures 4–11. In these cases, we present only the wedge growth behavior for $m = 0.4$, $n = 1$. In the auxiliary material, we include a series of figures that document the effect of changing $m$ and $n$ the wedge’s growth for all model parameters investigated.

[18] Large values of the power law exponents increase the efficiency of erosion, thus small changes in the surface slope of the wedge result in large changes in the erosional flux removed from the wedge. For example, when $m = 5/4$, $n = 5/2$, wedge propagation rates change rapidly with changing fault geometry. Because the erosional flux is high even at low surface slopes, steep fault geometries and hence narrow wedges lead to erosion that is efficient enough to remove all accreted material (Figure 2). As $m$ and $n$ increase, the response time and width of the wedge at each point in time decrease (Figure 3). In these cases, the short response time of the wedge leads them to rapidly adjust to changes in wedge or erosional parameters and reflect recent rates of frontal accretion, erosion by surface processes, and intrinsic wedge properties ($\mu$, $\mu_n$, $\lambda$, and $\lambda_b$).

4.3. Variation in $\nu T$

[19] The effect of the changing accreted flux per unit wedge width of material into the wedge is shown in Figures 4 and 5. Large values of $\nu T$ result in both larger $d\beta/dt$ and $dW/dt$ than their smaller counterparts (Figure 4). When $\nu T$ is large, higher wedge surface slopes are required to evacuate all of the accreted material; therefore, low angle decollements are favored when the accreted volume flux is large. Wedges whose initial surface slopes and basal decollements are initially steep and shallow, respectively, change their widths more rapidly than those in which orogenesis begins with subtle initial topography and steep decollement angles.

[20] Figure 5 shows the propagation rate and wedge width history of a wedge with no initial topography. The response time of the wedge varies little with $\nu T$, but instead is primarily dependent on $m$ and $n$ (auxiliary material items 1 and 2)\(^1\). After 10 Myr of model time, erosional processes with power law exponents of $m = 0.4$, $n = 1$ remove approximately all accreted material through erosive processes.

4.4. Variation in $D$

[21] Effects of changing the depth at which the basal decollement flattens ($D$) are shown in Figures 6 and 7. Small values of $D$ lead to reduced wedge cross-sectional areas for a constant decollement dip. These smaller wedge areas cannot store large volumes of material; therefore, they change their geometry more rapidly in response to a constant accreted flux than larger wedges. This results in large and small $d\beta/dt$ when $D$ is small and large, respectively. Likewise, wedge propagation rates are large when the decollement flattening depth is small.

[22] Figure 7 shows the complex temporal history of a series of initially flat wedges as $D$ is varied. All other parameters equal, wedges whose decollements flatten at lower depths have larger cross-sectional areas, and thus take longer to respond to changes in the erosion and/or accretion. Therefore, while larger values of $D$ result in smaller initial propagation rates, these decay slowly relative to the case when $D$ is small. As a result, wedges with large $D$ may become wider over time than wedges with small $D$.

4.5. Variation in $\lambda$ and $\lambda_b$

[23] We show how the wedge geometry changes as $\lambda$, $\lambda_b$ is changed in Figures 8 and 9. Large $\lambda$, $\lambda_b$ result in more negative values of $d\beta/dt$ for any given $\beta$. Large $\lambda$ results in lower surface slopes for a fixed decollement angle relative to their small $\lambda$ counterparts [Davis et al., 1983]. This reduction in the wedge’s surface slope and cross-sectional area allows storage of smaller volumes of accreted material and reduces the erosive power in the wedge at a given fault dip. Both these factors conspire to produce increased fault angle changes and wedge propagation rates for a given decollement angle.

[24] The effect of changes in $\lambda$ on the history of wedge development is shown in Figure 9. As $\lambda$ increases, so does

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the response time of the wedge. Finally, high fluid pressure ratios tend to ultimately form wider wedges, as shallower basal decollement geometries are required to produce surface slopes and erosive power sufficient to remove accreted material from the wedge.

4.6. Variation in $m_b$

Finally, we varied $m_b$ to determine its effect on the development of the wedge (Figures 10 and 11). Small $m_b$ results in a shallow decollement. These low decollement angles result in large wedge cross-sectional areas, even when the topography is initially flat. Simultaneously, lower decollement friction coefficients result in subtle surface slopes for each $\beta$ and hence low erosive power. These two effects act in opposite directions: the larger cross-sectional area of the wedge favors lower wedge propagation rates, while the lower surface slopes diminish the erosive flux and favor high propagation rates. In general, large values of $m_b$ lead to more negative values of $d\beta/dt$. Peak wedge propagation rates are equivalent for all values of $m_b$; however, high basal decollement friction coefficients sustain the propagation rates over a broader range of $\beta$ (Figure 10).

The history of an initially flat wedge (Figure 11) illustrates the changing importance of the effects of diminished slope for each value of $\beta$ on the wedge’s cross-sectional area and erosional capacity. At the beginning of orogenesis, wedges with low values of $\mu_b$ ($\mu_b = 0.2$) have lower propagation rates than those with large $\mu_b$ ($\mu_b = 0.85$), as the enhanced cross-sectional area of the low $\mu_b$ wedge must accrete large amounts of material to change its basal decollement angle. However, the rapidly increasing slopes of wedges with large $\mu_b$ increase the erosional capacity of the bedrock channels, resulting in a relatively rapid reduction in the propagation rate of the wedge. After ~2 Myr of model time, propagation rates of the low-$\mu_b$ wedges are significantly greater than their weak counterparts. Ultimately, the weak (low $\mu_b$) wedges become wider than those that are strong.

5. Application of Model to AFTB

5.1. Geologic Setting

The AFTB is a thin-skinned fold-and-thrust belt that forms an integral part of the Argentine Andean Cordillera. The study area is an ideal natural laboratory for studying the erosional impact on wedge growth because geologic studies in the area constrain the growth history of this fold-and-thrust belt [e.g., Giambiagi and Ramos, 2002; Ramos et al., 2002]. The AFTB lies between 32°30’S and 33°00’S, over a transition between flat and steep subduction [Barazangi and Isacks, 1976, 1979; Stauder, 1973] of the oceanic Nazca Plate. The area consists of three principal tectonic provinces that reflect the progressive widening and eastward migration of the AFTB orogenic wedge (Figure 12). From west to east, the three structural provinces are the Cordillera Prin-

Figure 7. Effect of changing sole-out depth ($D$) on the history of (left) $dW/dt$ and (right) wedge width $W$ over 10 Myr of model time.

Figure 8. Effect of changing Hubbert-Rubey fluid pressure ratio in the wedge and along the basal decollement ($\lambda$, $\mu_b$) on (left) $d\beta/dt$ and (right) $dW/dt$. See text and Table 1 for unvarying parameter values.
The Cordillera Principal (Figure 12) consists of a Jurassic-Neogene sequence that unconformably overlies pre-Jurassic basement rocks. The basement includes metamorphic rocks and partly metamorphosed black shales that are intruded by Carboniferous and Permian granitoids, which are in turn unconformably overlain by a thick section of Permo-Triassic volcanic rocks [Polanski, 1964]. The Jurassic-Paleocene strata mainly include deformed evaporites, red beds, platform limestones, shales, mudstones and siltstones [Alvarez et al., 1997; Alvarez and Ramos, 1999; Alvarez et al., 2000; Pángaro, 1995; Aguirre-Urreta, 1996; Tunik, 1999, 2001; Giambiagi and Ramos, 2002]. Deformation within the Cordillera Principal had begun at about 20 Ma [Ramos et al., 2002; Giambiagi and Ramos, 2002; Cegarra and Ramos, 1996] and ceased around 8.5 Ma [Giambiagi et al., 2001].

The Cordillera Frontal (Figure 12) mainly comprises pre-Jurassic basement rocks, with isolated remnants of Neogene sediments, which crop out east of its foothills. Uplift within the Cordillera Frontal had commenced by ~8.5 Ma and had ceased around 5.9 Ma [Ramos et al., 1999; Giambiagi et al., 2001]. Therefore uplift of the Frontal Cordillera likely took place between ~9 and 6 Ma.

Thrusting in the Precordillera has exposed Neogene synorogenic deposits derived from the eastward advancing orogenic wedge of the Cordillera Frontal and Cambro-Ordovician carbonates [e.g., von Gosen, 1992; Vergès et al., 2001]. In the area, uplift likely commenced at ~5 Ma [Ramos et al., 2002] and had ceased around 2 Ma [Godoy, 1999; Ramos et al., 1997, 2002]. Also, Quaternary deformation has been documented farther south [Sarewitz, 1988].

Deformation thus migrated eastward within the Cordillera Principal between 20 and 9 Ma, uplifted the Cordillera Frontal between ~9 and 6 Ma, and continued to migrate eastward into the Precordillera between ~5 and 2 Ma. We synthesized balanced cross sections and structural analyses to constrain the propagation rates of the AFTB [Ramos et al., 2002], convergence rate between the AFTB orogenic wedge and the foreland, and wedge accretion parameters throughout the three-stage history of the orogenic wedge (Figure 13). In section 5.2, shortening and wedge propagation rates are reported relative to the back of the wedge, following the convention of DeCelles and DeCelles [2001]. Balanced cross sections in the Cordillera Principal indicate convergence rates between 5.5 and 5.75 mm/yr [Cegarra...
and Ramos, 1996] during the first stage of deformation (~20–9 Ma). During this period, wedge propagation rates are on the order of 2.5 mm/yr [Ramos et al., 2002]. Retrodeformed structural sections within the Cordillera Principal indicate that a thickness of ~5 km of foreland material was accreted to the wedge as it advanced eastward [Giambiagi and Ramos, 2002]. Following this phase of deformation, convergence was accommodated along a deeper decollement level and deformation abruptly moved to the eastern margin of the Cordillera Frontal [Ramos et al., 2002] approximately 40 km to the east of the Cordillera Principal. This deeper decollement incorporated a thickness

Figure 11. Effect of changing basal decollement coefficient of friction ($\mu_b$) on the history of (left) $dW/dt$ and (right) wedge width $W$ over 10 Myr of model time.

Figure 12. Shaded relief map with principal lithotectonic units of the AFTB overlain. The Principal, Frontal, and Precordilleras represent the progressive migration of deformation from the west to east, respectively. Section A-A' shows location of schematic tectonic section shown in Figure 13. Geologic data are taken from Giambiagi and Ramos [2002]. Shaded relief topographic base is from GTOPO30 (~1 km pixel) digital elevation model.
of ~14 km of foreland basin and basement material into the orogenic wedge and exposed pre-Jurassic basement within the Cordillera Frontal. On the basis of these constraints, Ramos et al. [2002] calculate a wedge propagation rate of 13.3 mm/yr during uplift of the Frontal Cordillera between 9 and 6 Ma. Structural sections presented by Ramos et al. [1996] indicate that between 36 and 37 km of displacement was accommodated between the foreland and the back of the wedge in the Frontal Cordillera. Hence, during this time, convergence rates within the Frontal Cordillera were approximately 12.1 mm/yr [Giambiagi and Ramos, 2002]. Between 5 and 2 Ma [Giambiagi and Ramos, 2002, and references therein] relatively high wedge propagation rates of 9.1 mm/yr [Ramos et al., 2002] were observed. Thick sequences of the Neogene foreland basin and underlying rocks (~16 km) were incorporated into the wedge during this last important stage of deformation [Giambiagi and Ramos, 2002]. During this time, convergence rates between the wedge backstop and the foreland were ~10 mm/yr [Ramos et al., 1996; Giambiagi and Ramos, 2002].

5.2. Model Results

[32] We modeled the development of the Aconcagua orogenic wedge using the three stage history described above. The different phases of deformation exposed different rock types with disparate erodibilities. Our estimated model parameters for the wedge properties are presented in Table 2 and expanded upon in Appendix B. The range in widths of the orogenic wedge before each phase of deformation was calculated by subtracting the product of \( dW/dt \) and the time increment of the deformation from the current wedge width of 175 km. Using the history of the wedge geometry, we solved for the unknown rock erodibility parameter \( K \) by minimizing the difference between the

---

Figure 13. Schematic tectonic section highlighting three-stage growth of the AFTB orogenic wedge. Vertical black lines show inferred position of drainage divide. From 15 to 9 Ma, frontal accretion of foreland material incorporated a thickness of ~4–6 km of sediment into the wedge. Flattening of the subducting slab to the west around 9 Ma led to incorporation of greater thicknesses (16–18 km) of foreland material into the wedge. Finally, westward migration of deformation between 6 and 2 Ma uplifted the Precordillera, exposing Neogene basin sediments and Paleozoic rocks. During this last phase of deformation, the position of the drainage divide is poorly constrained and may lie up to 60 km east of the western extent of the AFTB. Section modified after Ramos et al. [2002] and Giambiagi and Ramos [2002].
Table 2. Model Parameters and Propagation Rates for the AFTB

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>20 – 9 Ma</th>
<th>9 – 6 Ma</th>
<th>5 – 2 Ma</th>
<th>20 – 2 Ma</th>
<th>Reference*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_e )</td>
<td>0.982</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( \mu_b )</td>
<td>0.85</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( \lambda_b )</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( k_s )</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( h )</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( v_r ), mm/yr</td>
<td>5.5 – 5.75</td>
<td>12.2</td>
<td>10</td>
<td>1,3,4</td>
<td>8</td>
</tr>
<tr>
<td>( T ), km</td>
<td>8</td>
<td>14</td>
<td>16</td>
<td>3,4,5</td>
<td>4</td>
</tr>
<tr>
<td>( dW/dt ), mm/yr</td>
<td>2.5</td>
<td>13.3</td>
<td>9.1</td>
<td>1,2,3</td>
<td>4</td>
</tr>
</tbody>
</table>

*References: 1, this study; 2, Byerlee [1978]; 3, Ramos et al. [2002]; and 4, Giambiasi and Ramos [2002]; 5, Ramos et al. [1996].

range of field-determined and model wedge widths after each period of deformation.

Our forward models indicate that \( m \) and \( n \) are important parameters in determining the growth of the wedge and so we explored the effect of different sets of these values on the growth of the AFTB. Table 3 shows the best fit \( K \) values for different combinations of \( m \) and \( n \). Errors in these values represent uncertainties introduced by ranges in wedge propagation rates reported by Ramos et al. [2002]. Generally, as \( m \) and \( n \) increase, \( K \) must decrease to produce equivalent wedge geometries. When \( m \) and \( n \) are low, mean rock-type erodibilities decrease and increase after the first and second phases of deformation, respectively. However, when \( m = 5/4, n = 5/2 \), a continual decrease in \( K \) over the wedge’s history is required to produce the inferred wedge geometry.

Using these ranges in \( K \), we calculated \( dW/dt \) over the history of the orogenic wedge and compared these values to those determined in the field. (Figure 14). As \( m \) and \( n \) increase, the propagation rate decreases more quickly than for low values of the exponents. In the case of low and moderate values of \( m \) and \( n \) \((1/3 < m < 0.4, 2/3 < n < 1)\), \( K \) is greatest during the first stage of deformation, decreases during the second, and increases slightly during the third. However, when \( m = 5/4, n = 5/2 \), \( K \) decreases during all three stages of deformation.

6. Discussion

6.1. Uncertainties in the Bedrock Power Law Incision Model

There are several important uncertainties in the bedrock power law incision model that must be addressed in any study that attempts to understand orogen-scale topography in this context. First, as Sklar and Dietrich [1998] point out, bedrock incision may be a complicated function of hydraulic geometry, uplift, basin parameters, climate, and hillslope sediment supply to the channel. Second, \( K \) encapsulates the effects of allometric changes in channel geometry with discharge, sediment flux entering the channel, the relation between effective discharge and drainage basin area, and the resistance of bedrock to fluvial erosion. Most of these factors likely change with \( m \), but to date, no comprehensive study has been conducted that defines this scaling relationship. Contrary to the work of others [e.g., Sklar and Dietrich, 1998; Whipple and Tucker, 1999], we chose not to scale \( U/K \) by \( 1/n \) in our forward models to maintain a constant topographic form for changing \( n \), as it is not apparent that the unknown scaling law of \( K \) and \( m \) supports this approach. Instead, we fixed \( K \) to values determined by Stock and Montgomery [1999] for \( n = 1, m = 0.4 \). If increasing power law exponent values reduce the value of \( K \), our models overestimate and underestimate the voracity of erosion in wedge development when the exponents are high and low, respectively. While these scaling problems lead us to view our conclusions as preliminary and subject to change upon further work, all studies that strive to understand orogen-scale relief using the power law bedrock incision model suffer from this deficiency. Nevertheless, our work is novel in that it includes the effects of realistic values of \( m \) and \( n \) (and their ratio) in a coupled model that investigates the feedbacks between deformation, rock erodibility, and climate.

6.2. General Model Results

Our results expand the work of Davis et al. [1983], Dahlen [1984], and DeCelles and DeCelles [2001]. In the original CCW formulation, a locus of \((\alpha, \beta)\) pairs was predicted for a set of mechanical wedge parameters \( \mu_e, \mu_b, \lambda \), and \( \lambda_b \). Any wedge at its critical failure limit falls on a line relating \( \alpha \) to \( \beta \) (line in Figure 15); however, the CCW theory did not predict wedge development over time or their ultimate \((\alpha, \beta)\) pair. Willett [1992] used a finite element method to model elastic-plastic wedge growth in response to convergence in the absence of erosion. We add the erosional component to the CCW theory to understand controlling factors and rates at which orogens progress from their initial states to those in which all material added to the wedge is evacuated by erosion. In this way, our model predicts both the line on which stable \( \alpha, \beta \) pairs of an orogenic wedge will lie and how a point representing the development of the orogen moves along that line in response to accretion of foreland material and erosion.

\[ [37] \beta \text{ and } dW/dt \text{ in our model decay with time as the wedge grows toward its stable configuration. Decrease in } \beta \text{ over time has been noted in many orogens [e.g., Mitra, 1997] and has been explained by the successive incorporation of increasingly structurally competent rocks into the wedge [Mitra, 1997; DeCelles and DeCelles, 2001]. Because } D \text{ relates the wedge area directly to the } dW/dt \text{ in our models, the change in wedge cross-sectional area for a given change in } \beta \text{ increases dramatically as } \beta \text{ decreases. In the absence of erosion, material accreted to the wedge causes its cross-sectional area to change at a constant rate, requiring } d\beta/dt \text{ to decrease as the wedge widens. This effect}

Table 3. Best Fit \( K \) Values for Observed History of AFTB

<table>
<thead>
<tr>
<th>Deformation Stage</th>
<th>( m = 1/3, n = 2/3 )</th>
<th>( m = 0.4, n = 1 )</th>
<th>( m = 5/4, n = 5/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage I (20 – 9 Ma)</td>
<td>( 1.1 \pm 0.04 \times 10^{-5} )</td>
<td>( 1.1 \pm 0.04 \times 10^{-5} )</td>
<td>( 8.6 \pm 0.3 \times 10^{-10} )</td>
</tr>
<tr>
<td>Stage II (9 – 6 Ma)</td>
<td>( 6.1 \times 10^{-6} )</td>
<td>( 5.2 \times 10^{-6} )</td>
<td>( 1.2 \times 10^{-10} )</td>
</tr>
<tr>
<td>Stage III (5 – 2 Ma)</td>
<td>( 7.1 \times 10^{-6} )</td>
<td>( 5.3 \times 10^{-6} )</td>
<td>( 6.4 \times 10^{-11} )</td>
</tr>
</tbody>
</table>
wedge will decrease as it grows. Therefore these two conspiring processes may provide alternative or complimentary hypotheses that explain decreases in $\beta$ and $d\beta/dt$ with time in orogenic wedges.

Finally, our model results show the important role of both climate and the $m$ and $n$ in determining the growth and lateral extent of orogenic wedges. As climate becomes dryer and/or rocks less erodible, wedge propagation rates increase and wedges widen. Conversely, where erosional efficiency is high, narrow orogens whose propagation rates decay rapidly in time might be expected. The magnitude of these effects is strongly controlled by the choice of $m$ and $n$. In our forward models, when these values are high (e.g., $m = 5/4$, $n = 5/2$), the wedge builds little topography, fault geometries remain close to their steep extremes throughout their history, and wedges adjust virtually instantaneously to changes in climate and/or wedge properties. However, low values (e.g., $m = 1/3$, $n = 2/3$) allow significant topography to be built before erosional efficiency becomes important and a wide range of wedge responses to typical erosional or mechanical perturbations may be observed.

### 6.3. AFTB Calibration Study

The validity of our modeling results and their applicability to natural systems is supported by the time-space relationships of wedge development in the AFTB. Between the first (20–9 Ma) and second (9–6 Ma) stage of deformation in the AFTB, deformation likely moved discretely toward the foreland in a series of steps that uplifted the

**Figure 14.** Modeled and observed propagation rates within the AFTB over time. Regions outlined by heavy black lines show field-determined ranges of the wedge propagation rate, while gray regions show range of model results. Model results are presented as fields to express the range of possibilities produced by geologic uncertainties in foreland-wedge convergence velocity ($v$). Results for power law erosion exponents of (top) $m = 1/3$, $n = 2/3$, (middle) $m = 0.4$, $n = 1$, and (bottom) $m = 5/4$, $n = 5/2$.

is augmented when erosion removes material from the wedge and decreases the rate of volume addition to the wedge. Because erosional efficiency increases as $\beta$ decreases, the rate of change in cross-sectional area of the

**Figure 15.** Schematic depiction of the development of a critical Coulomb wedge subject to bedrock incision. Line shows the conditions under which the wedge deforms at its Coulomb failure limit [Davis et al., 1983]. A wedge with no initial topography (A) will accrete material and increase its cross-sectional area and surface slope. As the surface slope increases, erosional processes remove larger amounts of accreted material, causing the change in cross section of the wedge to decelerate with time. Conversely, wedges with initially steep surface slopes (B) undergo more erosion than accretion. This causes a decrease in the wedge’s cross-sectional area and surface slopes until erosion balances accretion.
Cordillera Frontal. These discrete events led to wedge thickening and out-of-sequence thrusting in the higher portions of the wedge. These steps violate the assumptions of our model of critical growth of the wedge. However, they steps appear small and frequent enough to approximate a continuous migration of the wedge tip within the tolerance of our model assumptions. Also, the drainage divide may have shifted eastward during the last phase of deformation. Provenance studies [e.g., Giambiagi and Ramos, 2002; Giambiagi et al., 2001] suggest that most or all of the wedge drained to the east between 15 and 6 Ma (Figure 13). However, at an unknown time between 6 Ma and the present, the drainage divide migrated to the east. Because the timing of this migration is unknown, we assume for simplicity that the eastward movement of the divide took place after cessation of deformation in the AFTB. This assumption may cause a underestimation of the $K$ values inferred during the final stage of deformation. Lacking any direct information regarding the timing of divide migration and noting the uncertainties in the $K$ values during the final phase of deformation, we nevertheless regard our estimates of $K$ during this final stage as a representative range of values applicable to this wedge.

Despite the many simplifying assumptions of our orogenic wedge model, the determined $K$ values for the AFTB are within the range of those reported when $m = 0.4$, $n = 1$. Stock and Montgomery [1999] estimated $K$ values of for a variety of lithologies determined best fit power law exponents $m = 0.4$ and $n = 1$, and $K$ values to be between $4.8 	imes 10^{-5} - 4.7 	imes 10^{-4}, 3.8 	imes 10^{-6} - 7.6 	imes 10^{-6}$ and $4.4 	imes 10^{-7} - 1.1 	imes 10^{-6}$ for volcaniclastics and mudstones, basalts, and granitoid and metasedimentary rocks, respectively. While the combinations of rocks exposed in the AFTB are not directly comparable, the sandstones/mudstones and platform carbonates, metamorphic basement and combination of these and foreland sediments in our model yielded mean $K$ values of $1.1 	imes 10^{-5}, 5.2 	imes 10^{-6}$ and $5.3 	imes 10^{-6}$, respectively. These estimates are consistent with sediment/metasedimentary rocks with a crystalline component, metamorphic/crystalline basement, and a mixture of these rocks with foreland basin sediments, respectively. While Stock and Montgomery’s [1999] estimates span a wide range of $K$ values for each rock type, the wedge propagation rates and widths are quite sensitive to this value (Figures 2 and 3). Therefore it is not unreasonable to expect that if the effects of erosion were not properly captured in our models, our inverted $K$ values may be orders of magnitude different than those determined for similar rock types.

Our predicted propagation rates during development of the AFTB and $K$ values contain information about the potential range of power law exponents that may be applicable to the AFTB. First, as $m$ and $n$ increase in our models, $K$ must decrease to produce equivalent wedge geometries [e.g., Sklar and Dietrich, 1998; Whipple and Tucker, 1999]. Regardless of this decrease in $K$, the response time of the wedge also decreases as the result of increasing $m$ and $n$ (Figure 14) [Whipple and Tucker, 1999]. We can partially constrain potential $m$ and $n$ values by including qualitative observations of relative rock erodibilities exposed during different deformation stages. Deformation within the Cordillera Principal exposed sandstones, mudstones, and platform carbonates during the first stage of deformation. These rocks likely have the highest erodibility during the three stages of deformation. Extensive exposure of metamorphic basement in the Cordillera Frontal during the second stage of deformation likely represented the least erodible rocks during wedge growth. Finally, incorporation of foreland basin sediments into the Precordillera during the last deformation stage likely would increase the effective value of $K$ within the entire wedge. While $K$ convolves many factors including rock type climate, and/or degree of fracturing or foliation in the rocks [Whipple et al., 2000], Stock and Montgomery [1999] found that bulk lithologic type was the first order control on their determined values of $K$. Therefore we expect the value of $K$ to decrease, and slightly increase between the first and second and second and third stages of deformation, respectively. When $1/3 \leq m \leq 0.4$ and $2/3 \leq n \leq 1$, $K$ values undergo the qualitatively predicted transition during the deformation history of the wedge. However, when $m = 5/4, n = 5/2$, rock erodibility continually decreases, contrary to our qualitative observations. Our determination of propagation rates shows that when $m$ and $n$ are large, the shorter response time of the wedge moves its geometry closer to its time-invariant form at the end of each deformation phase. In this case, progressively lower values of $K$ are required to artificially maintain high propagation rates throughout the following phase of the wedge’s history. The continual decrease in $K$ observed in our models when $m = 5/4$ and $n = 5/2$ indicate that these values force the wedge to undergo these unrealistic decreases in $K$ over its history.

7. Conclusions

We developed a semianalytical model that couples the accretion of foreland material to a critically growing orogenic wedge that undergoes erosion. The wedge is required to be in its critical state during growth, grows by addition of material from the foreland, and is limited by fluvial bedrock incision. We used this model to examine the potential interaction between rates of foreland accretion of material into the wedge, erosional removal of material, and wedge widths and propagation rates over time. We made the following conclusions based on our model results:

1. The bedrock power law exponents $m$ and $n$ exert first-order controls on the response time of the wedge and its geometry through time. High values of $m$ and $n$ ($m = 5/4, n = 5/2$) cause the wedge to rapidly adjust to changes in erosional or accretion rates, or mechanical wedge parameters. However, wedge geometry is generally insensitive to these erosional and accretionary parameters when these exponents are high. Low values ($m = 1/3, n = 2/3$) allow a range of interaction between erosion, accretion of material, and wedge propagation rates and width over time. Commensurate changes in $K$ with $m$ and $n$ may partially offset this effect; however, lack of knowledge about scaling laws between these variables makes it difficult to evaluate such changes on wedge development. In any case, currently employed coupled thermomechanical-erosional models often use power law exponent values of $m = 1, n = 1$ to simulate erosion [e.g., Willett, 1999; Beaumont et al., 2000]. Our results indicate that these power law exponents play an important role in determining wedge geometry, and so their restriction to
these fixed, and perhaps unrealistic values, may complicate
the assessment of the role of erosion relative to other (i.e.,
rheology) factors.

2. When the bedrock power law exponents are low,
decreased erodibility (K) due to dry climates and/or
erotionally competent rock types, and/or increased foreland
accretion fluxes (vT) result in higher wedge propagation
rates and ultimately wider wedges than their higher K
and vT counterparts. The time between commencement of
deformation and full erosional evacuation of accreted material
increases as K decreases and vT increases. For the range in
K found in nature, a variety of wedge growth conditions is
possible. Importantly, this study is the first to document this
behavior for a realistic range in K inferred for various rock
types and climatic conditions.

3. Increasing sole-out depth of the basal decollement
decreases wedge propagation rates and increases wedge
response time. Wedges with deeper sole-out depths are
ultimately wider than those with shallow sole-out depths.

4. As the Hubbert-Rubey fluid pressure ratio within
the wedge and along the basal decollement is increased,
wedge width and response time increase. High wedge
propagation rates correspond to high fluid pressure ratios.

5. Low friction basal decollements generally lead to
higher propagation rates than when friction is low. Wedge
response time and wedge width increase with decreasing
basal friction.

To test our formulation, we modeled the development of
the Aconcagua fold-and-thrust belt in western Argentina.

Using field data to constrain the growth history of the
wedge, we solved for the unknown bedrock erodibility
constant (K) and compared this to previous published
estimates within similar rock types. When m = 0.4, n = 1,
K is within the range of these independently determined
estimates for rock types that might be expected to have
similar erodibilities to those exposed in the AFTB. Thus our
results are the first to determine the erodibility values
required to produce an observed wedge growth history
and to quantitatively test these models by comparing these
values to those determined independently.

As m and n increase, correspondence between
qualitatively determined relative rock erodibilities and those
determined in our analysis decreases. Therefore we consider
it likely that erosional processes are best represented in this
wedge by low to moderate power law exponents (1/3 ≤ m ≤
0.4, 2/3 ≤ n ≤ 1).

Appendix A: Detailed Model Formulation

Our model of orogenic wedge growth assumes that
the wedge accumulates mass by incorporating only material
of the foreland into the wedge, loses mass by erosion and
transport of material to the foreland, and changes its cross-
sectional area so as to maintain a mechanical balance
between tectonic and body stresses during its development.
Importantly, we assume that the cross-sectional area
changes continuously with time as material is accreted to
the wedge. However, the process of accretion usually takes
place by the periodic migration of deformation toward the
foreland on discrete faults. Therefore our model serves as an
approximation for this accretion process over longer (>1 Myr)
timescales. This simple model can be cast in terms of the
mass flux entering the wedge due to frontal accretion and
leaving the wedge due to erosion, and the change in volume
of the wedge. In the case that the density of material does not
change as material is incorporated into the wedge, the mass
balance of the wedge may be treated as a volume balance:

\[
\frac{dA}{dt} = F_a - F_e
\]  

(A1)

where dA/dt is the rate of change of the cross-sectional area
of the wedge, and F_a and F_e are the volume fluxes added to
and removed from the wedge by frontal accretion and ero-

where the subscript b indicates that this material is eroded
from the portion of the wedge above the foreland. Here, the
power law exponents (m, n) represent the processes acting to
erode the channel bed. In addition, rock type and climate are
captured in the K parameter [Howard et al., 1994]. To express
A in terms of the downstream profile position (x) we relate the
source area to the upstream profile length [Hack, 1957]:

\[
A = k_A x^h
\]  

(A4)

where k_A and h are constants that depend on the catchment
geometry. By combining equations A3 and A4, we express
the rate of change of the channel elevation in terms of the
downstream profile distance, x:

\[
\frac{dz}{dt_b} = -K x_m^m h_m^m
\]  

(A5)
Finally, the volume flux of material eroded from the wedge \( F_e \) is determined by integrating equation (A5) with respect to space:

\[
F_e = - \int_0^W \frac{dz}{dx} dx = \frac{K_b x^{(hm+1)}}{hm + 1} S^W_0
\]

where \( W \) is the width of the orogenic wedge (Figure 1).

Finally, we estimate how the difference between the eroded and accreted flux is expressed in the change in wedge geometry. In particular, we investigate the rates of change of the dip of the basal decollement, surface slope, and wedge width. For clarity, we treat the rate of change in the orogenic wedge area \( (dA/dt) \) as the sum of the rate of change of the portion below \( (dA/dt_b) \) and above \( (dA/dt_a) \) the foreland surface:

\[
\frac{dA}{dt} = \left( \frac{dA}{dt} \right)_a + \left( \frac{dA}{dt} \right)_b
\]

\[\text{(A7)}\]

DeCelles and DeCelles [2001] noted that \( dA/dt_b \) is geometrically related to the rate of change of the fault dip and wedge width by

\[
\left( \frac{dA}{dt} \right)_b = \left( W \tan \beta \frac{dW}{dt} \right) b + \frac{W^2}{2 \cos^2 \beta} \frac{d\beta}{dt}
\]

where \( \beta \) is the average basal fault dip of the wedge (Figure 1). As material is accreted to the wedge, this average decollement angle may decrease as deformation is taken up on structures within the foreland that widen the wedge. Conversely, in the case that erosion removes large amounts of material from the wedge, structures within the wedge may be reactivated while those at its foreland periphery may cease to accommodate convergence. This may result in an effective contraction in the wedge’s areal extent. Both the fault dip and the wedge width may be varied independently; however, many studies suggest that the basal decollement flattens as its strength becomes independent of the applied normal traction [Davis et al., 1983]. If the decollement depth exceeds that at which quartz and feldspar deform plastically due to increased temperature (typically between 12 and 16 km [Brace and Kohlstedt, 1980]), the model requirements of the wedge deforming as a brittle, elastic material may be violated. We relate \( \beta \) to \( W \) by assuming that the wedge soles out at a depth \( D \) below the foreland surface. Furthermore, because material only enters the wedge by frontal accretion, we set the rigid backstop at the location where the basal decollement intersects the brittle-plastic transition. Under these circumstances, the wedge width and its rate of change over time may be related to the fault dip as

\[
W = D \cot \beta
\]

\[\text{(A9a)}\]

\[
\frac{dW}{dt} = -D \csc^2 \beta \frac{d\beta}{dt}
\]

\[\text{(A9b)}\]

Substituting equations (A9a) and (A9b) into equation (A8), we can express the change in the lower portion of the wedge solely in terms of \( \beta \) and its rate of change:

\[
\frac{dA}{dt}_b = -D^2 \csc^2 \beta \frac{d\beta}{dt} b
\]

\[\text{(A10)}\]

The section of the orogenic wedge that lies above the foreland surface is a function of \( W_a \) and the wedge’s topographic slope (\( \alpha \)). As with the lower portion of the wedge, we express the change in area of this upper section as

\[
\frac{dA}{dt}_a = W \tan \alpha \frac{dW}{dt} + \frac{W^2}{2 \cos \alpha} \frac{d\alpha}{dt}
\]

\[\text{(A11)}\]

Substituting the expression for the wedge width as a function of fault dip (equation (A9a)) into this equation yields

\[
\frac{dA}{dt}_a = -D^2 \tan \alpha \frac{d\beta}{dt} + \frac{D^2 \cot^2 \beta}{2 \cos^2 \alpha} \frac{d\alpha}{dt}
\]

\[\text{(A12)}\]

According to equations (A10) and (A12), the rate of change of the area of the orogenic wedge depends on the sole-out depth, the surface and fault angles, and the rate of change of these angles. In our model, we relate the surface slope and its rate of change to the decollement geometry using the critical Coulomb wedge (CCW) theory [Davis et al., 1983; Dahlen et al., 1984; Dahlen, 1984]. In this formulation, body forces due to lithostatic loading increase with surface slope and fault dip. The theory predicts that the orogenic wedge is in mechanical equilibrium when the resolved tractions along the fault surface are equal to the failure limit along the surface as defined by the Coulomb Failure Criterion [Davis et al., 1983; Dahlen, 1984]. The relationship between the surface slope and fault dip at this critical condition is expressed most simply as [Dahlen, 1984]

\[
\alpha + \beta = \psi_b - \psi_o
\]

\[\text{(A13)}\]

For subaerial orogenic wedges, \( \psi_b \) and \( \psi_o \) depend on the internal angle of friction of the wedge (\( \phi_a \)), the internal angle of friction of the fault plane (\( \phi_o \)), the Hubbert-Rubey fluid pressure ratio within the orogenic wedge (\( \lambda \) ) and along its basal decollement (\( \lambda_b \)), and the surface slope (\( \alpha \) ). Complete expressions for these parameters are given by Dahlen [1984]. Hereafter, we denote the relation between surface slope and critical fault dip as

\[
\alpha = f(\beta, \psi, \psi_b, \lambda, \lambda_b)
\]

\[\text{(A14)}\]

and for a fixed set of mechanical wedge parameters, \( \alpha = f(\beta) \). Importantly, equation (A14) is meant only to represent the relationship between \( \alpha \) and \( \beta \) explicitly predicted by the critical Coulomb wedge theory [Davis et al., 1983] and are not necessarily meant to imply that \( \alpha \) is controlled by \( \beta \). The CCW theory predicts that in cases where \( \alpha < f(\beta) \) and \( \alpha > \)
f(β), the orogenic wedge will undergo internal deformation that acts on the surface slope, and extension within the wedge reduces the surface slope, respectively, until \( \alpha = f(\beta) \) [Davis et al., 1983]. In our model, we assume that mechanical adjustments within the wedge that maintain the surface slope and fault dip at their critical angles occur rapidly relative to the rates of accretion of material into and erosion from the wedge. In these cases, the orogenic wedge grows and shrinks in its critical state such that the relation between the surface slope and fault dip (equation (A13)) is maintained at all times.

[60] The rate of change of the surface slope is dependent on the rate of change of the fault dip and a function, \( g(\beta) \):

\[
\frac{d\alpha}{dt} = g(\beta, \psi, \psi_b, \lambda, \lambda_b) \frac{d\beta}{dt} \quad \text{(A15)}
\]

This allows us to express the change in total area of the wedge (equations (A10) and (A12)) solely terms of the two of the parameters in our model. First, we solve for the rate of change of the fault dip angle, we

\[
\frac{dA}{dt} = \frac{d\beta}{dt} \left[ -D^2 \tan f(\beta) + D^2 \cot^2 g(\beta) - D^2 \sec^2 \beta \right] \quad \text{(A16)}
\]

[61] The characteristics of the wedge geometry can be linked to frontal accretion of the foreland and surficial erosion processes of the orogen by combining equations (A2), (A6), and (A16) using equation (A1). First, we note that if the wedge is growing critically in its mechanically stable configuration and the basal decollement is planar, the surface slope does not vary as a function of distance along the wedge. If this slope is limited by fluvial bedrock incision, \( S \) in equation (A6) is invariant with position along the wedge and may be removed from the integration. In addition, we note that \( S = \tan(\alpha) = \tan(f(\beta)) \). Combining these equations and rearranging to solve for the rate of change of the fault dip angle, we find that

\[
\frac{d\beta}{dt} = \left[ -D^2 \tan \alpha + 2D^2 \cot^2 g(\beta) - D^2 \sec^2 \beta \right]^{-1} \cdot \frac{v_T}{K} \frac{h_m \mu_{hm+1} \cot(h_m+1) \beta^{80}}{h_m + 1} \quad \text{(A17)}
\]

[62] Using this expression with equation (A9b), the rate of change of the wedge width may also be computed. In addition, where an initial wedge geometry is prescribed, integration of equation (A17) with respect to time yields the temporal history of the change in basal decollement dip, which may in turn be converted into changes in \( W \) (equation (A9b)) over time. Because the wedge starts and grows in its mechanically critical state, initial conditions must consist of a stable orogenic wedge geometry in which \( \alpha = f(\beta) \).

[63] We chose a numerical solution method to determine two of the parameters in our model. First, we solve for \( g(\beta) \) at a given value of \( \beta \) by calculating the change in \( \alpha \) that results from a small \( (1 \times 10^{-3} \) degrees) increment in the value of \( \beta \). The resulting change in \( \alpha \) was normalized by this increment to determine the change in \( \alpha \) resulting from a unit change in \( \beta \). When this value is scaled by \( d\beta/dt \), \( g(\beta) \) is approximated. Second, we chose to integrate equation (A17) numerically when calculating the temporal development of the modeled wedge. We used a simple, first-order explicit integration scheme with \( \Delta t = 1000 \) years in the models. To assess the accuracy of our solutions, we performed several trials on rapidly changing wedge geometries in which the time step was reduced by an order of magnitude and found no significant difference between the model results. Therefore this explicit integration method provides reliable results within the range of model parameters investigated.

Appendix B: Wedge Parameter Estimates

[64] In this appendix, we present our estimations of the orogenic wedge parameters (\( \mu, \mu_b, \lambda, \lambda_b \)) and erosion parameters (\( K, k_w, h \)). We have no direct measures of the fluid pressures or friction coefficients within or along the wedge and wedge base, respectively; however, measurements of the current and initial geometry of the wedge partially constrain these parameters. To estimate the wedge widths over time, we used the propagation rates of Ramos et al. [2002] of 2.5, 13.3, and 9.1 mm/yr over the last 15–9, 9–6, and 5–2 Myr, respectively, to compute 103.8 km of widening over the history of the wedge. We subtracted this from the currently observed 175 km wedge width to obtain an initial width of 71.2 km. On the basis of regional cross sections through the AFTB [Giambiagi and Ramos, 2002], we assumed that the flattening depth of the wedge was around 20 km throughout most of its deformation history. Using this depth, the initial fault angle (\( \beta_0 \)) was likely around 13.6. This range agrees well with restored sections of the westernmost AFTB, which estimate the average fault dip to be \( \sim 5 \) [Giambiagi and Ramos, 2002].

[65] When the Hubbert–Rubey fluid pressure ratio within the wedge and along the basal decollement are equal and \( \alpha = 0^\circ \), \( \beta \) is primarily dependent on \( \mu \) and \( \mu_b \) [Davis et al., 1983]. We assumed that the basal decollement obeyed Byerlee’s friction law during deformation (\( \mu_b = 0.85 \)). In addition, we assumed that the fluid pressure within the wedge and along the decollement are equal. Finally, we considered the surface of the AFTB to have no topography prior to 20 Ma. Under these assumptions, we computed \( \mu \) to be 0.982 when the initial fault dip was 13.6°. The higher coefficient of friction values within the wedge relative to Byerlee’s law likely reflects the contribution of cohesion to the strength of the wedge [Dahlen et al., 1984]. Using these estimates of \( \mu, \mu_b, \lambda, \lambda_b \) with the current wedge geometry, we estimated pore fluid pressures within the wedge to be \( \lambda = \lambda_b = 0.7 \). These pore fluid pressures agree well with estimates for the Himalaya and Taiwan (\( \lambda = \lambda_b = 0.76 \) and \( \lambda = \lambda_b = 0.675 \), respectively) [Davis et al., 1983]. Because we have no direct evidence for or estimates of changes in \( \mu_b, \mu_b, \lambda, \lambda_b \) over time, we assume that they remained constant over the lifetime of the orogen.

[66] Geomorphic parameters were constrained by DEM analysis and estimation of unit erodibility exposed in the AFTB wedge. We used the Hydro1K digital elevation model (DEM) and flow direction grids to constrain \( k_w \) and
$h$. A number of combinations of $k_n$ and $h$ yield similar root-mean-square (RMS) misfits between values measured in the Hydro1K DEM and predicted by equation (A4). We selected the combination $k_n = 4$, $h = 1.4$. Similar RMS values were obtained by increasing $h$; however, best fit values for $k_n$ rapidly became unreasonably small ($k_n < 0.1$) with larger $h$ values.

**Notation**

- $\alpha$: surface slope of the wedge [L/L].
- $\beta$: fault decollement angle [L/L].
- $D$: depth to the decollement at wedge backstop [L].
- $A_a$: area of wedge above horizontal datum (foreland elevation) [L$^2$].
- $A_b$: area of wedge below horizontal datum (foreland elevation) [L$^2$].
- $z$: elevation above sea level [m].
- $v$: convergence velocity of the foreland with respect to the rear of the wedge [L/t].
- $T$: thickness of accreted foreland material [L].
- $K$: coefficient of erosion [L$^{1-2m}$].
- $m$: area exponent, erosion rule.
- $n$: slope exponent, erosion rule.
- $k_a$: area-length coefficient [L$^{2-n}$].
- $h$: exponent in area-length relationship (reciprocal of Hack's exponent) [L$^{2-n}$].
- $f(\beta)$: function relating wedge surface slope to decollement angle at Coulomb failure limit.
- $g(\beta)$: function relating rate of change in surface slope to rate of change in decollement angle at Coulomb failure limit.

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