Seismic Wave Modeling in Poroelastic Media Using the Generalized Reflection/Transmission(R/T) Coefficients Method

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Summary

The generalized reflection and transmission (R/T) coefficients method is presented as a stable and effective method for modeling seismic wave propagation in saturated porous media. Several simple examples are calculated and compared with those given in Zhu and McMechan(1991) using the finite difference method. In addition, we apply the method to a realistic carbonate reservoir for crosswell seismic simulation, and compare the results with the field data and those calculated with a viscoelastic model.

Introduction

The generalized reflection and transmission(R/T) coefficients method is used widely in modeling elastic waves in vertically layered half-space because of its computational stability and efficiency over the Thomson-Haskell method, especially for high-frequency problems (Luco and Apsel, 1983; Kennett, 1983; Chen, 1993). By means of the method, Van Schaack et al.(1995) presented the detailed analyses of the field crosswell data collected in a producing carbonate oilfield in west Texas. With the help of modeling, nearly every event appearing in the measured seismograms can be identified and separated. Nevertheless, its reservoir properties such as porosity, permeability and viscosity can not be directly analyized with the method, because the method is based on a viscoelastic model rather than the poroelastic model. In this study, the generalized R/T coefficients method is extended to porous media. A stable and effective method for modeling seismic wave propagation in saturated porous media is presented. For testing our formulation and the code, several simple examples are calculated and compared with those given in Zhu and McMecham(1991) using the finite difference method. In addition, we apply the method to a realistic carbonate reservoir for crosswell seismic simulation, and compare the results with the field data and those calculated with a viscoelastic model.

Solution of Biot's equations

Figure 1 shows the model geometry of a layered half-space. Each layer is a porous medium described by Biot's theory (Biot, 1962a). It consists of a solid phase

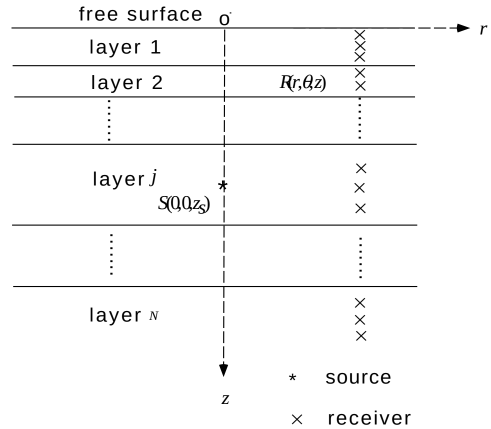

Figure 1: The multilayered half-space

and the liquid phase, i.e., so-called two-phase media. The solid matrix is elastic, locally homogeneous, and statistically isotropic, and fully filled with the liquid phase such as gas, oil, or water. The liquid is of viscous. When seismic waves propagate through porous media, part energy is dissipated due to the relative motion between a viscous pore fluid and the solid matrix. Both theoretical and experimental research show that the energy dissipation is related to permeability of the matrix, viscosity of the pore fluid, and structure of the pore space. Due to space limitations, only important steps are given in the following derivation. We will present a complete derivation in the future publication.

In the frequency domain, omitting the time factor of $e^{-i\omega t}$, Biot's equations can be expressed (Biot, 1962a) as

$$\begin{cases} \mu\nabla^2\mathbf{u} + H\nabla e - L\nabla\zeta + \omega^2(\rho\mathbf{u}+\rho_f\mathbf{w}) = 0 \\ \nabla(Le - M\zeta) + \omega^2(\rho_f\mathbf{u}+\rho_c\mathbf{w}) + \frac{i\omega\eta}{\kappa}\mathbf{w} = 0 \end{cases} \quad (1)$$

where $\mathbf{u}$ is the displacement of the matrix or frame and $\mathbf{w}$ is the permeable displacement of the pore-fluid defined as $\mathbf{w} = \phi(\mathbf{u}_f - \mathbf{u})$ with $\mathbf{u}_f$ the pore-fluid displacement, ϕ as porosity, η as viscosity, κ as permeability, $\rho_c = T\rho_f/\phi$, T is tortuosity, $\rho = \rho_s(1-\phi) + \phi\rho_f$, ρ_s and ρ_f are the densities of the grain and the pore-fluid, respectively; $e = \nabla\cdot\mathbf{u}$ and $\zeta = -\nabla\cdot\mathbf{w}$; $\alpha = 1 - k_b/k_s$;

$L = \alpha M$; $H = k_c + \mu/3$; $M = 1/[\phi/k_f + (\alpha - \phi)/k_s]$; k_s, k_b and k_f are the bulk moduli of the grain, the matrix and the pore-fluid, respectively; k_c and μ are the bulk and shear moduli of the dry matrix. The total stress tensor τ and the pore-fluid pressure P_f associated to equation (1) are

$$\tau = [(H-\mu)\,e - L\zeta]\,\mathbf{I} + \mu\,(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\star}) \quad (2)$$
$$P_f = -Le + M\zeta \quad (3)$$

$\mathbf{I}$ is the unit tensor and "$\star$" stands for the transpose of matrix. To solve equation (1), we take $\mathbf{u}$ and $\mathbf{w}$ as

$$\mathbf{u} = \nabla\varphi_u + \nabla\times(\nabla\times\psi_u\mathbf{e}_z) + \nabla\times(\chi_u\mathbf{e}_z) \quad (4)$$
$$\mathbf{w} = \nabla\varphi_w + \nabla\times(\nabla\times\psi_w\mathbf{e}_z) + \nabla\times(\chi_w\mathbf{e}_z) \quad (5)$$

where φ_u, ψ_u, χ_u, φ_w, ψ_w, χ_w are the displacement potential functions, $\mathbf{e}_z$ is the unit vector in z-direction. Substituting equations (4-5) into equation (1), we can obtain the displacement fields in porous media. For the sake of brevity, we omit the expressions.

To simplify the treatment below, it is convenient to apply the following transform (Ben-Menahem and Singh, 1968) to the cylindrical coordinates(r, θ, z),

$$\begin{cases} \mathbf{B}_m = (\mathbf{e}_r\frac{\partial}{\partial kr} + \mathbf{e}_\theta\frac{1}{kr}\frac{\partial}{\partial\theta})Y_m \\ \mathbf{C}_m = (\mathbf{e}_r\frac{1}{kr}\frac{\partial}{\partial\theta} - \mathbf{e}_\theta\frac{\partial}{\partial kr})Y_m \\ \mathbf{P}_m = \mathbf{e}_z Y_m \end{cases}$$

where $Y_m = J_m(kr)e^{im\theta}$ and J_m is the Bessel function of order m. In coordinates ($\mathbf{B}_m$, $\mathbf{P}_m$, $\mathbf{C}_m$), the displacements and stresses can be expressed in matrix form as

$$\left[u^j_{Pm}, u^j_{Bm}, \mathrm{w}^j_{Pm}, P^j_{fm}, \tau^j_{Pm}, \tau^j_{Bm}, u^j_{Cm}, \mathrm{w}^j_{Cm}, \tau^j_{Cm}\right]^{\star} = \begin{bmatrix} I^j_{11} & I^j_{12} \\ I^j_{21} & I^j_{22} \\ I^j_{31} & I^j_{32} \end{bmatrix} \begin{bmatrix} E^j_d(z) & 0 \\ 0 & E^j_u(z) \end{bmatrix} \begin{Bmatrix} \eta^j_{dm}(z) \\ \eta^j_{um}(z) \end{Bmatrix} \quad (6)$$

where $z^{j-1} < z < z^j$, the superscript "j" is layer index. The 3×3 matrices $I^j_{mn}(m = 1,2,3; n = 1,2)$ are related to the medium parameters of layer j. The diagonal matrices $E^j_d(z)$ and $E^j_u(z)$ are given by

$$E^j_d(z) = diag(e^{[a^j_1(z^{j-1}-z)]}, e^{[a^j_2(z^{j-1}-z)]}, e^{[b^j(z^{j-1}-z)]})$$
$$E^j_u(z) = diag(e^{[a^j_1(z-z^j)]}, e^{[a^j_2(z-z^j)]}, e^{[b^j(z-z^j)]})$$

where a^j_1, a^j_2 and b^j are the vertical wavenumbers of fast and slow P-waves and S-wave of layer j. The amplitude vectors η^j_{dm} and η^j_{um} are associated with downgoing and upgoing fast and slow P-waves and S-wave propagating in layer j. Equation (6) represents the displacement and the stress/pressure fields for porous media. It has 9 components and is different from that for the viscoelastic case where it has 6 components (Luco and Apsel, 1983).

Generalized R/T coefficients matrices for porous media

To determine the unknown amplitude vectors η^j_{dm} and η^j_{um} for each layer, we need to impose the boundary conditions at each interface. For P-SV waves, the displacements and the stresses/pressure at each interface must satisfy the following equations

$$(P^1_{fm}, \tau^1_{Pm}, \tau^1_{Bm}) = 0 \qquad (\text{ on } z = 0) \quad (7)$$
$$(u^j_{Pm}, u^j_{Bm}, \mathrm{w}^j_{Pm}, P^j_{fm}, \tau^j_{Pm}, \tau^j_{Bm}) = (u^{j+1}_{Pm}, u^{j+1}_{Bm}, \mathrm{w}^{j+1}_{Pm}, P^{j+1}_{fm}, \tau^{j+1}_{Pm}, \tau^{j+1}_{Bm}) \qquad (\text{on } z = z^j, j = 1, ..., N-1) \quad (8)$$
$$\eta^N_{um} = 0 \qquad (\text{the first two components}) \quad (9)$$

For SH wave, the boundary conditions are,

$$\tau^1_{Cm} = 0 \qquad (\text{on } z = 0) \quad (10)$$
$$(u^j_{Cm}, \tau^j_{Cm}) = (u^{j+1}_{Cm}, \tau^{j+1}_{Cm}) \qquad (\text{no } z = z^j, j = 1, ..., N-1) \quad (11)$$
$$\eta^N_{um} = 0 \qquad (\text{the third component}) \quad (12)$$

Using these boundary conditions and following the procedures given in Luco and Apsel(1983), the generalized R/T coefficients matrices for porous media can be structured. They are similar to those for elastic media except for the size of the matrices. Using the recurrence relations of the generalized R/T coefficients, the fields in any layer can be determined once the fields in the medium containing the source is known. In this study, the source fields are directly added to the amplitude vectors $\eta^l_{um}(z)$ and $\eta^l_{dm}(z)$ of layer l where source is located.

Numerical examples

1. Comparison with the previous work

To test our formulation and code, several simple examples are calculated. One is a single-layer half-space, the other is a 3-layer half-space. The pore is filled with gas, oil and water, respectively. The seismic responses calculated here show good agreement with those given in Zhu and MeMechan(1991) using finite differences. In addition, because of anisotropy of the finite difference grid, weak shear events in homogeneous medium is also generated even for an explosive force(Figure 2, Zhu and MeMechan,1991). No shear wave is visible in our result in Figure 2, thus illustrating the advantage of the R/T method for 1-D model.

2. Crosswell seismic simulaltion

Figure 3 is a common receiver gather collected in

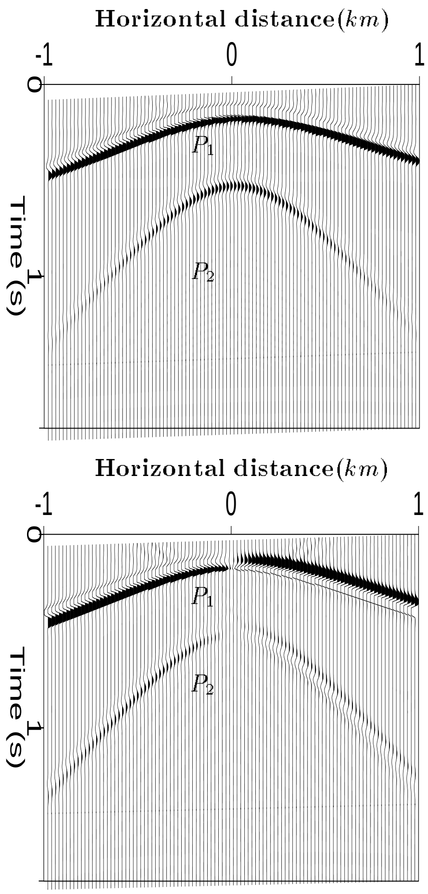

Figure 2: Synthetic poroelastic seismograms (The upper is the vertical component, the lower is the horizontal component). An explosive force is located at (r, θ, z)=$(0, 0, 400m)$, its main frequency is $5Hz$. Receivers are located at the surface. The medium parameters used here are the same as given in Table 1 in Zhu and MeMechan(1991). The P_1 and P_2 stand for fast and slow P-waves. No shear wave is visible in two components. The slow P-wave appears only if viscosity is very small, in this case $10^{-9}cp$.

a producing carbonate oilfield in west Texas(1991). Two cased boreholes, one a producing well and the other a CO_2 monitoring well, were used in the survey; the nominal separation of the two wells is $184ft$. A common receiver gather was recorded at a receiver depth of $2880ft$ with a source sampling interval of $2.5ft$. The data were collected over a depth interval of 2650-3150ft. From Figure 3, we see that the wavetrains are very complex.

Figures 4-5 show the crosswell seismic simulations calculated using the generalized R/T coefficients method. The former is based on a viscoelastic model, the later is based on the poroelastic model. A horizontally single-force source is located at $2880ft$ with main frequency of $1000Hz$. The medium parameters are taken from the corresponding logs. To save computational time, the logs were blocked into 45 thin beds of varying thickness(see the logs plotted lines on the left of Figure 4). A constant intrinsic attenuation is introduced through changing logging velocities into complex velocities. For modeling poroelastic media, the logging velocities are taken as velocities of the saturated media. The pore fluid is taken as water with viscosity $1.cp$. r is 0.5 for calculating tortuosity $T = 1 - r(1 - 1/\phi)$. The moduli k_s and k_b are determined from k_s and k_f through the relation(Raymer et al., 1980)

$$\mathrm{k}_c = \mathrm{k}_s(1-\phi) + \mathrm{k}_f\phi \tag{13}$$

$$\mathrm{k}_b = \mathrm{k}_s(1-\phi) \tag{14}$$

Permeabilities are roughly estimated from porosities through (Dutta, 1983)

$$\kappa = 0.087\phi^6 \tag{15}$$

where κ is in darcies, the porosities and permeabilities used are shown on the left of Figure 5. Comparing Figures 4-5 with Figure 3, the main events are in agreement. Here, what we take care of is the variations of the amplitudes of the wavetrains when different models are used. From Figures 4 and 5, we can see that the attenuations of the seismograms calculated with the poroelastic model are stronger than those with a viscoelastic model although the poroelastic and viscoelastic media are assumed to have the same intrinsic attenuations (Q_p and Q_s). Especially, at depth interval of 2850-2950ft where permeabilities and porosities are relatively higher, the attenuations are apparently strong, and the field data also show relatively strong attenuation.

Conclusions

A stable and effective method for modeling seismic wave propagation in saturated porous media is presented. Using the method, effects of reservoir parameters such as porosity, permeability and viscosity on seismic wave propagation in porous media can be directly investigated. The code can be used for the surface seismic surveys, the vertical seismic profiles, and crosswell seismic profiles.

Acknowdgement

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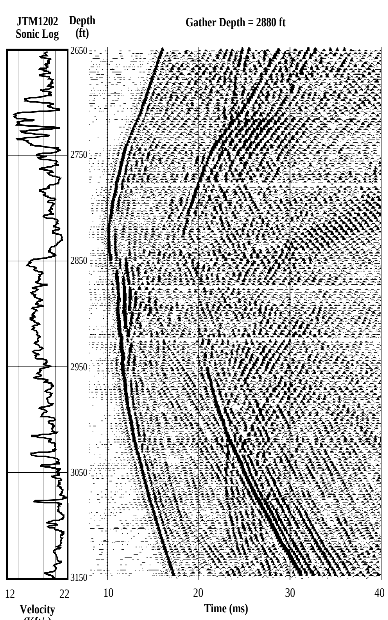

Figure 3: A common receiver gather for receiver depth of $2880ft$. Source sampling interval is $2.5ft$. This Figure is taken from Van Schaack et al.(1995).

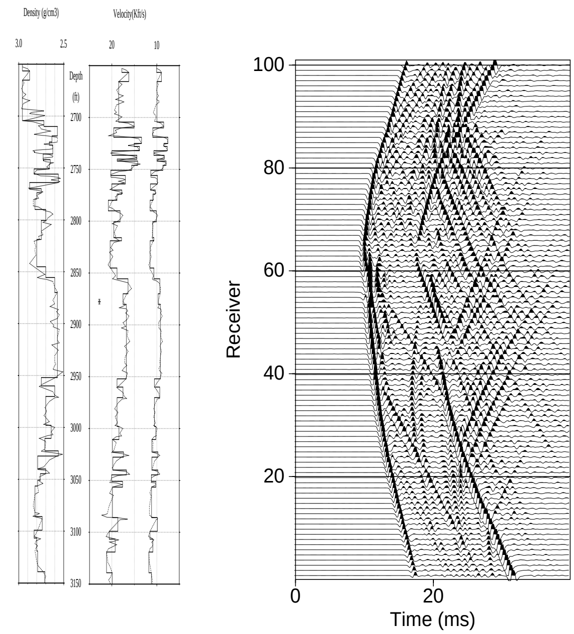

Figure 4: Crosswell seismograms are calculated with the generalized R/T coefficients method which is based on a viscoelastic model. The left is the logs for density and velocities of P- and S-waves. The blocked lines are used as the multilayered model for our calculation.

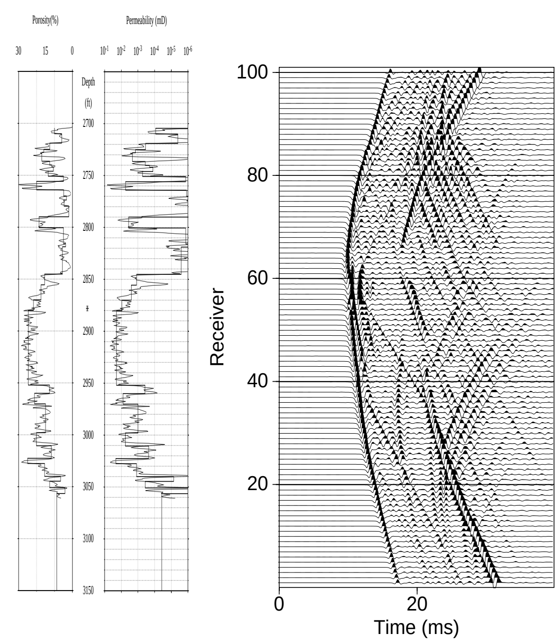

Figure 5: Crosswell seismograms are calculated with the generalized R/T coefficients method which is based on a poroelastic model. The logging velocities are taken as velocities of the saturated porous media. The pore fluid is taken as water. Permeabilities are roughly estimated from logging porosities through equation (15). No measured porosities are given within the intervals where its depth is less than $2700ft$ or greater than $3060ft$. For modeling, they are taken as low-porosity and low-permeability formations.