PAPER F

DIFFERENTIAL ACOUSTIC RESONANCE SPECTROSCOPY

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ABSTRACT

A new method of measuring the velocity and Q of sound, i.e., compressional waves, in rocks is proposed. The new method, called Differential Acoustic Resonance Spectroscopy (DARS), is based on measuring the change in resonant frequencies of a cavity perturbed by the introduction of a small sample of rock. The cavity's fundamental resonance can be used to measure acoustic velocity and Q over a broad band of frequencies, including the low frequency seismic band, e.g., 100s to 1000s of Hz. The dispersion of multiple resonances can be used to accurately estimate other properties, e.g., density, of the rock sample. Acoustic spectroscopy (ARS) is an established methodology that is used by the National Bureau of Standards to measure the velocity and Q of fluids. For rock measurements, DARS is readily adapted to high pressures and high temperatures in order to simulate in situ conditions. Moreover, DARS can be used to estimate the acoustic properties of samples of unconsolidated sediments, samples of irregular shape such as well cuttings, and small samples (low frequencies). In this paper I present the theoretical basis for the DARS measurement.

INTRODUCTION

The velocity of sound in rocks and fluids is important diagnostic information of a variety of important physical properties of materials. Applications in the Earth sciences abound from the basic sciences (e.g., development of equations of state and motion) to the commercial (e.g., exploration, development and monitoring of hydrocarbon reservoirs). A problem that has bothered the petroleum community for years is the large discrepancy in frequency used in laboratory measurements (e.g., 1 MHz) and field measurements (i.e., 10 Hz - 10 KHz). Issues of frequency dispersion are often raised but most often go unresolved.

In order to better understand frequency dispersion and to aid with the interpretation of field data, rock physicists have developed numerous theoretical and empirical models for "homogeneous" porous rocks with fluids. Many of these models have been conditioned to high frequency lab data, but may have been inappropriately used to interpret low frequency field data. Why don't geophysicists make "routine low frequency" measurements of the velocity of sound in rocks and fluids. The problem is largely due to availability of small samples only. The most popular laboratory technique is the time of flight of waves through acoustically large samples. When the sample is only a few centimeters long, this demands wavelengths of millimeters or megahertz frequencies. Low frequency measurements are possible (e.g. the resonant bar) but are more difficult, expensive, and measure fundamentally different properties (i.e., extensional wave velocity vs. compressional wave velocity). Issues of heterogeneity and sampling are also important but are not the subject of this paper. What is needed is a broadband and low frequency measurement that can be applied on dimensionally small samples of rocks.

Cavity resonators have been shown to provide a remarkably accurate means of measuring the velocity of sound in fluids (Mehl and Moldover, 1981). The idea is simple. The resonant frequency of a cavity is dependent on the velocity of sound in the contained fluid. The frequency can be measured with an accuracy of one part in a million and the velocity of sound easily determined to an accuracy better than 0.05%. Measurements with precision of 0.003% have been reported. Furthermore, the velocity is easily measured as a function temperature and pressure. The measurement is routinely made at a few kilohertz, e.g., 4 KHz, in the laboratory. This technique has been called Acoustic Resonance Spectroscopy - ARS.

In this paper, I propose a variation on ARS. First measure the resonant frequency of the fluid-filled cavity. Next, introduce a small sample of the test material, i.e., rock, into the cavity and measure the change in frequency. Through a combination of calibration and modeling, determine the velocity of the sample from the change in frequency. I call this method Differential Acoustic Resonance Spectroscopy or DARS. Accurate velocity measurements can be measure for acoustically small samples at frequencies as low as a few hundred Hertz in the laboratory, i.e., at seismic frequencies. As with ARS, the velocity can be determined at different temperatures and pressures. Perhaps more importantly, small samples with irregular shapes can be measured. In summary, DARS is capable of measuring the velocity of compressional waves of rocks under the following conditions:

(1) Low frequencies - 100's of Hertz
(2) Broad measurement bandwidth - 10,000's of Hertz
(3) narrow band measurement at a particular center frequency, e.g., 1200 Hz
(2) Small samples, also implying low frequencies
(3) Samples with irregular shapes, such as borehole cuttings

All that has been said about velocity can also be said about attenuation or Q. ARS provides an accurate measurement of the Q of the fluid, and through DARS the Q of a rock sample as well.

In this paper I present the methodology of Differential Acoustic Resonant Spectroscopy and some preliminary theoretical modeling. I reiterate that DARS is simple yet a very sophisticated method for measuring the acoustic properties of fluids and rocks. Through a combination of modeling and calibration, accurate and precise measurements of velocity and Q can be made fast and with low costs. The development of the technique a systematic development effort that will probably take 3-5 years. Nevertheless, useful and accurate results can be made during the first year of study if sufficient funding is available. This is not an STP level activity, but will require major support, by possibly a combination of government and private sources.

DIFFERENTIAL ACOUSTIC RESONANCE SPECTROSCOPY

The method is illustrated in Figures 1 and 2. The unperturbed resonant frequency f_0 of a cavity resonator is known from calibrated measurements. The enclosed fluid can be a gas or liquid, depending on the desired frequency of operation as discussed below. The cavity should be of simple shape to facilitate accurate description of the acoustic fields, but any shape can be used if properly calibrated in the laboratory. The immediately obvious shapes are the sphere and the right circular cylinder illustrated in Figure 1. The low order modes of the cavity are "long wavelength" modes. that is, they represent standing waves in the cavity over a narrow band of frequencies, e.g., high Q. In fact the Q of a gas filled cavity often exceeds 1000, so very accurate measurement of resonant frequency is easily made using conventional techniques such as electronic frequency counters. If a more accurate measurement is required, phase quadrature techniques yield frequency accuracies to 10^{-7}.

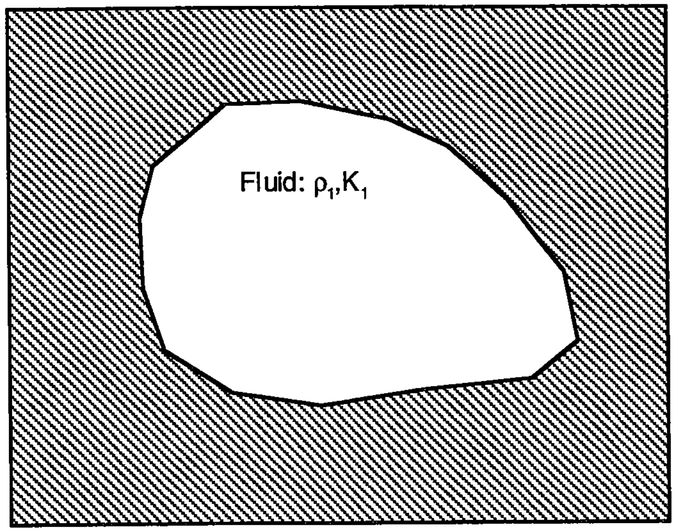

Figure 1. A closed fluid-filled cavity with rigid walls has a spectrum of high Q resonances $f_0, f_1, f_2, \ldots f_N$.

When a small sample of rock is placed in the cavity as shown in Figure 2, the resonant frequency is perturbed to $f_0 + \Delta f_0$. We'll see later that small means percentage by volume, e.g., <1%. The location of the sample is also important.

Because the sample is small compared to the wavelength, it can be thought that the sample perturbs the "average" velocity of the media, but this idea is too simplistic. We will see in the theory section how the perturbation occurs.

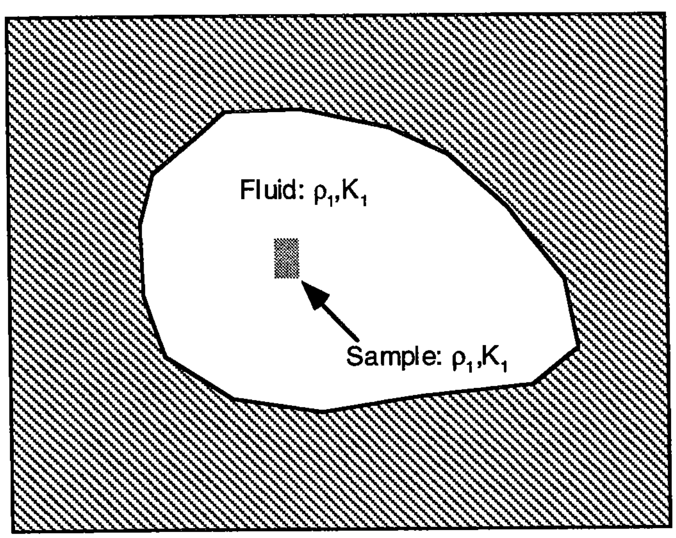

Figure 2. The fluid-filled cavity with a test sample has perturbed resonances $f_0 + \Delta f_0, f_1 + \Delta f_1, \ldots f_N + \Delta f_N$.

For example, for long and thin right circular cylinders, the resonant frequency f_0 of the first longitudinal mode is dependent simply on the length of the cylinder L and the velocity of the enclosed fluid c, i.e., $f_0 = c/2L$. The frequency can be adjusted by changing either the length or the fluid. There is a variety of well characterized liquids with velocities of the order of 1000 m/s to 1500 m/s, thus a cylinder 0.5 m long can be easily made to resonate near 1000 - 1500 Hz. If the liquid is replaced by a gas (e.g., velocities approximately 300 m/s), the same 0.5 m cavity resonates near 300 Hz. It is possible to synthesize fluids with designer velocities ranging by more than a factor of two.

AN EMPTY SPHERICAL CAVITY WITH RIGID WALLS

In this section I'll review the model for the spherical resonator. We first consider oscillations of a lossless fluid in a cavity with rigid walls without the sample. The spherical resonator with a test sample at its center may be the ideal choice for DARS. The geometry is illustrated in Figure 3. A spherical cavity of radius a, volume V_0 and inside surface area S_0. The resonator is fully filled fluid with wave velocity of c_0 The acoustic wave fields for the spherical cavity can be described analytically. This simple model for the real cavity that originally treated by Rayleigh in the 1800's. According to Bureau of Standards measurements, this simple model is sufficient for obtaining the velocity of sound in the fluid with an accuracy of 0.02%.

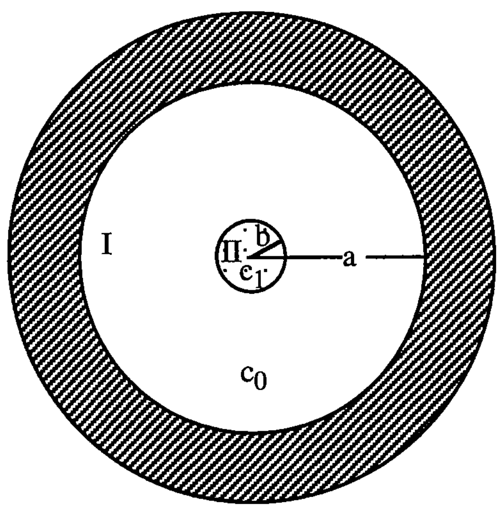

Fig. 3. A rigid spherical cavity resonator
with a spherical perturbation at its center.

The time-harmonic acoustic fields for waves may be described by the velocity potential $\varphi(r)e^{i\omega t}$, where $\varphi(r)$is a solution of the reduced wave equation

$$(\nabla^2 + \frac{\omega^2}{c_0^2})\varphi(r) = 0 \quad \text{within R.} \tag{1}$$

$$\iiint \varphi^2 dv = V\Pi \quad \frac{\partial}{\partial n}\varphi(r) = 0 \quad \text{on S} \tag{2}$$

The acoustic particle velocity $\boldsymbol{u}(r)$, pressure $p(r)$, and density ρ are related to the potential by

$$\boldsymbol{u}(r) = \nabla\varphi(r) \tag{3}$$

and

$$p(r) = -i\omega\rho\varphi(r) \tag{4}$$

In spherical coordinates the solutions of (1) take the form of spherical Bessel functions of nth order $j_l(kr)$ and spherical harmonics. For the moment we are concerned with radial modes. The radial motion *R(r)* satisfies an eigenvalue equation whose eigenvalues v_{nl} are determined by requiring that the normal component of velocity of the "l" mode vanishes at the rigid walls at $r = a$, i.e.,

$$\frac{d}{dr}j_l(kr) = 0, \quad r = a. \tag{5}$$

The eigenvalue index n is used to delineate successive roots of equation (5) for the lth radial mode. Only the $l = 0$ mode for which $j_0(kr) = \sin(kr)/kr$ is of immediate interest; therefore equation (5) reduces to

$$\tan(ka) = ka. \tag{6}$$

From the eigenvalues v_{n0}, we immediately find the eigenfrequencies $f_n = c_0 v_{n0}/(2\pi a)$. Eigenfrequencies for the first mode ($n = 1$) in a laboratory-sized cavity of different dimensions filled with water are listed in Table I below. When the cavity dimensions are known, the solution given by equation (6) for a rigid cavity is accurate enough to determine the velocity of the fluid to within 0.02% in a real and practical cavity.

a (m)	Freq. (Hz)
0.05	21,454
0.10	10,727
0.25	4,291
0.50	2,145
1.00	1,073

Table I. Eigenfrequencies of a spherical cavity of radius a filled with water (1500 m/s) for the first radial mode with eigenvalue $\nu_{10} = 4.493409$.

A RIGID CAVITY CONTAINING A SMALL ROCK SAMPLE

The effect of a small sample of rock or other material inside the cavity can be analyzed by considering scattering by the object. The acoustic properties of the sample are expressed in terms of its density $\delta\rho$ and compressibility $\delta\kappa$ relative to the fluid. The sample occupies a volume V_1. In the absence of the sample, the solutions of the wave equation for the eigenfunctions φ_l and eigenvalues ν_{nl} are given above.

If we assume that the sample volume is small compared to the cavity volume, i.e., $V_1 \ll V$, a pair of integral equations for the new eigenfunctions ψ_l and eigenvalues η_{nl} can be found in terms of the unperturbed eigenfunctions φ_l and eigenvalues ν_{nl} using the calculus of variations:

$$\eta_{nl}^2 - \nu_{nl}^2 \approx \frac{A}{V\Pi} \iiint_V k^2 \delta\kappa \varphi_l^2 + \delta\rho |\nabla \varphi_l|^2 dr. \tag{7}$$

$$\psi_l(r) \approx \varphi_l(r) + A \sum_{k \neq l} \frac{\iiint_V k^2 \delta\kappa \varphi_l \varphi_k + \delta\rho \nabla \varphi_l \nabla \varphi_k dr}{V\Pi(\eta_k^2 - \eta_l^2)} \varphi_k(r). \tag{8}$$

where $A\varphi_l$ is the modified wavefield inside the rock sample. The eigenvalue estimation can be further simplified when the sample is very small and by writing equation (7) in terms of the velocity:

$$\eta_{nl}^2 \approx v_{nl}^2 - \frac{k^2V_1}{V\Pi}\langle \rho^2 c_0^2 u_l^2 \rangle \delta\rho - \frac{k^2V_1}{V\Pi}\langle \varphi_l^2 \rangle \delta\kappa, \quad V_1 << \lambda^3, \tag{9}$$

where $u_l = (1/k\rho c_0)\nabla\varphi_l$ is the velocity and $\langle \alpha^2 \rangle = (1/V_1)\iiint_{V_1} \alpha^2 dr$ is the RMS field (velocity and pressure) inside the sample. Equation (9) is value for small samples whose maximum linear dimension is less than about one-third wavelength.

DISCUSSION

From equation (9), we see that the change in the eigenvectors and eigenfrequencies is proportional to the ratio of the volume of the sample to the volume of the cavity, (V_1 / V. A part of the change is proportional to the perturbation in the density $\delta\rho = (\rho_1 - \rho) / \rho_1$ times the mean-square velocity of the unperturbed wavefield. A second part of the change is proportional to the perturbation in the compressibility $\delta\kappa = (\kappa_1 - \kappa) / \kappa$ times the mean square of the pressure at the sample.

According to equation (9), if the rock sample is massive but incompressible $(\rho_1 > \rho, \kappa_1 < \kappa)$, the first term in (9) decreases the frequency because of the sample's immobility causing the fluid near it to move faster, thus increasing the effective mass of the standing wave. The second term in (9) increases the frequency of resonance because the sample is stiffer than the displaced fluid and increases the effective stiffness of the wave.

Furthermore, we see that the change in frequency depends on the location of the sample. If placed near a pressure minimum where the velocity is high (i.e., $\rho cu > \varphi$) the frequency generally decreases. If the sample is placed near a pressure maximum, the frequency increases. This situation is similar to the electromagnetic case where the effect of conducting object depends on the magnitude of the electric or magnetic fields at the location on of the object.

The change in frequency depends on the location of the sample relative to the maxs and mins of the pressure and velocity fields. Therefore, the sample will disperse the frequencies of different modes because each mode has a different spatial structure. This multi-mode dispersion will be useful for estimating multiple properties of the sample, such as density and compressibility simultaneously.

Finnaly I point out that the model developed here can be applied to porous media or attenuating media as well. In principle this provides a method for estimating the permeability of the sample.

THE GENERAL ELASTIC RESONATOR WITH A SMALL PERTURBATION

For the general resonator with an elastic sample of arbitrary shape we may apply the Boltzmann-Ehrenfest adiabatic invariant theorem which states that if the state of any oscillating system is changed adiabatically, the ratio of the time-average stored energy $\overline{E}$ to the resonant frequency ω remains invariant, i.e.,

$$\frac{\overline{E}}{\omega} = \text{const.} \tag{10}$$

An adiabatic change is defined as a perturbation which does not cause a change from the unperturbed mode to an adjacent mode. Usually this means that the sample (perturbation) is small. When a sample is placed in a cavity the energy change is $\delta\overline{E}$, and the resonant frequency shift $\delta\omega$ is related to $\delta\overline{E}$:

$$\frac{\delta\omega}{\omega_o} = \frac{\delta\overline{E}}{\overline{E}_o}. \tag{11}$$

The application of this theorem to an electromagnetic cavity resonator is discussed by Johnson (1965). In acoustics, the energy is

$$E = T - U, \tag{12}$$

where T is the kinetic energy and U is the potential energy. We may express T as

$$T = \frac{1}{2}\iiint_V (\rho \dot{u}_i \dot{u}_i) dV, \tag{13}$$

and U as

$$U = \frac{1}{2}\iiint_V (\sigma_{ij}\varepsilon_{ij}) dV, \tag{14}$$

where u_i, σ_{ij} and ε_{ij} are displacement, stress and strain components, respectively. We may use the boundary integral equation

$$u_i(\mathbf{r}) = u_i^o(\mathbf{r}) + \iint_{S'} [u_j(\mathbf{r}') \frac{\partial}{\partial n} G_{ij}(\mathbf{r}, \mathbf{r}') - G_{ij}(\mathbf{r}, \mathbf{r}') \frac{\partial}{\partial n} u_j(\mathbf{r}')] dS' \tag{15}$$

to find the wave field. Here, G_{ij} is the Green's function, S' is the surface of the sample, and u_i^o is the unperturbed field. In general we need to use numerical techniques to solve equation (15). After finding the displacement u_i we may calculate the strain ε_{ij} and stress σ_{ij} in Cartesian coordinates from the equation

$$\varepsilon_{ij} = \frac{1}{2}(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}) \tag{16}$$

and

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}. \tag{17}$$

Averaging the energy obtained from equations (12)–(14) over a period T we have

$$\overline{E} = \frac{1}{T} \int_0^T E(t) dt. \tag{18}$$

We need to calculate the energy for the unperturbed and the perturbed wave fields in order to find the change $\delta \overline{E}$. Then from equation (2) we get

$$\delta\omega = \omega_o \delta \overline{E} / \overline{E}_o. \tag{19}$$

The change in the resonant frequency, $\delta\omega$, is a function of elastic parameters λ, μ, ρ. By forward simulation we can understand how the elastic parameters affect the frequency.

SUMMARY

I have presented the general idea of Differential Acoustic Resonance Spectroscopy for measuring the acoustical properties of rocks at low frequencies. The theory indicates that DARS can be an accurate and precise technique when calibrated. Such calibration is required of all existing techniques as well, e.g. time of flight measurements. A considerable amount of additional theoretical work and simulation is required.

Preliminary laboratory results (not reported here) indicate that the frequency shift is easily observed in the laboratory.

ACKNOWLEDGMENTS

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