# *TRAVELTIME EQUATIONS IN CROSSWELL SEISMIC PROFILING*

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## *ABSTRACT*

The traveltime equations for direct arrival, primary reflections, and multiple reflections are derived. When the depth of the source approaches the depth of the reflector, the traveltimes of the direct arrival and primary reflection coalesce into each other. Transmission multiple reflections are equivalent to primary reflections, whose virtual depths have a linear relationship with the source depth.

## *INTRODUCTION*

In this paper, we first derive the traveltime equations for direct arrival and primary reflections in cross-well seismic profiling. Then we derive the traveltime equations for multiple reflections.

Finally, we analyse the kinematic relationships of these different arrivals, and their effects in seismic imaging.

## *DIRECT ARRIVAL*

In the following study of deriving traveltime equations for cross-well seismic survey profiling, I assume that the two boreholes are straightly vertical and the lateral separation distance is $L$. A single source is placed in one borehole, the legend and the depth of the source are denoted by $S$. A series of receivers are placed in another borehole, the legend and the depth of a receiver are denoted by $R$. I further assume the interwell medium has a constant velocity $v$.

The ray path linking the source $S$ and the receiver $R$ is straight, Figure 1. The traveltime for the direct arrival is:

$$t = \frac{1}{v}\sqrt{(S-R)^2 + L^2} \qquad (1)$$

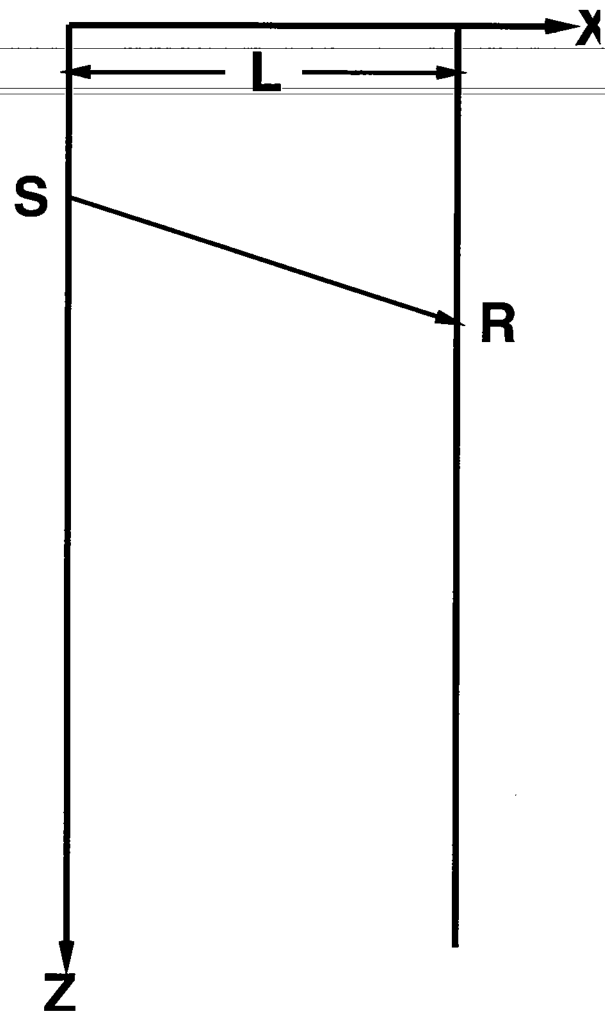


Figure 1: Direct arrival

***PRIMARY REFLECTIONS***

There are two cases of primary reflections, i.e., upgoing primary reflection when both the source and the receiver are above the reflector (Figure 2), and downgoing primary reflection when both the source and the receiver are below the reflector (Figure 3). When the reflector is flat at a depth of $H$, the image source is in the source borehole. In both cases the depth of the image source $S^*$ could be described by $H+(H-S)=2H-S$. Employing the traveltime equation of direct arrival, the traveltime equation for the primary reflection is:

$$t = \frac{1}{v}\sqrt{(2H-S-R)^2 + L^2}, \text{ for } S<H, R<H; \text{ or } S>H, R>H \qquad (2)$$

For a common shot profiling with S as a constant, it is hyperbola with the apex at H=2H-S. For R<H or R>H , the traveltime equation represents only a far limb of the hyperbola.

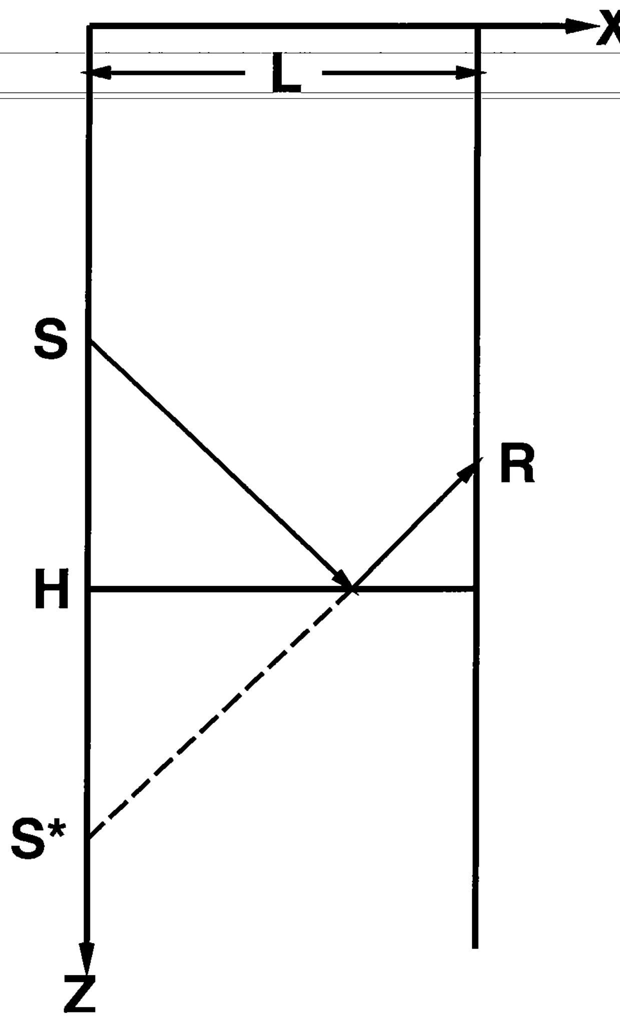


Figure 2: Primary upgoing reflection.

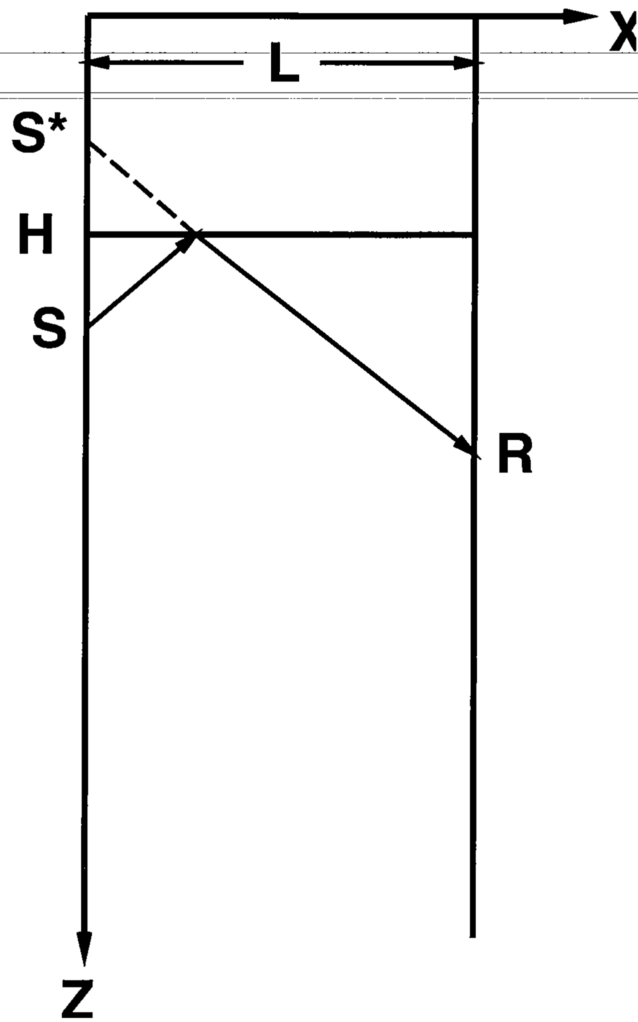


Figure 3: Primary downgoing reflection.

## *DOWNGOING (TRANSMISSION) MULTIPLE REFLECTIONS*

First order transmission multiple reflection

The depth of the upper reflector is $H_1$, that of the lower reflector $H_2$, $H_2 > H_1$. For first order transmission multiple reflection, there are two cases, i.e., the receiver is located between the two reflectors or the receiver is located below the lower reflector.

When the receiver is located between the two reflectors (Figure 4), the depth of the equivalent primary reflector is $H_2 + (H_2 - H_1) = 2H_2 - H_1$. The depth of the image receiver $R*$ is $2H_2 - H_1 - (R - H_1) = 2H_2 - R$. Employing the traveltime equation of primary reflection, the traveltime equation for the first order transmission multiple reflection is:

$$t = \frac{1}{v}\sqrt{\left(2(2H_2 - H_1) - S - (2H_2 - R)\right)^2 + L^2}$$

$$= \frac{1}{v}\sqrt{\left(2(H_2 - H_1) - S + R\right)^2 + L^2}, \text{ for } S < H_2, R > H_1 \qquad (3)$$

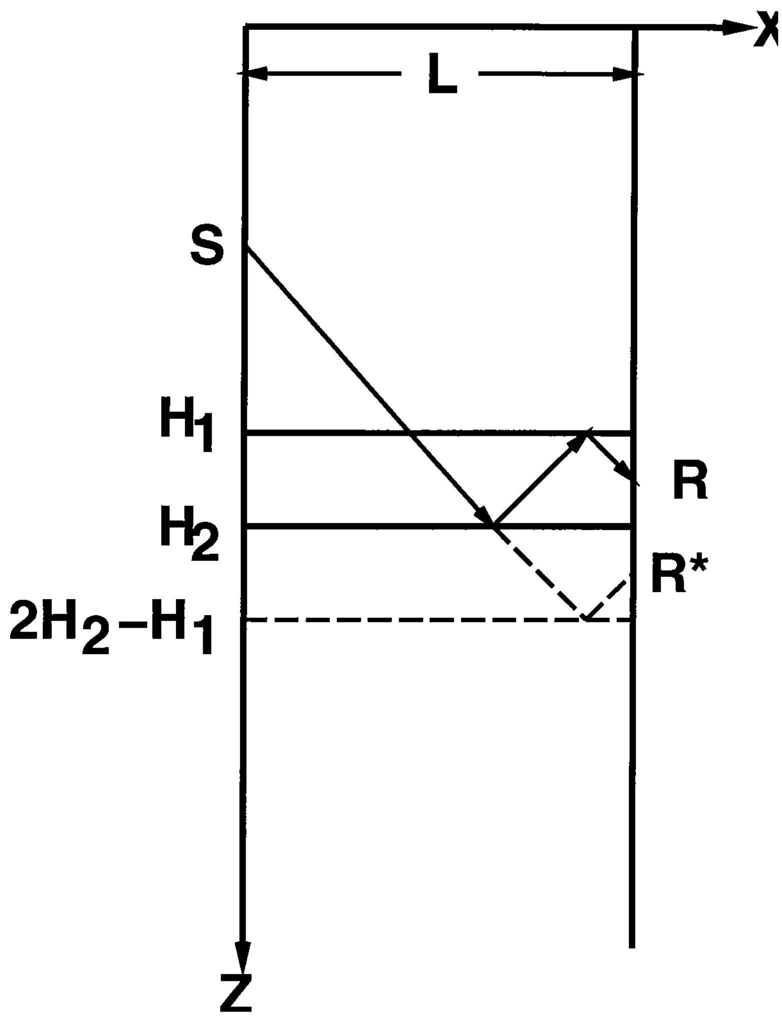


Figure 4: First order downgoing multiple reflection.

When the receiver is located below the lower reflector (Figure 5), the depth of the equivalent primary reflector is $H_2 + 2(H_2 - H_1) = 3H_2 - 2H_1$. The depth of the image receiver $R*$ is $3H_2 - 2H_1 - (R - H_2) = 4H_2 - 2H_1 - R$. The traveltime equation is:

$$t = \frac{1}{v}\sqrt{\left(2(3H_2 - 2H_1) - S - (4H_2 - 2H_1 - R)\right)^2 + L^2}$$

$$= \frac{1}{v}\sqrt{\left(2(H_2 - 2H_1) - S + R\right)^2 + L^2}, \text{ for } S < H_2, R > H_1 \qquad (4)$$

We see that it is the same as equation (3).

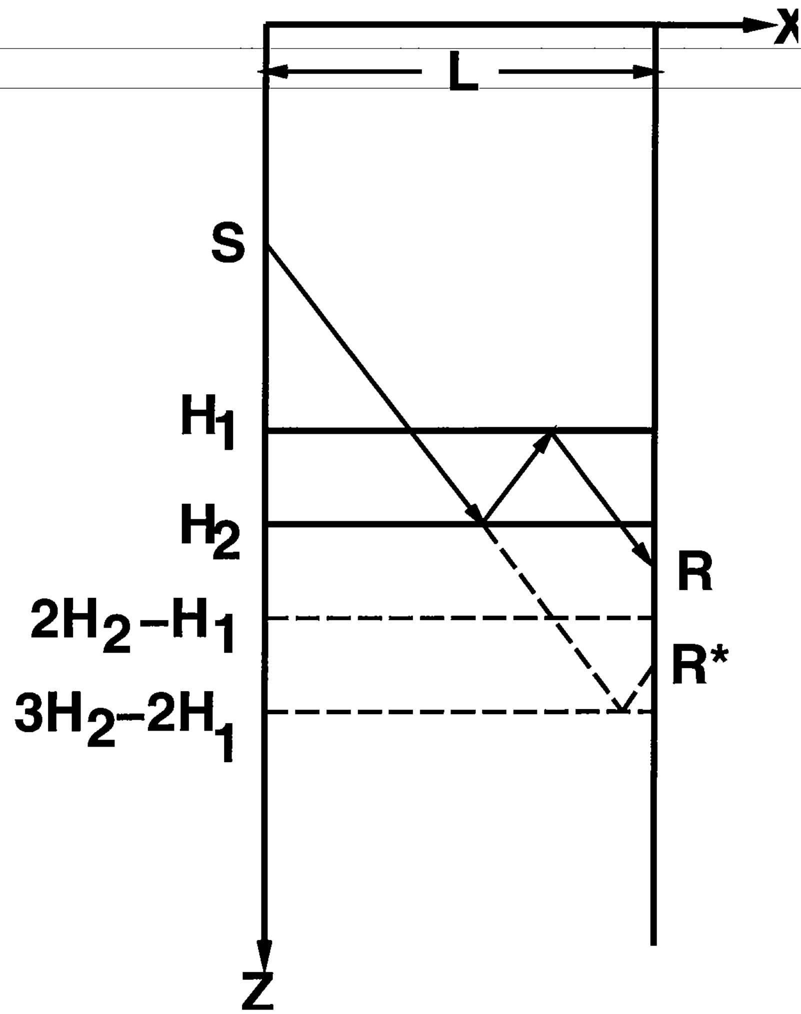


Figure 5: First orderdowngoing multiple reflection.

<u>Second order transmission multiple reflection</u>

The depth of the equivalent primary reflector is $H_2+3(H_2-H_1)=4H_2-3H_1$, Figure 6. The depth of the image receiver is $4H_2-3H_1-(R-H_1)=4H_2-2H_1-R$. The traveltime equation is:

$$t=\frac{1}{v}\sqrt{\left(2\left(4H_2-3H_1\right)-S-\left(4H_2-2H_1-R\right)\right)^2+L^2}$$

$$=\frac{1}{v}\sqrt{\left(4\left(H_2-H_1\right)-S+R\right)^2+L^2},\ \text{for } S<H_2,R>H_1 \qquad (5)$$

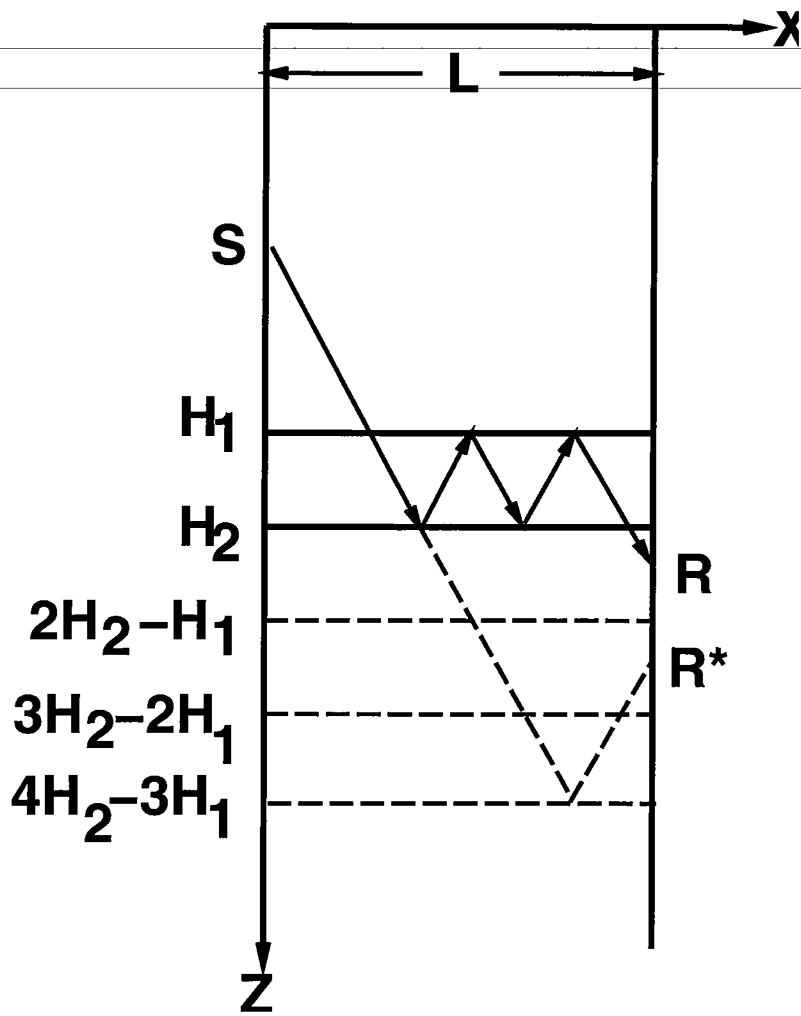


Figure 6: Second order downgoing multiple reflection.

n-th order transmission multiple reflection

We can generalize the above traveltime equations of low order transmission multiple reflections to the n-th order. The traveltime of the n-th order transmission multiple reflection is

$$t = \frac{1}{v}\sqrt{\left(2n\left(H_2 - H_1\right) - S + R\right)^2 + L^2}, \text{ for } S < H_2, R > H_1 \qquad (6)$$

### *UPGOING (TRANSMISSION) MULTIPLE REFLECTIONS*

When the source is deeper than the two reflectors, the equations (3), (4), (5), and (6) are still valid under the different conditions $H_2 < H_1, S > H_2$ and $R < H_1$.

### *UPGOING MULTIPLE REFLECTIONS*

First order upgoing multiple reflection

For the first order upgoing multiple reflection, Figure 7, the depth of the equivalent primary reflector is $H_2 + (H_2 - H_1) = 2H_2 - H_1$. The traveltime equation is:

$$t = \frac{1}{v}\sqrt{\left(2(2H_2 - H_1) - S - R\right)^2 + L^2}, \text{ for } S < H_2, R < H_2 \tag{7}$$

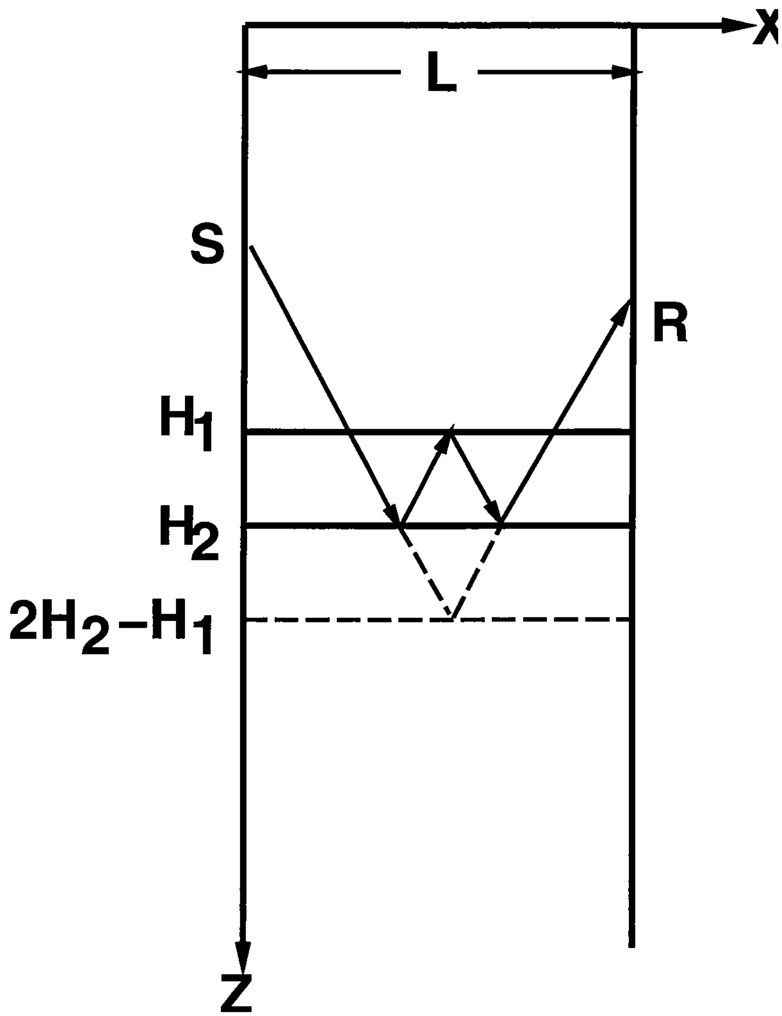


Figure 7: First order upgoing multiple reflection.

Second order upgoing multiple reflection

For the second order upgoing multiple reflection, Figure 8, the depth of the equivalent primary reflector is $H_2 + 2(H_2 - H_1) = 3H_2 - 2H_1$. The traveltime equation is:

$$t = \frac{1}{v}\sqrt{\left(2(3H_2 - 2H_1) - S - R\right)^2 + L^2}, \text{ for } S < H_2, R < H_2 \tag{8}$$

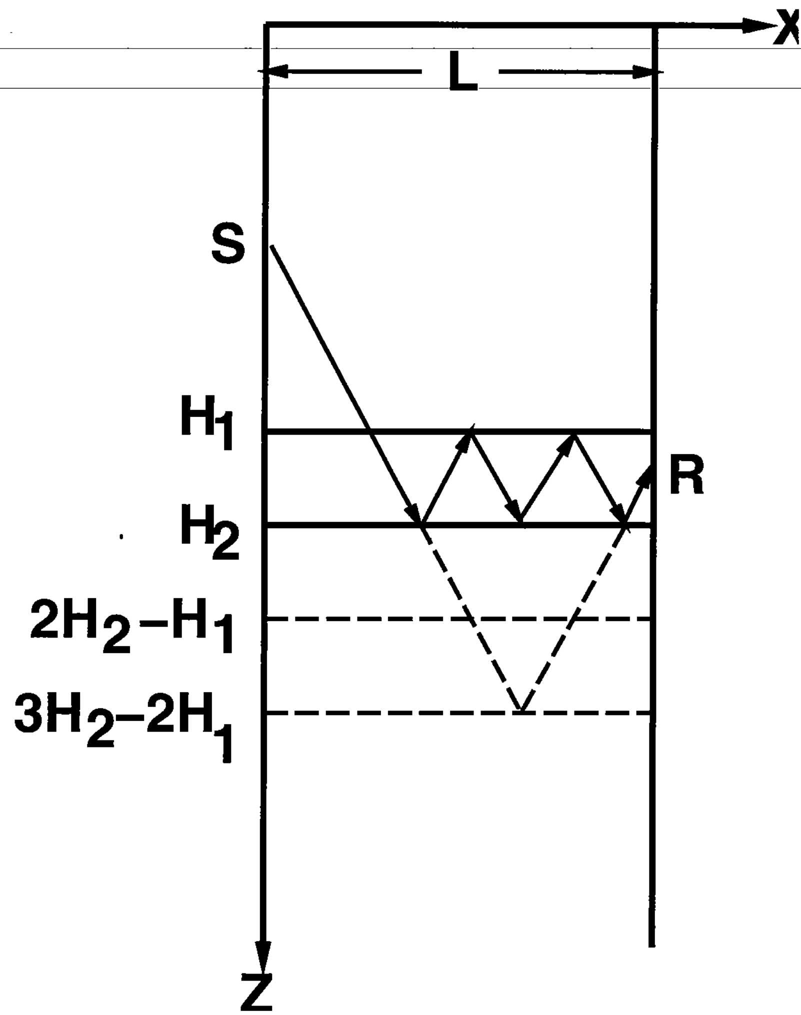


Figure 8: Second order upgoing multiple reflection.

n-th order upgoing multiple reflection

As after any one additinal reflection between the two reflectors, the depth of the equivalent primary reflector increases by $H_2 - H_1$. For the n-th order upgoing multiple reflection, the traveltime equation is:

$$t = \frac{1}{v}\sqrt{\left(2\left((n+1)H_2 - nH_1\right) - S - R\right)^2 + L^2}, \text{ for } S < H_2, R < H_2 \quad (9)$$

***DOWNGOING MULTIPLE REFLECTIONS***

When the source is deeper than the two reflectors, the equations (7), (8), and (9) are still valid under the conditions $H_2 < H_1, S > H_2$ and $R > H_2$.

***DIRECT ARRIVAL AND PRIMARY REFLECTIONS***

The traveltime equation (2) for the primary reflection can be rewritten as:

$$t = \frac{1}{v}\sqrt{(H + H - S - R)^2 + L^2} \tag{10}$$

Comparing this equation to the traveltime equation (1) for direct arrival shows that when the depth of the source approaches the depth of the reflector, these two traveltime equations coalesce into each other. Taking wave propagation directions into account, we know that when the depth of the source approaches the depth of the reflector from above (below),the traveltime of the upgoing (downgoing) primary reflection approaches that of the upgoing (downgoing) direct arrival.

## *MULTIPLE REFLECTIONS*

Travel time equations (2) and (9) shows that the multiple reflection is equivalent to a primary reflection from a virtual depth, and as the source depth $S$ changes the virtual depth stays fixed. This is detrimental to seismic imaging. Before using the data for seismic imaging, a multiple attenuation scheme, e.g., predictive deconvolution should be applied to attenuate the multiples. But multiple reflections are orders of magnitude weaker than transmission multiple reflections. So we will concentrate on studying and attenuating transmission multiple reflections.

## *DOWNGOING (TRANSMISSION) MULTIPLE REFLECTIONS AND DOWNGOING PRIMARY REFLECTION*

Travel time equations (2) and (6) shows that for

$$2H - S = -(2n(H_2 - H_1) - S)$$

$$H = S - n(H_2 - H_1) \tag{11}$$

Downgoing transmission multiple reflection is equivalent to a downgoing primary reflection from the depth $H = S - n(H_2 - H_1)$, which has a linear relationship with the source depth S. For downgoing transmission multiples that $S < H_2, H < H_1$; for upgoing transmission multiples that $S > H_2, H > H_1$.

## *REFERENCES*

Mo, L., and Harris, J. M., 1995, Analysis of noises after prestack migration in cross-well seismic profiling: STP-5.