Stochastic Seismic Inversion using both Waveform and Traveltime Data and Its Application to Time-lapse Monitoring

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Summary

A stochastic approach to seismic inversion using the ensemble Kalman filter (EnKF) is proposed. Seismic depth and time image data are used as the input for EnKF stochastic seismic inversion. The sonic log is used to estimate source wavelet and create initial models for the inversion, which provides an efficient integration of sonic log data and seismic data. We use both travel time and waveform data for the inversion and obtain the absolute seismic velocity instead of the relative impedance. EnKF can continuously update the model using time-lapse data. A synthetic example is used to demonstrate the possible application to seismic monitoring.

Introduction

The purpose of seismic inversion is to recover the subsurface elastic properties (e.g., acoustic impedance and velocity) from seismic data. For example, Oldenburg et al. (1983) discussed the deterministic impedance inversion; Hass and Dubrule (1994) introduced a stochastic impedance inversion; Cao et al. (1989) presented an inversion method to estimate background velocity and impedance simultaneously. Francis (2005) and Sancevero et al. (2005) compared deterministic and stochastic impedance inversion using examples. In general, stochastic seismic inversion has higher vertical resolution than deterministic inversion.

The stochastic seismic inversion proposed in this study is an implementation of ensemble Kalman filter (EnKF). A complete introduction to EnKF can be found in Evensen (2007). EnFK can perform linear and non-linear stochastic inversion. It can also integrate different types of data for the inversion. Taking advantage of these features, we combine waveform data and travetime data for the seismic inversion. The waveform inversion in our study is a non-linear inversion. The use of travetime data improves the estimation of the absolute seismic velocity.

This study is motivated by seismic monitoring for geological CO_2 sequestration. CO_2 sequestration provides a possible solution for reducing the green gas emission to the atmosphere. For safety and operational reasons, we need to monitor the containment of the CO_2 storage in the subsurface. The monitoring is a dynamic process. EnKF is naturally suitable for dynamic inversion. We will use the CO_2 monitoring as an example to demonstrate our method, though it can also be used for general stationary reservoir characterization using surface reflection seismic data and sonic logs.

Method

Let us consider the seismic signal **d** recorded at surface that is a function of subsurface model parameters **m**. In this seismic inversion problem, **d** is normal incidence reflection data obtained after all necessary signal processing, and **m** is the 1-D seismic velocity directly below the receiver. Data **d** and model **m** are related through an observation matrix **G** for a linear case:

$$\mathbf{d} = \mathbf{Gm}, \qquad (1)$$

or a general observation function g including non-linear cases:

$$\mathbf{d} = g(\mathbf{m}). \qquad (2)$$

We want to estimate model **m** from observed data **d** by a stochastic inversion procedure implemented with the ensemble Kalman filter.

We here follow the derivation in Evensen (2003) and apply the general EnFK theory to our problem, i.e., joint seismic inversion using both waveform and traveltime data. In our case, **m** is an n-dimensional model vector composed with discretized 1-D velocity below the receiver; **d** is an m-dimensional data vector having m_1 waveform data points and m_2 traveltime data points, where $m=m_1+m_2$. A proper scaling factor is needed to normalize the two types of data.

Assume that model **m** has Gaussian probability distribution with mean $\mathbf{m}_0$ and covariance **C**, and data **d** also has Gaussian probability distribution with mean $\mathbf{d}_0$ and covariance **R**. We create a model ensemble

$$\mathbf{M} = [\mathbf{m}_1, \ldots, \mathbf{m}_N] \qquad (3)$$

that has the mean $\mathbf{m}_0$ and the covariance **C**, and a data ensemble

$$\mathbf{D} = [\mathbf{d}_1, \ldots, \mathbf{d}_N] \qquad (4)$$

that has the mean $\mathbf{d}_0$ and the covariance **R**. Here, $\mathbf{m}_i$ and $\mathbf{d}_i$ are ensemble members; N is the ensemble size that should be large enough in order to provide a good approximation to the probability distribution for the model and the data. The EnKF gives the statistical solution for a linear problem shown in equation 1 as

$$\hat{\mathbf{M}} = \mathbf{M} + \mathbf{K}(\mathbf{D} - \mathbf{GM}), \qquad (5)$$

where

$$\mathbf{K} = \mathbf{CG}^T(\mathbf{GCG}^T + \mathbf{R})^{-1} \qquad (6)$$

is called Kalman gain. The EnKF solution for a non-linear equation 2 will be discussed in next section. $\hat{\mathbf{M}}$ is an $n \times N$ matrix; each column represents a realization from the posterior probability distribution. The average of all columns (or realizations) forms the solution for the model estimation. In a time-lapse inversion problem, new data are coming continuously, and the model can be continuously updated by repeating the procedure above (equations 3-5) using the estimated model obtained in current step as the initial model for next time step.

Implementation

We start with an initial model $\mathbf{m}_0$ created from prior knowledge, e.g., sonic logs and their interpolations, or just a constant model in the worst case. Then we construct the model ensemble in equation 3 as

$$\mathbf{m}_i = \mathbf{m}_0 + \boldsymbol{\varepsilon}_i$$

where $\boldsymbol{\varepsilon}_i$ is an n-dimensional random vector from Gaussian distribution. Convolution is used as the observation function for waveform data modeling. We calculate reflection coefficients from 1-D velocity and convolve the reflection profile with a wavelet extracted from the normal incidence seismogram and a sonic log. The observation function g in this study is not a linear function, and we cannot directly use equation 6, because it is difficulty to find an observation matrix $\mathbf{G}$ for this convolution modeling operation. We have to use an observation matrix-free implementation (Mandel, 2006) for this inversion.

The model covariance $\mathbf{C}$ in equation 6 can be approximated by the ensemble covariance as

$$\mathbf{C} = \mathbf{A}\mathbf{A}^T/(N-1), \qquad (7)$$

where

$$\mathbf{A} = \mathbf{M} - E(\mathbf{M}) = \mathbf{M} - \frac{1}{N}\sum_{i=1}^{N}\mathbf{m}_i .$$

Then model update (equation 5) can be done with

$$\hat{\mathbf{M}} = \mathbf{M} + \frac{1}{N-1}\mathbf{A}(\mathbf{G}\mathbf{A})^T\mathbf{P}^{-1}[\mathbf{D} - g(\mathbf{M})], \qquad (8)$$

where

$$\mathbf{P} = \frac{1}{N-1}\mathbf{G}\mathbf{A}(\mathbf{G}\mathbf{A})^T + \mathbf{R}, \qquad (9)$$

and the i^{th} column of matrix $\mathbf{GA}$ can be obtained from

$$[\mathbf{GA}]_i = g(\mathbf{m}_i) - \frac{1}{N}\sum_{j=1}^{N} g(\mathbf{m}_j). \qquad (10)$$

For the data ensemble $\mathbf{D}$, we perturb the observed data $\mathbf{d}$ and have

$$\mathbf{d}_i = \mathbf{d} + \boldsymbol{\gamma}_i .$$

Here, $\boldsymbol{\gamma}_i$ is an m-dimensional random vector from Gaussian distribution. Then the data covariance $\mathbf{R}$ required in equation 9 can be obtained from the ensemble covariance

$$\mathbf{R} = \boldsymbol{\gamma}\boldsymbol{\gamma}^T/(N-1).$$

We next apply the procedure above to a synthetic example.

An Example of Time-lapse Seismic Monitoring

We have utilized a simulation study for seismic monitoring on CO_2 sequestration in coalbeds. This study is part of the Global Climate and Energy Project (GCEP) at Stanford University.

Time-lapse Models

We first build a 2-D reservoir flow model according to the geology and flow parameters of unmineable coalbeds in the Powder River Basin. The primary goal of this flow simulation is to create a series of relatively realistic CO_2 storage models for monitoring tests. For a period of 10 years, 175 time-lapse models are generated using the flow simulator GEM. Various cases, e.g., CO_2 storage with or without leakage, are simulated. In the coalbed, matrix porosity = 5%, cleat porosity = 1-5%, matrix permeability = 0.5md and cleat permeability = 100md.

We then convert the flow simulation results to time-lapse *P*-wave velocity models with the help of a rock physics model. Figure 1 shows four velocity models at time =0, 3 months, 1 year, and 3 years. It can be seen that the *P*-wave velocity decreases due to the CO_2 saturation. The method discussed in previous sections is applied to these models to test if we can track the CO_2 front using EnKF.

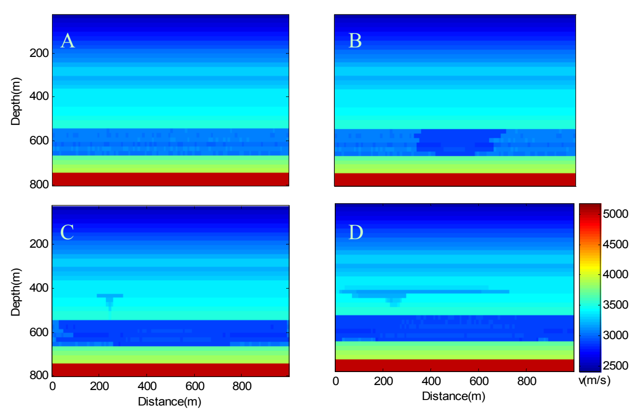

Figure 1: Four time-lapse *P*-wave velocity modes created based on CO_2 flow simulation in the coalbeds. A: time=0; B: time=3 months; C: time=1 year; D: time=3 years.

Seismic Data

A finite difference method is used to calculate the relatively realistic seismic data (served as observed data) for all 4 time-lapse models. 40 shot gathers are calculated for each model. The source peak frequency is 50 Hz. Figure 2 just gives a few samples of the shot gathers calculated using model D.

Prestack depth migration is used to image the calculated seismic data and one of the resulting depth images is shown in Figure 3. The time image shown in Figure 4 is the zero-offset traces. The reflection waveform in the depth images plus the reflection picks from time and depth images are used for joint seismic inversion. Table 1 lists the reflectors picked from depth and time images (Figures 3 & 4) at distance=500 m, which is the travetime data used for the joint inversion.

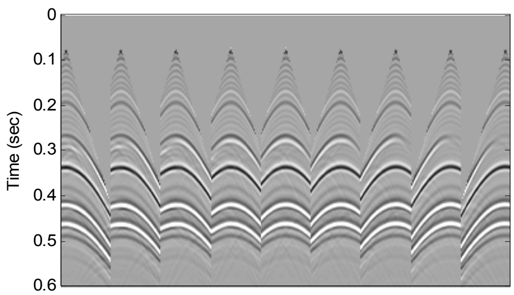

Figure 2: Samples of the shot gathers calculated using the finite difference.

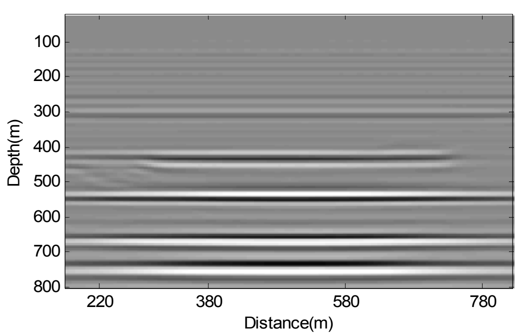

Figure 3: Depth image of model D.

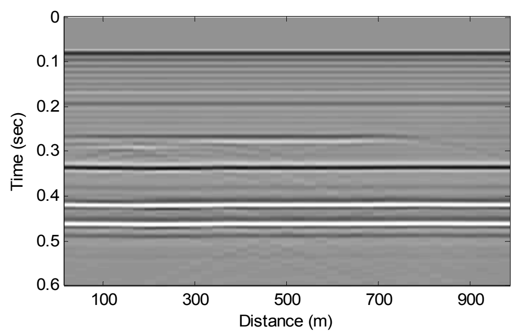

Figure 4: Time image of model D.

Table 1: Samples of traveltime picks used for the inversion.

Reflector	1	2	3	4	5
Depth (m)	270	310	550	670	750
Time (sec)	0.1675	0.1918	0.3340	0.4173	0.4595

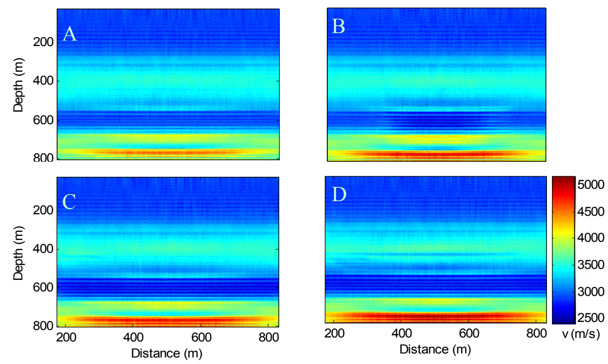

Figure 5: Time-lapse velocity models inverted using EnKF. Models A-D correspond to time=0, 3 months, 1 year, and 3 years, respectively.

Seismic Inversion with EnKF

Fast forward modeling tools are essential for EnKF inversion, because we have to calculate $g(\mathbf{m}_i)$ (see equation 10) for each sample of the ensemble that usually has a size of hundreds. There are two types of forward modeling are involved in this joint inversion. For waveform data, we assume a sonic log is available for source wavelet estimation and use the source wavelet for convolution modeling. In this study, we just simply use the true velocity profile for the wavelet estimation. Constant density is assumed for impedance calculation. The forward modeling in the inversion for traveltime t is a summation down to a given reflector, i.e.,

$$t = 2\sum_i 1/v_i \,,$$

where v_i is the 1-D velocity of i^{th} depth pixel.

Applying the procedure described in previous section to the "observed" seismic data, we obtain the inverted velocity models shown in Figure 5. In order to see the velocity changes more clearly, the velocity difference between models B-D and base model A are shown in Figure 6. A constant initial model is used in this test. It can be seen that the overall absolute velocity structure and the velocity drop due to CO_2 injection are sufficiently recovered. Profiles in Figure 7 give a close comparison between the given model and the inverted model. Figure 8 compares the "observed" (or given data) and the data calculated with inverted velocity. The given data and modeled data are virtually identical, though the given velocity model and the inverted velocity model exhibit some difference, which may be caused by the amplitude distortion in the depth imaging. True amplitude imaging is very important for this seismic inversion.

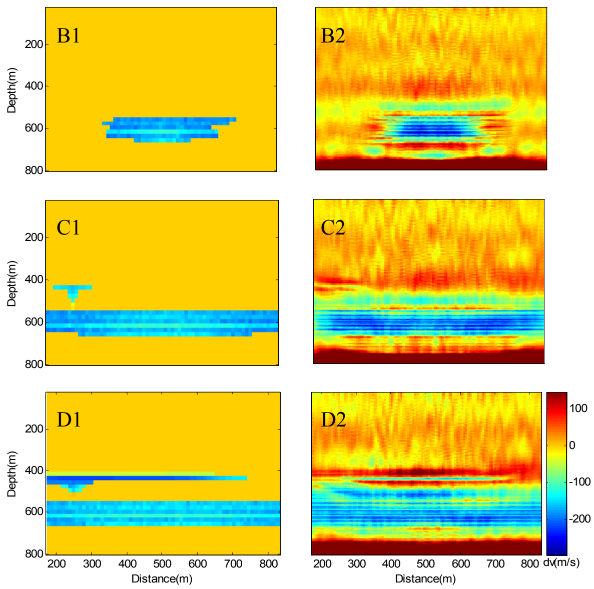

Figure 6: Velocity differences between time-lapse models B-D and base model A. Left: given models. Right: Inverted models.

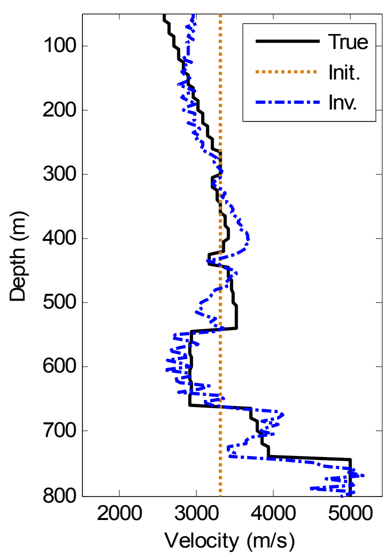

Figure 7: A comparison between true model (solid black line) and inverted model (Dashdot blue line) at distance=500 m. Dotted yellow line is the initial model.

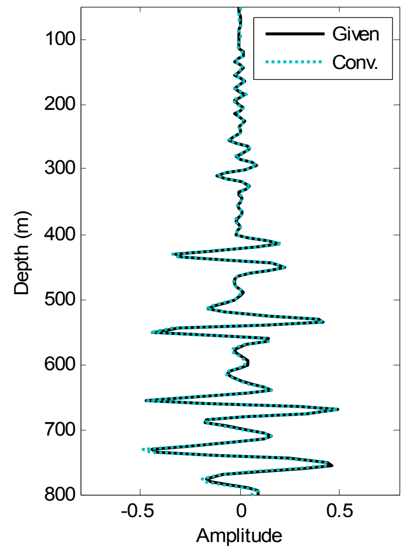

Figure 8: A comparison between "observed" data (solid line) and modeled data (dotted line). Solid line is sampled from distance=500m from depth image and the dotted line is calculated from inverted velocity at the same location.

Conclusions

The ensemble Kalman filter provides a powerful tool for stochastic seismic inversion, especially for dynamic inversion in seismic monitoring. Integrating travetime data into the inversion makes the estimation of absolute velocity possible. Waveform data used in the joint inversion gives the high resolution components of inverted velocity.

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EDITED REFERENCES

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