PAPER L

SEISMIC WAVE PROPAGATION AND ATTENUATION IN HETEROGENEOUS POROUS MEDIA

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ABSTRACT

To unveil a reservoir's properties, we need to know how time-domain seismic waves propagate in saturated porous media. In this paper we derive a new and effective formula for modeling seismic wave propagation and attenuation in 1-D heterogeneous fluid-saturated media, by extending the generalized reflectivity theory to the porous media. At low frequencie, we neglect the second kind compressional wave (i.e., slow *P* wave), then we obtain the quasi-viscoelastic solution for seismic wave propagation and attenuation in vertically layered saturated porous media. Presently, we have finished coding the quasi-viscoelastic formulation, and the computation code for general vertically layered saturated porous media case is under development. As a preliminary application, we use quasi-viscoelastic approach to investigate the seismic wave propagation and attenuation in a vertically layered saturated porous media.

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INTRODUCTION

Last year we reported a rigorous analysis on the reflection and transmission coefficients between two saturated porous media, and discussed the possible influence on the AVO analysis. In conventional AVO analysis, the properties of saturated porous media are inferred from seismic sounding data based on the reflection coefficient formulas of plane wave (i.e., single wave number). There is an intrinsic problem with this analysis strategy: the seismic data used in AVO analysis are seismograms, i.e., time-space domain data; while the basic formula for analysis is for wave number-frequency domain data. Strictly speaking, to reveal rigorously the reservoir's properties, we need to be aware of the knowledge of reflection of seismic waves in saturated porous media in time domain. In this paper, we shall report our effort in this problem. To investigate the time domain response to a point source in heterogeneous saturated porous media, we develop a set of semi-analytic formulation by extending the generalized reflectivity theory (Luco and Apsel, 1983; Kennett, 1983; Chen et al. 1996) to the porous media case. In the following sections, we shall first briefly summarize some of the basic equations and properties of the field in the porous media. We then derive the generalized reflectivity formulation for solving the viscoelastic waves in the porous media excited by a point source. Finally we report some preliminary numerical results.

WAVE FIELDS IN HOMOGENEOUS SATURATED POROUS MEDIA

Denoting by **u** and **U** the displacements in the mineral and liquid phases, the frequency domain equations of motion in a saturated porous media are of the form (Biot, 1956a, b):

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho f_i = -\omega^2(\rho_{11}u_i + \rho_{12}U_i) + i\omega b(u_i - U_i), \tag{1}$$

and

$$\frac{\partial \Theta}{\partial x_i} = -\omega^2(\rho_{21}U_i + \rho_{22}u_i) - i\omega b(u_i - U_i), \tag{2}$$

where

$$\sigma_{ij} = 2Ne_{ij} + [F(\nabla\cdot\boldsymbol{u}) + G(\nabla\cdot\boldsymbol{U})]\delta_{ij} \tag{3}$$

$$\Theta = G(\nabla\cdot\boldsymbol{u}) + T(\nabla\cdot\boldsymbol{U})\_ = -p_f \tag{4}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{5}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i}\right), \tag{6}$$

and the effective Lame coefficients F, G, T and N are defined in terms of the properties of pore fluid, mineral framework, and bulk mineral from which the framework is constructed:

$$F = \frac{(1-\beta)(1-\beta-K_b/K_s)K_s + \beta K_s K_b/K_f}{1-\beta-K_b/K_s+\beta K_s/K_f} - \frac{2}{3}N, \tag{7a}$$

$$G = \frac{(1-\beta-K_b/K_s)\beta K_s}{1-\beta-K_b/K_s+\beta K_s/K_f}, \tag{7b}$$

$$T = \frac{\beta^2 K_s}{1-\beta-K_b/K_s+\beta K_s/K_f}, \tag{7c}$$

and

$$N = (1-\beta)\mu_s\ . \tag{7d}$$

Here, β is the porosity, K is the bulk modulus, and μ is the shear modulus; the subscripts s, f and b denote properties of the mineral, fluid, and porous framework. In equations (1) and (2), ρ_{11}, ρ_{12} and ρ_{22} are mass balance coefficients giving the inertial coupling between fluid and framework motions. $b(\omega)$ is a function of frequency that couples viscous forces in the fluid to the surrounding framework. According to homogenization theory (Auriault, 1980; Auriault et al., 1985), these coefficients can be related by a complex frequency-dependent permeability $\kappa(\omega)$ in the following way:

$$b(\omega) = H_R(\omega), \tag{8a}$$

$$\rho_{22}(\omega) = \frac{\beta^2}{\omega} H_I(\omega), \tag{8b}$$

$$\rho_{12}(\omega) = \beta\rho_s - \rho_{22}(\omega), \tag{8c}$$

and

$$\rho_{12}(\omega) = \beta\rho_f - \rho_{12}(\omega), \tag{8d}$$

where, $H_R(\omega) + iH_I(\omega) = 1/\kappa(\omega)$, and $\rho_s$and ρ_f are, respectively, the densities of the solid and pore fluid. In this study, we shall use Auriault's formula of complex permeability (Auriault, 1985), i.e.,

$$\kappa(\omega) = i\left(\frac{\beta}{\omega\rho_f}\right) J_2\left[i\sqrt{\frac{i8\omega\kappa_0\rho_f}{\eta\beta}}\right] \Big/ J_0\left[i\sqrt{\frac{i8\omega\kappa_0\rho_f}{\eta\beta}}\right], \tag{8}$$

where, η is the viscosity of pore fluid, and κ_0 is the intrinsic permeability of the mineral framework. Introducing scalar and vector displacement potentials relative to each phase in the equations of motion, three kinds of body waves can be found in a saturated porous media (Biot, 1956a, b): two compressional waves, P1 wave and P2 wave; and a shear wave, S wave. In frequency domain, the phase velocities of two dissipative compressional waves are the solutions of the biquadratic equation,

$$\left(V_p\right)^4\left(g_{22}g_{11} - g_{12}{}^2\right) - \left(V_p\right)^2\left[g_{22}(F+2N) + g_{11}T - 2g_{12}G\right] + (F+2N)T - G^2 = 0, \tag{9}$$

and the phase velocity of dissipative shear wave is given by

$$V_s = \sqrt{g_{22}N/\left(g_{11}g_{22} - g_{12}{}^2\right)}, \tag{10}$$

where,

$$g_{pq}(\omega) = \rho_{pq}(\omega) - i(-1)^{p+q}\frac{b(\omega)}{\omega} \quad \text{for } p,q = 1,2. \tag{11}$$

Following Helmholtz theorem, we can demonstrate that the displacement vectors of solid and fluid phases can be expressed by the supposition of the three kinds body waves in the form of

$$\boldsymbol{u} = \nabla\phi_1 + \nabla\phi_2 + \nabla\times(\psi\hat{y}), \tag{12}$$

and

$$\boldsymbol{U} = \chi_1(\omega)\nabla\phi_1 + \chi_1(\omega)\nabla\phi_2 + \chi_3(\omega)\nabla\times(\psi\hat{y}), \tag{13}$$

where, ϕ_1, ϕ_2 and ψ are the scalar and vector potentials for two compressional waves and shear wave, respectively. The partition coefficients χ_j are given by

$$\chi_{1,2}(\omega) = -\frac{1}{(V_{p1,2})^2}\frac{(F+2N)T - G^2}{g_{22}G - g_{12}T} + \frac{g_{11}T - g_{12}G}{g_{22}G - g_{12}T}, \tag{14a}$$

and

$$\chi_3(\omega) = -\frac{g_{12}(\omega)}{g_{22}(\omega)}\ . \tag{14b}$$

WAVE FIELDS IN VERTICALLY LAYERED SATURATED POROUS MEDIA

Let us now consider the wave fields in vertical layered porous media. As shown in Figure 1, the physical model considered here consists of M plane homogeneous and isotropic saturated porous layers in which the top boundary is a traction free surface and the bottom boundary extends to the infinity. Seismic source is embedded in the s-th layer. Each saturated porous layer consists of the solid mineral grains and pore fluid, and is described by the Lame coefficients of the mineral and pore fluid, and the porosity and permeability of the solid framework. As shown above, the wave fields in such media can be decomposed into three types of body-wave, and the corresponded potentials are ϕ_1, ϕ_2 and ψ. Due to the effects of reflection and/or transmission effects by the up and down plane boundaries, wave fields inside each layer consist of the up-going and down-going waves, i.e.,

$$\phi_1^{(j)}(\boldsymbol{x},\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{c_{p1u}^{(j)}e^{i\gamma_1^{(j)}(z^{(j)}-z)}+c_{p1d}^{(j)}e^{i\gamma_1^{(j)}(z-z^{(j-1)})}\right\}e^{ikx}\,dk, \tag{15}$$

$$\phi_2^{(j)}(\boldsymbol{x},\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{c_{p2u}^{(j)}e^{i\gamma_2^{(j)}(z^{(j)}-z)}+c_{p2d}^{(j)}e^{i\gamma_2^{(j)}(z-z^{(j-1)})}\right\}e^{ikx}\,dk, \tag{16}$$

and

$$\psi^{(j)}(\boldsymbol{x},\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{c_{su}^{(j)}e^{i\nu^{(j)}(z^{(j)}-z)}+c_{sd}^{(j)}e^{i\nu^{(j)}(z-z^{(j-1)})}\right\}e^{ikx}\,dk, \tag{17}$$

where,

$$\gamma_{1;2}^{(j)}=\sqrt{\left(\omega/\mathrm{V}_{p1;2}\right)^2-k^2}, \qquad \text{with } \mathrm{Im}\left\{\gamma_{1;2}^{(j)}\right\}\geq 0,$$

$$\nu^{(j)}=\sqrt{\left(\omega/\mathrm{V}_s\right)^2-k^2}, \qquad \text{with } \mathrm{Im}\left\{\nu^{(j)}\right\}\geq 0,$$

and $c_{p1u}^{(j)}$, $c_{p2u}^{(j)}$, $c_{su}^{(j)}$, $c_{p1d}^{(j)}$, $c_{p2d}^{(j)}$ and $c_{sd}^{(j)}$ are unknown coefficients to be determined by imposing boundary conditions and source radiation. The boundary conditions at each plane interface between two saturated porous media are:

- Continuity of total displacement (normal & shear components),
- Continuity of fluid motion (normal component of fluid displacement),
- Continuity of total traction (normal & shear components),
- Continuity of the pressure of pore fluid.

To apply these boundary condition to determining those unknown coefficients, we define a motion-stress vector as

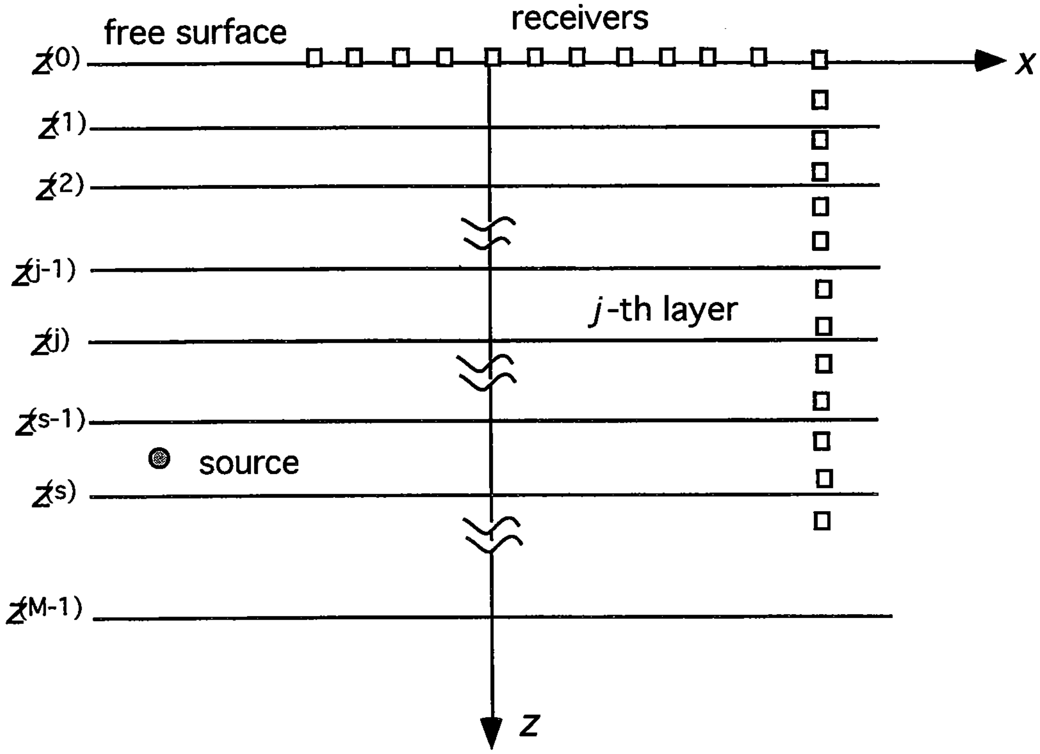

Figure 1. The physical model considered in this study. A seismic source is embedded in the s-th layer, receivers can be anywhere within the media.

$$\boldsymbol{Y}^{(j)}(z,k)=\int_{-\infty}^{+\infty}\left[W_x^{(j)},W_z^{(j)},U_z^{(j)},\Sigma_x^{(j)},\Sigma_z^{(j)},p_f^{(j)}\right]^T(x,z)\,e^{-ikx}\,dx, \tag{18}$$

where,

$$\boldsymbol{W}^{(j)}(x)=(1-\beta^{(j)})\boldsymbol{u}^{(j)}(\boldsymbol{x})+\beta^{(j)}\boldsymbol{U}^{(j)}(\boldsymbol{x}), \tag{19}$$

and

$$\Sigma^{(j)}(\boldsymbol{x})=\left\{\sigma^{(j)}(\boldsymbol{x})+\Theta^{(j)}(\boldsymbol{x})\boldsymbol{I}\right\}\cdot\hat{z} \tag{20}$$

are the total displacement and traction on a plane interface, respectively. Substituting equations (15) through (17) into (18), we obtain

$$Y^{(j)}(z) = A^{(j)}\Lambda^{(j)}(z)c^{(j)}, \tag{21}$$

where, $c^{(j)} = \left[c^{(j)}_{p1u}, c^{(j)}_{p1u}, c^{(j)}_{su}, c^{(j)}_{p1d}, c^{(j)}_{p1d}, c^{(j)}_{sd}\right]^T$ is unknown coefficient vector; $\Lambda^{(j)}(z)$; is a diagonal matrix function given by

$$\Lambda^{(j)}(z) = diag\left\{e^{i\gamma_1^{(j)}(z^{(j)}-z)}, e^{i\gamma_2^{(j)}(z^{(j)}-z)}, e^{i\nu^{(j)}(z^{(j)}-z)}, e^{i\gamma_1^{(j)}(z-z^{(j-1)})}, e^{i\gamma_2^{(j)}(z-z^{(j-1)})}, e^{i\nu^{(j)}(z-z^{(j-1)})}\right\}; \tag{22}$$

and $A^{(j)}$ is a 6x6 constant coefficient matrix whose elements, $\{ a^{(j)}_{nm};\ n;m=1\ 2,\ 3\}$, are given in Appendix A.

In terms of motion-stress vector $Y^{(j)}$, the boundary conditions at each plane interface between two saturated porous media can be expressed as

$$Y^{(j)}(z^{(j)}) = Y^{(j+1)}(z^{(j)}), \qquad \text{for } j = 1,2,3,\ldots,M; \tag{23}$$

$$Y^{(j)}(z)\Big|_{z=+\infty} = \text{finite}; \tag{24}$$

and

$$\begin{bmatrix} Y_4^{(j)}(0) \\ Y_5^{(j)}(0) \\ Y_6^{(j)}(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{on free surface.} \tag{25}$$

REFLECTION AND TRANSMISSION MATRICES

To effectively determine the unknown coefficients for each layer, we introduce modified and generalized reflection and transmission (R/T) matrices and derive their explicit expressions by imposing the boundary conditions.

Modified R/T matrices, which describe the reflection and transmission effects about an interface between two homogeneous saturated porous media, are defined by the relations:

$$\begin{cases} c_u^{(j)} = T_u^{(j)}\left(c_u^{(j+1)} + s_u\delta_{s(j+1)}\right) + R_{du}^{(j)}\left(c_d^{(j)} + s_d\delta_{sj}\right) \\ c_d^{(j+1)} = R_{ud}^{(j)}\left(c_u^{(j+1)} + s_u\delta_{s(j+1)}\right) + T_d^{(j)}\left(c_d^{(j)} + s_d\delta_{sj}\right) \end{cases}, \tag{26}$$

for $j = 1,2,3,\ldots,M$. Here, $T_u^{(j)}$, $R_{du}^{(j)}$, $R_{ud}^{(j)}$, $T_d^{(j)}$ are the modified R/T matrices for the j-th interface, they all are 3x3 matrices; and $c_u^{(j)} = \left[c^{(j)}_{p1u}, c^{(j)}_{p1u}, c^{(j)}_{su}\right]^T$ and $c_d^{(j)} =$

$\left[c_{p1d}^{(j)}, c_{p1d}^{(j)}, c_{sd}^{(j)}\right]^T$ are coefficient vectors for up-going and down-going waves in the j-th layer; s_u and s_d are seismic source vectors related to seismic source radiation. Inserting the explicit expression of $Y^{(j)}(z)$, eq.(21), into then boundary conditions, eq.(23), we obtain

$$\begin{bmatrix} T_u^{(j)} & R_{du}^{(j)} \\ R_{ud}^{(j)} & T_d^{(j)} \end{bmatrix} = \begin{bmatrix} A_{11}^{(j)} & -A_{12}^{(j+1)} \\ A_{21}^{(j)} & -A_{22}^{(j+1)} \end{bmatrix}^{-1} \begin{bmatrix} A_{11}^{(j+1)} & -A_{12}^{(j)} \\ A_{21}^{(j+1)} & -A_{22}^{(j)} \end{bmatrix} \begin{bmatrix} E^{(j+1)} & \\ & E^{(j)} \end{bmatrix}, \tag{27}$$

for $j = 1, 2, 3, \ldots, M$. Here $A_{11}^{(j)}$, $A_{12}^{(j)}$, $A_{21}^{(j)}$ and $A_{22}^{(j)}$ are four 3x3 sub-matrices of $A^{(j)}$; and $E^{(j)}$ is a diagonal matrix given by

$$E^{(j)} = diag\left\{ e^{i\gamma_1^{(j)}(z^{(j)}-z^{(j-1)})},\ e^{i\gamma_2^{(j)}(z^{(j)}-z^{(j-1)})},\ e^{i\nu^{(j)}(z^{(j)}-z^{(j-1)})} \right\}.$$

On the free surface, the modified reflection matrix, $R_{ud}^{(0)}$, is defined by

$$c_d^{(1)} = R_{ud}^{(0)} c_u^{(1)}.$$

Using the traction free condition, eq.(25), we obtain

$$R_{ud}^{(0)} = -\left(A_{22}^{(j)}\right)^{-1} A_{21}^{(1)} E^{(1)}. \tag{28}$$

Equations (27) and (28) provide the explicit formulas for computing the modified R/T matrices for each plane interface and free surface.

The generalized R/T matrices, $\hat{T}_u^{(j)}$, $\hat{R}_{du}^{(j)}$, $\hat{R}_{ud}^{(j)}$ and $\hat{T}_d^{(j)}$, are defined by the following relations:

$$\begin{cases} c_u^{(j)} = \hat{T}_u^{(j)}\left(c_u^{(j+1)} + s_u \delta_{s(j+1)}\right) \\ c_d^{(j+1)} = \hat{R}_{ud}^{(j)}\left(c_u^{(j+1)} + s_u \delta_{s(j+1)}\right) \end{cases} \quad \text{for } j = s-1,\ s-2,\ \ldots,\ 2,\ 1\ ; \tag{29}$$

and

$$\begin{cases} c_d^{(j+1)} = \hat{T}_d^{(j)}\left(c_d^{(j)} + s_d \delta_{sj}\right) \\ c_u^{(j)} = \hat{R}_{du}^{(j)}\left(c_d^{(j)} + s_d \delta_{sj}\right) \end{cases} \quad \text{for } j = s,\ s+1,\ \ldots,\ M-2,\ M-1. \tag{30}$$

Substituting eqs.(29) and (30) into eq.(26), we derive the following recursive formulas:

$$\begin{cases} \hat{T}_u^{(j)} = \left[I - R_{du}^{(j)} \hat{R}_{ud}^{(j-1)}\right]^{-1} T_u^{(j)} \\ \hat{R}_{ud}^{(j)} = R_{ud}^{(j)} + T_d^{(j)} \hat{R}_{du}^{(j-1)} \hat{T}_u^{(j)} \end{cases} \quad \text{for } j = s-1,\ s-2,\ \ldots,\ 2,\ 1\ ; \tag{31}$$

and

$$\begin{cases} \hat{T}_d^{(j)} = \left[I - R_{ud}^{(j)} \hat{R}_{du}^{(j+1)} \right]^{-1} T_d^{(j)} \\ \hat{R}_{du}^{(j)} = R_{du}^{(j)} + T_u^{(j)} \hat{R}_{du}^{(j+1)} \hat{T}_d^{(j)} \end{cases} \quad \text{for } j = s,\ s+1,\ \ldots,\ M-2,\ M-1. \tag{32}$$

where I is a 3x3 unit matrix, $\hat{T}_u^{(j)}$, $\hat{R}_{du}^{(j)}$, $\hat{R}_{ud}^{(j)}$ and $\hat{T}_d^{(j)}$, are all 3x3 matrices. In the bottom layer ($j = M$), only down-going waves exist, i.e., $c_u^{(M)} = 0$, therefore

$$\hat{R}_{du}^{(M)} = 0. \tag{33}$$

In the top layer ($j = 1$), the down-going waves only come from the free surface ($z = z^{(0)} = 0$), thus

$$\hat{R}_{ud}^{(0)} = R_{ud}^{(0)}. \tag{34}$$

Equations (31) and (32) along with the initial conditions given by eqs. (33) and (34) provide an efficient recursive scheme to calculate the generalized R/T matrices from the modified R/T matrices given in equations (27) and (28).

SOURCE RADIATION

The seismic source vectors s_u and s_d can be determined by a special integration consideration. In the source layer, the $\omega - k$ domain wave fields directly radiated from the source have the following form:

$$S(z) = A^{(s)} \Lambda^{(s)}(z) s(z), \tag{35}$$

where,

$$s(z) = \begin{bmatrix} s_u H(z - z_s) \\ s_d H(z_s - z) \end{bmatrix}. \tag{36}$$

In terms of motion-stress vector $S(z)$, the basic equations of motion [eqs.(1) and (2)) and constitutive equations (eq.(3) and (4)] can be rewritten as

$$\frac{dS(z)}{dz} = i\Gamma S(z) - q\delta(z - z_s), \tag{37}$$

where,

$$\Gamma = diag\left\{ -\gamma_1^{(j)}, -\gamma_2^{(j)}, -v^{(j)}, \gamma_1^{(j)}, \gamma_2^{(j)}, v^{(j)} \right\},$$

and

$$q = [0, 0, 0, f_x, f_z, 0]^T e^{-ikx_s}.$$

Integrating eq.(37) over an infinite small region around the source point $z = z_s$ yields

$$S(z_s^+) - S(z_s^-) = -q. \qquad (38)$$

Substituting eq.(35) into equation (38), we obtain

$$s_u = -\left[\Lambda_u^{(s)}(z_s)\right]^{-1}\left(A_{21}^{(s)} - A_{22}^{(s)}A_{12}^{(s)-1}A_{11}^{(s)}\right)^{-1} q_2, \qquad (38a)$$

and

$$s_d = -\left[\Lambda_d^{(s)}(z_s)\right]^{-1}\left(A_{21}^{(s)}A_{11}^{(s)-1}A_{12}^{(s)} - A_{22}^{(s)}\right)^{-1} q_2, \qquad (38b)$$

where, $q_2 = [f_x, f_z, 0]^T e^{-ikx_s}$, and

$$\Lambda_u^{(s)}(z_s) = diag\left\{e^{i\gamma_1^{(s)}(z^{(s)}-z_s)}, e^{i\gamma_2^{(s)}(z^{(s)}-z_s)}, e^{i\nu^{(s)}(z^{(s)}-z_s)}\right\},$$

$$\Lambda_d^{(s)}(z_s) = diag\left\{e^{i\gamma_1^{(s)}(z_s-z^{(s-1)})}, e^{i\gamma_2^{(s)}(z_s-z^{(s-1)})}, e^{i\nu^{(s)}(z_s-z^{(s-1)})}\right\}.$$

SOLUTION SYNTHESIS

Having the generalized R/T matrices, we can compute the unknowns $c_u^{(j)}$ and $c_d^{(j)}$ for any layer. Therefore, we can determine the displacements and stresses for any layer. From eqs. (29) and (30) we obtain

$$\begin{cases} c_d^{(s)} + s_d = \left[I - \hat{R}_{ud}^{(s-1)}\hat{R}_{du}^{(s)}\right]^{-1}\left(s_d + \hat{R}_{ud}^{(s-1)}s_d\right) \\ c_u^{(s)} + s_u = \left[I - \hat{R}_{du}^{(s)}\hat{R}_{ud}^{(s-1)}\right]^{-1}\left(s_u + \hat{R}_{du}^{(s)}s_d\right) \end{cases}. \qquad (39)$$

Plugging the eq.(39) into, again, eqs.(29) and (30), we can calculate the unknowns $c_u^{(j)}$ and $c_d^{(j)}$ by the following formulas

$$\begin{cases} c_u^{(j)} = \hat{T}_u^{(j)}\hat{T}_u^{(j+1)}\ldots\hat{T}_u^{(s-1)}\left[I - \hat{R}_{du}^{(s)}\hat{R}_{ud}^{(s-1)}\right]^{-1}\left(s_u + \hat{R}_{du}^{(s)}s_d\right) \\ c_d^{(j)} = \hat{R}_{ud}^{(j-1)}c_u^{(j)} \end{cases}, \qquad (40)$$

for *j=1, 2, 3, ..., s-1*; and

$$\begin{cases} c_d^{(j)} = \hat{T}_d^{(j-1)}\hat{T}_d^{(j)}\ldots\hat{T}_d^{(s)}\left[I - \hat{R}_{ud}^{(s-1)}\hat{R}_{du}^{(s)}\right]^{-1}\left(s_d + \hat{R}_{ud}^{(s-1)}s_u\right) \\ c_u^{(j)} = \hat{R}_{du}^{(s)}c_d^{(j)} \end{cases}, \qquad (41)$$

for $j = s+1, s+2, \ldots, M$. Incorporating $c_u^{(j)}$ and $c_d^{(j)}$ into eq.(21), we obtain the $k-\omega$ domain solutions in any layer:

$$\left[W_x^{(j)}, W_z^{(j)}, U_z, \Sigma_x^{(j)}, \Sigma_z^{(j)}, p_f\right]^T(x,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Y^{(j)}(z,k,\omega)\Lambda^{(j)}(z)c^{(j)}e^{ikx}dk. \tag{42}$$

Finally, by taking inverse Fourier transforms over ω we can obtain the final solutions in time domain.

QUASI- VISCOELASTIC SOLUTIONS

In the most circumstances we encountered, the second kind compressional wave (P2) has negligible contribution to the observable wave fields, because of its high attenuation in the saturated porous media. In such case we can neglect the second kind compressional wave, only consider the coupling between the first kind compressional wave and shear wave. Then the formulation for general saturated porous media derived in previous sections can be simplified, and are reduced to the quasi-viscoelastic formulation. Such reduction is simple and straightforward: neglect the unknowns $c_{p2u}^{(j)}$ and $c_{p2d}^{(j)}$, and accordingly only impose the continuities of total displacements and tractions at each boundary. As result, the formulation of quasi-viscoelastic solution will be in the same form as those of general saturated porous solution presented in prevoius sections, except the 6x6 constant matrcies $A^{(j)}$ become 4x4 ones; the 3x3 R/T matrices become 2x2 ones.

PRELIMINARY NUMERICAL RESULTS

At the present stage, we have finished coding the computer program for quasi-viscoelastic formulation. The computational code for general multi-plane layered porous media problem is still under developping, and expect to be finished in the near future. Therefore, in this report, we shall only show some preliminary numerical results of modeling the seismic waves in multi-layered porous media by using the quasi-viscoelastic formulation. We shall consider two examples to investigate seismic waves propagation and attenuation in this 1-D saturated porous media. The first example is three layer model which consists of a high porosity low-velocity zone. The second example is a 10-layer saturated porous media model which consist several high porosity zones. The parameters of these models are given in tables 1 and 2, respectively. The simulation results are shown in Figures 2 through (7). Figure 2 is a gather of seismograms of the horizontal and vertical displacements from which we can see that the waves propagating in satuated porous media Figure2a shows stronger attenuation than the elastic waves (Figure 2b). Figure 3 shows the effects of permeability on the attenuation, where the permeabilities are taken as 0.01D, 100D and 100000D for Figure3a, b and c, respectively. Figure 3d is an elastci case for comparisons. From these comparisons, we can see that the effect of permeability on the attenuation is noticeable. Figures 4 to 5 show the results for a 10-layer saturated porous media model. Once again, we can observe the attenuation effects caused by the saturated porous media .

Table 1 Paramters of a three layer model

Layer number	Depth (m)	ρ_s	V_s (km/s)	V_p (km/s)	Fluid Type	β	κ (Darcy)	ρ_f	V_f (km/s)	η (Poise)
1	0.00	2.677	3.22	5.45	Water	0.05	1.0E+3	1.00	1.50	1.0E-2
2	100.	2.628	2.84	4.30	oil	0.05	1.0E+5	0.88	1.45	1.8
3	120.	2.900	4.20	8.00	Water	0.10	1.0E+3	1.00	1.50	1.0E-2

Table 2 Parameters of a ten layer model

Layer number	Depth (m)	ρ_s	V_s (km/s)	V_p (km/s)	Fluid Type	β	κ (Darcy)	ρ_f	V_f (km/s)	η (Poise)
1	0.00	2.677	3.22	5.45	Water	0.05	1.0E+3	1.00	1.50	1.0E-2
2	100.	2.600	2.80	4.20	Water	0.05	1.0E+3	1.00	1.50	1.0E-2
3	110.	2.660	3.50	5.90	Water	0.10	1.0E+4	1.00	1.50	1.0E-2
4	120.	2.570	2.85	4.30	Gas	0.16	1.0E+4	0.14	0.63	2.2E-4
5	130.	2.628	2.60	4.10	Oil	0.19	1.0E+5	0.88	1.45	1.8E+0
6	140.	2.630	2.85	4.30	Oil	0.16	1.0E+4	0.88	1.45	1.8E+0
7	160.	2.600	3.22	5.45	Oil	0.20	1.0E+5	0.88	1.45	1.8E+0
8	170.	2.728	2.80	4.2	Water	0.15	1.0E+4	1.00	1.50	1.0E-2
9	180.	2.800	3.60	6.00	Water	0.10	1.0E+3	1.00	1.50	1.0E-2
10	200.	3.000	4.40	8.00	Water	0.05	1.0E+2	1.00	1.50	1.2E-2

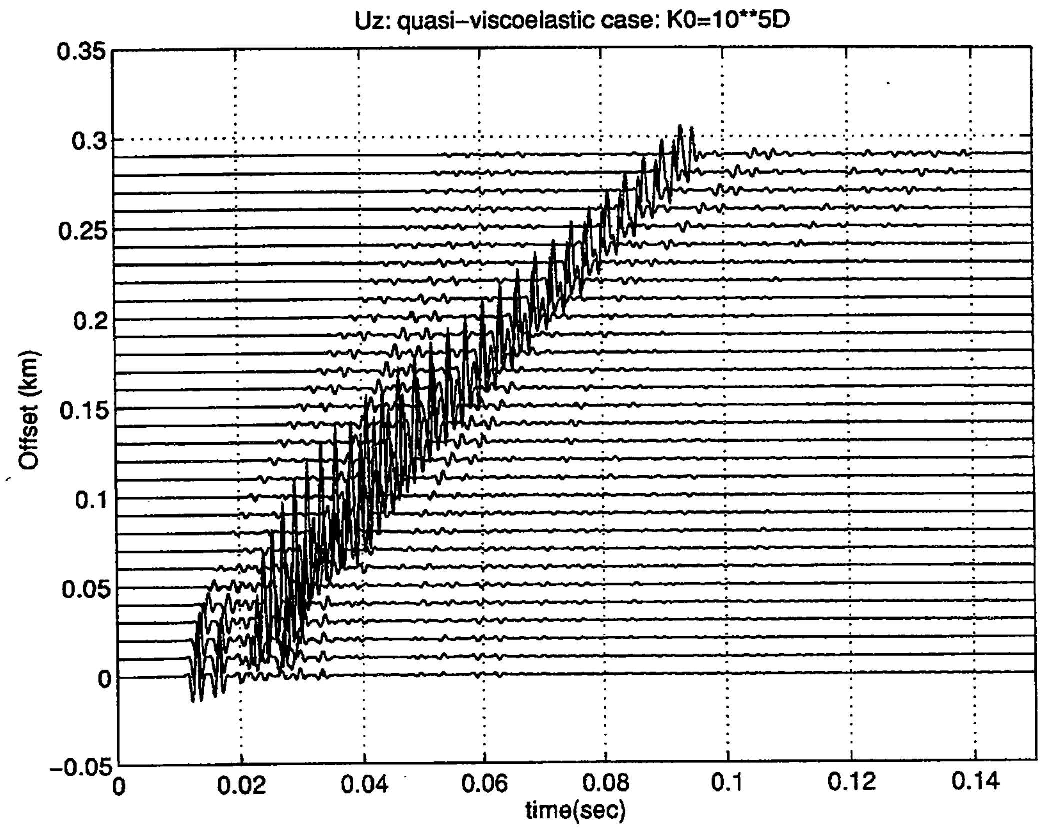

Figure 2a The synthetic seismograms for a three layer porous media. Compared with the elastic solution shown in Figure 2b, we can see the attenuation caused by the porous properties.

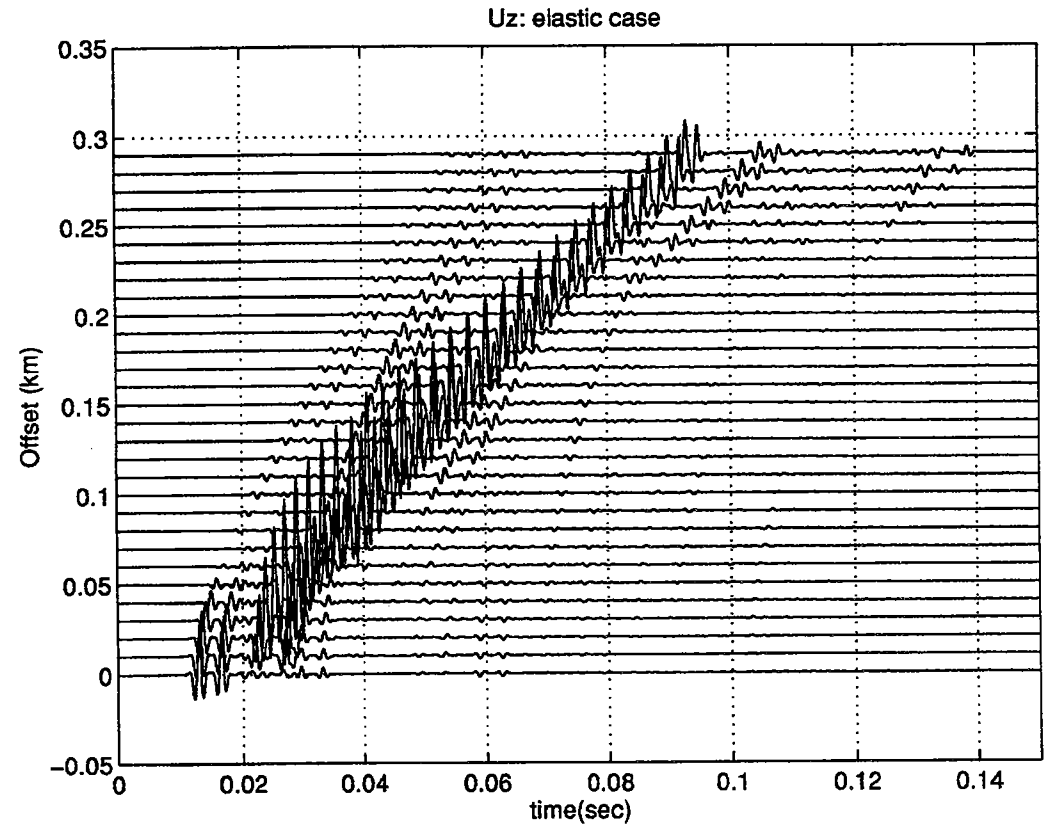

Figure 2b Snythetic seismograms for the elastic model.

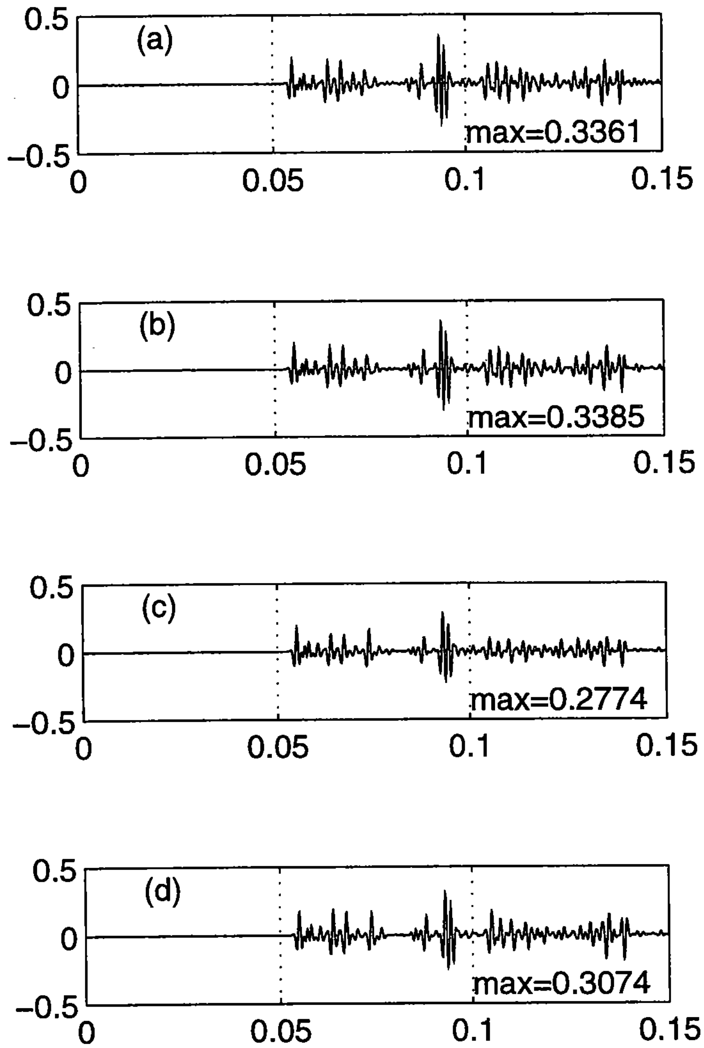

Figure 3 Comparisons of synthetic seismograms for different permeabilites: (a). K0=0.01d; (b). K0=1.D; (c). K0=10**5d; (d). Elastic case.

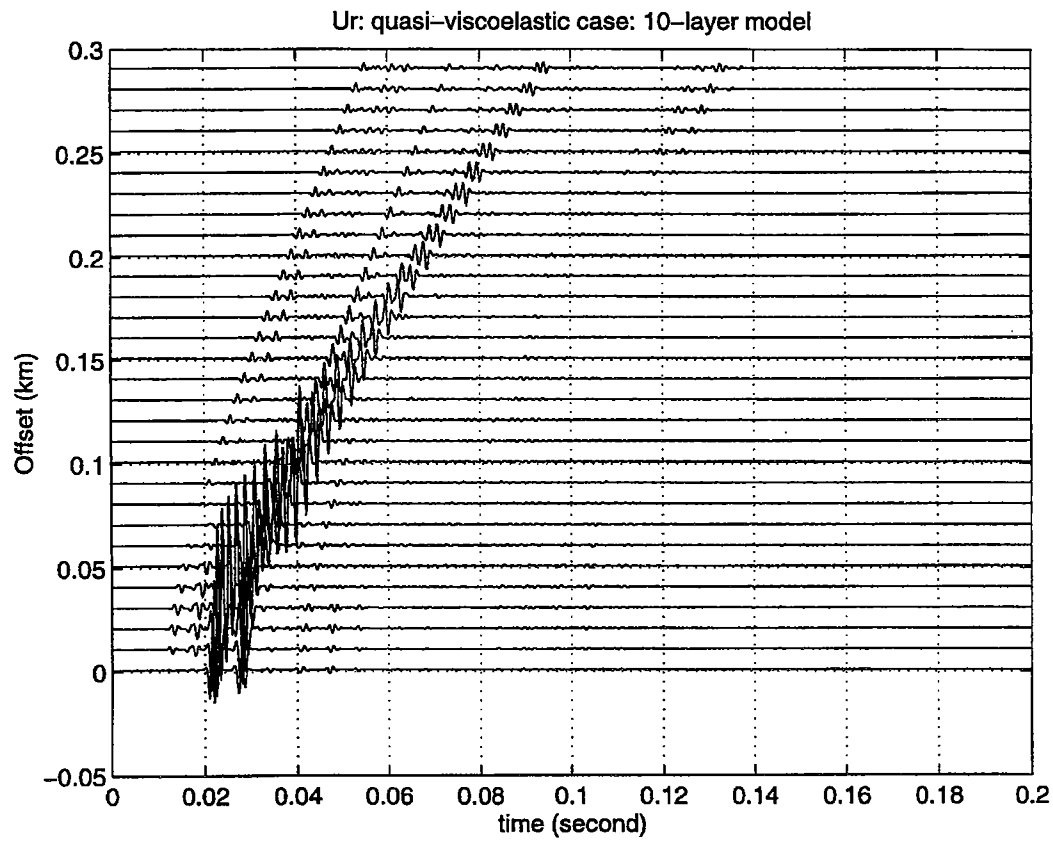

Figure 4a Synthetic seismograms for a 10-layer porous media model. Compared with Figure 4b for the elastic media, a noticeable attenuation can been seen.

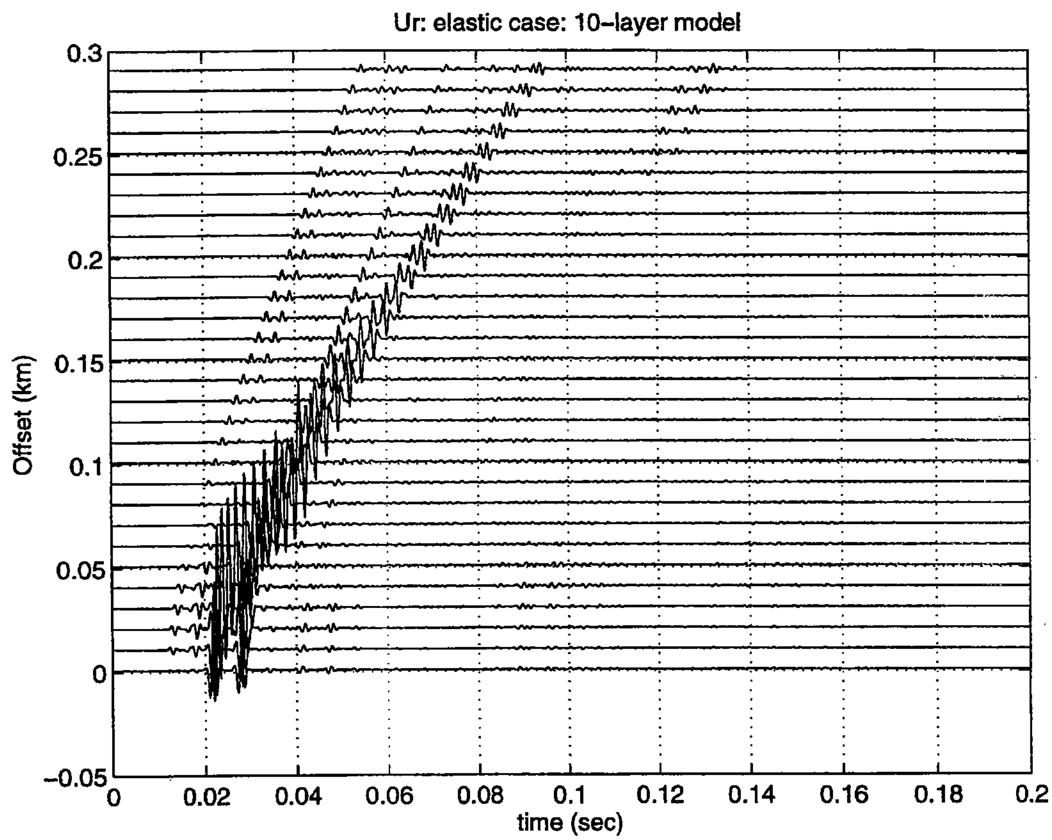

Figure 4b Synthetic seismograms for the model with the same structure but of elastic case.

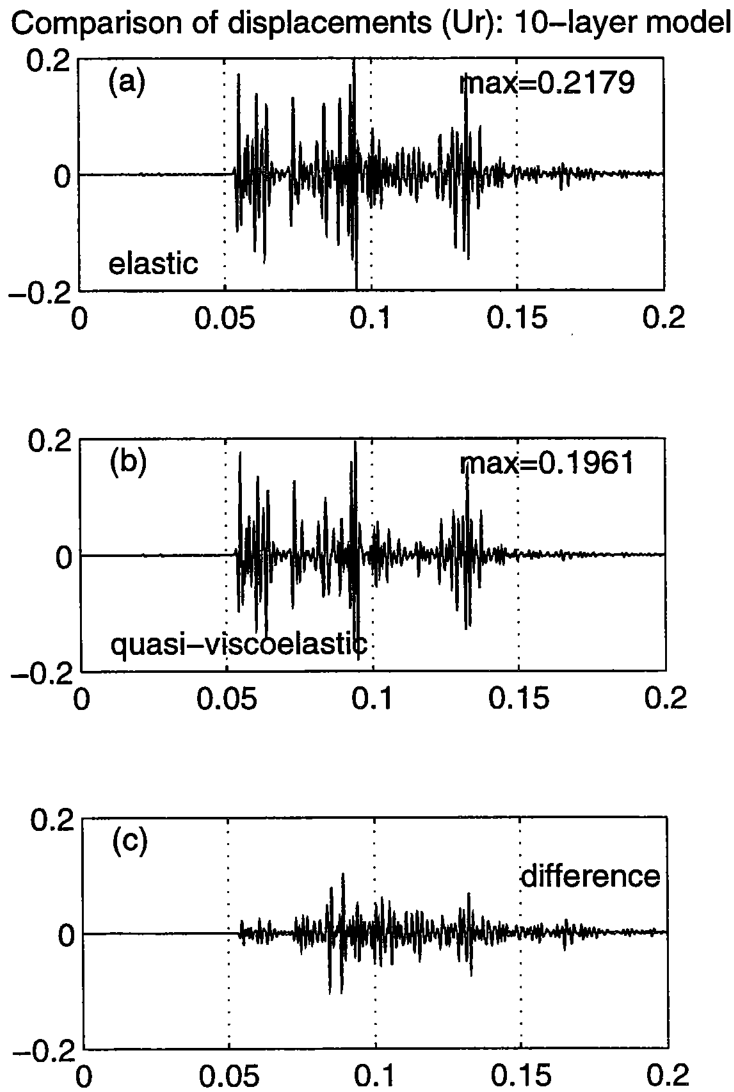

Figure 5. A close compariisons of synthetic seismograms for a 10-layer model with that of elastic model.

CONCLUSIONS

To effectively modeling the seismic waves propagation through the vertically layered saturated porous media, we have derived a set of efficient formulation by extending the reflectivity theory to our porous media problem. This formulation is useful in the reservoir characterization and discription, and it also can be used to asses the AVO analysis. If we neglect the slow wave, our formulation is reduced to the quasi-viscoelastic solution. As preliminary results, we have used the quasi-viscoelastic solution to investigate seismic wave's attenuation in saturated porous media. Our results show the attenuation shows up when the peremability become lager.

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APPENDIX

The elements of constant coefficient matrix $A^{(j)}$ are defined as

$$a_{11}^{(j)} = a_{14}^{(j)} = \alpha_1^{(j)} k,$$

$$a_{12}^{(j)} = a_{15}^{(j)} = \alpha_2^{(j)} k,$$

$$a_{13}^{(j)} = -a_{16}^{(j)} = \alpha_3^{(j)} v^{(j)},$$

$$a_{21}^{(j)} = -a_{24}^{(j)} = -\alpha_1^{(j)}\gamma_1^{(j)},$$

$$a_{22}^{(j)} = -a_{25}^{(j)} = -\alpha_2^{(j)}\gamma_2^{(j)},$$

$$a_{23}^{(j)} = a_{26}^{(j)} = \alpha_3^{(j)}k,$$

$$a_{31}^{(j)} = -a_{34}^{(j)} = -\chi_1^{(j)}\gamma_1^{(j)},$$

$$a_{32}^{(j)} = -a_{35}^{(j)} = -\chi_2^{(j)}\gamma_2^{(j)},$$

$$a_{63}^{(j)} = a_{66}^{(j)} = \chi_3^{(j)}k,$$

$$a_{41}^{(j)} = -a_{44}^{(j)} = 2\gamma_1^{(j)}N^{(j)}k,$$

$$a_{42}^{(j)} = -a_{45}^{(j)} = 2\gamma_2^{(j)}N^{(j)}k,$$

$$a_{43}^{(j)} = a_{46}^{(j)} = \chi_3^{(j)}N^{(j)}\left[\left(v^{(j)}\right)^2 - k^2\right],$$

$$a_{51}^{(j)} = a_{54}^{(j)} = -\left\{2N^{(j)}\left(\gamma_1^{(j)}\right)^2 + \left[F^{(j)} + G^{(j)} + \left(T^{(j)} + G^{(j)}\right)\chi_1^{(j)}\right]\left(\frac{\omega}{V_{p1}^{(j)}}\right)^2\right\},$$

$$a_{52}^{(j)} = a_{55}^{(j)} = -\left\{2N^{(j)}\left(\gamma_2^{(j)}\right)^2 + \left[F^{(j)} + G^{(j)} + \left(T^{(j)} + G^{(j)}\right)\chi_2^{(j)}\right]\left(\frac{\omega}{V_{p2}^{(j)}}\right)^2\right\},$$

$$a_{53}^{(j)} = -a_{56}^{(j)} = 2\chi_3^{(j)}v^{(j)}N^{(j)}k,$$

$$a_{61}^{(j)} = -a_{64}^{(j)} = -\chi_1^{(j)}\gamma_1^{(j)},$$

$$a_{62}^{(j)} = -a_{65}^{(j)} = -\chi_2^{(j)}\gamma_2^{(j)},$$

$$a_{63}^{(j)} = a_{66}^{(j)} = 0,$$

where, $\alpha_n^{(j)} = 1 - \beta^{(j)}\left(1 - \chi_n^{(j)}\right)$, for n=1, 2, 3.