Conditions Allowing Self-healing vs. Crack-like Rupture Propagation in Presence of Thermal Weakening Processes Based on Realistic Physical Properties

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Abstract

We have conducted rupture propagation simulations allowing for the combined effects of thermal pressurization of pore fluid and flash heating of microscopic contact asperities. Rapid, large slip as in earthquakes produces a large amount of frictional heat, and probably activates such thermal weakening mechanisms. They act until macroscopic fault temperature T nears the melting point, and they are coupled through T and macroscopic fault surface pore pressure p. Because we consider two mechanisms, the characteristic slip displacements for each of them is physically important; a compromise on one of them can eventually nullify its effect compared to the other. We use a range of realistic hydraulic properties for thermal pressurization (hydro-thermal diffusivity factor 20-450 mm^2/s; factor corresponds to $f^2VL_*/4$ [Rice, 2006] where f is the friction coefficient, V is slip rate, and L_* is a characteristic slip defined in the thermal pressurization analysis), and few-microns contact evolution distance L for flash heating with a slip law formulation and the direct effect. We use the spectral implementation of the BIE method for elastodynamic calculations, and set a one-dimensional FD grid perpendicular to the fault plane at each node in order to calculate local heat and fluid transport assuming an infinitesimally thin slipping plane. We also use a multi-step time increment procedure by setting longer steps for slip history storage and shorter steps for integration of state variable, T, and p. Elasticity and the constitutive relation are solved simultaneously at every shorter time step with linearly interpolated stress transfer. This method reduces the amount of memory but produces numerical stability.

We nucleated rupture by adding a sudden perturbation to the shear stress, which is initially uniform and much lower than the static friction level. Our calculations show that the effect of evolving changes in T and p is to extend the crack-like (vs. self-healing) solution regime in the parameter space, although we are still examining the way nucleation may interact with rupture mode. Given a steady state shear stress which is a function only of slip rate, Zheng and Rice [1998] derive a critical background shear stress below which a growing crack-like solution does not exist for mode III rupture. In our case, such a steady-state function cannot be defined due to strength dependency on T and p on the fault. We then can obtain crack-like solutions with background shear stress lower than the critical value defined by the ZR concept based on steady-state flash heating only, at the initial T and p. With decreasing background shear stress, the type of solution changes from crack-like to self-healing after a clear threshold. The flash heating constitutive relation has very steep velocity weakening around its critical slip rate, which effectively decelerates fault motion in the cases of self-healing solutions. By changing hydraulic parameters, the threshold background shear stress between the two types of solutions changes so that low hydro-thermal diffusivity favors crack-like solutions. The size of perturbation also matters and a larger (in length or amplitude) perturbation also favors crack-like solutions.

Introduction

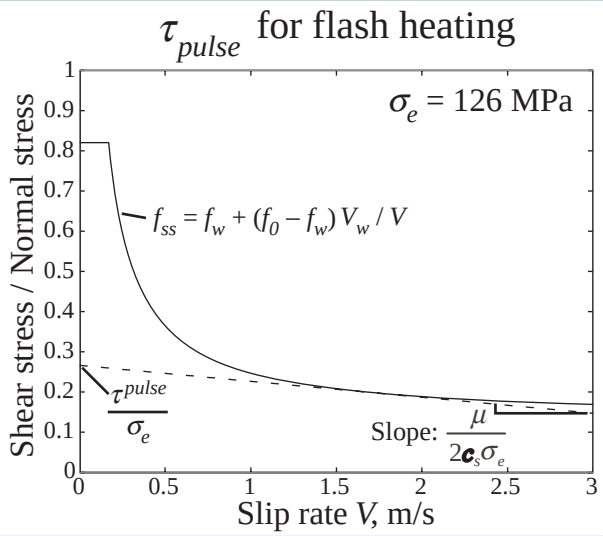
τ_{pulse}: Critical background shear stress below which crack-like solution does not exist for mode III rupture [Zheng and Rice, 1998]

True when steady state shear stress is *a function only of slip rate*

How does introduction of thermal weakening processes (T and p dependency), affect the type of solution?

We focus on
- Thermal pressurization
- Flash heating

Coupled through T and p



Set of equations

Elastodynamic spectral BIE method [*Perrin et al.*, 1994; *Geubelle et al.*, 1995]

$$\tau = \tau_0 - \frac{\mu}{2c_s} V + \phi \quad \text{where} \quad \Phi = C_s(t) D(t) + \int_0^t C(t-t') \dot{D}(t') dt'$$

Fourier t.

Effective stress law

$$\tau = \sigma_e f = (\sigma - p)\, f$$

Rate- and state-dependent frictional law (Slip law like state evolution)

$$f = a \ln \frac{V}{V_{ref}} + \Psi \qquad \frac{d\Psi}{dt} = -\frac{V}{L}(f - f_{ss})$$

[*Dietrich*, 1979; *Ruina*, 1983]

Flash heating, Rice's model

$$f_{ss} = \begin{cases} f_0 & V \le V_w \\ f_w + (f_0 - f_w) V_w / V & V \ge V_w \end{cases} \quad \text{where} \quad V_w = \pi \frac{\alpha_{th}}{L} \left(\frac{T_w - T}{\tau_c / \rho c} \right)^2$$

[*Rice*, 2006]

Thermal pressurization assuming infinitesimally thin fault

$$\frac{\partial T}{\partial t} = \alpha_{th} \frac{\partial^2 T}{\partial z^2}$$

$$\frac{\partial P}{\partial t} = \alpha_{hy} \frac{\partial^2 P}{\partial z^2} + \Lambda \frac{\partial T}{\partial t}$$

Boundary condition

$$\alpha_{th} \rho c \left. \frac{\partial T}{\partial z} \right|_{z=0+} = -\frac{1}{2} f \sigma_e V$$

$$\left. \frac{\partial P}{\partial z} \right|_{z=0} = 0$$

[*Sibson*, 1973; *Lachenbruch*, 1980; *Mase and Smith*, 1987; *Andrews*, 2002; *Rice*, 2006]

Numerical implementation

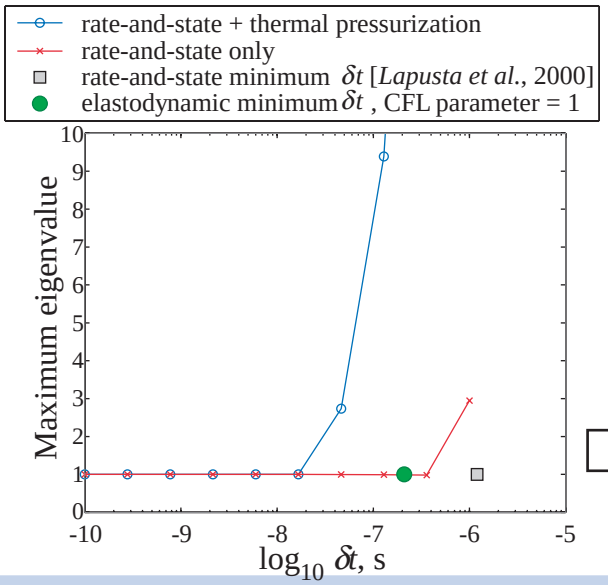
Boundary integral method is used with ***multiple time steps***.

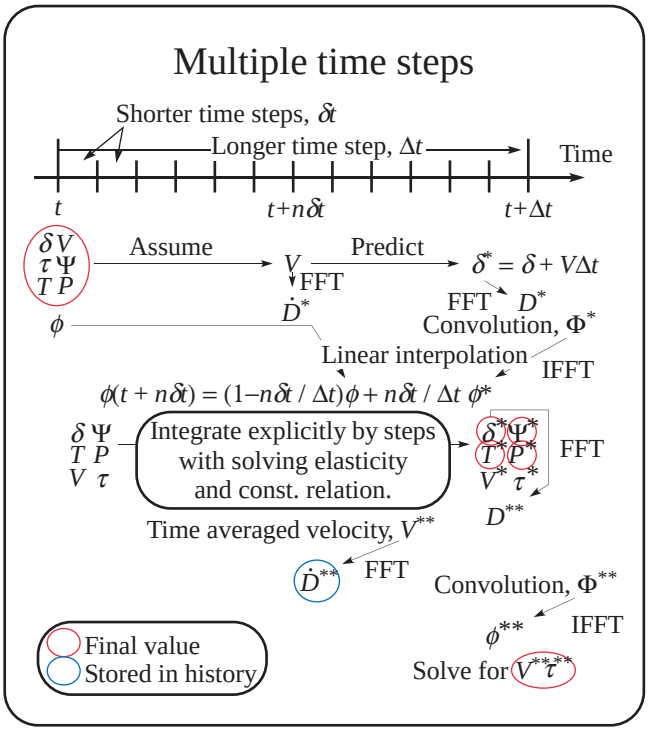
- Longer time step for storage of slip rate history
  Chosen to satisfy stability for the wave equation

- Shorter time step for integration of constitutive relation
  Stability analysis for discretized system around $V = V_w$
  where $-\partial f_{ss}/\partial V$ is the largest

$$\left[\delta^{n+1}, \Psi^{n+1}, \{T\}^{n+1}, \{p\}^{n+1}\right]^T = Q\left[\delta^{n}, \Psi^{n}, \{T\}^{n}, \{p\}^{n}\right]^T$$

Numerically obtained maximum eigen value of $Q \leqq 1$





Coupling of processes makes numerics less stable compared with each process alone.

Initial condition / parameters

We have two evolution distances

- State evolution distance L ~ asperity size (microns)
- Themal pressurization L_* defined in [*Rice*, 2006]

$$f^2 V L_* / 4 = \left(\frac{\rho c}{\Lambda}\right)^2 \left(\sqrt{\alpha_{th}} + \sqrt{\alpha_{hy}}\right)^2$$

The ratio of them is *physically important*.

So, we stick to L = 5 μm and realistic thermo-hydraulic parameters,

Hydro-thermal diffusivity factor: $\left(\frac{\rho c}{\Lambda}\right)^2 \left(\sqrt{\alpha_{th}} + \sqrt{\alpha_{hy}}\right)^2$ = 20 – 460 mm²/s (r = 0 – 1)

a little wider range than [*Wibberey*, 2002; *Wibberey and Shimamoto*, 2003]

by setting damage index, r ∈ [0, 1] $\alpha_{hy} = 0.86 + 2.66r$ mm²/s and $\Lambda = 0.98 - 0.64r$ MPa/K

Other physical parameters

μ	30 GPa
c_s	3 km/s
T_w	900 °C
τ_c	3 GPa
f_0	0.82
f_w	0.13
ρc	2.7 MJ/m³K

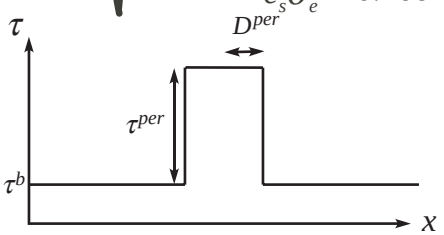
Initial conditions

σ	196 MPa
p_0	70MPa
σ_{e0}	126 MPa
T_0	210 °C
V_0	10^{-9} m/s

$$\tau_0 = \begin{cases} \tau^b & |x| > D^{per} \\ \tau^b + \tau^{per} & |x| < D^{per} \end{cases}$$

=> $\tau^{pulse} / \sigma_{e0}$

$$= f_w + \sqrt{2(f_0 - f_w)\frac{V_w \mu}{c_s \sigma_e}} = 0.266$$



frictional coefficients from
Tullis and Goldsby, 2003; *Rice*, 2006

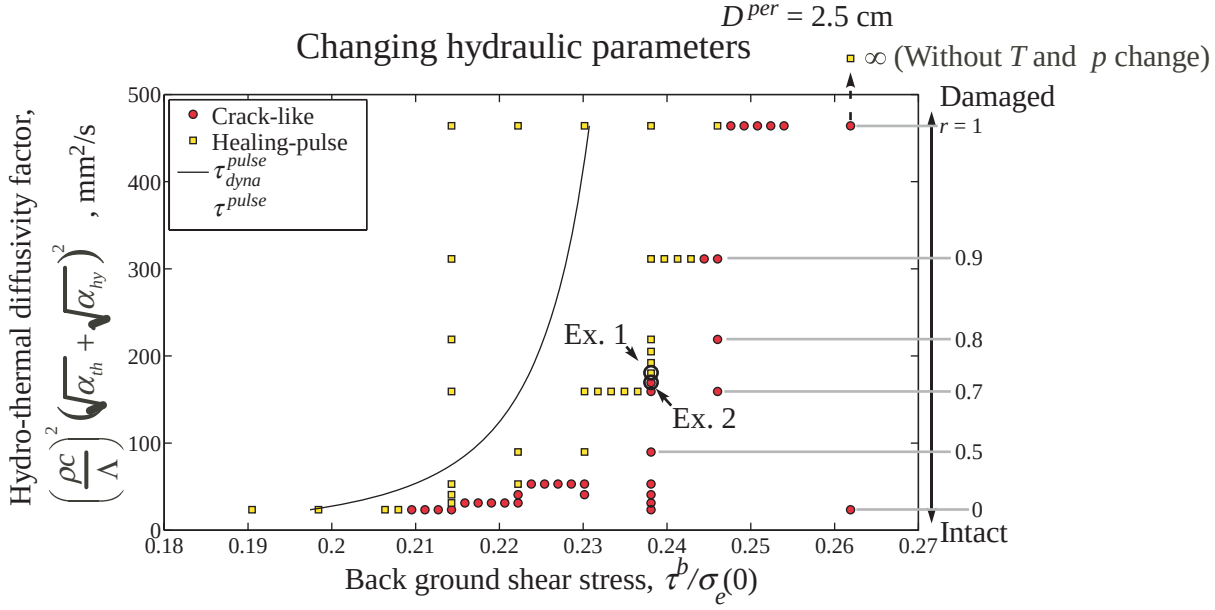
Results 1

Effect of hydraulic parameters

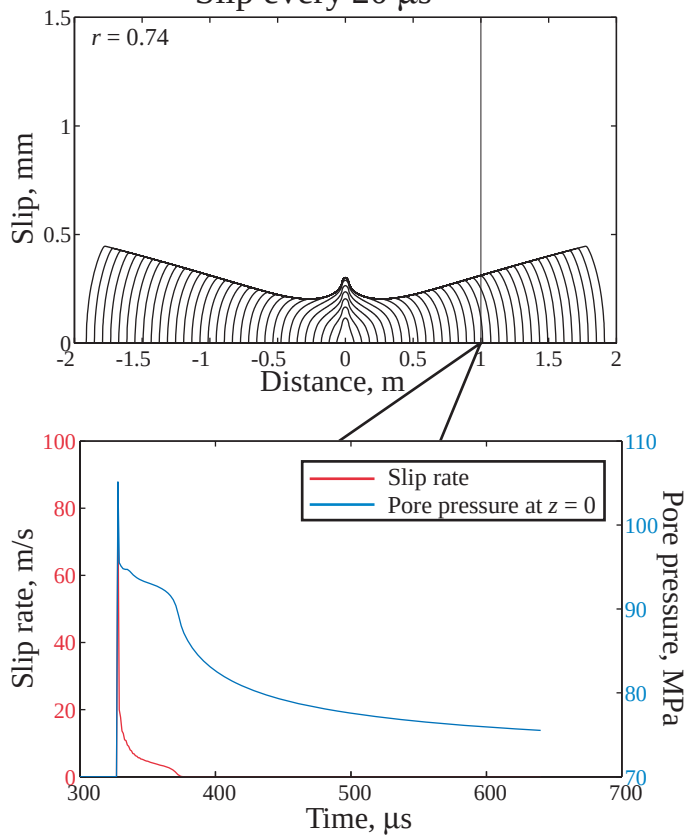
$\tau^{per} / \sigma_{e0} = 0.238$

$D^{per} = 2.5$ cm

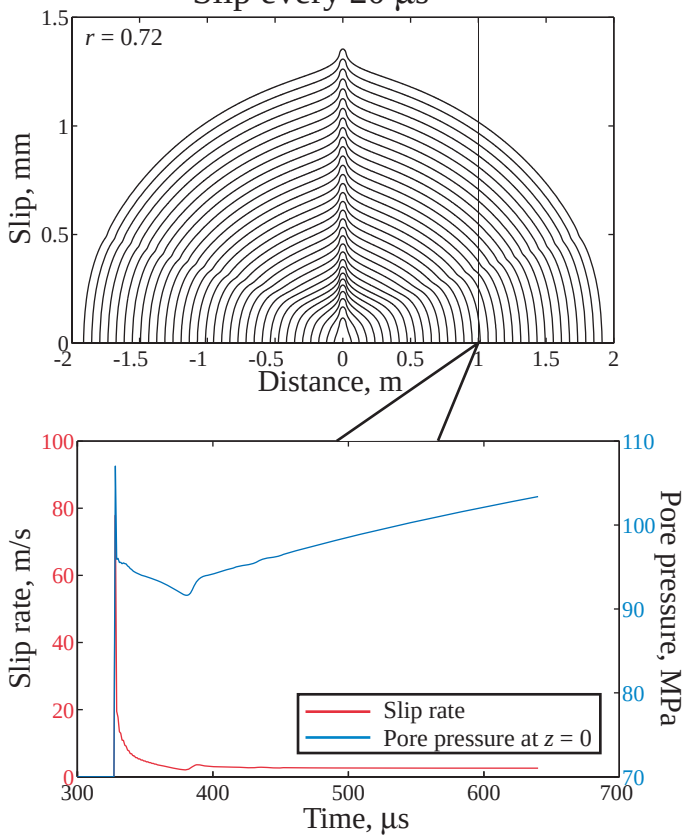
Changing hydraulic parameters



Ex. 1 Slip every 20 μs



Ex.2 Slip every 20 μs



Results 2

Effect of perturbation size

Characteristic slip rate, V_{dyna} in the dynamic motion due to perturbation is given so that (V_{dyna}, $\sigma_{e0} f_{dyna}$) is the rightmost intersection of

$$\tau = \tau^b + \tau^{per} - V\frac{\mu}{2c_s} \quad \text{and} \quad \tau = \sigma_{e0}\left(f_w + (f_0 - f_w)\frac{V_w}{V}\right)$$

Characteristic time length:

$$t_{dyna} = \frac{D^{per}}{c_s}$$

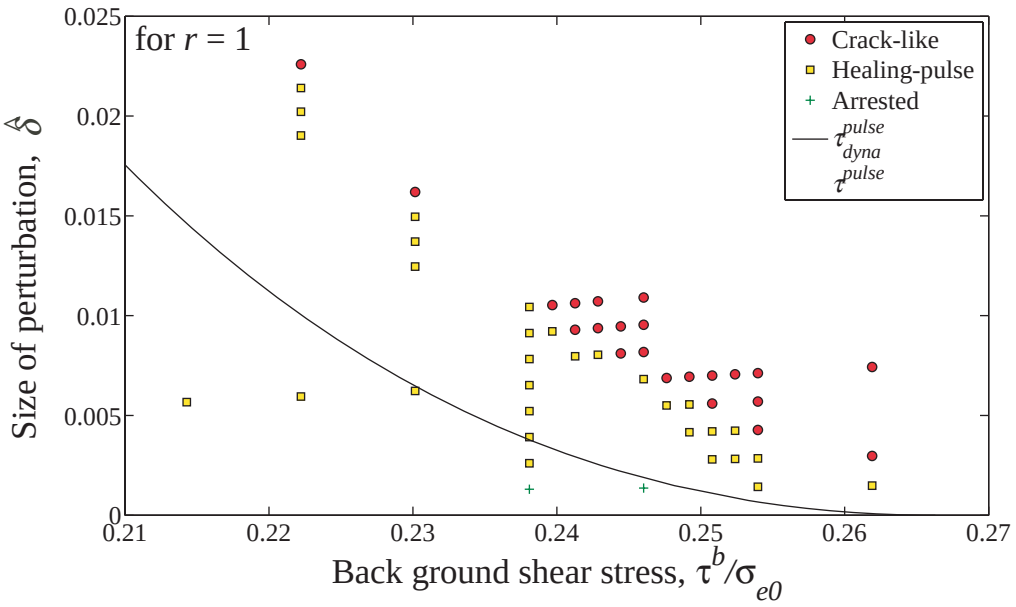
Characteristic thermal pressurization evolution distance:

$$L_{*dyna} = L_*\big|_{V_{dyna},\, f_{dyna}}$$

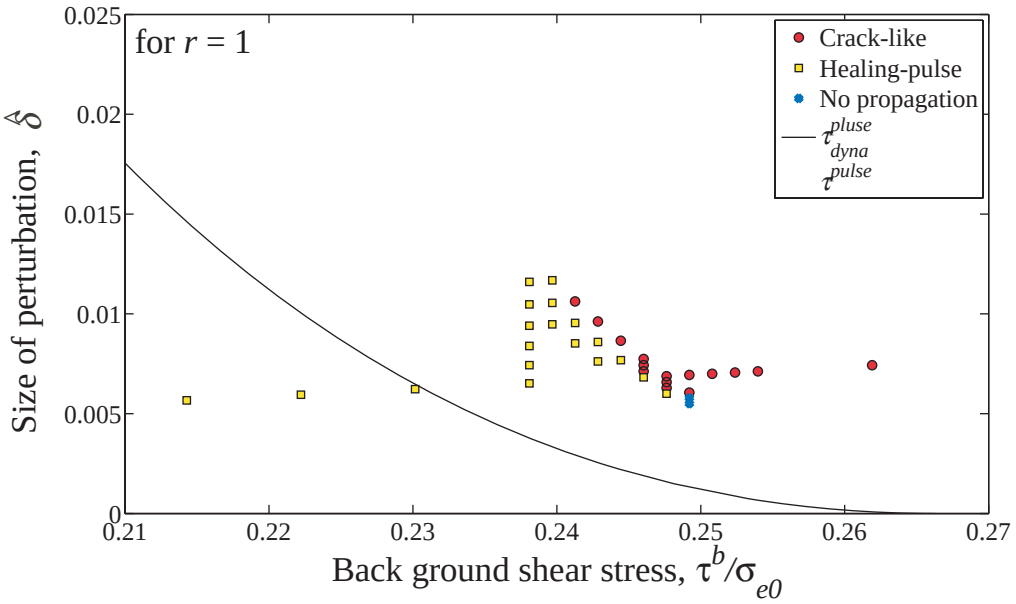
Measure of the size of perturbation in terms of thermal pressurization:

$$\hat{\delta} = t_{dyna} V_{dyna} / L_{*dyna}$$

Changing length of perturbation



Changing amplitude of perturbation



Discussion

Criteria for crack-like and slip-pulse solutions

Due to T and p change, V_w and σ_e, and thus steady state shear stress decrease.
Given characteristic slip rate and time, we can roughly estimate change in σ_e and T.

$$\sigma_{e\,dyna} = \alpha\sigma_{e0} \quad \text{and} \quad T_{dyna} = T_0 + \left(1+\sqrt{\frac{\alpha_{hy}}{\alpha_{th}}}\right)\frac{\sigma_{e0}}{\Lambda}[1-\alpha]$$

$$\text{where} \quad \alpha = \exp(\hat{\delta})\,\mathrm{erfc}\left(\sqrt{\hat{\delta}}\right)$$

[Rice, 2006]

Critical background shear stress level for dynamic case, similar to τ^{pulse} is:

$$\frac{\tau^{pulse}_{dyna}}{\sigma_{e0}} = \alpha f_w + \sqrt{2\alpha(f_0 - f_w)\frac{V_{w\,dyna}\mu}{c_s\sigma_{e0}}}$$

$$\text{where} \quad V_{w\,dyna} = V_w\big|_{T=T_{dyna}}$$

$\frac{\tau^b}{\sigma_{e0}} = \frac{\tau^{pulse}_{dyna}}{\sigma_{e0}}$ is also plotted in the figures with solid lines, which is not an awful criteria.

"$\frac{\tau^b}{\sigma_{e0}} < \frac{\tau^{pulse}_{dyna}}{\sigma_{e0}}$ => Healing pulse solution unless arrested"
is true for cases thus for studied.

Dependency on the size of perturbation

It is shown that the size of perturbation and the nucleation procedure influence the rupture mode. This means that it is important to investigate more realistic methods of nucleation.

Summary

Multi time stepping procedure has been implemented to spectral boundary integral code.

Rupture propagation calculation accounting for flash heating and thermal pressurization has been performed with realistic physical properties.

Introduction of temperature and pore pressure changes promotes crack-like solution

There is a clear boundary between crack-like and healing-pulse solutions

A critical background shear stress level below which solution is always healing-pulse type is proposed although it depends on the method of nucleation.

Further study concerning a realistic way of nucleation is needed to develop a criteria between the two types of solutions.

List of symbols

a	Direct effect
C	Kernel for slip rate history
C_s	Kernel for current slip
c_s	Shear wave velocity
D	Fourier t. of slip
$\dot{D}$	Fourier t. of slip rate history
D^{per}	Perturbation length
f	Frictional coefficient
f_{ss}	Steady state frictional coefficient
f_0	Unweakened frictional coefficient
f_w	Weakened frictional coefficient
L	State evolution distance
L_*	Evolution distance for thermal pressurization [Rice, 2006]
$L_{*\,dyna}$	L_* in dynamic slip process
p	Pore pressure, subscript 0 means initial value
r	Damage index
t	Time
T	Temperature, subscript 0 means initial value
T_w	Weakening temperature
V	Slip rate, subscript 0 means initial value
V_{dyna}	Characteristic slip rate in dynamic slip process
V_{ref}	Reference slip rate
V_w	Weakening slip rate
x	Distance along fault
z	Distance normal to a fault
α_{hy}	Hydraulic diffusivity
α_{th}	Thermal diffusivity
ϕ	Stress transfer
$\hat{\delta}$	Characteristic displacement due to perturbation in terms of thermal pressurization
Φ	Fourier t. of ϕ
Λ	Pore pressure rise per temperature change in undrained condition
μ	Shear modulus
ρc	Specific heat
σ	Normal stress
σ_e	Effective normal stress, subscript 0 means initial value
τ	Shear stress
τ_0	Loading stress
τ^b	Background loading stress
τ_c	Contact strength, assumed to be 0.1μ
τ^{per}	Perturbation stress
τ^{pulse}	Critical shear stress for impossibility of crack-like solution [Zheng and Rice, 1998]
τ^{pulse}	Critical shear stress accounting for dynamic weakening
Ψ	State variable

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