A finite element algorithm for 3-D transient electromagnetic modeling

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Summary

We present a 3-D finite-element time-domain (FETD) algorithm for the simulation of electromagnetic (EM) diffusion phenomena. The algorithm simulates transient electric fields and time derivatives of the magnetic fields for a general anisotropic earth. In order to compute transient fields, the electric field wave equation is transformed into a system of ordinary differential equations (ODE) via a Galerkin method with Dirichlet boundary conditions. To ensure both numerical stability and an efficient time step size, the system of ODE is discretized in time using the implicit backward Euler scheme. The resultant FETD matrix-vector equation is solved using a sparse direct solver with a fill-in reducing algorithm. When advancing the solution in time, the algorithm adjusts the tine step by examining if or not a current step size can be doubled without affecting the accuracy of the solution. Instead of directly solving another FETD matrix-vector equation for transient magnetic fields, Faraday's law is employed to compute time-derivatives of magnetic fields only at receiver positions. The accuracy and efficiency of the FETD algorithm are demonstrated using time-domain controlled source EM (TD-CSEM) simulations.

Introduction

Transient electromagnetic (TEM) methods are used in both near-surface and deep exploration geophysics. Since interpretation of TEM data in complex geological environments increasingly resort to forward/inverse modeling, the numerical simulation of TEM fields is of particular interest. Among the variety of numerical simulation techniques, finite-difference time-domain (FDTD) algorithms have become the most popular for TEM simulations (Wang and Hohmann, 1993; Commer and Newman, 2004). Their popularity is due to the fact that they are relatively simple to implement, efficient, and can provide accurate solutions to a wide range of TEM simulations.

However, the FDTD method also has well known drawbacks. From a modeling point of view, its biggest weakness is that large complex geological structures (e.g. bathymetry), which do not conform to rectangular grids, need to be captured by stair-step approximations. The stair-step approximation might seem to adequately model significantly-irregular topography using a series of very small grids in parallel computing environments. However, such a stair-step modeling approach can introduce errors into numerical modeling results especially when sources and receivers are placed on the complex surface described by the fine stair steps. Furthermore, the stair-step modeling approach can introduce unnecessarily small grid spacing in the computational domain, resulting in inefficiently small time steps when the Du Fort-Frankel method is used.

We present herein a 3-D FETD algorithm as an alternative to FDTD for diffusive EM simulation in complex geological environments. In contrast to finite difference (FD) methods, finite element (FE) algorithms are based on a geometry-conforming unstructured mesh which allows precise representations of complex geological structures in computationally economic and elegant ways. The price paid is the development cost of the finite-element simulation code.

Theory and Method

In a given computational domain $\mathbf{V}$, the full electric field wave equation is given as

$$\nabla \times [\frac{1}{\mu}\nabla \times \mathbf{e}(\mathbf{r},t)] + \boldsymbol{\varepsilon}\frac{\partial^2 \mathbf{e}(\mathbf{r},t)}{\partial t^2} + \boldsymbol{\sigma}\frac{\partial \mathbf{e}(\mathbf{r},t)}{\partial t} = -\frac{\partial \mathbf{j}_s(\mathbf{r},t)}{\partial t}, \quad (1)$$

where $\mathbf{e}(\mathbf{r},t)$ is the electric field at time t at position $\mathbf{r}\in\mathbf{V}$, μ, $\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$ and $\mathbf{j}_s(\mathbf{r},t)$ are magnetic permeability, a 3x3 dielectric permittivity tensor and a 3x3 electric conductivity tensor, and an electric current source term, respectively.
First, a residual vector for eqn. (1) is defined as

$$\mathbf{p}(\mathbf{r},t) \equiv \nabla \times [\frac{1}{\mu}\nabla \times \mathbf{e}(\mathbf{r},t)] + \boldsymbol{\varepsilon}\ \frac{\partial^2 \mathbf{e}(\mathbf{r},t)}{\partial t^2} + \boldsymbol{\sigma}\frac{\partial \mathbf{e}(\mathbf{r},t)}{\partial t} + \frac{\partial \mathbf{j}_s(\mathbf{r},t)}{\partial t}. \quad (2)$$

The residual vector must be zero everywhere within V in order to satisfy eqn. (1). However, from a numerical point of view, it is practical to discretize the computational domain into a number of finite elements. Then, the residual vector for each element is forced to be zero in a weighted-integral sense (Jin, 2002):

$$\iiint_{V^e} \mathbf{n}_i^e(\mathbf{r}) \cdot \mathbf{p}^e(\mathbf{r},t)dV = 0\ , \quad (3)$$

where the superscript e denotes the eth tetrahedral element, $\mathbf{n}_i^e(\mathbf{r})$ with i varying from 1 to n is a set of weighting functions, and V^e is the volume of the eth tetrahedral element.

If the set of $\mathbf{n}_i^e(\mathbf{r})$ functions used in eqn. (3) is also chosen as the set of basis functions for the electric field, the electric field is expanded as

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$$\mathbf{e}^e(\mathbf{r},t)=\sum_{j=1}^{n}\mathbf{e}_j^e(\mathbf{r},t)=\sum_{j=1}^{n}u_j^e(t)\mathbf{n}_j^e(\mathbf{r}), \tag{4}$$

where $u_j^e(t)$ is an amplitude of the electric field on the j^th edge of the e^th element and needs to be determined using the FETD method. In this study, edge-based Whitney functions (Whitney, 1957) are chosen as the basis functions as well as weight functions for eqns. (3) and (4).

Substituting eqn. (4) into eqn. (3) and dropping the displacement current term yield the following system of 1^st-order ODEs:

$$\mathbf{B}^e\frac{d\mathbf{u}^e(t)}{dt}+\mathbf{C}^e\,\mathbf{u}^e(t)+\mathbf{s}^e=0, \tag{7}$$

where

(i,j) element of $\mathbf{B}^e=\iiint_{V^e}\cap^e\,\mathbf{n}_i^e(\mathbf{r})\cdot\mathbf{n}_j^e(\mathbf{r})\,dV$; (8)

(i,j) element of $\mathbf{C}^e=\iiint_{V^e}\frac{1}{\mu}\nabla\times\mathbf{n}_i^e(\mathbf{r})\cdot\nabla\times\mathbf{n}_j^e(\mathbf{r})dV$; (9)

i element of $\mathbf{s}^e=\iiint_{V^e}\mathbf{n}_i^e(\mathbf{r})\cdot\frac{\partial\mathbf{j}_\mathbf{s}(\mathbf{r},t)}{\partial t}dV$; (10)

$$\mathbf{u}^e=[u_1^e\quad u_2^e\quad \ldots\quad u_n^e], \tag{11}$$

and n is the number of the basis functions for the e^th tetrahedron.

The system of ODEs is considered local because it results from each individual tetrahedral element. Using connectivity information about finite elements and applying Dirichlet boundary conditions to the boundaries of the computational domain **V**, the local systems of diffusion equations from individual elements are assembled into a single global system of diffusion equations:

$$\mathbf{B}\frac{d\mathbf{u}(t)}{dt}+\mathbf{C}\mathbf{u}(t)+\mathbf{s}=0 \tag{13}$$

Using an implicit 2^nd-order backward Euler scheme, eqn. (13) is discretized in time into

$$\mathbf{D}\mathbf{u}^{n+2}=(3\mathbf{B}+2\Delta t\mathbf{C})\mathbf{u}^{n+2}=\mathbf{B}(4\mathbf{u}^{n+1}-\mathbf{u}^n)-2\Delta t\mathbf{s}^{n+2}, \tag{14}$$

where $\mathbf{u}(t)=\mathbf{u}(n\Delta t)=\mathbf{u}^n$, and Δt is the time step size.

The most expensive part in the FE computation is advancing the solution in time. Our primary choice of the numerical solver for eqn. (14) is a direct solver. Matrix **D** is explicitly factorized into the product of lower and upper triangular matrices **L** and **U**. Because Matrix **D** is a function of Δt in eqn. (14), the factorization is performed only when Δt is changed. Before the factorization starts, matrix **D** is re-ordered to minimize fill-in in the resulting triangular matrices. Finally, forward and backward substitution completes the solution process at a given time. When models are too large for the memory of a given computer, we use an iterative solver. In this case, the solution at the previous time step is used as the initial guess at the current time step. A preconditioner also needs to be re-constructed only when Δt is changed.

EM diffusion simulations require a very small Δt in early time to resolve the broad frequency spectrum of the induced TEM fields. However, because the high frequency component of the TEM field is more rapidly attenuated in time, one can take increasingly larger time steps and thus advance the solution quickly without affecting the accuracy. Therefore, our FETD algorithm tries to double Δt every *m* time steps, where *m* is an input parameter. If an earth model is conductive, a smaller *m* is chosen; if an earth model is rather resistive, a larger *m* needs to be chosen. When the FETD algorithm attempts to switch a time step size from Δt to 2Δt, the electric fields are computed using both time steps. If the difference between the two solutions is smaller than a specified tolerance, 2Δt is accepted as a new time step. If the tolerance criterion is not satisfied, the FETD algorithm rejects 2Δt and continues using the current Δt. However, the byproduct matrices (e.g. the triangular matrices or preconditioner) for 2Δt are stored for future uses after another *m* time steps. For brevity, we call this approach the adaptive time step doubling method.

In order to advance eqn. (14), the initial electric fields must be provided. When an earth model is excited using a step-on or Gaussian source waveform, the initial fields are set to zero. However, when a step-off source waveform is employed, the initial DC electric fields need to be calculated via the Poisson equation. Therefore, we also solve the Poisson equation using the FE method. The FE method is based on secondary potential approach since it provides more accurate solutions near sinks and sources (Li and Spitzer, 2002). Once the electric potentials are determined at every FE node in the computational domain, the electric fields along the edges of the elements can be directly calculated using the gradients of the potentials.

After the transient electric fields are calculated in the computational domain using the FETD algorithm, the magnetic fields are determined exploiting the fact that most magnetic receivers do not measure amplitude of magnetic fields, but rather the time derivative of magnetic fields (Commer and Newman, 2004). The time derivatives can be easily determined via Faraday's law by directly applying the curl operator to the basis function in eqn. (4). In this way, we compute the time derivatives of magnetic fields only at receiver positions and avoid having to solve another matrix-vector equation for the transient magnetic field diffusion.

Time-Domain CSEM Simulation Examples

To demonstrate the accuracy and performance of our FETD algorithm, a serial implementation named

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FETDEM3D is written in MATLAB, from where several external routines are called. The MATLAB portion of FETDEM3D mainly includes FE pre-processing tasks, whereas the external routines are responsible for main FE computations. The FETD modeling was carried out on Sun V40z with 4 Opteron dual-core CPUs with 32 GB memory running Red Hat Linux. The results are compared with the 1D analytical or the 3D FDTD solution of Commer and Newman (2004). Although our FETD algorithm can simultaneously handle multiple arbitrarily-configured electric dipoles with various source waveforms over anisotropic media, single step-off electric dipole responses over isotropic media are considered in this section for comparison and verification purposes.

The first example is a simple marine TD-CSEM model. The model consists of a 0.7 Ohm-m resistive homogeneous seafloor and a 400 m deep, 0.3 Ohm-m resistive seawater column. To ensure numerical stability, the resistivity of the air is set to 10,000 Ohm-m. A 250 m long, x-oriented electric dipole is placed 50 m above the seafloor. Its ramp-off time is set to 1e-2 (seconds). Three EM receivers are placed on the seafloor at x= 2, 4 and 6 km source-receiver offsets. The model is discretized into 108,540 tetrahedral elements, generating 125,883 unknowns. The FETD solutions are plotted in **Figure** 1, showing excellent agreement with the analytical solutions.

Figure 2 summarizes the performance of the adaptive time step doubling method for the seafloor model above. Without the method, it took 16.2 hours with 50,000 time steps to complete the simulation. In contrast, when the doubling method was employed, the simulation was completed in 36 minutes with 1,393 time-steps. The time step doubling procedures were performed 8 times.

The next example is a 3-D resistive gas reservoir model shown in **Figure** 3a. The inline TD-CSEM responses over the gas reservoir are simulated using both 3-D FDTD and FETD algorithms. A 250 m long electric dipole whose ramp-off time is set to 1E-4 seconds is placed at the center of the model. The 3-D FDTD solutions for the model were imported from Um (2005). The FETD algorithm discretizes the model into 114,116 tetrahedral elements, generating 131,741 unknowns. It took 53 minutes to complete the FETD simulation with a total of 1,559 time steps when the adaptive time step doubling method is employed. The solutions from both the FETD and FDTD methods are plotted together in **Figures** 3b and 3c. The curves for each receiver position agree well with each other at most times except at very early times where slight differences in the electric fields are observed because the employed FD grid does not handle high frequency EM signals very well.

The final example is a gently dipping (4 degrees) two-dimensional (2D) seafloor with and without a 3-D hydrocarbon reservoir illustrated in **Figure** 4a. In order to elucidate the effects of the slope on the marine TD-CSEM method, a flat seafloor model with and without the same hydrocarbon reservoir is also simulated. The flat seafloor model has a uniform 400 m thick seawater column. The dipping and flat seafloor models are discretized into 165,528 tetrahedral elements with 191,780 unknowns and 127,046 tetrahedral elements with 146,871 unknowns, respectively. The simulations were completed in 65 and 41 minutes, respectively. The inline electric field responses at 4 km source-receiver offset are plotted in **Figures** 4b and 4c. The differences observed in **Figure** 4 can be thought of as the combination of the following factors: 1) the airwave effect varies as the thickness of the seawater column above the receiver changes due to the bathymetry; 2) the receiver coordinate is tilted towards the slope; 3) the receiver on the slope measures stronger galvanic effects than that on the flat seafloor because of its shorter distance from the hydrocarbon reservoir. In short, a gently-dipping simple seafloor structure can cause significant effects on the TD-CSEM measurements and, as demonstrated above, seafloor bathymetry needs to be modeled with special care.

Conclusions

We have presented an efficient 3-D FETD algorithm to simulate diffusive electromagnetic phenomena. The algorithm is especially useful for modeling complex topography and reservoir geometry. The FETD algorithm uses an implicit backward Euler scheme to retain numerical stability with a larger time step size that helps accelerate FETD solution processes especially in late time. The inherent high-computational efforts associated with solving the resultant FETD matrix-vector equation at every time step are mitigated by re-factorizing the FETD matrix only when a time step size is changed. By adaptively doubling the time step at intervals, the FETD algorithm trades off the computational cost of re-factorizing the FETD matrix for the faster advance in FETD solutions. The adaptive time step doubling method plays an important role in speeding up the FETD computation especially in a marine TD-CSEM simulation where an EM diffusion process occurs slowly until very late time due to the high electrical conductivities.

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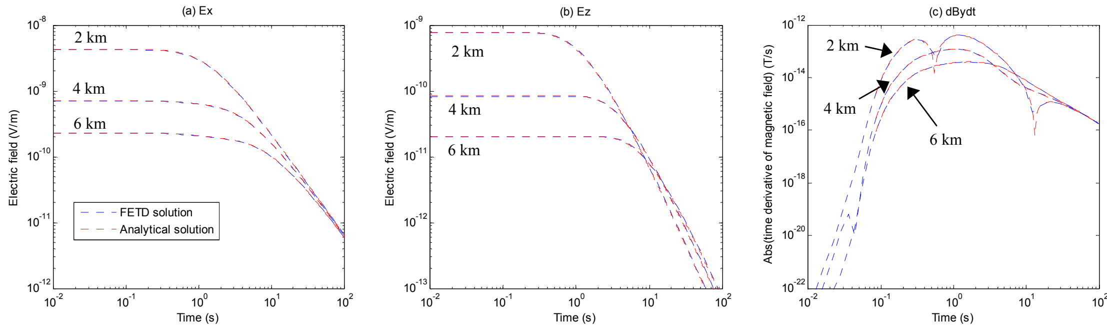

Figure 1. In-line TD-CSEM responses at 2, 4 and 6 km source-receiver offsets over the homogeneous seafloor model.

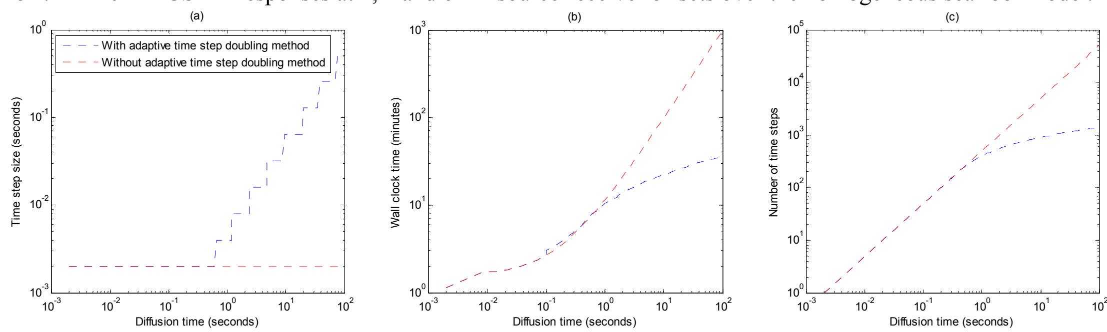

Figure 2. Comparison of computational efficiency with and without the time step doubling method.

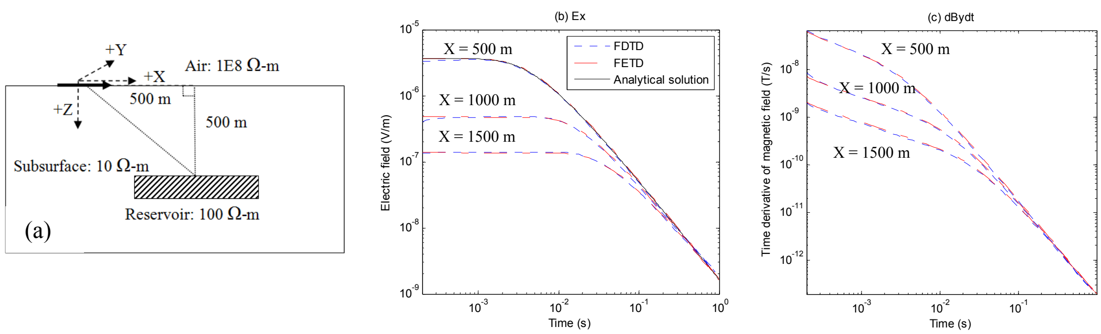

Figure 3. (a) The 3-D gas reservoir model. (b) Ex fields from FETD and FDTD. (c) dBydt fields from FETD and FDTD.

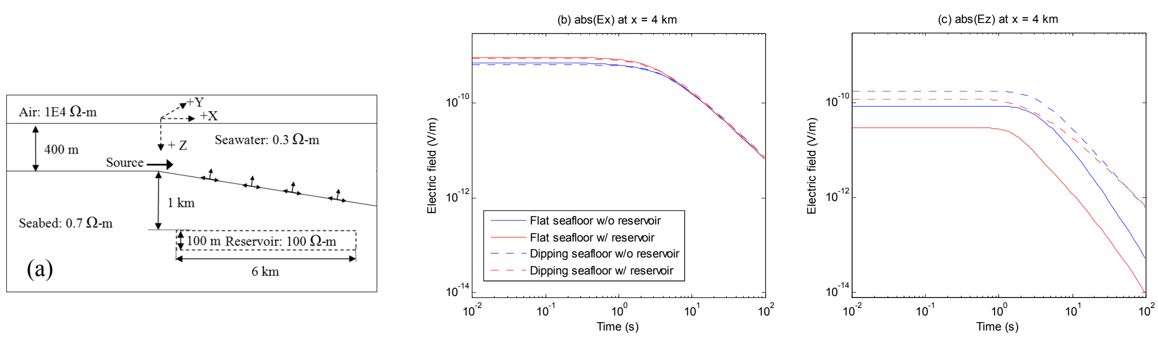

Figure 4. (a) The 2D seafloor model with and without the 3D reservoir. (b) Ex fields. (c) Ez fields. The size of the 3D reservoir is 6 (km)-by-6 (km)-by 0.1 (km) in the x-, y- and z- directions, respectively. Its axis base point is (1 km, -3 m, 1500 m).

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