PAPER D

MULTIRESOLUTION SHAPING FILTERS WITH APPLICATION TO GEOSTATISTICS

Youli Quan and An-Ping Yang*

ABSTRACT

A shaping filter is an inverse convolution operator that modifies the waveform of an input signal to a desired waveform under the least-squares sense. The wavelet transform is used to design a set of multiresolution filters to shape a waveform more efficiently. We first decompose a signal to different resolutions using wavelet transform. Then, the least-squares filtering technique is applied to design shaping filters for each resolution in the wavelet domain. Since the signal is processed independently under different resolutions, the multiresolution inverse filtering is more powerful than the regular shaping filtering. We apply the multiresolution shaping filters for the integration of seismic data and well logs in geostatistics. In this application, we introduce the concept of "transfer function." The transfer function transforms the seismic data to petrophysical properties (e.g., porosity and permeability) based on log data, that is, the well log data is treated as the desired signal, and we shape or calibrate the waveform of seismic traces to the well logs. A field data example is presented, which shows that the transformation of seismic data can significantly increase the resolution of the cokriging porosity imaging.

---

* E&P Technology Department, Texaco, Houston

INTRODUCTION

A shaping filter converts the waveform of a signal into a desired shape. The shaping filter has many appllications in signal processing (Oppenheim, 1978). It has been used for seismic data deconvolution (Yilmaz, 1987). The integration of 3-D seismic data with well log data is an important topic in geostatistics (e.g., Journel et al. 1992; Doyen, 1988; Haas and Dubrule, 1994). In order to obtain the best integration, we need to find a relationship that converts a seismic trace to petrophysical parameters (e.g., porosity and permeability). We propose to use the shaping filter for this conversion. The conventional shaping filter directly convert the original signal, which is not powerful enough for our purpose. Therefore, we introduce a more efficient and powerful technique, multiresolution shaping filtering, to do this conversion. We first use the wavelet transform (Daubechies, 1992; Mallat, 1989) to decompose a seismic trace and the desired signal (e.g., porosity log, impedance log, or permeability log) into different resolutions. Then, we design shaping filters in the wavelet transform domain for each resolution. Since the signal is converted at each resolution, the conversion is more meaningful and the shaping effects are more significant. The conventional shaping filter can be viewed as a single resolution shaping filter.

In this report, we first introduce the concepts of the single resolution shaping filter (i.e., the conventional shaping filter), then introduce the concepts of wavelet transform and generalize the single resolution shaping filter to the multiresolution shaping filters. As an application, we use the multiresolution shaping filters to calibrate the seismic data to well log data for geostatistics.

SHAPING FILTER

The conventional shaping filter directly converts a signal into a desired signal, without decomposing these signals into various resolutions. Figure 1 shows the filter model of this type. Let $x(t)$ be an input signal, for example, a seismic trace, and $d(t)$ a desired signal, for example, a well log trace. The output signal of this system is $y(t)$. Our purpose is to design a filter $f(t)$ that makes the least-square error between the actual output $y(t)$ and the desired output $d(t)$ to be minimum, that is, to convert the input signal $x(t)$ to be desired signal $d(t)$ under the least-square criterion. The residual error L is defined as

$$L = \sum_n (d_n - y_n)^2 . \tag{1}$$

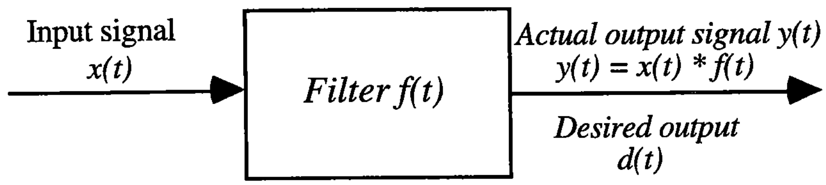

Figure 1. A shaping filter model.

The actual output *y(t)* can be written as

$$y_n = \sum_k x_k f_{n-k} . \tag{2}$$

Therefore, equation (1) becomes

$$L = \sum_n (d_n - \sum_k x_k f_{n-k})^2 . \tag{3}$$

To find the filter coefficients $(f_o, f_1, \dots, f_{N-1})$ that gives the minimum *L*, we set

$$\partial L / \partial f_i = 0, \qquad i = 0, 1, \dots, N\text{-}1 \tag{4}$$

and obtain a system of equations (Yilmaz, 1987)

$$\begin{bmatrix} r_o & r_1 & r_2 & \cdots & r_{N-1} \\ r_1 & r_o & r_1 & \cdots & r_{N-2} \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ r_{N-1} & r_{N-2} & r_{N-3} & \cdots & r_o \end{bmatrix} \begin{bmatrix} f_o \\ f_1 \\ . \\ . \\ . \\ f_{N-1} \end{bmatrix} = \begin{bmatrix} g_o \\ g_1 \\ . \\ . \\ . \\ g_{N-1} \end{bmatrix} \tag{5}$$

for the shaping filter $(f_o, f_1, \dots, f_{N-1})$. Here,

$$r_{i-k} = \sum_l x_{l-k} x_{l-i} , \tag{6a}$$

and

$$g_i = \sum_l d_l x_{l-i} \quad . \tag{6b}$$

Solving equation (5) gives the shaping filter *f(t)*= $(f_0, f_1, ..., f_{N-1})$. In general, a larger N gives a smaller residual error L.

MULTIRESOLUTION DECOMPOSITION OF A SIGNAL USING WAVELET TRANSFORM

A signal can be expressed as a sum of basis functions with different resolutions. Such a multiresolution decomposition is meaningful, because each resolution corresponds to certain information in the signal. The goal of the multiresolution shaping filtering is to analysize and shape a signal at each resolution. The multiresolution decomposition of a signal can be done by using the wavelet transform (Mallat, 1989; Daubechies, 1992).

A wavelet is a function $\psi(x)$ whose translations and dilations can used for the expansion of a signal. The translation and dilation operations are defined as

$$\psi_{ab}(x) = \frac{1}{\sqrt{a}} \psi(\frac{x-b}{a}). \tag{7}$$

Here, parameter b represents translation, and parameter a is for dilation. The factor $1/\sqrt{a}$ in equation (7) ensures the norm of $\psi(x)$ remain unchanged during the dilation operation. Discretization of parameters a and b such that $a = 2^j$ and $b = 2^j k$ gives

$$\psi_{jk}(x) = \frac{1}{\sqrt{2^j}} \psi(\frac{x}{2^j} - k), \tag{8}$$

where integer j is the dilation (or resolution) index, and k is the translation index. $\psi_{jk}(x)$ forms a basis for function expansions. It has been shown (Mallat, 1989) that one can interpret the decomposition of a signal in orthonormal wavelet basis as multiresolution decomposition of this signal.

The decomposition can be efficiently calculated with a pyramidal algorithm of complexity *O(N)*, i.e., the computation is linearly proportional to the data points *N*. Figure 2 shows the implementation of this wavelet decomposition. The numerical equivalent of this decomposition process is the successive, discrete convolution of the signal with digital filters $\tilde{H}$ (low-pass) and $\tilde{G}$ (high-pass), followed by subsampling at every other data point. Here, $\tilde{H}$ and $\tilde{G}$ are the reflected version of filters H and G, i.e., $\tilde{h}_k = h_{-k}$ and $\tilde{g}_k = g_{-k}$, where, h, g, $\tilde{h}$, and $\tilde{g}$ are the impulse responses of H , G, $\tilde{H}$, and, $\tilde{G}$, respectively. Figure 3 shows an example of filters H and G in time domain and frequency domain.

The wavelet representation is complete and it is possible to reconstruct the original signal S_o from its wavelet decomposition. The reconstruction can be computed with a similar algorithm based on the convolution with the digital filters H (low-pass) and G (high-pass), which is illustrated in Figure 4. At each stage, we recombine the decomposed signal S_j and D_j by adding one zero in between two samples, convolving the resulting signals respectively with H and G, and summing the results.

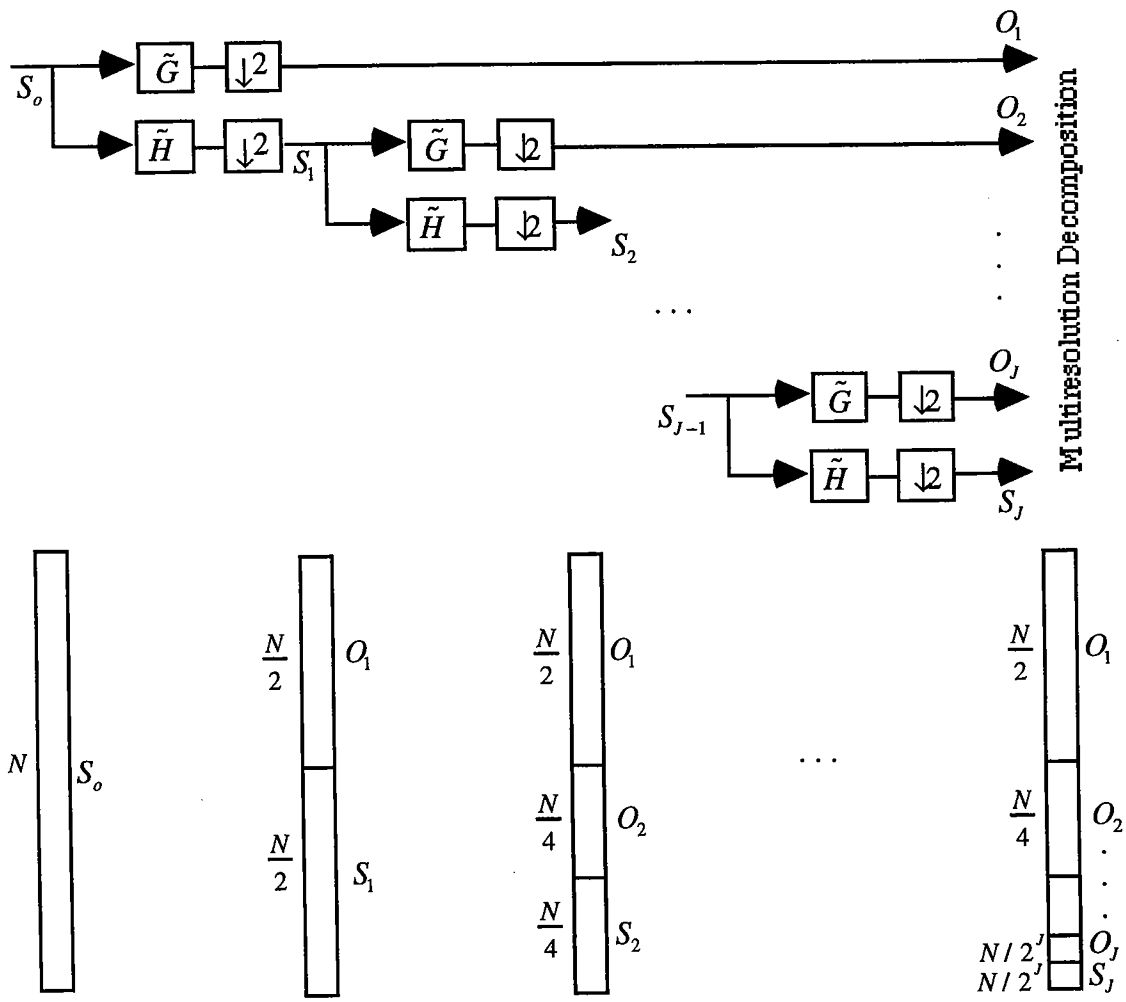

Figure 2. Pyramidal architecture for the multiresolution decomposition of a 1-D signal S_o of N points long. Symbol $\downarrow 2$ means down-sampling at each other data point. The original signal S_o is input to a high-pass filter $\tilde{G}$ and a low-pass filter $\tilde{H}$, and the outputs are subsampled by 2. The resulting O_1 of $N/2$ points long is a final output with highest resolution. The output S_1 of $N/2$ points long is an intermediate result which may be further decomposed to be O_2 and S_2 of $N/4$ points long. This procedure can be stooped at step.

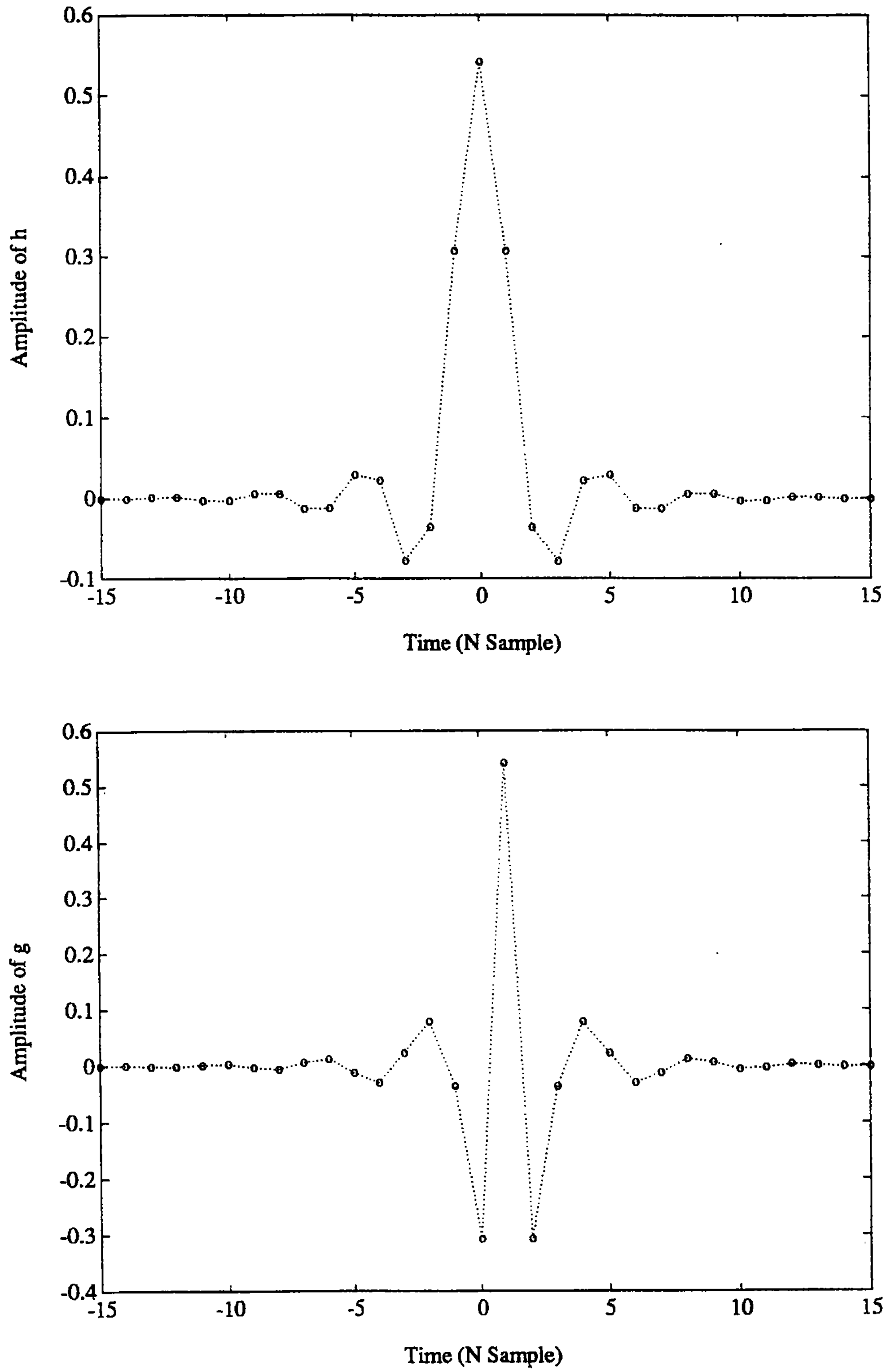

Figure 3a. The impulse response of filters *h* and *g*. This is an example of wavelet.

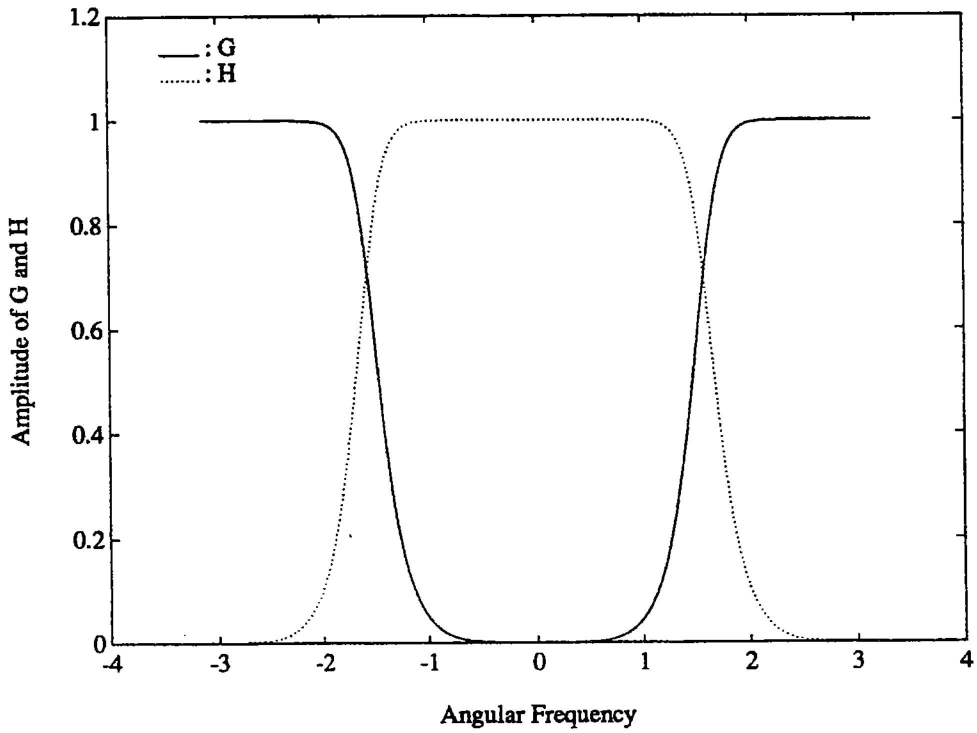

Figure 3b. The frequency response of filters H and G.

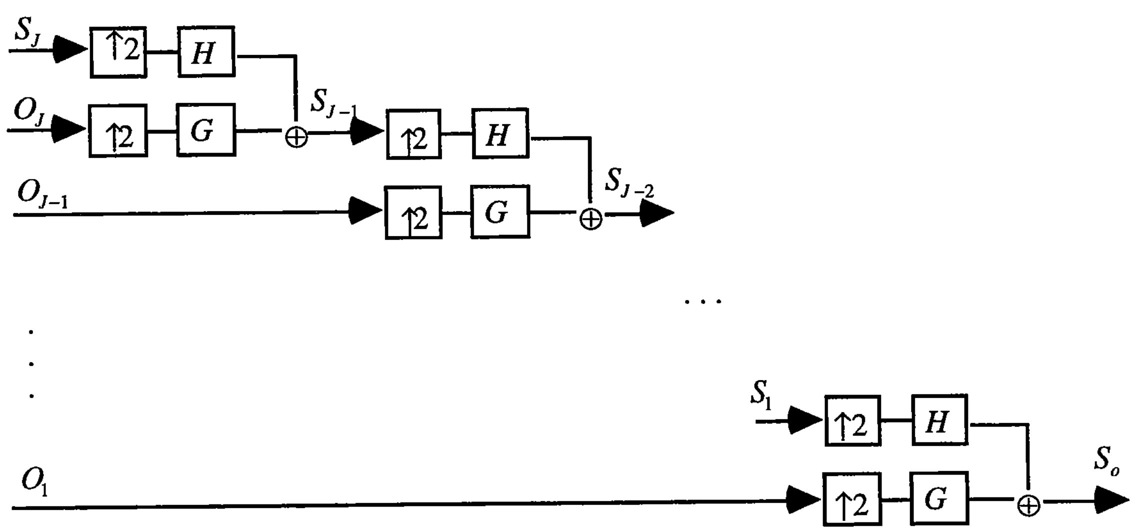

Figure 4. The reconstruction of a signal S_o from its multiresolution decomposition (see Figure 2 for the forward process). Symbol ↑2 means up-sampling by 2.

MULTIRESOLUTION SHAPING FILTERS

After we cut up a signal into different resolutions or frequency components, we can investigate the signal at each resolution. Since different resolutions correspond to different details of a signal, it is useful to process a signal at different resolutions. We propose to convert an input signal to a desired signal at each resolution, that is, to perform multiresolution shaping filtering.

Let $X=(O_1^{(x)},...,O_J^{(x)},S_J^{(x)})$, $Y=(O_1^{(y)},...,O_J^{(y)},S_J^{(y)})$, and $D=(O_1^{(d)},...,O_J^{(d)},S_J^{(d)})$ (see Figure 2) be the wavelet multiresolution representations of the input signal $x(t)$, the actual output signal $y(t)$, and the desired signal $d(t)$ (see Figure 1), respectively. We want to find a series of filters, $F=(F_{O_1},F_{O_1},...,F_{O_J},F_{S_J})$, which convert

$$O_j^{(x)} \rightarrow O_j^{(d)}, \qquad \text{for } j=1, ..., J, \tag{9a}$$

$$S_J^{(x)} \rightarrow S_J^{(d)}. \tag{9b}$$

The actual outputs are

$$O_j^{(y)} = O_j^{(x)} * F_{O_j}, \qquad \text{for } j=1, ..., J, \tag{10a}$$

$$S_J^{(y)} = S_J^{(x)} * F_{S_J}, \tag{10b}$$

where, sign "$*$" means convolution. The design of the shaping filters F is to make the least-square error between the actual output Y and the desired output D to be minimum. Replacing the shaping filter f, the input signal x, and the desired signal d in equations 5 and 6 by F_{O_j} / F_{S_J}, $O_j^{(x)}/S_J^{(x)}$, and $O_j^{(d)}/S_J^{(d)}$, respectively, we can obtain the shaping filter for each resolution. The implementation of the multiresolution shaping filtering can be summarized as:

(a) decompose the original signal $x(t)$ into $(O_1^{(x)},...,O_J^{(x)},S_J^{(x)})$;

(b) decompose the desired signal d(t) into $(O_1^{(D)},...,O_J^{(D)},S_J^{(D)})$;

(c) repeatedly use equations (5) and (6) to calculate the shaping filters $(F_{O_1},F_{O_1},...,F_{O_J},F_{S_J})$ for all resolutions;

(d) convert the multiresolution representation of the original input signal $(O_1^{(x)},...,O_J^{(x)},S_J^{(x)})$ to be $(O_1^{(y)},...,O_J^{(y)},S_J^{(y)})$ using equation (10);

(e) reconstruct multiresolution representation of the shaped signal $(O_1^{(y)},...,O_J^{(y)},S_J^{(y)})$ to get $y(t)$.

APPLICATION TO GEOSTATISTICS

One of the strengths of geostatistics is that it can quantitatively combine different types of data. An example is the integration of seismic data and well log data for geostatistics. Seismic data are considered as soft data which are usually massive but not very accurate. On the other hand, well log data are hard data. Compared to seismic, there are much fewer log data, but they are substantially more accurate. In geostatistics, these two types of data can be integrated using the method of cokriging.

In order to increase the crosscorrelation between seismic data and well log data to make the seismic data more suitable for the integration, we need to convert the seismic data to rock properties represented by the well logs. The conventional way for this conversion is linear transform (e.g., Doyen and Guidish, 1992). We introduce the concept of calibration of seismic traces to well log traces illustrated in Figure 5. Assume that the surface seismic data are zero-incident and the travel time has been correctly converted into depth already. At a well drilled within the seismic survey, the seismic trace at this well and the well log go through the same formation. Therefore, the seismic trace and well log trace should exhibit certain correlation. We empirically determine a relationship or calibrator that converts the seismic trace to petrophysical properties based on the well logs. Having obtained the calibrator, we convert other seismic traces off the wells using this calibrator.

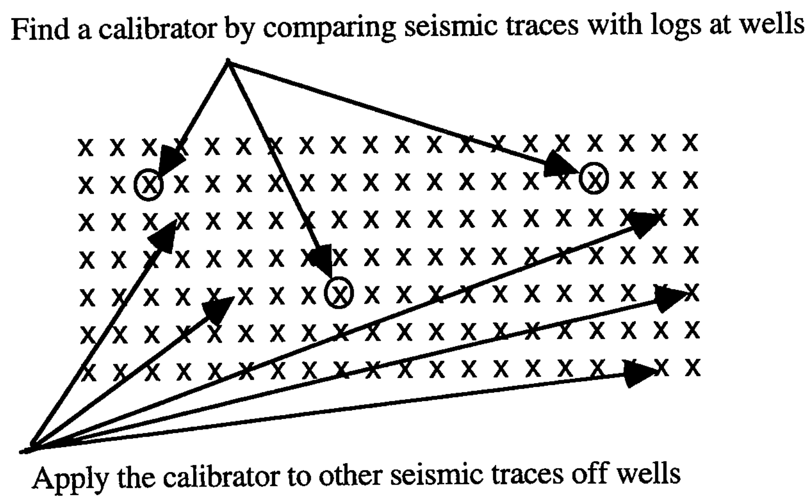

Figure 5. A map view of 3-D seismic survey. Seismic traces "X" and well logs "O" are used for geostatistics.

In this application, the original signal is the seismic trace at a well, and the desired signal is the well log. The resulting shaping filters are the calibrator. Before applying this technique to field data, let us first examine a synthetic example. We use a *P* wave impedance log to calculate a synthetic seismic trace. Then, we use seismic trace as the input signal, and the impedance log as the desired signal for the multiresolution shaping test. in Figure 6, Curve A is the original impedance log, Curve C is the synthetic seismic trace, and Curve B is the "pseudo impedance log" transformed or shaped from Curve C based on Curve A. In this shaping test, Curves A and B are decomposed into 5 resolutions, respectively. The shaping filtering is performed to each resolution. Both Curves A and C are 1024 points long. From the highest resolution to the lowest resolution, the lengths of the shaping filters are 15, 8, 5, 3, and 2 points, respectively. The correlation coefficient between A and C is 0.01. After shaping, the correlation coefficient between A and B increases to 0.80. For comparison, Figure 7 shows the result using the conventional shaping filter. The correlation coefficient between A and B in Figure 7 is 0.42. The length of this conventional shaping filter is 75 points.

Figures 8-10 are a field data example. Porosity logs from two wells are used as "hard data" for geostatistics to produce a porosity cross section between these two wells. Seismic traces between these two wells are selected from a 3-D seismic survey as "soft data" for the geostatistics. Figure 8 shows the two porosity logs (Curves A_1 and A_2), and two seismic traces (Curves C_1 and C_2) collected at these two wells. We simultaneously convert C_1 into A_1, and C_2 into A_2 to calculate a set of multiresolution shaping filters (5 resolutions; from the highest resolution to the lowest resolution, the filter lengths are 20, 11, 6, 4, 3). Curves B_1 and B_2 are the "pseudo porosity logs" converted from seismic traces C_1 and C_2 using the same shaping filters. Curves B_1 and B_2 exhibit more high frequency components than the original seismic traces C_1 and C_2. Certain features in the porosity logs are transferred to Curves B_1 and B_2. Having obtained the multiresolution shaping filters, we use them as the calibrator to convert other seismic traces between wells. Figure 9 shows the cokriging porosity cross section using a linear transform to convert the seismic trace to porosity. Figure 10 is the same cross section, but uses the multiresolution shaping technique. Comparing Figures 9 and 10, we find that the calibration based on the multiresolution shaping filtering can significantly increase the resolution of the cross section, which enables us to see more details of the formation structures.

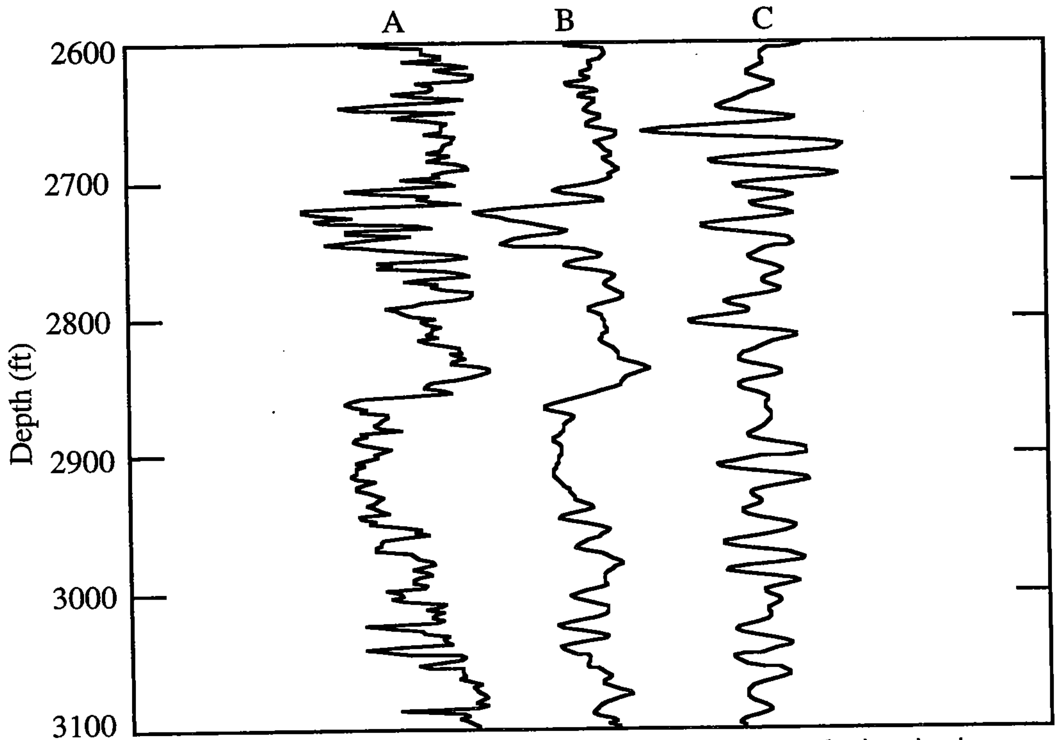

Figure 6. Multiresolution shaping filters are used to convert synthetic seismic trace, Curve C, into Curve B based on the impedance log, Curve A.

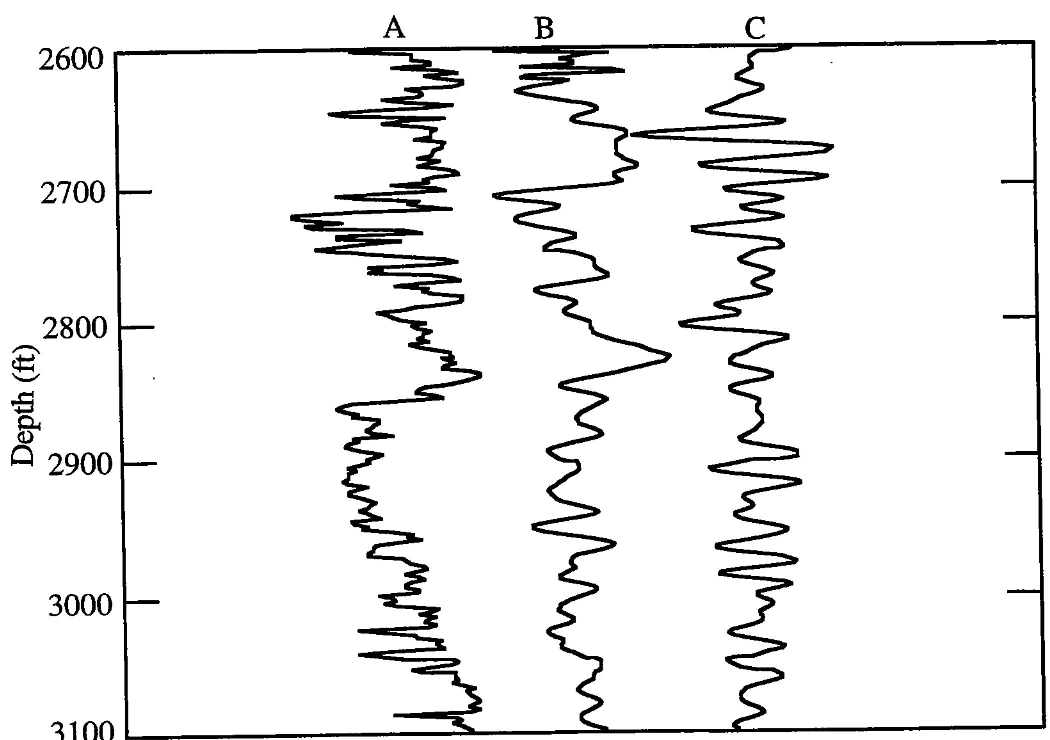

Figure 7. A conventional shaping filter is used to convert synthetic seismic trace, Curve C, into Curve B based on the impedance log, Curve A.

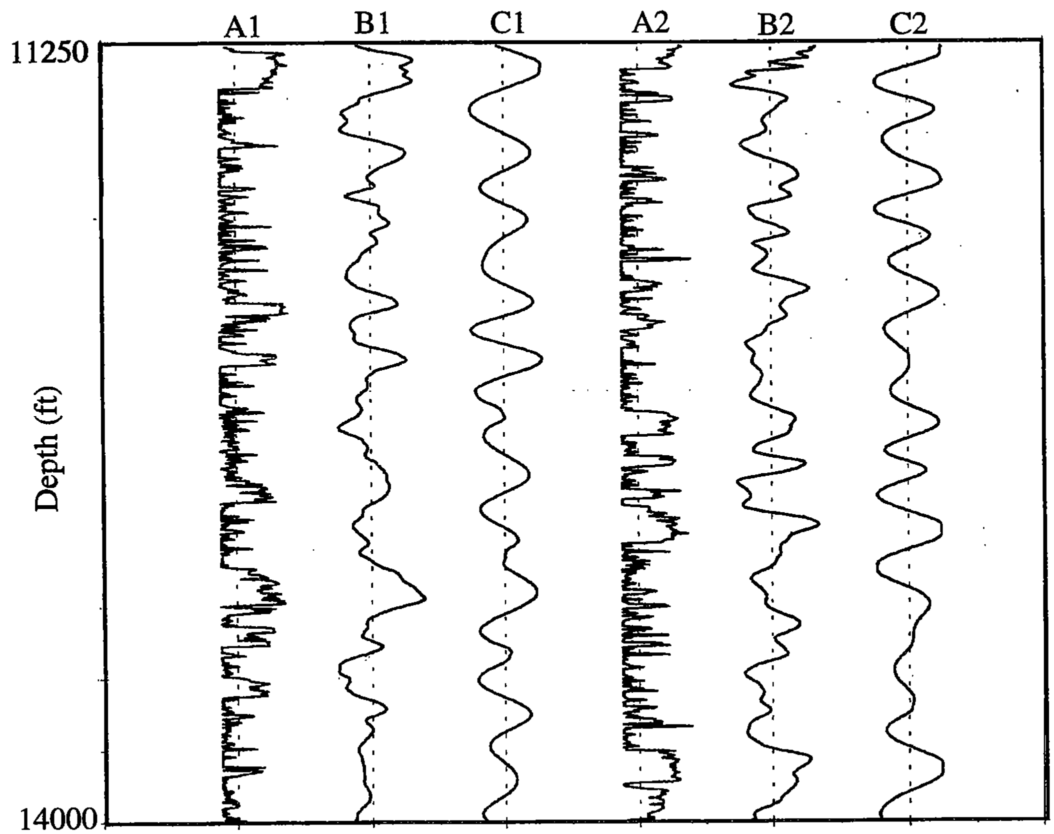

Figure 8. Multiresolution shaping filtering example of field data. Seismic traces, C_1 and C_2 are converted into B_1 and B_2 based on the porosity logs, A_1 and A_2.

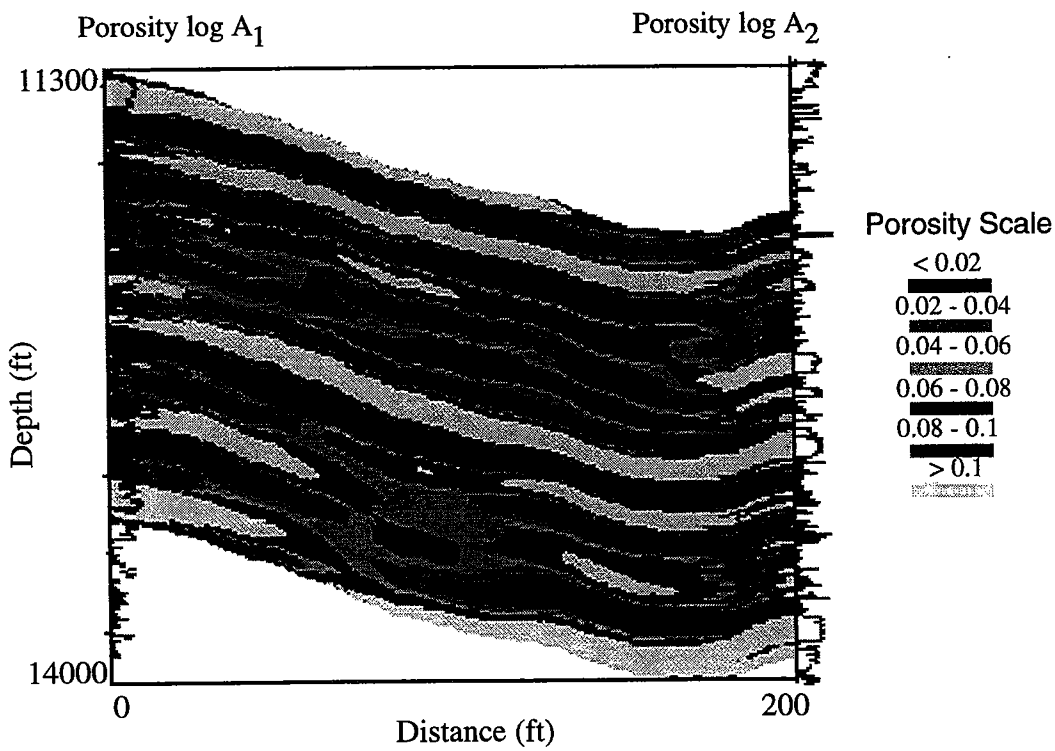

Figure 9. Porosity cross section from integration of porosity logs and seismic data. Porosity logs are plotted at two ends of this cross section. The seismic amplitude is converted into porosity using the conventional approach – linear transform.

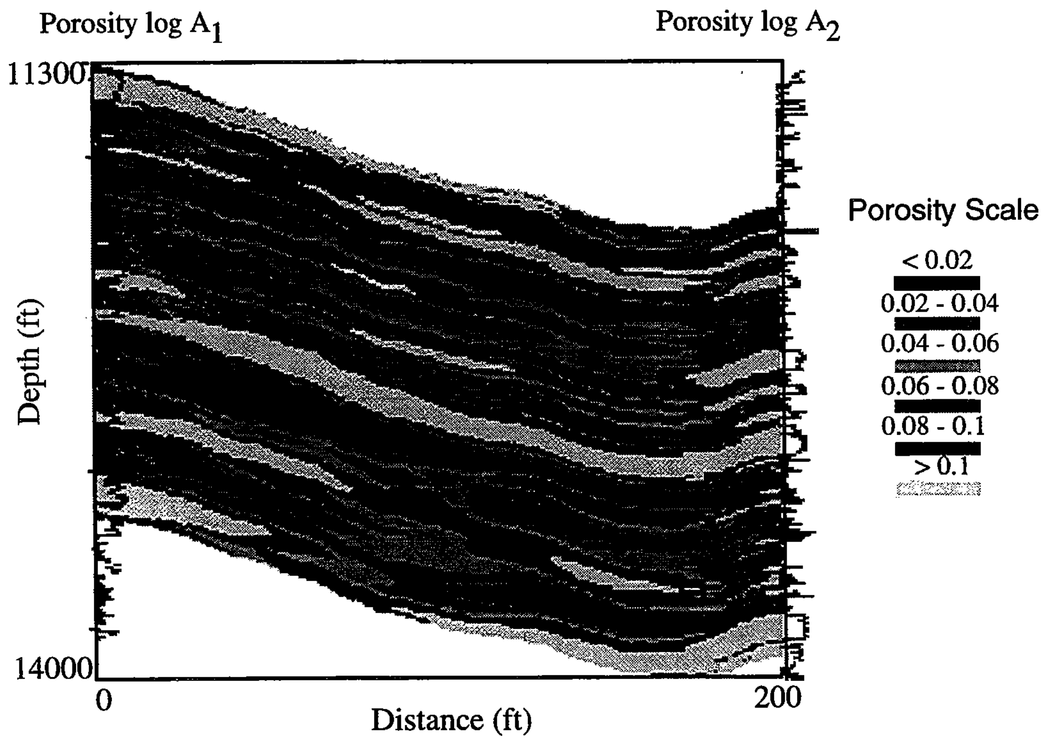

Figure 10. For this porosity cross section, the seismic amplitude is converted into porosity using a transfer function – the multiresolution shaping filters. This transfer function may more properly describe the relationship between the well logs and seismic data than the conventional linear transform. This cross section shows higher resolution than Figure 9.

CONCLUSIONS

When a signal contains details of different resolutions or scales, it is more meaningful to decompose it to different resolutions for signal processing. We use the wavelet transform to decompose a signal, and design multiresolution shaping filters in wavelet domain. The conventional shaping filter is a signal resolution filter. We have successfully applied this multiresolution filtering technique to integrate the well logs and 3-D seismic data for geostatistics. In this application, the seismic traces are empirically converted to well logs by multiresolution shaping filters. We need to further investigate the physical insight of this conversion.

REFERENCES

Daubechies, I., 1992, Ten lectures on wavelets: SIAM, Capital City Press.

Doyen, P., 1988, Porosity from seismic data: A geostatistical approach: Geophysics, 53, No 10.

Doyen, P. M. and Guidish, T. M., 1992, Seismic discrimination of lithology, a Monte Carlo approach: Reservoir Geophysics, Investigations in Geophysics, No 7, R. E. Sheriff (Ed.) 243 – 250, SEG, Tulsa, OK.

Haas, A. and Dubrule, O., 1994, Geostatistical inversion - a sequential method of stochastic reservoir modeling constrained by seismic data: First Break, **12,** No 11.

Journel, A. G., Xu, W. and Tran, T., 1992, Integrating seismic data in reservoir modeling: the collocated cokriging alternative: SCRF Report **5**, Stanford University, California.

Mallat, S. G., 1989, A theory for multiresolution signal decomposition, the wavelet representation: IEEE Transactions PAMI, **11**, 674-693

Oppenheim, A., Ed., 1978, Applications of Digital Signal Processing: Prentice-Hall.

Yilmaz, O., 1987, Seismic Data Processing: Society of Exploration Geophysicists.