Investigators

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Abstract

Data substitution as a means of evolving time lapse sparsely acquired data has been tested and is presented in this report. Results show that it is a viable option for analyzing data acquired for monitoring subsurface flow such as monitoring sequestered CO_2. Results however, also show that artifacts are bound to occur because changes in the old data that may have been caused by changes in the subsurface are not taken into account. A more efficient method is a one that takes these changes into account and methods with this property are currently being researched.

Introduction

The primary objective of this study is the development of data and model evolution methods for use in 4-d dynamic tomography. In carrying out a time lapse monitoring experiment, given the same resources, two scenarios are possible; 1) acquiring complete dataset at every large time step and 2) acquiring incomplete (sparse) dataset at every closely spaced time step. Scenario 2 gives the opportunity for tracking the changes in the reservoir more frequently. Throughout the lifetime of the reservoir being monitored, a few sources and receivers can be used to acquire quasi-continuous data. A complete dataset may have been acquired during site characterization and can be used as the "base" dataset. Having acquired a dense dataset and several sparse datasets, the challenge becomes how to use the sparse data efficiently. One potential quandary is the development of an under-determined tomographic inversion problem, in which case, we have less data than the number of unknowns. A solution to this queue is to predict what the data would have been if additional sources and receivers were used.

A lot of work has been done on image reconstruction from incomplete data especially in the field of medical imaging where data may have been acquired over incomplete aperture (e.g. [1], [2]) as well as in the field of astronomy (e.g. [3]). The studies in the stated examples reconstructed spatially incomplete data. In this study, my intention is to reconstruct temporally incomplete data. Techniques considered include the implementation of the Kalman filter, locally varying mean kriging and simple data substitution. So far, only data substitution has been successfully implemented. Because of the efficiency of diffraction tomography in terms of speed, I have decided to use it in testing data/model evolutions techniques. An alternative approach is conventional traveltime tomography which will involve the generation of synthetic seismic data that simulates data acquired by a specific source – receiver geometry, followed by traveltime picking and then iterative traveltime inversion. The advantage of using diffraction tomography is that significantly less amount of time is required to generate an ideal recoverable model. The inherent idea behind diffraction tomography is that every geologic model occupies a certain portion of Fourier space. In like manner, every source – receiver geometry can recover a certain portion of the Fourier space [4]. The portion of the geologic model that can be reconstructed is the portion of Fourier space where both the acquisition geometry and the geologic models overlap [4].

Background

This section gives the mathematical backing for data substitution in the Fourier domain. Data substitution is a method that uses the linearity property of Fourier transforms. The Fourier transform is linear which means that if we consider two periodic sequences $f_1(x, y)$ and $f_2(x, y)$, with periods of *N*, and combined as shown in the equation

$$f_3(x, y) = af_1(x, y) + bf_2(x, y) \tag{1}$$

then the discrete Fourier series (DFS) representation of (1) is given as

$$F_3(k_x, k_y) = aF_1(k_x, k_y) + bF_2(k_x, k_y) \tag{2}$$

all with period *N* [5].

Following this line of argument, we can define a given 2-D geologic model, $f_m(x, z)$, at time t_0, as

$$f_m^0(x, z) = a_0 f_0(x, z) \tag{3}$$

and at time t_1, as

$$f_m^1(x, z) = f_m^0(x, z) - b_1 g_1(x, z) + c_1 f_1(x, z) \tag{4}$$

where $b_1 g_1(x, z)$ and $c_1 f_1(x, z)$ are zero everywhere except at the points where the models $f_m^0(x, z)$ and $f_m^1(x, z)$ are different. There, $b_1 g_1(x, z)$ is equal to $f_m^0(x_i, z_i)$ and $c_1 f_1(x_i, z_i)$ is equal to $f_m^1(x_i, z_i)$. In the same vain, the geologic model at time t2, $f_m^2(x, z)$ can be expressed as

$$f_m^2(x, z) = f_m^1(x, z) - b_2 g_2(x, z) + c_2 f_2(x, z) \tag{5}$$

In a generalized form,

$$f_m^n(x, z) = f_m^{n-1}(x, z) - b_n g_n(x, z) + c_n f_n(x, z) \tag{6}$$

where $n = 1, 2, 3....$ is the time step.

From the linearity principle explained with equations (1) and (2), the DFS representation of (6) is

$$F_m^n(k_x, k_y) = F_m^{n-1}(k_x, k_y) - b_n G_n(k_x, k_y) + c_n F_n(k_x, k_y) \quad (7)$$

It is noteworthy to state that the same logic will apply if we start with the Fourier representation. The outlined steps above govern the implementation of data substitution in Fourier space presented in the report.

Results

Data substitution was tested using "hand drawn" geologic models. One of the units in the geologic model was chosen to be the reservoir for storing the sequestered CO_2. A fault is allowed to exist in the model. This fault permits the upward flow of CO_2 to another geologic unit with reservoir characteristics. There are a total of 221 models representing 221 snapshots in time. Each one of the geologic models is 250 by 250 elements in dimension and represents an earth unit 1000m by 1000m. Figure 1 shows the geologic model at different states and its corresponding flow model (difference model). Figure 2 shows the 2-D Fourier transforms of the geologic model at time t_0 and selected difference models. Parts of the Fourier transform corresponding to the fault structure as well as the curved shape of geologic units are readily identifiable.

Filters corresponding to surface data acquisition, crosswell data acquisition and vertical seismic profiling (VSP) acquisition were used. 21 shots and 21 receivers were used in all three cases; surface acquisition frequency range was 5 – 50 Hz, crosswell acquisition frequency range was 100 – 1000 Hz and VSP acquisition frequency range was 5 – 100 Hz. I applied an acquisition strategy which involved full/complete coverage at time t_0 and sparse/ incomplete coverage at every other time. Excluding the first time step, an equivalent of a full coverage was acquired every 42 consecutive time steps. The advantages of such a strategy include the fact that old data are frequently replaced and also, it gives room for data tracking which is useful for any prediction scheme. Figures 3, 4 and 5 show filter states at time t_0 and at selected time steps.

In this analysis, data refers to the function upon which the inverse Fourier transform is applied in the equation

$$O(x,z) = \frac{1}{(2\pi)^2} \iint \widetilde{O}(K_x, K_z) \cdot \exp\left[i\left(K_x x + K_z z\right)\right] dK_x \; dK_z \quad (8)$$

as derived by Harris[6] and Wu & Toksöz [4] for diffraction tomography, where $O(x,z)$ is the reconstructed image. Substitution was done in Fourier space. Figures 6, 7 and 8 show reconstructed images (with and without data substitution) after they have been subtracted from the reconstructed image at time t_0 i.e. difference images. See the appendix for an illustration of the procedural steps followed. More accurate results were produced from inversions that involved data substitution when compared to those without data substitution. In fact, without data substitution, the results were not interpretable. There are however, some artifacts in the reconstructed images obtained when data substitution was used. These artifacts may be a result of replacing only a partial spectrum of the changes that occurred in each time step, keeping in mind that not all changes were captured due to the limited aperture used. If this is truly the cause, a better approach is to predict the current state of old data at each time step. Kriging the time axis as well as applying Kalman filter schemes are methods that are being considered to be used to predict data.

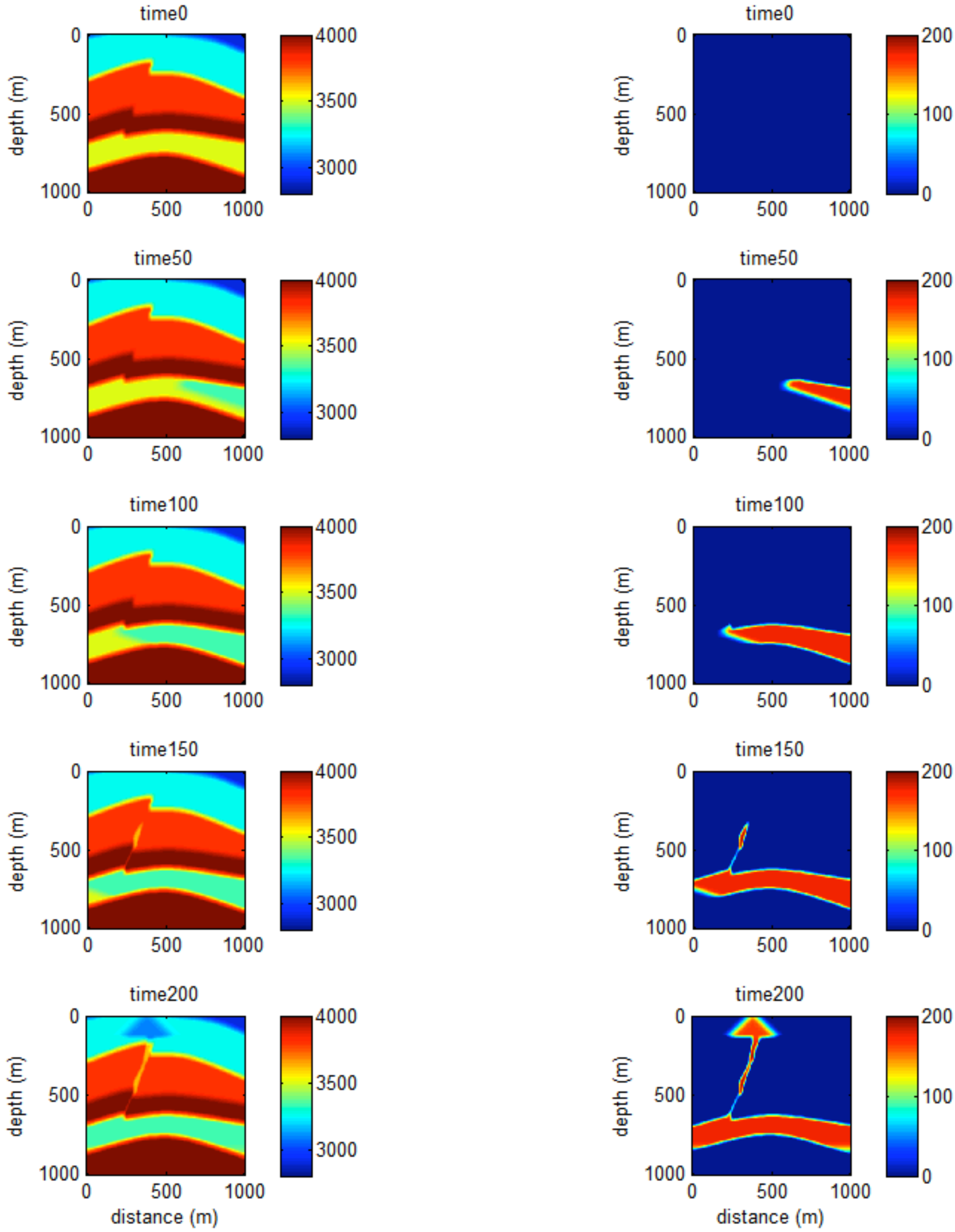
Progress

Progress has been made in the area of understanding the diffraction tomography concept and its implementation. Some previously (earlier quarters) identified problems have been solved. Mathematical formulations to support presented ideas have been developed. Results presented in this paper have increased my confidence in the ability to use diffraction tomography in developing data evolution schemes. This work is meant to culminate in the development of data/model evolution techniques applicable to monitoring sequestered CO_2. Data evolution techniques can be applied to quasi continuous seismic data which is best suited for time lapse imaging. Quasi continuous data acquisition is economical when sparse, hence the need for a data evolution technique.

Future Plans

Over the summer, I intend to continue my literature review to better understand the problem and to equip myself with the skills required to obtain a solution to the problem. Of particular interest are the field of data reconstruction and the applications and implementation of Kalman filters. I plan to implement the Kalman filter in the Fourier domain. I also plan to apply a locally varying mean kriging algorithm to my data. These two are deliverables I intend to have at the end of the summer.

Figures



(a) **(b)**

Figure 1. a) The geologic model at different states. b) Its corresponding difference model (flow model). A difference model is obtained by subtracting the model at time t_n from the model at time t_0.

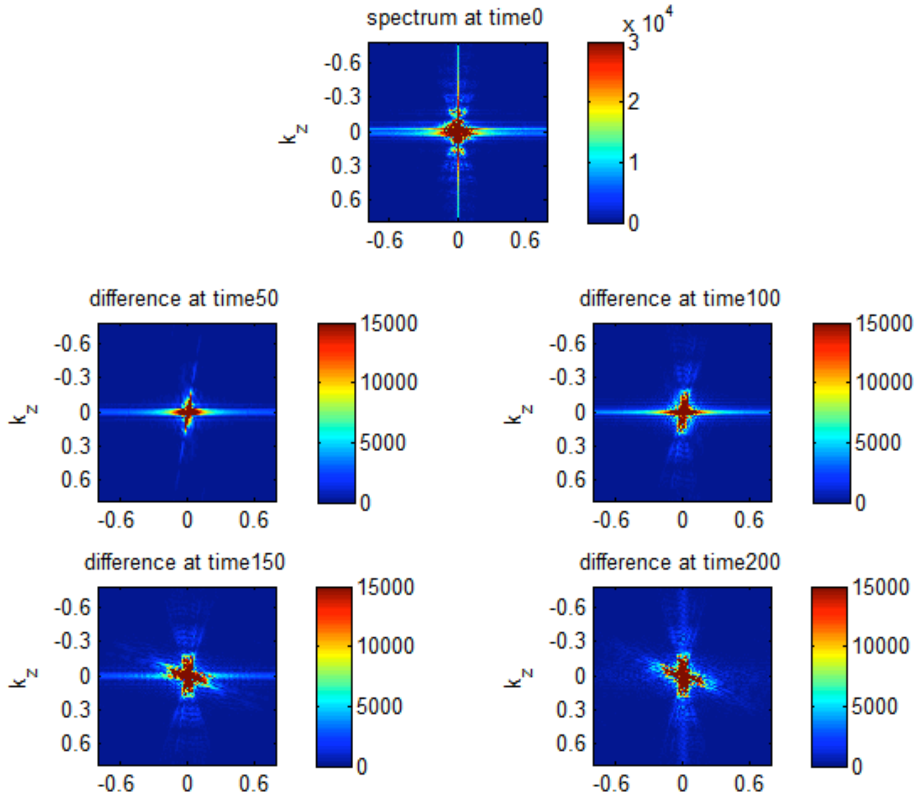
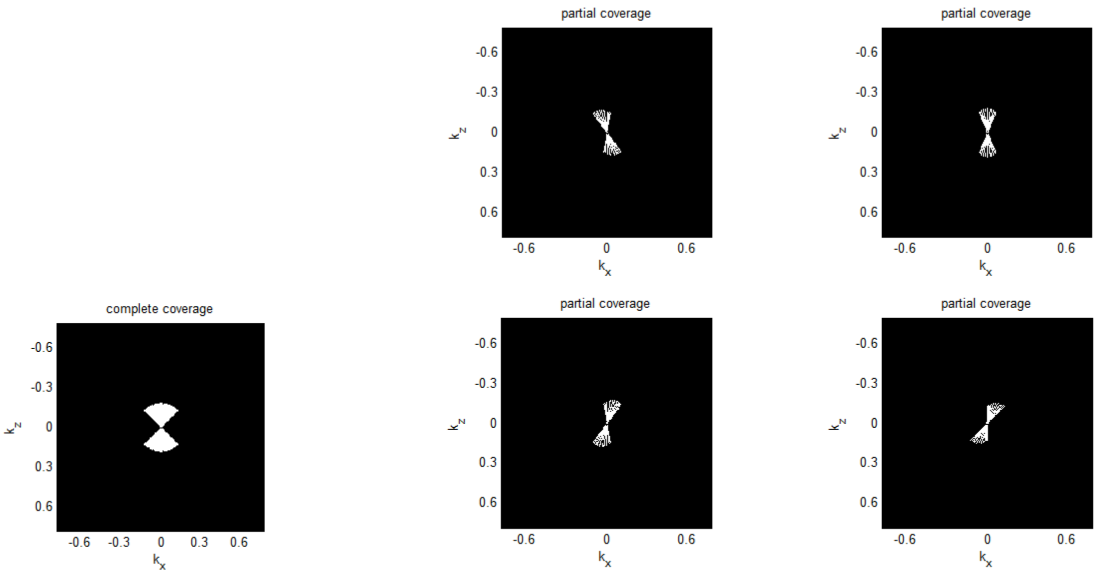
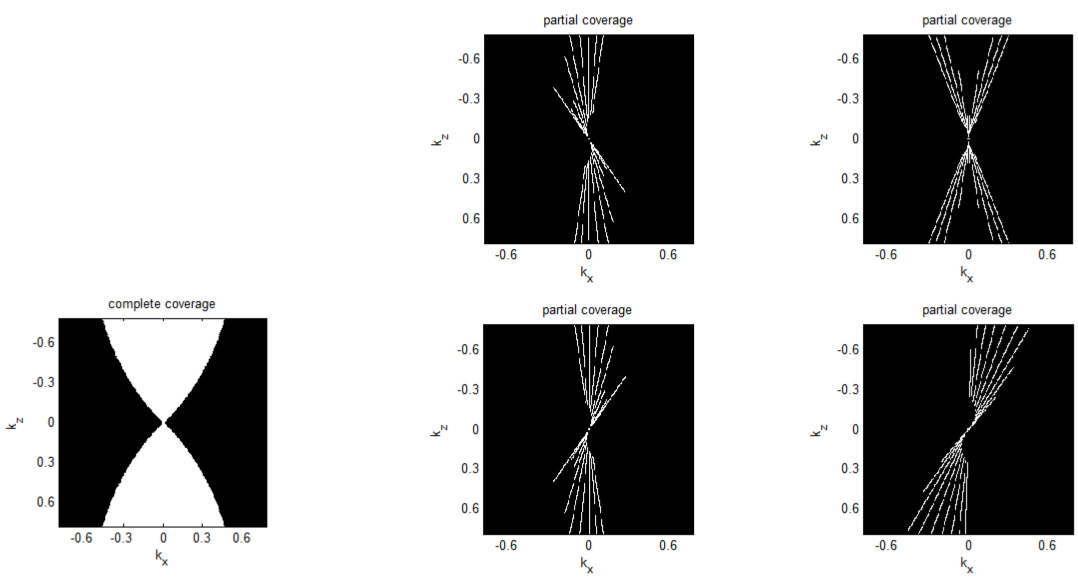


Figure 2. The absolute values of the 2-D Fourier transform of the geologic model at time t_0 and selected 2-D Fourier transform difference models. A 2-D Fourier transform difference models is obtained by subtracting the absolute values of the 2-D Fourier transform of the model at time t_n from the absolute value of the 2-D Fourier transform of the model at time t_0.



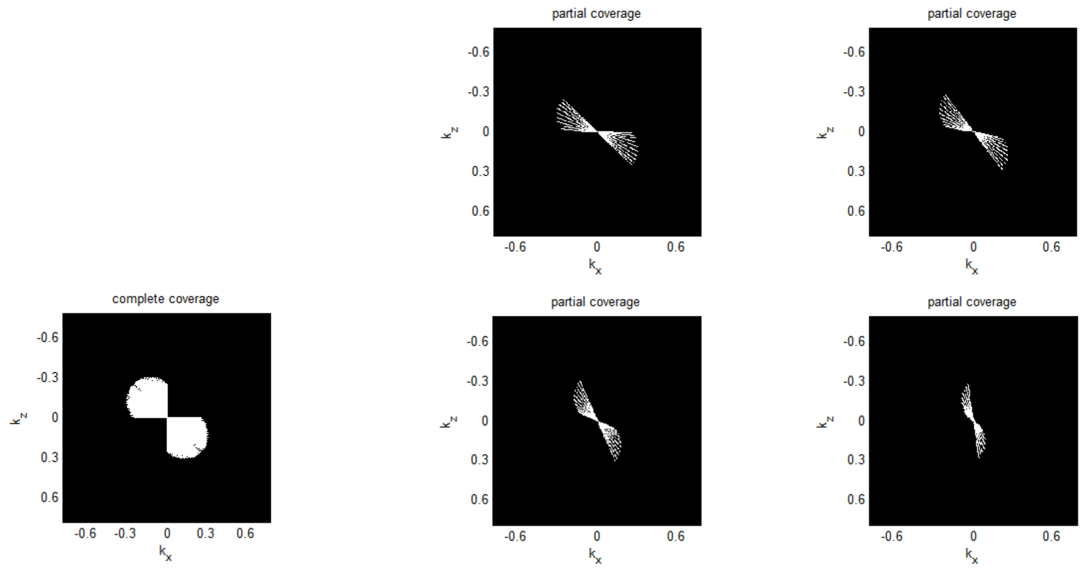
(a) **(b)**

Figure 3. Surface acquisition geometry filter states at a) time t_0 and at b) selected time steps.



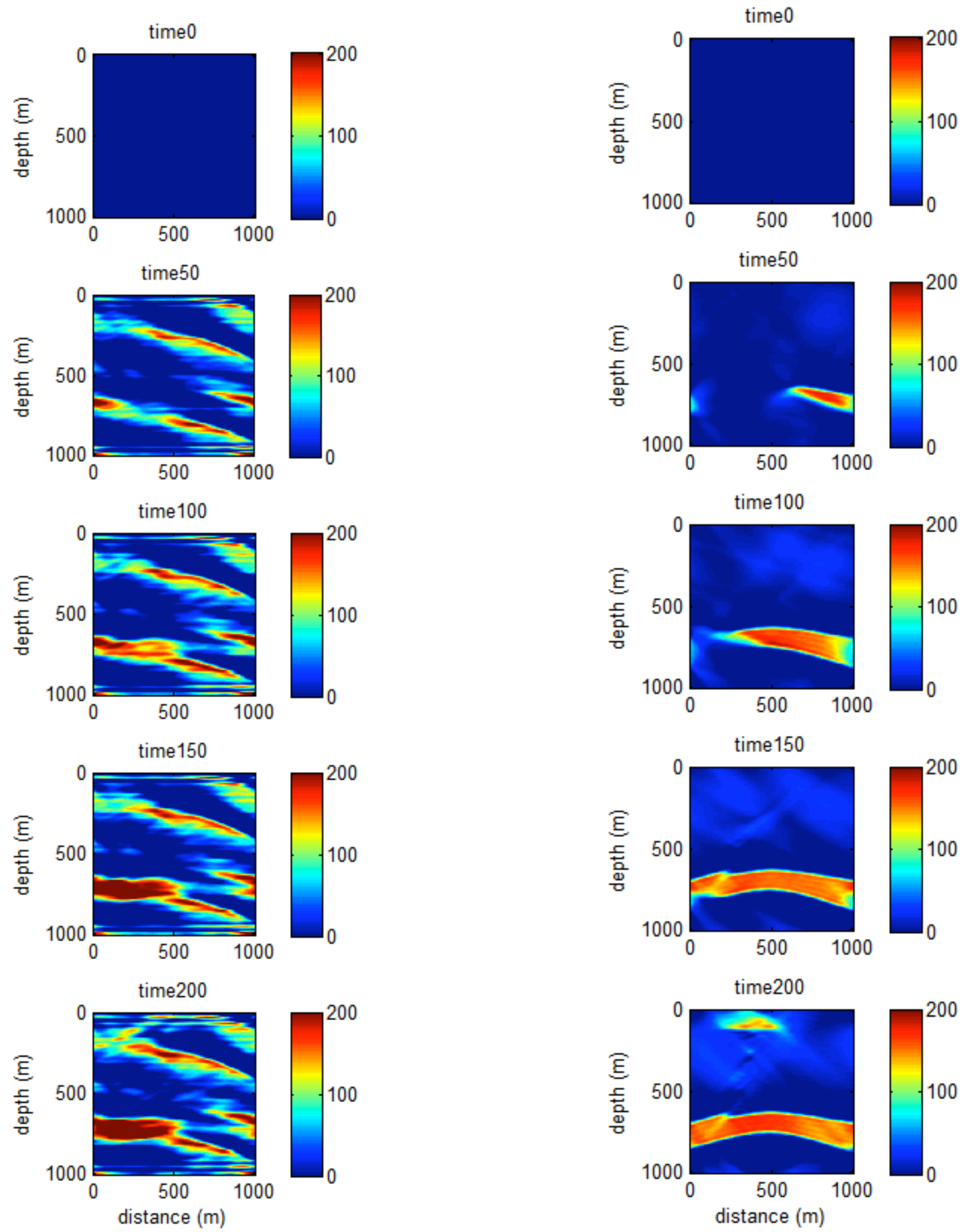
(a) **(b)**

Figure 4. Crosswell acquisition geometry filter states at a) time t_0 and at b) selected time steps.



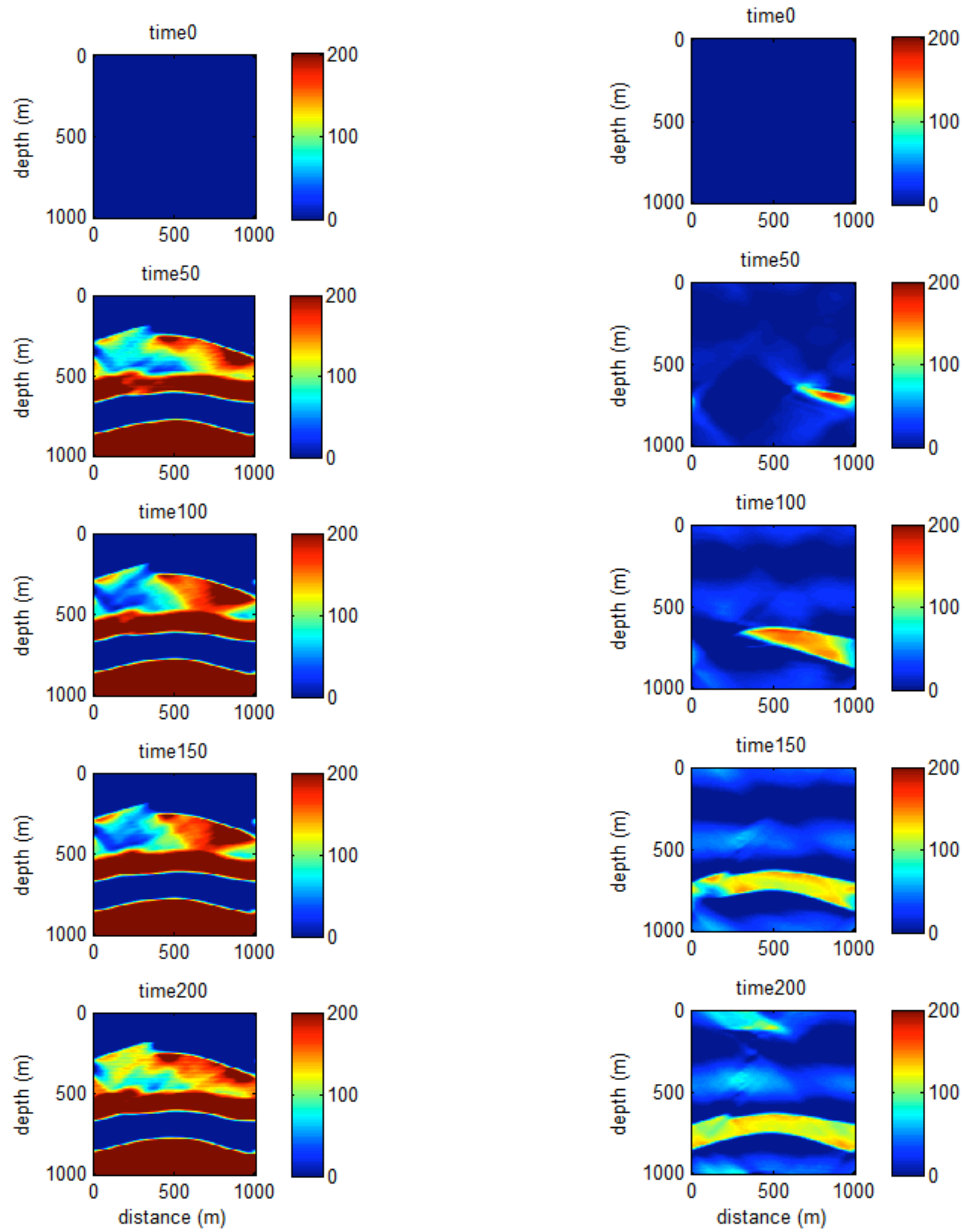
(a) **(b)**

Figure 5. VSP acquisition geometry filter states at a) time t_0 and at b) selected time steps.



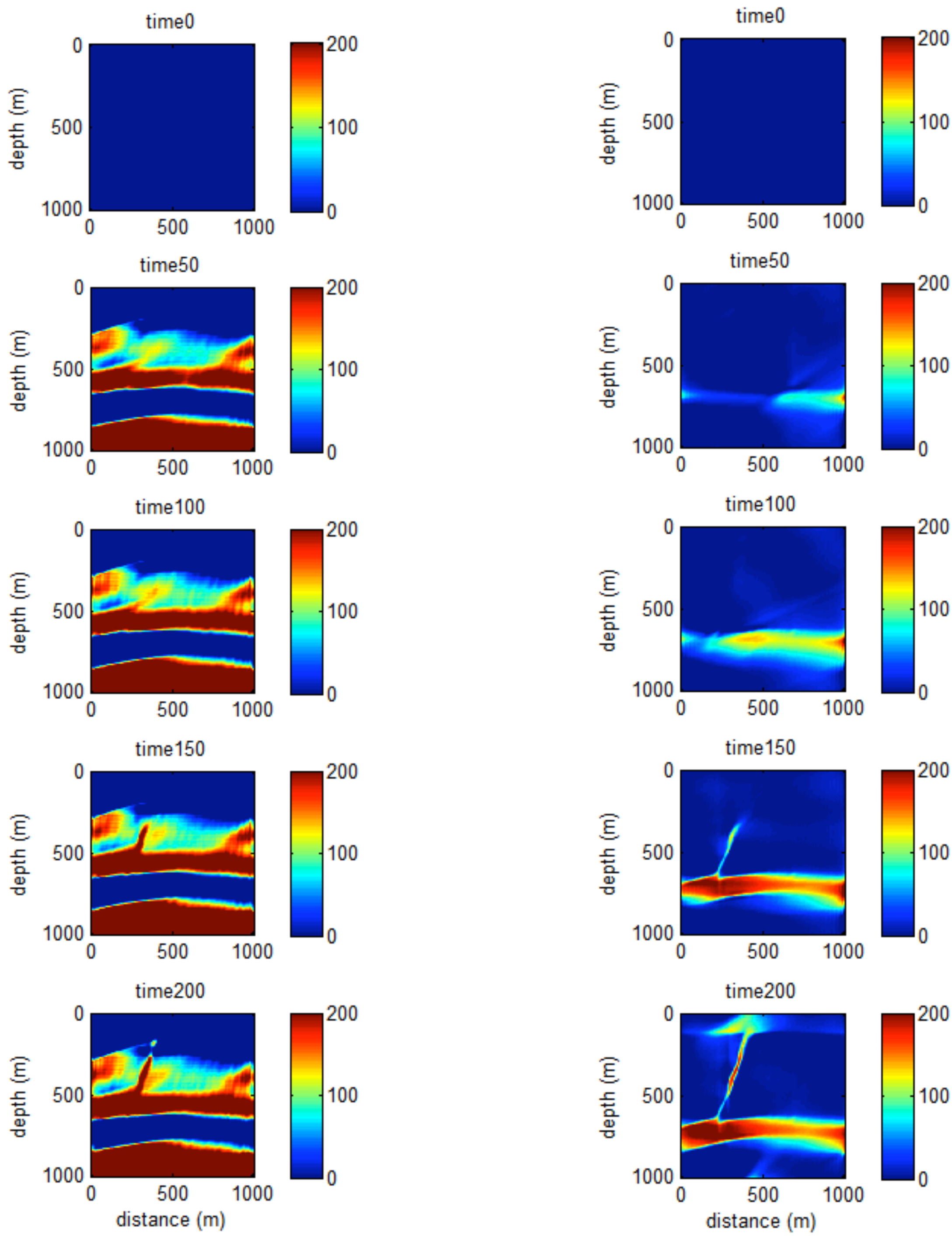
(a) **(b)**

Figure 6. Reconstructed difference images from surface acquisition geometry. a) Without data substitution. b) With data substitution. The reconstructed image at time t_n was subtracted from the reconstructed image at time t_0 to produce a difference image.



(a) **(b)**

Figure 7. Reconstructed difference images from crosswell acquisition geometry. a) Without data substitution. b) With data substitution. The reconstructed image at time t_n was subtracted from the reconstructed image at time t_0 to produce a difference image.



(a) **(b)**

Figure 8. Reconstructed difference images from VSP acquisition geometry. a) Without data substitution. b) With data substitution. The reconstructed image at time t_n was subtracted from the reconstructed image at time t_0 to produce a difference image.

References

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