Beyond Elasticity: New Directions in Earthquake Modeling

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- Ruptures on geometrically complex faults: Zijun Fang and Lucile Bruhat
- Earthquake+tsunami simulations: Gabe Lotto
- Earthquake sequence simulations in sedimentary basins: Brittany Erickson, Kali Allison
State of Stress in Earth’s Crust

Motivating questions:
• At what stress levels do faults become capable of hosting earthquakes?
• How can we have weak faults (like San Andreas) in a strong crust, if that crust is also riddled with faults, and dynamic weakening acts on all faults?
• What is the origin of complexity in faulting process (variability in rupture size, branching, jumping, rupture velocity)?
• Can we quantify the likelihood of different earthquakes?
Stress in the Earthquake Cycle

- What sets upper and lower limits on stress?
- What is absolute stress?
Simplest View

Could this work?
Effective stress \((\sigma - p) \sim 100\) MPa (lithostatic–hydrostatic at seismogenic depth)
From experiments, static friction \(\approx 0.8^*\)
...but dynamic friction more complicated

*with a few exceptions like clayey gouge in SAFOD and JFAST drilling of Japan Trench
Constraints from Lab Experiments

experiments by Greg McLaskey with 2 m granite block biaxial apparatus at USGS

2 s before quake

50 ms snapshots as slip accelerates toward instability

static friction ≈ 0.8
dynamic friction ≈ 0.77

shear/normal stress, $\tau/\sigma$

experiments at $\sigma \approx 6$ MPa
Could this work?
Effective stress \((\sigma - p) \sim 100 \text{ MPa}\) (lithostatic–hydrostatic at seismogenic depth)
From experiments, static friction \(\approx 0.8\), dynamic friction \(\approx 0.77\)

\(\Rightarrow\) stress drops \(\sim (0.8-0.77) \times 100 \text{ MPa} \sim 3 \text{ MPa}\), exactly as observed, and slip velocities \(\sim 1 \text{ m/s}\) (producing realistic ground motion)
The Catch: Dynamic Weakening

dramatic reduction of frictional resistance at coseismic slip rates
from thermal pressurization, flash heating, thermal decomposition, and many other processes

[Di Toro et al., 2011]
Simple View

Could this work?
Effective stress $(\sigma-p) \sim 100$ MPa (lithostatic–hydrostatic at seismogenic depth)
From experiments, static friction $\approx 0.8$, dynamic friction $\approx 0$

$\rightarrow$ stress drops $\sim 80$ MPa, not observed in large earthquakes, and slip velocities $\sim 30$ m/s (ground motion we don’t want to imagine)
background or overall driving stress
\[ \tau_b/(\sigma-p) \]

\[ \Delta \tau \approx 100 \text{ MPa} \]

\(~6000 \text{ yr}~

Time
This is the world we live in.

background or overall driving stress

$$\tau_b/(\sigma-p)$$

$$\Delta \tau \approx 100 \text{ MPa}$$

$$\sim 6000 \text{ yr}$$

This is the world we live in.

absolute stress unconstrained by seismic wave amplitude

$$\Delta \tau \approx 3 \text{ MPa}$$

$$\sim 200 \text{ yr}$$

time
How Can We Resolve This?

We first have to recognize that we are discussing two related, but subtly different stresses:

- local tractions on fault surface, \( \tau \)
- overall or average stress driving slip, \( \tau_b \) (remote tectonic stress)

These coincide only for planar faults with spatially uniform loading and slip.

Similarly, slip initiates at propagating rupture front when stress concentration transiently raises fault to static friction.
Next, suppose lower limit is dynamic friction ($\approx 0$ from dynamic weakening). To obtain reasonable stress drops, faults must operate at very low stress levels.
And that’s how the San Andreas (and other mature faults) appear to operate – “stress / heat flow paradox”

maximum compressive stress at high angle to San Andreas, requiring $\frac{\tau_b}{(\sigma-p)} < 0.3$

absence of heat flow anomaly requires low dynamic friction

[Townend, 2006; see also ongoing work in Community Stress Model project]
And that’s how the San Andreas (and other mature faults) appear to operate – “stress / heat flow paradox”

dynamic rupture simulations on planar faults with thermal pressurization and flash heating [Noda, Dunham, Rice, 2009] show self-healing slip pulses at low $\tau_b$

*threshold stress level* $\tau_{pulse}$

Arresting pulse $\rightarrow$ Growing pulse $\rightarrow$ Crack

Background stress, $\tau^b/\bar{\sigma}_0$

![Graph showing stress and velocity over time](image)

- Low initial stress at rupture front (0.8)
- Typical static friction $\tau^b = 0.2302\bar{\sigma}_0$
- Low stress during slip

(a)
...but almost all other faults operate at higher stress

Collettini and Sibson [2001] compilation of active ($M_w$ 5.5-6) dipping faults, showing fault operation at $\tau_b/(\sigma-p) \approx 0.6$, based on Sibson [1985] lock-up angle concept

+ many other lines of evidence, including stress measurements in deep boreholes [Townend and Zoback, 2001]
Two possibilities, both with identical background stress levels, but quite different dynamics

1. **Dynamic weakening is unique to mature faults** having well-developed cores prone to shear localization, heating, and thermal weakening.

\[ \tau_b/(\sigma-p) \approx 0.6 \]

2. Dynamic weakening eliminates frictional resistance, but **some other mechanism provides resistance to slip in addition to friction**.

\[ \tau_b/(\sigma-p) \approx 0.6 \]

Second explanation appealing because dynamic weakening is ubiquitous in experiments and localization develops rapidly in sheared gouge.
Faults are Geometrically Complex

...in many ways: surface roughness, segmentation, branching, etc.

This geometric complexity introduces resistance to slip, even when faults are frictionless.

Corona Heights fault, San Francisco
(former quarry, front face stripped away)

1992 Landers earthquake

[Yann Klinger, IPGP]

[Candela et al., 2012]
Shear Resistance from Fault Geometric Complexity

slip on rough fault causes massive near-fault strains, and (elastic) restoring stresses push back on fault in direction opposing average slip

that “backstress” or “roughness drag” $\tau^{\text{drag}}$ increases with structural complexity, but is ultimately bounded by finite strength of off-fault material

(in damage zones extending $\sim 100 \text{ m}$ from major faults)

regardless of whether damage zones are created by inelastic deformation during rupture, or are relic of fault maturation process, further inelastic deformation is expected in this weakened region during earthquakes

[Dieterich and Smith, 2009] rough (fractal) fault

[Mitchell and Faulkner, 2009, study of faults in Atacama desert, Chile]
Dynamics Ruptures on Geometrically Complex Faults with Extreme Dynamic Weakening

**structural complexity:** idealized here as fractal roughness of single fault surface, quantified with *amplitude-to-wavelength ratio* $\alpha$

$\alpha \sim 10^{-2}$ for immature faults, $\alpha \sim 10^{-3}$ for mature faults

[Power and Tullis, 1991; Brodsky et al., 2011; Candela et al., 2013]

**hypothesis:** roughness introduces resistance to slip, in addition to friction, and *"roughness drag"* $\tau^{\text{drag}}$ is dominant resistance for all but smoothest faults
Inelastic Deformation using Drucker-Prager Plasticity
(very similar to Mohr-Coulomb)

plastic strain in rough fault simulations

plasticity essential to prevent otherwise unrealistically large stresses in vicinity of fault

[Dunham et al., 2011]
Vary roughness, measure threshold stress for self-sustaining rupture...but no unique threshold stress for rupture on rough faults

**Figure 4.** Sequences of cumulative slip $\Delta$ every 0.28 s for ruptures occurring on the same fault but at three different $\tau^b$ levels. The extent of rupture and the amount of fault slip increase with background stress.

[Fang and Dunham, 2013]
Remarkable variability, despite statistically identical fault surfaces (same roughness $\alpha$) and same background stress $\tau_b$.

$\alpha = 0.006$, $\sigma_{\text{rms}}/\sigma^2 = 0.2817$, $\tau^2/\sigma^2 = 0.2976$

red=fault profile, blue=slip snapshots

[Fang and Dunham, 2013]
Remarkable variability, despite statistically identical fault surfaces (same roughness $\alpha$) and same background stress $\tau_b$

One interpretation of these ensemble simulations is *different hypocenters along single fault*

![Graph showing fault profile and slip snapshots](image)

Extreme sensitivity to local conditions helps explain why $M_w 7.3$ foreshock two days prior to $M_w 9.0$ Tohoku earthquake remained “small” despite its proximity to mainshock hypocenter (similar behavior observed at Parkfield and elsewhere)

[Fang and Dunham, 2013]
Ensemble Dynamic Rupture Simulations

instead of just running a few simulations on one realization of a random fractal fault, we ran \textit{>1000 simulations} (100 realizations for each $\tau_b$, $\alpha$ combination)

while limited to 2D, we can begin to ask (and address) new types of questions
If earthquake nucleates at some $\tau_b/(\sigma-p)$, will it become large?

...but note that stress levels have not yet reached observed $\tau_b/(\sigma-p) \approx 0.6$

Roughness drag substantially increases threshold stress for self-sustaining rupture.

Figure 6. Probabilities of rupture to reach an extent of 20 km (in either direction) from the hypocenter as a function of background stress level $\tau^b$. Rougher faults require larger $\tau^b$ to reach the same extent.

[Fang and Dunham, 2013]
Roughness-induced stress perturbations and geometric resistance largest at short wavelengths, but computational expense limits us to $\lambda > \lambda_{\text{min}} \sim 100$ m.
Bringing Simulations Closer to Reality (with additional computational resources)

predicted threshold stresses $\tau^b$ based on analytical estimate of $\tau^{\text{drag}}$ and hypothesis that $\tau^b \approx \tau^{\text{pulse}} + \tau^{\text{drag}}$

![Graph showing predicted threshold stresses $(\tau^b/\sigma^0)$ vs. $\Delta u/\lambda_{\min}$ for different values of $\alpha$. The graph shows the off-fault material strength and parameters covered in current simulations.](image)

threshold for flat faults with extreme dynamic weakening

$\tau^b/\sigma^0 = (\tau^{\text{pulse}} + \tau^{\text{drag}})/\sigma^0$

Parameters covered in current simulations

[ Fang and Dunham, 2013 ]
Bringing Simulations Closer to Reality
(with additional computational resources)

predicted threshold stresses $\tau^b$ based on analytical estimate of $\tau^{drag}$ and hypothesis that $\tau^b \approx \tau^{pulse} + \tau^{drag}$

- smooth faults: $\tau^{drag}$ negligible, so $\tau^b \approx \tau^{pulse} \approx 0.25(\sigma - p)$ (like SAF)
- rough faults: $\tau^{drag}$ dominates resistance, ultimately bounded by finite strength of off-fault material, so $\tau^b \approx 0.6(\sigma - p)$

[Fang and Dunham, 2013]
Bringing Simulations Closer to Reality  
(with additional computational resources)

predicted threshold stresses $\tau^b$ based on analytical estimate of $\tau^{\text{drag}}$ and hypothesis that $\tau^b \approx \tau^{\text{pulse}} + \tau^{\text{drag}}$

another possibility is development of additional subparallel fault strands, oriented so as to provide “cooperative strain accommodation” [Shaw, Richards-Dinger, Dieterich] and less overall resistance

[Fang and Dunham, 2013]
Can we distinguish between these two possibilities?

1. **dynamic weakening is unique to mature faults**
   having well-developed cores prone to shear localization, heating, and thermal weakening

2. dynamic weakening eliminates frictional resistance, but **some other mechanism provides resistance to slip in addition to friction**

Second explanation appealing because dynamic weakening is ubiquitous in experiments and localization develops rapidly in sheared gouge.
Possible Discriminants

geometric resistance model predicts:

local near-fault stress changes ~ absolute stress (~100 MPa)

→ diversity of aftershock focal mechanisms [Smith and Dieterich, 2010]
as observed in Loma Prieta, Landers, and other events

(but seismically inferred stress drop, linking moment and fault
dimension and controlling ground motion amplitude, is only
~3 MPa << absolute stress)

rough faults produce realistically flat acceleration spectra at high
frequencies [Dunham et al., 2011];
see also comparison to GMPEs from
3D simulations [Shi and Day, 2013]
Possible Discriminants

generic resistance model predicts:

\( net \ heat \ generation \ far \ less \ than \ during \ sliding \ at \ high \ \tau \)

• along fault, \( \tau \) reduced by dynamic weakening during slip, so minimal \( \Delta T \) and no melting
• away from fault surface, throughout 100 m wide damage zone, heating with \( \Delta T \approx 0.01 \) K accompanies plastic deformation

total energy converted to heat in two models:

generic resistance: \( W_{off} = \int \int \sigma_{ij} \dot{e}_{ij}^p \, dt \, d^3x \sim \tau_{high} \int \int \epsilon^p \, d^3x \sim \tau_{high} \frac{P_{volume}}{P_{fault}} \)

high dynamic friction: \( W_{on} = \int \int \tau V \, dt \, d^2x \sim \tau_{high} \int \Delta u \, d^2x \sim \tau_{high} \frac{P_{volume}}{P_{fault}} \)

ratio of dissipated energy ~ ratio of potency release off and on fault = \( \frac{P_{volume}}{P_{fault}} \ll 1 \)
Moving Toward Probabilistic Quantification of Rupture Style (and Ground Motion) using Ensemble Dynamic Rupture Simulations

*Supershear ruptures occur only on rougher faults*, exactly opposite of common statement [Bouchon et al., 2010] that supershear requires smooth fault segments!

[Bruhat, Fang, Dunham, work in progress, 2014]
Moving Toward Probabilistic Quantification of Rupture Style (and Ground Motion) using Ensemble Dynamic Rupture Simulations

Threshold $\tau^{\text{pulse}}$ for flat faults could also quantify likelihood of ruptures:
- taking branches
- jumping step-overs
- linking segments in multifault event

...but this is still incomplete, as we also need to know probability that nucleation will occur some $\tau^b$

Unlikely fault will reach these high $\tau^b$ because, as tectonic loading increases $\tau^b$, earthquakes likely occur earlier and relax $\tau^b$

[Bruhat, Fang, Dunham, work in progress, 2014]
Vision for SCEC5

Simulation of earthquake sequences on geometrically complex faults (with plasticity, inertial rupture dynamics, and in 3D, of course!)

- Single-event dynamic rupture codes
- Earthquake simulators (boundary elements)

We've taken some initial steps toward this goal, and in the process stumbled on something quite interesting.
Earthquake Sequences in Sedimentary Basins

Current modeling capabilities:
- 2D quasi-dynamic simulation
- rate-and-state friction
- using finite difference method to accommodate 
  *elastic heterogeneity* (no plasticity)

sedimentary basins like Salton Trough in southern CA

not possible with BEM simulators

[Lindsey and Fialko, 2013] shear modulus cross-section from SCEC CVM-H 6.3
Earthquake Sequences in Sedimentary Basins

Current modeling capabilities:
• 2D quasi-dynamic simulation
• rate-and-state friction
• using finite difference method to accommodate elastic heterogeneity (no plasticity)

not possible with BEM simulators

2D antiplane shear model: vertical strike-slip fault through center of basin, driven by remote displacements on lateral boundaries

sedimentary basins like Salton Trough in southern CA

[Erickson and Dunham, 2014; Allison, Erickson, Dunham, work in progress]
Earthquake Cycles in Sedimentary Basins

- Blue lines = 5 year interval, black lines = 1 second interval

- Rate-and-state friction:
  - Velocity-weakening
  - Velocity-strengthening

[Ericksom and Dunham, 2014; Allison, Erickson, Dunham, work in progress]
Earthquake Cycles in Sedimentary Basins

Compliant basins give rise to alternating sequences of *sub-basin* and *surface-rupturing events*.

**Diagram:**
- Blue lines = 5 year interval, black lines = 1 second interval.
- Surface-rupturing events are much larger, and feature extensive shallow coseismic slip.
- No basin.

**Graph:**
- Rate-and-state friction:
  - Velocity-weakening
  - Velocity-strengthening

**Table:**
- Basin depth = 4 km
- \( \mu_{basin} = 12 \text{ GPa} \)

[Erickson and Dunham, 2014; Allison, Erickson, Dunham, work in progress]
Why do Compliant Basins Inhibit Rupture?

+ other causes, not modeled here, including velocity-strengthening in sedimentary rocks and lower $\sigma-p$ near surface (future work)

[Shear Stress $\tau$ (MPa)]

lower stress in basin

stressed concentration left behind by sub-basin event (facilitates subsequent ruptures)

[0 5 10 15 20] Depth (km)

[20 years prior to event]

prior to surface-rupturing event

prior to sub-basin event

transition to aseismic creep

[Erickson and Dunham, 2014; Allison, Erickson, Dunham, work in progress]
Does This Happen in Nature?

[Rockwell and Klinger, 2013]
slip at surface (m) in 1940 $M_w$ 7.0 Imperial Valley earthquake

rather similar to our models, with implications for seismic hazard, paleoseismology, shallow-slip deficit

modified from Archuleta [1984]

contours of slip (m) in 1979 $M_w$ 6.5 Imperial Valley earthquake
Uncovering these fascinating behaviors is only the start, as we begin to incorporate elastic heterogeneity and ultimately inelastic deformation in earthquake simulators.

We can also take simulations in other new directions, such as adding gravity...
Tsunamis from Dynamic Ruptures

With SCEC’s new focus on the Ventura-Pitas Points fault system, the need to understand *tsunami generation* has risen to a new prominence

[Hubbard et al., 2014]
It’s easy to extend wave propagation codes to simulate tsunamis!

\[ y = \eta(x, t) \]

method rigorously solves linearized problem, including surface gravity wave dispersion at short wavelengths

free surface BC on moving surface (Eulerian description): \[ p = 0 \quad \text{on} \quad y = \eta(x, t) \]

apply approximate free surface BC on unperturbed surface (linearization): \[ p(\eta) \approx p(0) + \frac{\partial p}{\partial y} \bigg|_{y=0} \eta \approx p(0) - \rho g \eta \]

\[ \Rightarrow p = \rho g \eta, \quad \frac{\partial \eta}{\partial t} = v_y \quad \text{on} \quad y = 0 \]

with no other changes to governing equations! [Lotto and Dunham, 2014]
Fig. 5 Space-time plot of sea surface $\eta$, illustrating the variety of waves generated by an earthquake on a thrust fault beneath the ocean. The black lines are drawn at the tsunami speed in the shallow water limit, $\sqrt{gH}$; the fluid sound speed, $c = \sqrt{K/\rho}$; and the Rayleigh wave speed, $0.919 c_s$. After about 300 s, the bulk of the seismic and ocean acoustic waves have propagated out of the domain and only slower-traveling surface gravity waves remain. Note the dispersive tail to the tsunami.
While our method is not useful for estimating run-up and inundation, it can provide self-consistent initial conditions for nonlinear tsunami simulations (unresolved issues with complex bathymetry)

...perhaps even more exciting would be modeling initiation of submarine landslides (gravity+inelastic failure)

landslide near Santa Barbara, possible 10 m local tsunami – could such events be triggered by seismic shaking?
Conclusions and Vision for SCEC5

**Simulation of earthquake sequences on geometrically complex faults**
*(with plasticity, inertial rupture dynamics, and in 3D, of course!)*

- Consideration of fault geometric complexity resolves many outstanding issues in earthquake science (stress/heat flow, high-frequency ground motion), but we must move **beyond elasticity**.
- Ensemble dynamic rupture simulations allow us to quantify variability of earthquake behavior.
- Earthquakes cause tsunamis and landslides – we can potentially model these hazards, too, in a unified framework.