PAPER J

KOND ALGORITHM FOR NUMERICAL SIMULATION OF 3-C SEISMOGRAMS IN HETEROGENEOUS ANISOTROPIC MEDIA

Zhongjie Zhang*, Youli Quan, Xiaofei Chen, and J. M. Harris

ABSTRACT

A new and effective forward modelling technique - KOND (Kernel Optimum Nearly analytical Discretization) algorithm has been developed and implemented into the numerical simulation of multi-component seismic wave-fields in 2-D heterogeneous and anisotropic media, The main idea for constructing such an algorithm is to use the source equations and their branch equations as many as possible, to find a higher order approximate solution as analytically as possible, to use more local continuous solutions around the grid points as frequently as possible, and to use the connection relations as many elements as possible to compensate for the loss of information when the source equation is discretized. Computing seismic wave-fields, reflections on artificial boundaries have been suppressed with the use of absorbing boundary conditions, and interior interfaces have been treated with the FBI (Forward and Backward Iteration) technique. The results for model computations reveal that the algorithm is stable, precisive and shows less dispersion than conventional methods; can save more computer memory and time than conventional methods to reach the same accuracy; is feasible to simulate the seismic propagation in complicated media, and extendible easily to 3-D and other sophisticated problems in Geophysics.

---

* Institute of Geophysics, Chinese Academy of Sciences, China

INTRODUCTION

Recently, many observatory experiments and theoretic studies have demonstrated that seismic anisotropy is widespread, whether it is interstice or apparent which is induced by thin layering or oriented cracks or fractures. The information of seismic anisotropy has an important meaning in the exploration and production of oil and gas reservoirs, in the study of Geodynamics, and for the prediction of natural disasters (Crampin, 1980; Schonberg, 1989; Backus, 1962; etc.), which has been absorbing the research interests of seismologists and explorational geophysicists (Thomson, 1986; Crampin, 1991; Schonberg, 1988; Helbig, 1984; Teng et al., 1992; etc.).

Numerical simulation is a useful technique for increasing our understanding of seismic propagation in heterogeneous and anisotropic media, for interpreting of practical multi-component seismic observations and for testing the feasibility of software of lately developed forward, inverse and migration schemes(Reshef, and Kosoloff, 1985;).

So far, many numerical methods have been developed for forward modelling of single or multi-component seismic wave-fields in acoustic or elastic, isotropic or anisotropic media, such as the reflectivity method for modelling seismic propagation in layered media (Booth and Crampin, 1983a, b; Fryer, 1984; Schoenberg, et al., 1989; Chen, 1990, 1996; etc.), the Ray tracing technique (Cerveny, 1985; Chapman, 1991; etc.), the Fourier transformation method (Kosoloff, 1988; etc.), the Finite difference method (Verierx, 1986; Faria, 1994; etc.), the Finite element method (Kunhua Chen, 1984; etc.), and others (Vidale, 1988; Qin et al., 1993; Tessmer and Kosloff, 1994; Hornby et al., 1994; etc.). They have their merits and drawbacks. Generally, the wave equation is directly discretized by the finite-difference or finite element method, which leads to some problems, namely, the two types of information embedded in the wave equations (source equation) are lost more or less. One type of information loss is the relations that are embedded in the source equation and connecting the local values and their time evolution, the other one is the information on the functional 'values' embedded in the solution of wave-fields. The greatest problem may be that the conventional method used to process wave equations would yield a finite error compared with the source equation, which may be the main source of the diffusive error. And that is the starting point of the KOND algorithm. Kondoh(1991) used such a Thought Analysis method to develop KOND scheme to solve one- and two-order equation in the fusion sciences, and obtained good results (1991).

In this paper, we utilized a similar thought process to solve the wave (source) equations in anisotropic media as directly as possible, avoiding the use of the finite difference equation. In order to develop the KOND algorithm successfully in the modelling of seismic records in 2-D heterogeneous anisotropic media, we review the main points for constructing the KOND algorithm in general, in the first, the detail may refer to Kondoh (1991). Then introduce the new numerical scheme for the simulation of seismic wave-fields in complicated media. In the computation of wave-fields, the reflection on artificial boundaries is suppressed with the use of an absorbing boundary condition for each component, the interior interfaces in the computational region are approached with the FBI technique (Zhongjie, et al., 1996) . Some computational results are given and analyzed. The results show that the KOND algorithm developed here is stable, precisive with less dispersion than other methods, and can be easily extended to 3-D and other complicated problems in Geophysics.

BACKGROUND

Assuming there is a two-order equations to be solved in the following form:

$$\mathrm{L(X)U(X)}=\rho\frac{\partial^2 U(X)}{\partial t^2} \tag{1}$$

we use Eq.(1) as source equation.

From Eq.(1), we know the following equation is valid always:

$$F_X\left[LU-\rho\frac{\partial^2 U(X)}{\partial t^2}\right]=0 \tag{2}$$

Where X=(x1, x2, x3, ...). F_X denotes the derivative function(s) of source equation(s) with respect to X. we use Eq.(2) as a set of branch equations.

The optimum solution of Eq.(1) is to include the information embedded in Eq.(1) and Eq.(2), that is, the best solution is shown in the following set on the functional values of the analytic solution and its derivatives:

$$\{U(X),\partial_i U(X),\partial_{ij}U(X),\cdots\} \tag{3}$$

The conventional finite-difference equations for Eq.(1) has finite error compared with the source equation Eq.(1) and therefore it has finite loss of information on the relations which are embedded in the source equation and connecting the local values and their time evaluations.

From discrete values of solutions at grid points:

$$\{U_n,\partial_i U_n,\partial_{ij}U_n,\cdots\} \tag{4}$$

interpolation curves around grid points:

$$\left\{U_n(s), \partial_i U_n(s), \partial_{ij} U_n(s), \cdots\right\}, \qquad s \equiv x - x_n^h \tag{5}$$

and connection relations at the neighboring grid points:

$$\begin{aligned} U_n(-h_d) &= U_{n-1} & U_n(h_d) &= U_{n+1} \\ \partial_i U_n(-h_d) &= \partial_i U_{n-1} & \partial_i U_n(h_d) &= \partial_i U_{n+1} \\ \partial_{ij} U_n(-h_d) &= \partial_{ij} U_{n-1} & \partial_{ij} U_n(h_d) &= \partial_{ij} U_{n+1} \\ & \cdots \end{aligned} \tag{6}$$

Each element of the set (4) should be the piecewise segment of the corresponding analytic solutions of Eq.(3); and the combination of Eq.(4), (5) and (6) is exactly equivalent to the set of the true solution of Eq.(1).

Using the Taylor expansion, Eq.(6) can be written as follows:

$$U_n(s) = U_n + \sum_i \partial_i U_n s_i + \sum_{i,j} \partial_{ij} U_n \, {}^{s_i s_j}\!/_2 + \cdots \tag{7a}$$

$$\partial_i U_n(s) = \partial_i U_n + \sum_j \partial_{ij} U_n s_j + \sum_{j,k} \partial_{ijk} U_n \, {}^{s_j s_k}\!/_2 + \cdots \tag{7b}$$

$$\partial_{ij} U_n(s) = \partial_{ij} U_n + \sum_k \partial_{ijk} U_n s_k + \sum_{k,l} \partial_{ijkl} U_n \, {}^{s_k s_l}\!/_2 + \cdots \tag{7c}$$

$$\cdots$$

Obviously, the discrete values of the solution are the coefficients of the interpolation curves by the Taylor expansion and therefore they can induce good approximate and locally continuous solutions around the given grid point. In other words, the infinite set of the discrete values of Eq.(4) itself becomes one of the best discretizations for the whole information on the functional values of the continuous true solutions in Eq.(1) based upon the interpolation curves by the Taylor expansions.

Since we can use two or three elements, for example $U_n, \partial_i U_n, \partial_{ij} U_n, \cdots$. some finer information included in the rest infinite terms beyond the terms $\partial_{ij} U_n$ would be lost. but the rest infinite terms in the Taylor expansions can be folded up approximately in the finite additional terms by using the connection relations of Eq.(6). From such a way, we can suppress effectively the loss of information.

We can see from the above analysis that in order to suppress the loss of information by the discretization of the source equation, we should find higher-order approximate analytic solutions for the source and its branch equations as analytically as possible. or say, if we've a method which is nearly analytical for obtaining better approximate solutions for the more elements of the set of Eq.(4), we would get the more accurate and denser information for the set of true solution Eq.(1).the accuracy of the information for the solutions by this method is optimum at the grid points, the discretization by this method is kernel optimum nearly analytic.

The four basic and key points of the KOND algorithm based on the above thought analysis is as follows:

a. use the source equations and their branch equations as many as possible.

b. find higher-order approximate analytic solutions as analytically as possible.

c. find the set of discrete solutions $\{U_n, \partial_i U_n, \partial_{ij} U_n, \cdots\}$ using as many elements as possible.

d. use the connection relations as many elements as possible in order to include the semiglobal information for the curved regions between the neighboring grid points and to find the additional higher order Taylor coefficients which represent approximately the rest of the infinite terms of the Taylor expansion.

Each part of the KOND algorithm seems to be rather simple and abstract to develop new schemes with higher numerical accuracy. The following section will demonstrate how the four elements of the KOND algorithm give us novel numerical schemes which yield great accuracy and therefore significantly reduce CPU time to attain the same accuracy.

KOND ALGORITHM FOR MODELLING IN HETEROGENEOUS ANISOTROPIC MEDIA

In heterogeneous anisotropic media, we have :

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i \tag{8}$$

The relationship between stress and strain:

$$\sigma_{ij} = C_{ijkl} \varepsilon_{jl} \tag{9}$$

Where $\sigma_{ij}, \varepsilon_{ij}$ mean the stress and strain component, respectively, and $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$, C_{ijkl} is elastic parameter in anisotropic media, we will designate it as C_{ij} in the following discussion.

According to the first point, to use the source equations and the branch equation as many as possible, we set the following as 1st branch equations:

$$\dot{u}_i = w_i \qquad\qquad \dot{p}_i = w_i \tag{10}$$

Where, $U = [u_i]^T$, $W = [w_i]^T$, $P = [p_i]^T$, $F = [f_i]^T$. u_i is a displacement component , $\dot{u}_i$ is a velocity component, and $\ddot{u}_i$ is an accelaration component.

The following wave equations in 2-D anisotropic media can be considered as source equations and part of branch functions:

$$\dot{W} = P = (L_1 + L_2)U + \frac{F}{\rho} \tag{11}$$

Where differentiator operators L_1 and L_2 take the following forms:

$$L_1 = \frac{1}{\rho}\frac{\partial}{\partial x}\left(C_1\frac{\partial}{\partial x} + C_2\frac{\partial}{\partial z}\right) \qquad L_2 = \frac{1}{\rho}\frac{\partial}{\partial z}\left(C_3\frac{\partial}{\partial x} + C_4\frac{\partial}{\partial z}\right)$$

$$C_1 = \begin{bmatrix} C_{11} & C_{15} & C_{16} \\ C_{15} & C_{55} & C_{56} \\ C_{16} & C_{56} & C_{66} \end{bmatrix} \qquad C_2 = \begin{bmatrix} C_{15} & C_{13} & C_{14} \\ C_{55} & C_{35} & C_{45} \\ C_{56} & C_{36} & C_{46} \end{bmatrix}$$

$$C_3 = \begin{bmatrix} C_{15} & C_{55} & C_{56} \\ C_{13} & C_{35} & C_{36} \\ C_{14} & C_{45} & C_{46} \end{bmatrix} \qquad C_4 = \begin{bmatrix} C_{55} & C_{35} & C_{45} \\ C_{35} & C_{33} & C_{34} \\ C_{45} & C_{34} & C_{44} \end{bmatrix}$$

As there are two types of variables namely spatial and temporal variables x, z and t. so too can we get the other part of the branch equations from the derivative equation of source equation with respect to spatial variables and make the following setting:

$$\bar{U} = \left[U, \frac{\partial}{\partial x}U, \frac{\partial}{\partial z}U\right]^T \tag{12}$$

and

$$\bar{P} = \left[P, \frac{\partial}{\partial x}P, \frac{\partial}{\partial z}P\right]^T \tag{13}$$

and

$$\bar{W} = \left[W, \frac{\partial}{\partial x}W, \frac{\partial}{\partial z}W\right]^T \tag{14}$$

According to the second point, we should solve the above equations locally around one given point (x_i, z_j, t_{n+1}) as analytically as possible. Using the Taylor expansion around the time t=ndt, we can get the following formula:

$$\bar{U}_{i,j}^{n+1}=\bar{U}_{i,j}^{n}+\Delta t\left(\frac{\partial \bar{U}}{\partial t}\right)_{i,j}^{n}+\frac{(\Delta t)^2}{2}\left(\frac{\partial^2 \bar{U}}{\partial t^2}\right)_{i,j}^{n}+\frac{(\Delta t)^3}{6}\left(\frac{\partial^3 \bar{U}}{\partial t^3}\right)_{i,j}^{n}+\frac{(\Delta t)^4}{24}\left(\frac{\partial^4 \bar{U}}{\partial t^4}\right)_{i,j}^{n} \tag{15}$$

and

$$\bar{W}_{i,j}^{n+1}=\bar{W}_{i,j}^{n}+\Delta t\left(\frac{\partial \bar{W}}{\partial t}\right)_{i,j}^{n}+\frac{(\Delta t)^2}{2}\left(\frac{\partial^2 \bar{W}}{\partial t^2}\right)_{i,j}^{n}+\frac{(\Delta t)^3}{6}\left(\frac{\partial^3 \bar{W}}{\partial t^3}\right)_{i,j}^{n}+\frac{(\Delta t)^4}{24}\left(\frac{\partial^4 \bar{W}}{\partial t^4}\right)_{i,j}^{n} \tag{16}$$

According to the definition of U, W and P, we can get the following relation:

$$\frac{\partial^{m+k+l+1}\bar{U}}{\partial t^{m+1}\partial x^k \partial z^l}=\frac{\partial^{m+k+l}\bar{W}}{\partial t^{m}\partial x^k \partial z^l}=\frac{\partial^{m+k+l-1}\bar{P}}{\partial t^{m-1}\partial x^k \partial z^l} \tag{17}$$

Considering the above relations , the equations (15) and (16) can be rewritten as follows:

$$\bar{U}_{i,j}^{n+1}=\bar{U}_{i,j}^{n}+\Delta t\,\bar{W}_{i,j}^{n}+\frac{(\Delta t)^2}{2}\bar{P}_{i,j}^{n}+\frac{(\Delta t)^3}{6}\left(\frac{\partial \bar{P}}{\partial t}\right)_{i,j}^{n}+\frac{(\Delta t)^4}{24}\left(\frac{\partial^2 \bar{P}}{\partial t^2}\right)_{i,j}^{n} \tag{18}$$

and:

$$\bar{W}_{i,j}^{n+1}=\bar{W}_{i,j}^{n}+\Delta t\,\bar{P}_{i,j}^{n}+\frac{(\Delta t)^2}{2}\left(\frac{\partial \bar{P}}{\partial t}\right)_{i,j}^{n}+\frac{(\Delta t)^3}{6}\left(\frac{\partial^2 \bar{P}}{\partial t^2}\right)_{i,j}^{n}+\frac{(\Delta t)^4}{24}\left(\frac{\partial^3 \bar{P}}{\partial t^3}\right)_{i,j}^{n} \tag{19}$$

Now, we proceed to the third point. In order to find the set of discrete solutions of $\bar{U}_{i,j}^{n+1}$ and $\bar{W}_{i,j}^{n+1}$ from the above equations, we should use the terms at right sides of above equations with the values at the time $t=n\Delta t$ and $t=(n-1)\Delta t$. Using the definitions from eq.(11), we obtain $\frac{\partial}{\partial x}P, \frac{\partial}{\partial z}P, \frac{\partial}{\partial t\partial x}P, \frac{\partial}{\partial t\partial z}P, \cdots$ as follows:

$$\frac{\partial}{\partial x}P=(L_3+L_4)U+\frac{1}{\rho}\frac{\partial}{\partial x}F-\frac{1}{\rho^2}\frac{\partial\rho}{\partial x}F \tag{20a}$$

$$\frac{\partial}{\partial z}P=(L_5+L_6)U+\frac{1}{\rho}\frac{\partial}{\partial z}F-\frac{1}{\rho^2}\frac{\partial\rho}{\partial z}F \tag{20b}$$

$$\frac{\partial^2}{\partial t\partial x}P=(L_3+L_4)W+\frac{1}{\rho}\frac{\partial^2}{\partial t\partial x}F-\frac{1}{\rho^2}\frac{\partial\rho}{\partial x}\frac{\partial F}{\partial t} \tag{20c}$$

$$\frac{\partial^2}{\partial t\partial z}P=(L_5+L_6)W+\frac{1}{\rho}\frac{\partial^2}{\partial t\partial z}F-\frac{1}{\rho^2}\frac{\partial\rho}{\partial z}\frac{\partial F}{\partial t} \tag{20d}$$

$$\frac{\partial^2}{\partial t^2}P=(L_1+L_2)^2U+(L_1+L_2)\frac{F}{\rho}+\frac{1}{\rho}F_t \tag{20e}$$

$$\frac{\partial^3}{\partial t^2\partial x}P=L_7(L_1+L_2)U+L_7\frac{F}{\rho}+\frac{1}{\rho}\frac{\partial^3}{\partial t^2\partial x}F-\frac{1}{\rho^2}\frac{\partial\rho}{\partial x}\frac{\partial^2 F}{\partial t^2} \tag{20f}$$

$$\frac{\partial^3}{\partial t^2\partial z}P=L_8(L_1+L_2)U+L_8\frac{F}{\rho}+\frac{1}{\rho}\frac{\partial^3}{\partial t^2\partial z}F-\frac{1}{\rho^2}\frac{\partial\rho}{\partial z}\frac{\partial^2 F}{\partial t^2} \tag{20g}$$

Where differentiator $L_3, L_4, L_5, L_6, L_7, L_8$ take the following forms:

$$L_3=\left(\frac{1}{\rho}\frac{\partial\rho}{\partial x}+\frac{\partial}{\partial x}\right)L_1 \qquad L_4=\left(\frac{1}{\rho}\frac{\partial\rho}{\partial z}+\frac{\partial}{\partial z}\right)L_2$$

$$L_5=\left(\frac{1}{\rho}\frac{\partial\rho}{\partial z}+\frac{\partial}{\partial x}\right)L_1 \qquad L_6=\left(\frac{1}{\rho}\frac{\partial\rho}{\partial x}+\frac{\partial}{\partial z}\right)L_2$$

$$L_7=\frac{\partial}{\partial x}(L_1+L_2) \qquad L_8=\frac{\partial}{\partial z}(L_1+L_2)$$

Proceed to the fourth point in the next. Since we have to determine the values of U(i,j, n+1) and W(i, j, n) with higher numerical accuracy from those of U(i, j, n), W(i, j, n) etc., we need the values of the following terms:

$$\left(\frac{\partial^2}{\partial x^2}U\right)_{i,j}^n,\left(\frac{\partial^2}{\partial z^2}U\right)_{i,j}^n,\left(\frac{\partial^2}{\partial x\partial z}U\right)_{i,j}^n$$

So, we use the following interpolation curves to determine the above terms with Taylor expansions:

$$G(X,Z)=\sum_{r=0}^{N}\frac{1}{r}\left(X\frac{\partial}{\partial x}+Z\frac{\partial}{\partial z}\right)^r U \tag{21}$$

The derivatives with respect to x and z can be written as:

$$G_x(X,Z)=\sum_{r=0}^{N-1}\frac{1}{r}\left(X\frac{\partial}{\partial x}+Z\frac{\partial}{\partial z}\right)^r\frac{\partial}{\partial x}U \tag{22a}$$

$$G_z(X,Z)=\sum_{r=0}^{N-1}\frac{1}{r}\left(X\frac{\partial}{\partial x}+Z\frac{\partial}{\partial z}\right)^r\frac{\partial}{\partial z}U \tag{22b}$$

So, we have the following connection function set to compute the above terms:

$$[G(x-\Delta x,z)]_{i,j}^n=U_{i-1,j}^n \qquad [G(x+\Delta x,z)]_{i,j}^n=U_{i+1,j}^n \tag{23a}$$

$$[G_x(x-\Delta x,z)]_{i,j}^n=\left(\frac{\partial}{\partial x}U\right)_{i-1,j}^n \qquad [G_z(x-\Delta x,z)]_{i,j}^n=\left(\frac{\partial}{\partial z}U\right)_{i-1,j}^n \tag{23b}$$

$$[G_x(x+\Delta x,z)]_{i,j}^n=\left(\frac{\partial}{\partial x}U\right)_{i+1,j}^n \qquad [G_z(x+\Delta x,z)]_{i,j}^n=\left(\frac{\partial}{\partial z}U\right)_{i+1,j}^n \tag{23c}$$

With the above relations, we can obtain:

$$\left(\frac{\partial^2}{\partial x^2}U\right)_{i,j}^n=\frac{2}{(\Delta x)^2}\left(U_{i+1,j}^n-2U_{i,j}^n+U_{i-1,j}^n\right)-\frac{1}{2\Delta x}\left(\left(\frac{\partial}{\partial x}U\right)_{i+1,j}^n-\left(\frac{\partial}{\partial x}U\right)_{i-1,j}^n\right) \tag{24a}$$

$$\left(\frac{\partial^2}{\partial z^2}U\right)_{i,j}^n=\frac{2}{(\Delta z)^2}\left(U_{i,j+1}^n-2U_{i,j}^n+U_{i,j-1}^n\right)-\frac{1}{2\Delta z}\left(\left(\frac{\partial}{\partial z}U\right)_{i,j+1}^n-\left(\frac{\partial}{\partial z}U\right)_{i,j-1}^n\right) \tag{24b}$$

$$\left(\frac{\partial^2}{\partial x\partial z}U\right)_{i,j}^n=\frac{1}{4\Delta x\Delta z}\left(U_{i+1,j+1}^n-U_{i-1,j+1}^n-U_{i+1,j-1}^n+U_{i-1,j-1}^n\right)+$$
$$\frac{1}{2\Delta x}\left(\left(\frac{\partial}{\partial z}U\right)_{i+1,j}^n-\left(\frac{\partial}{\partial z}U\right)_{i-1,j}^n\right)+\frac{1}{2\Delta x}\left(\left(\frac{\partial}{\partial z}U\right)_{i+1,j}^n-\left(\frac{\partial}{\partial z}U\right)_{i-1,j}^n\right) \tag{24c}$$

$$\left(\frac{\partial^3}{\partial x^3}U\right)_{i,j}^n=\frac{15}{2(\Delta x)^3}\left(U_{i+1,j}^n-U_{i-1,j}^n\right)-\frac{3}{2(\Delta x)^2}\left(\left(\frac{\partial}{\partial x}U\right)_{i+1,j}^n+8\left(\frac{\partial}{\partial x}U\right)_{i,j}^n+\left(\frac{\partial}{\partial x}U\right)_{i-1,j}^n\right) \tag{24d}$$

The other terms can be similarly obtained.

Meanwhile, in the computation of wave-fields at the time of the (n+1)th step, we must consider the following relation:

$$\frac{\partial^{k+l}\bar{W}}{\partial x^k\partial z^l}=\frac{\partial^{k+l+1}\bar{U}}{\partial t\partial x^k\partial z^l} \tag{25}$$

in order to compute the following term:

$$\left(\frac{\partial^{k+l}\bar{W}}{\partial x^k \partial z^l}\right)_{i,j}^n = \frac{\left(\frac{\partial^{k+l}\bar{U}}{\partial x^k \partial z^l}\right)_{i,j}^n - \left(\frac{\partial^{k+l}\bar{U}}{\partial x^k \partial z^l}\right)_{i,j}^{n-1}}{\Delta t} \tag{26}$$

So, with the initial values of U(i, j,0),we use Eq.(24a)-(24d) to compute $\frac{\partial^{k+l}\bar{U}}{\partial x^k \partial z^l}$, and $\frac{\partial^{k+l}\bar{W}}{\partial x^k \partial z^l}$ with Eq.(26).We can get $\bar{P}_{i,j}, \frac{\partial \bar{P}}{\partial t}, \frac{\partial^2 \bar{P}}{\partial t^2}, \cdots$ with Eq.(20a)-(20g),furthermore to compute the wave-fields U(i, j, n+1) with Eq.(18) and (19).

TREATMENT OF ARTIFICIAL BOUNDARY

In the computation, as the computational region is limited by the size of the computer's storage space, meaningless reflections will occur at the artificial boundaries. In this work, we use the following method to suppress reflections at the artificial boundaries:

$$\left(I\frac{\partial}{\partial t} + \rho^{-\frac{1}{2}} A^{\frac{1}{2}} \frac{\partial}{\partial x}\right) U = 0 \qquad \text{for the right boundary} \tag{27}$$

$$\left(I\frac{\partial}{\partial t} - \rho^{-\frac{1}{2}} A^{\frac{1}{2}} \frac{\partial}{\partial x}\right) U = 0 \qquad \text{for the left boundary} \tag{28}$$

$$\left(I\frac{\partial}{\partial t} + \rho^{-\frac{1}{2}} C^{\frac{1}{2}} \frac{\partial}{\partial x}\right) U = 0 \qquad \text{for the bottom boundary} \tag{29}$$

Where matrices A and C have the following forms:

$$A = \begin{pmatrix} C_{11} & C_{16} & C_{15} \\ C_{16} & C_{66} & C_{56} \\ C_{15} & C_{56} & C_{55} \end{pmatrix} \qquad C = \begin{pmatrix} C_{55} & C_{45} & C_{35} \\ C_{45} & C_{44} & C_{34} \\ C_{35} & C_{34} & C_{33} \end{pmatrix}.$$

The above absorbing conditions work well at the artificial boundaries. The interior interfaces are treated with the Forward-Backward Interation techniques to maintain the stress at the interfaces (Zhongjie et al., 1996).

COMPUTATION

One five layer model has been calculated with the above algorithm. Fig.1-3 represent Ux, Uy and Uz component seismic records, Fig.4-6 are snapshots at 400th step of the computation of the Ux, Uy and Uz components for the five layers model . We can see from the seismograms calculated that less dispersion effects were shown on the seismic records, which makes this method is much more accurate than conventional methods.

CONCLUSION

We have developed a new algorithm for numerical simulation of multi-component seismic wave-fields in heterogeneous and anisotropic media. the method is stable and precisive, and less disperse and to be easily extendible to 3D and other kind of problems in Geophysics.

The first author is grateful for the financial support of the Chinese Natural Sciences Foundation and the President's Foundation of Chinese Academy of Sciences.

REFERENCE

Alan R. Levander, 1988, Fourth order finite-difference P-SV seismograms, Geophysica, 53, 1425-1436.

Alford, R. M. , Kelly, K. R. , and Boore, D. M. , 1972, Accuracy of finite-difference modelling of the acoustic wave equation, Geophysics, 39, 834-852.

Atterman, Z., and Karal F. C., 1968, Propagation of elastic waves in layered media by finite difference methods, Bull. Seis. Am., 58, 367-398.

Backus, G. E., 1962, Long-wave elastic anisotropy in a continental layering, J. Geophy. Res., 67, 4427-4440.

Banik, N. C., 1987, An effective anisotropy parameters in transversely isotropic media, Geophysics, 52, 1654-1664.

B. E. Hornby, Schwartz L. M., and Hudson J. A., 1994, Anisotropic effective-medium modelling of the elastic properties of shales, Geophysics, 59, 1570-1583.

Booth, D. C., and Crampin, S., 1983a, The anisotropic reflectivity technique: theory, Geoph. J. Roy. Astr. Soc., 72, 755-756.

--------------------------------------, 1983b, The anisotropic reflectivity technique: anomalous arrives from an anisotropic upper mantle, Geoph. J. Roy. Astr. Soc., 72, 767-782.

Carcione, P., 1988, Wave propagation simulation in a linear-viscoelastic medium, Geoph. J. Astr. Soc., 93, 393-407.

Cerveny, V., Molotkov, I. A., and Psencik, I., 1977, Ray method in seismology, Univ. of Karlova Press.

Champan, C. H., and Shearer, P. M., 1989, Ray tracing in azimuthally anisotropic media - II. quasi-shear-wave coupling, Geophys. J. Int., 96, 65-83.

Chen, X., 1990, Seismogram synthesis for multi-layered media with irregular interfaces by the global generalized reflection/transmission matrices method--part I, theory of 2-D SH case, Bull. Seis. Soc. Am., 86, 389-405.

Chen, X., 1996, Seismogram synthesis for multi-layered media with irregular interfaces by the global generalized reflection/transmission matrices method--part III, theory of 2-D P-SV case, Bull. Seis. Soc. Am., 80, 1694-1724.

Crampin, S., Evan, R., et al., 1980, Observation of dilantancy-induced polarization anomalies and earthquake prediction: a new physical basics, Nature, 286, 874-877.

Crampin, S., and Lovell, J. H., 1991, A decade of shear-wave splitting of the Earth's crust: what does it mean? what use can we make of it? and what should we do next? , Geophys. J. Int., 107, 387-407.

Fryer, G. J., 1984, Seismic waves in stratified anisotropic media, Geoph. J. Roy. Astr. Soc., 85, 329-347.

Gajewiski, D., and Psencik, I., 1987, computation of high frequency seismic wave-fields in 3-D laterally inhomgeneous anisotropic media., Geoph. J. Roy. Astr. Soc., 91, 383-411.

Helbig, K., 1984, Transverse Isotropy in exploration seismic, Geoph. J. Roy. Astr. Soc., 76, 79-88.

Kosloff, D., and Baysal, E., 1982, Forward modelling by a fourier method, Geophysics, 47, 1402-1412.

Qin, F., and Schuster, G. T., 1993, First-arrival traveltime calculation for anisotropic media, Geophysics, 58, 1349-1358.

Reshef, M., and Kosloff, D. D., 1985, Applications of elastic forward modelling to seismic interpretation, Geophysics, 50, 1226-1272.

Schoenberg, M., and Douma, J., 1988, Elastic wave propagation in media with parallel fractures and aligned cracks, Geophysical Prospecting, 36, 571-590.

Schoenberg, M., and Muir, F., 1989, A calculus for finely layered anisotropic media, Geophysics, 54, 581-589.

Thomsen, L., 1986, Weak elastic anisotropy, Geophysics, 51, 1954-1966.

Vidale, J., 1988, Finite-difference calculation of traveltimes, Bull. seis. Soc. Am., 78, 2062-2076.

Virieux, J., and Farra, V., 1991, Ray tracing in 3D complex isotropic media: an analysis of the problem, Geophysics, 56, 2057-2069.

Teng J. W. , Zhang. Z. J., Wang, A. W., Wei, J. C., and Ji., L. X.,1992, Study of seismic anisotropy: evolution, situation and programs, Progress of Geophysics, 7, 14-28. (in Chinese)

T. Kondoh, and Y. Hosaka, 1991, Kernel Optimum Nearly-analytical Discretization (KOND) method applied to parabolic equation (KOND-P scheme), Research Rep., National Institute for fusion sciences, Nagoya, Japan, NIFS-118.

Zhang Zhongjie, Quan Youli, Chen Xiaofei, and Jerry, M. Harris, 1996, FBI for field continuity at interior interface in heterogeneous media, STP Report.

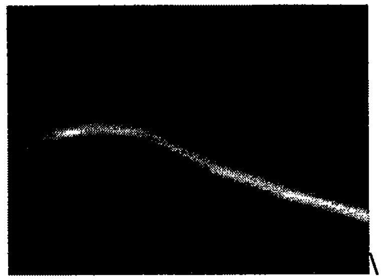

Fig. 1 VSP seismic record of y-component.

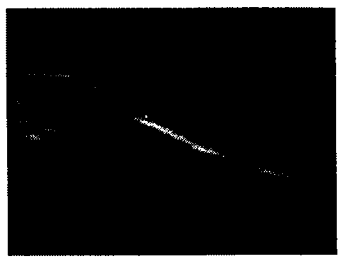

Fig. 2. VSP seismic record of x-component.

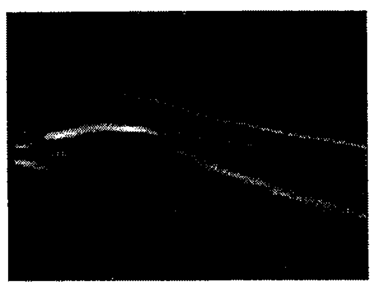

Fig. 3 VSP seismic record of z-component.

Fig. 4 Snapshot of y-component displacement at 400th step.

Fig. 5 Snapshot of x-component displacement at 400th step.

Fig. 6 Snapshot of z-component displacement at 400th step.