PAPER J

WAVE EQUATION TOMOGRAPHY WITH PHASE CORRECTION FROM THE FIRST ARRIVAL TRAVELTIMES

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SUMMARY

We use the first arrival traveltime to correct the phase distortion in a nonlinear wave equation inversion scheme. This improves the precision of tomographic reconstruction of a velocity structure with large variations and helps solve the ill-posed problem of wave equation inversion. When the variation of the velocity distribution is large, general non-linear wave equation inversions are very ill-posed and for such strong nonlinear we can not obtain a correct inversion. One of main reasons is that the calculated and observed phase of the wavefield differs greatly if the initial model is far from the true model. This leads to highly mismatched phase between the calculated and the observed wave field. The phase mismatch is even more pronounced if a high operating frequency is employed in order to increase resolution. To address this problem, we use the first arrival to "demodulate" the wave field in the frequency domain with a goal of restoring the phase of wave field. Then we minimize a objective function consisting of so called "demodulated" wave field to solve wave equation inversion problem. In this way, we find that the inversion is much improved, and when the velocity perturbation in a complicated model reaches 35%, we can still obtain a good inversion. A computer simulation shows that our method is a very robust for acoustical wave inversion with good reconstruction precision.

INTRODUCTION

Many wave equation tomography methods have been proposed. Some use Born or Rytov approximation (Harris, 1987, Harris and Wang, 1995, Wu and Toksöz, 1987). Others use the non-linear iterative inversion theory to solve wave equation inversion problem, and develop the inversion method in frequency domain (Pratt and Worthington, 1990, Harris and Yin, 1993) or in time domain (Tarantola etal, 1990,, Luo and Schuster, 1990). But when the variation of velocity is large, all of these methods face a challenge due to the intrinsic non-linearity and ill-posed nature of the problem of wave equation inversion. One reason is the large phase mismatch between the calculated and observed wave fields when the variation of the velocity of wave propagation in the media is large: suppose the observed phase is $\Phi^o = \Phi_0^o + \Delta\Phi^o$, where $\Phi_0^o = \omega t^o$, and t^o is the first arrival traveltime observed from the waveform; and, the calculated phase is $\Phi^c = \Phi_0^c + \Delta\Phi^c$, where $\Phi_0^c = \omega t^c$, and t^c is the first arrival traveltime from the synthetic waveform. If the difference $t^o - t^c$ between observed and calculated first arrival of an event is 0.001 second, and if we use the wave field with 500 Hz to invert for velocity, the phase difference will be larger than π , and located outside the range $(-\pi/2, \pi/2)$. Moreover, the higher the operating frequency, the more serious the phase mismatch will be. Therefore, it is very difficult to get satisfactory results form wave equation tomography. In order to address this non-linear and ill-posed problem in wave equation tomography, we use the first arrival traveltime to make a phase correction or "demodulate" the wave field in frequency domain. Then, we use the general non-linear inversion method to minimize the objective function which contains the demodulated wave field. The phase difference between the observed and calculated wave fields after demodulation will be $\Delta\Phi^o - \Delta\Phi^c$ instead of the original $(\Phi_0^o - \Phi_0^c) + (\Delta\Phi^o - \Delta\Phi^c)$ in all iterative inversion from starting to the end. We match $(\Phi_0^o - \Phi_0^c)$ first, then, $(\Phi_0^o - \Phi_0^c) + (\Delta\Phi^o - \Delta\Phi^c)$. In addition, we can see in the next section that the first arrival phase is employed as a implicit matching condition in our algorithm. In this way, the phase mismatch problem can be resolved in each iterative step of the inversion.

The paper is organized as follows, the problem of wave equation inversion method and its solution are described in the theory and method section. Then, we apply our method to the crosswell geometry to conduct a forward and inverse simulation. We compare the inversion results using the new method and old inversion method in the numerical test section. We find that a complicated model with 35% velocity perturbation can be reconstructed very well by our new method while standard nonlinear inversion method in frequency domain has failed. We believe that the method we propose here will be useful when applied to real data. The general idea proposed in this paper can be extended to inversion in the time domain case.

THEORY AND METHOD OF WAVE EQUATION INVERSION WITH PHASE CORRECTION

Let us revisit the general inversion method in the frequency domain and check its solution first. We know that the goal in the non-linear inversion is to minimize the objective function

$$Q(\boldsymbol{m}) = J(\boldsymbol{m}) + \alpha H(\boldsymbol{m}) \longrightarrow min \tag{1}$$

Where $J(\boldsymbol{m}) = \sum_{g=1}^{G} \sum_{s=1}^{S} \| U^o(\omega) - U^c(\omega) \|_2$, and $U^o(\omega)$ and $U^c(\omega)$ are the observed and calculated wave fields in the frequency domain, H is a smoothing operator, and $\boldsymbol{m}$ is a model vector, G and S are total number of the receiver and source, the turning parameter α regulates the amount of smoothing versus the fit to the data of the model. Suppose that we have a waveform $u(t)$ in time domain, if we apply the Fourier transform to the waveform $u(t)$, then we have $U(\omega) = \mathrm{F}\ (u(t))$, where F denotes Fourier transform. If the signal were shifted to $u_1(t) = u(t + t_0)$, then we have $U_1(\omega) = e^{j\omega t_0} U(\omega)$. We here call $U_1(\omega)$ a "demodulated" signal of $U(\omega)$ in frequency domain. We can see the oscillation of $U(\omega)$ is much quicker than $U_1(\omega)$. The fact is illustrated in Figures (1)-(4). Suppose we have a time domain signal $u(t)$ as shown in Figure (1) with a frequency band of (0,500) Hz. Then the spectrum of it can be derived by Fourier transform, which is shown in Figure (2). From Figure (2), we can see, if we want to match this spectrum, we much have good initial model so that the waveform is very close to the original one; otherwise, the iterative inversion will probably be divergent. But if we shift the signal to $u(t + t_0)$ as shown in Figure (3), then we obtain the spectrum of the signal $u(t + t_0)$ as shown in Figure (4). By comparing Figure (2) with Figure (4), it is obvious that the oscillation of the spectrum in Figure (2) is much stronger than that shown in Figure (4). Therefore, the inversion implemented in the domain as shown in Figure (4) is much easier than that as shown in Figure (2). So we believe that the non-linear problem in the demodulated domain easier than before.

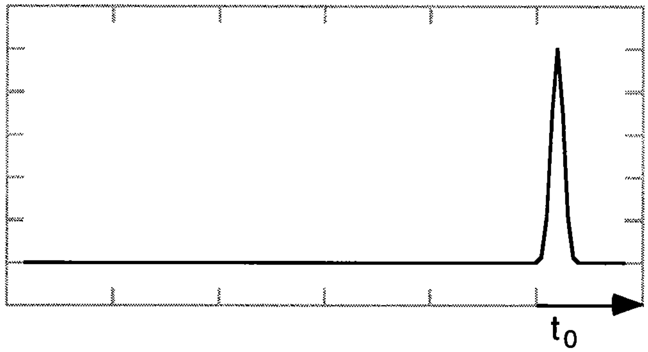

Figure 1: A time domain signal u(t)

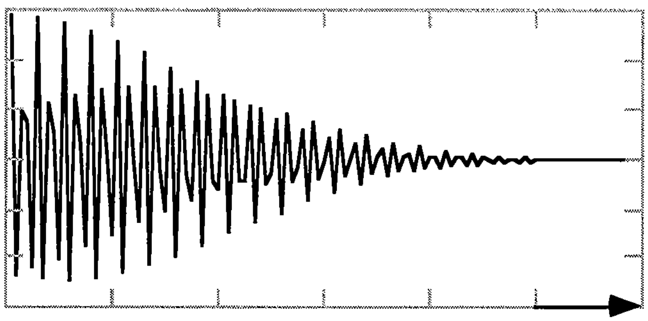

Figure 2: The real part of the spectrum of u(t)

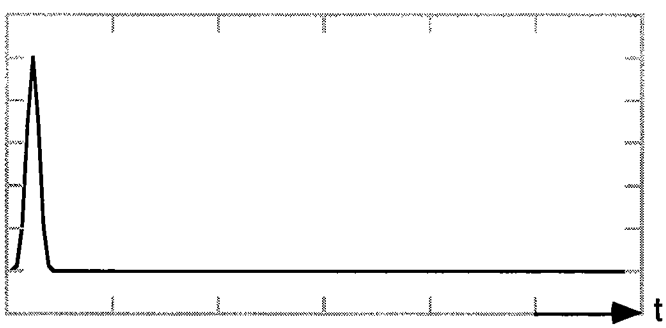

Figure 3: The time domain signal of $u(t+t_0)$

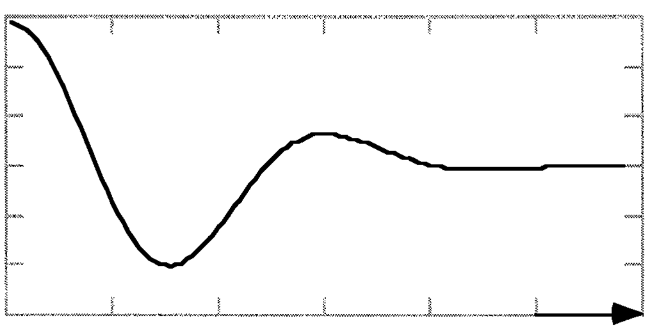

Figure 4: The real part of spectrum of u(t+t0)

Therefore, we have the suggestion that the inversion will be better inversion by matching the demodulated waveform rather than the original waveform. According to nonlinear inversion theory, we establish a new object function as follows:

$$Q(\boldsymbol{m}) = J(\boldsymbol{m}) + \alpha H(\boldsymbol{m})$$

$$= \sum_{g=1}^{G} \sum_{s=1}^{S} \| U_d^o(\omega) - U_d^c(\omega) \|_2 + \alpha H(\boldsymbol{m}) \longrightarrow min \tag{2}$$

where $U_d^o(\omega) = \mathrm{F}\ (u^0(t+t_0))$, and $U_d^c = \mathrm{F}\ (u^c(t+t_c))$, F denotes the Fourier transform, and subscript (superscribpt) o , c and d represent the observed and calculated wave field data and the demodulated in frequency domain, respecttively. In this paper, we restrict ourselves to inversion for velocity using acoustical wave equation. Therefore, we use the following integral equation

$$U(r,\omega) = -k^2 \iint_{\Omega} U_t(r', r_s, \omega) m(r') G(k|r' - r|) dr' \tag{3}$$

where $U(r,\omega)$ are the scattering field in the background media with velocity V_0, $U_t(r,\omega)$ is the total pressure field in the media with velocity V, ω is the angular frequency, $m(r) = 1 - \frac{V^2}{V_0^2}$, $k = \frac{\omega}{V_0}$, and $G(k|r - r'|)$ is the Green's function in the background media. The total field in equation (3) can be solved by the moment method (Harris and Yin, 1993) or finite difference method in frequency domain (Pratt and Worthington, 1990). So we have $U_d^o(\omega) = e^{j\omega t_o} U^o(\omega)$ and $U_d^c(\omega) = e^{j\omega t_c} U^c(\omega)$.

Therefore, by using the iterative inversion shown below to invert for model $\boldsymbol{m}(r)$, we have

$$\boldsymbol{m}^{(q+1)} = \boldsymbol{m}^{(q)} + \delta \boldsymbol{m}^{(q)} \tag{4}$$

$$\delta \boldsymbol{m}^{(q)} = x_1 \vec{e}_1 + x_2 \vec{e}_2 + x_3 \vec{e}_3 \tag{5}$$

where

$$(\vec{e}_1)_i = (\frac{\partial J}{\partial \boldsymbol{m}})_i,\ (\vec{e}_2)_i = (\frac{\partial H}{\partial \boldsymbol{m}})_i,\ (\vec{e}_3)_i = \sum_j (\frac{\partial^2 J}{\partial m_i \partial m_j} \frac{\partial Q}{\partial m_j}) \tag{6}$$

and from equation (2), we have

$$(\frac{\partial J}{\partial \boldsymbol{m}})_i = \sum_{g}^{G} \sum_{s}^{S} R[e^{-j\omega t^c} (-j\omega \frac{\partial t^c}{\partial m_i} U^c + \frac{\partial U^c}{\partial m_i})] \cdot R(e^{-j\omega t^c} U^c - U_d^o)$$

$$+ I[e^{-j\omega t^c} (-j\omega \frac{\partial t^c}{\partial m_i} U^c + \frac{\partial U^c}{\partial m_i})] \cdot I(e^{-j\omega t^c} U^c - U_d^o) \tag{7}$$

$$(\frac{\partial^2 J}{\partial m_i \partial m_j}) =$$
$$\sum_g^G \sum_s^S R[e^{-j\omega t^c}(-j\omega \frac{\partial t^c}{\partial m_i} U^c + \frac{\partial U^c}{\partial m_i})] \cdot R[e^{-j\omega t^c}(-j\omega \frac{\partial t^c}{\partial m_i} U^c + \frac{\partial U^c}{\partial m_i})]$$
$$I[e^{-j\omega t^c}(-j\omega \frac{\partial t^c}{\partial m_i} U^c + \frac{\partial U^c}{\partial m_i})] \cdot I[e^{-j\omega t^c}(-j\omega \frac{\partial t^c}{\partial m_i} U^c + \frac{\partial U^c}{\partial m_i})] \tag{8}$$

From equation (3) , we have

$$\frac{\partial U^c}{\partial m_i} = -k^2 U_t^c(\omega, r', r_s) G(k|r - r'|) \Delta s_i, \; r' \subset \Delta s_i \tag{9}$$

where Δs_i is the area of i-th pixel and $\frac{\partial t^c}{\partial m_i}$ can be derived by variable methods. Under the ray approximation, we have

$$\frac{\partial t^c}{\partial m_i} = -\frac{\Delta l_i V_i}{2V_0^2} \tag{10}$$

where Δl_i and V_i are ray length and velocity in i-th cell. The coefficients $x_i (1 \le i \le 3)$ can be derived by the method described in the paper (Harris and Yin, 1993).

From equation (7), we see that there are two derivatives making contribution to the Frechet derivative of object function, one is the derivative of first arrival to the model, another is the derivative of total field to the model. Therefore, the phase of first arrival is used as a implicit constraint condition which should be matched in all iterative steps, that is, the variation of the first arrival phase is also employed to correct the current model. Thus, this will be helpful for the iterative procedure to be convergent to the true model. So the inversion steps can be summarized as follows:

1. pick the first arrival time from the sacttering waveform *u(t);*
2. shift the waveform $u(t+t_0)$;
3. transform $u(t+t_0)$ to $U_d^o(\omega)$;
4. calculate searching directions e_1, e_2, e_3;
5. updating the model using equation (4);
6. if $Q(m) \le \varepsilon$, ε, stop iterative inversion, otherwise, go to 4.

INVERSION RESULTS USING SIMULATION DATA

Figure (5) and (6) are two synthetic models. In Figure (5), there is a two layer perturbation. Figure (6) is a more complicated model. The velocity in each region is shown. The minimum velocity is 5000 m/s, the maximum 6750 m/s, and the range of the variation is 35%. In the forward calculation, the image region is divided into 20×50 pixels, where the width of each pixel is 2.5 m. In the frequency we use is 200 Hz. The experiment consists of 50 sources and receivers spaced at intervals of 2.5 m apart. We use moment method to produce the observed wave field. Figure (7) and Figure (8) are the tomography reconstruction results of the model (5) and model (6) by the demodulation method presented herein,. The starting models are constant with a velocity of 5000 m/s. Figure (9) and Figure (10) are the tomography reconstruction of the model (5) and model

(6) by minimizing equation (1) using the standard nonlinear method, in which the starting models are also constant with a velocity of 5000 m/s .

CONCLUSIONS

We put forward a new wave equation iterative tomography method, in which the first arrival traveltime is used to "demodulate" the wave field in frequency domain to reach the goal of resolving the large phase mismatch between the observed and calculated wave fields. Then the velocity is inverted by minimizing a new objective function which describes the mismatch between the observed "demodulated" wave field and corresponding calculated data below a threshold determined by the errors in the observation. In addition, in our algorithm, the first arrival phase is automatically used as a implicit constraint condition to help the algorithm to be convergent to the true model. Through this way, we can avoid the ambiquity problem of the phase between the observed and calculated wave field and obtain a robust wave equation inversion method. The computer simulation results show that when the perturbation of the velocity is as large as 35%, our inversion method still works while the old nonlinear inversion has already failed.

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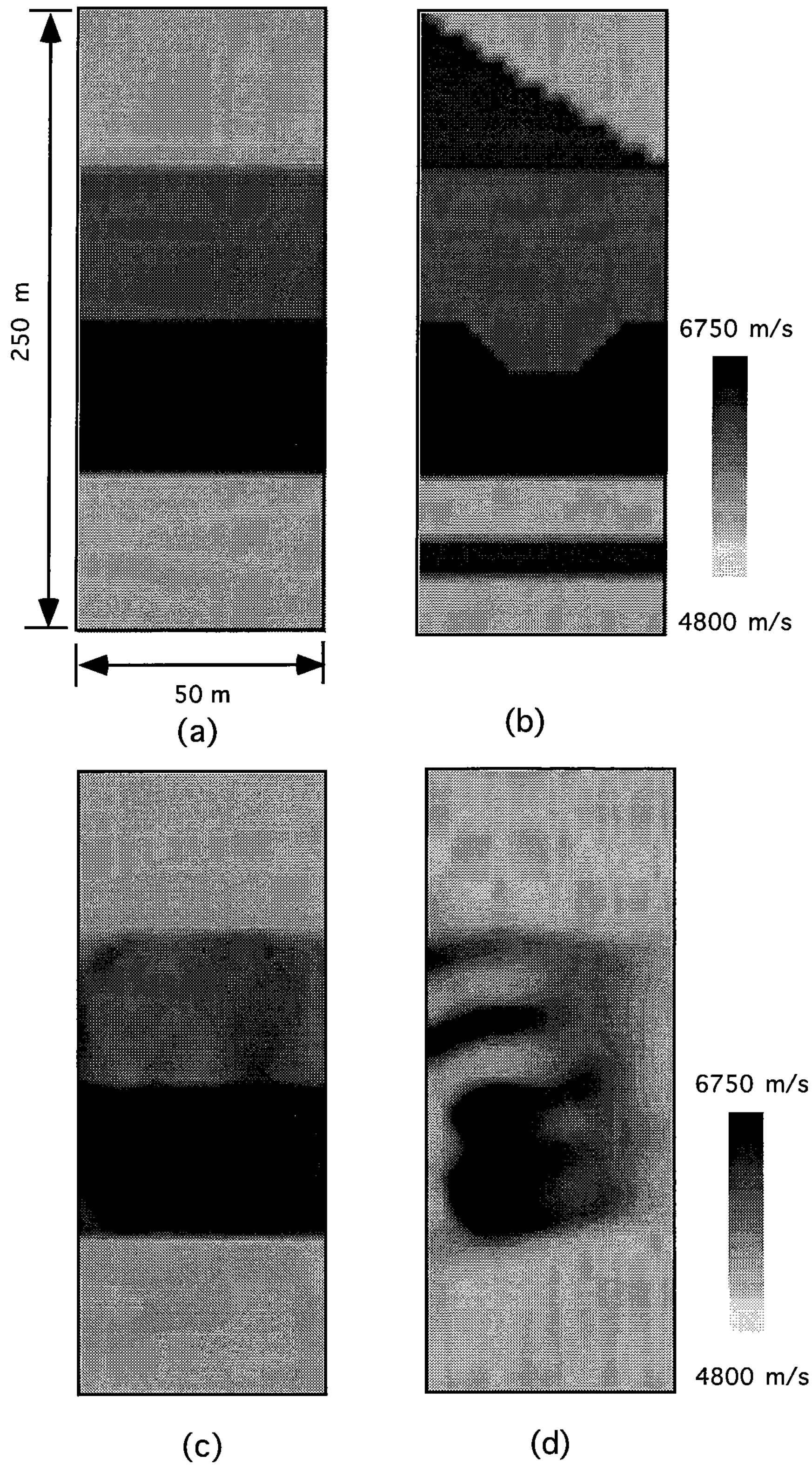
250 m
50 m
(a)
(b)
6750 m/s
4800 m/s
(c)
(d)
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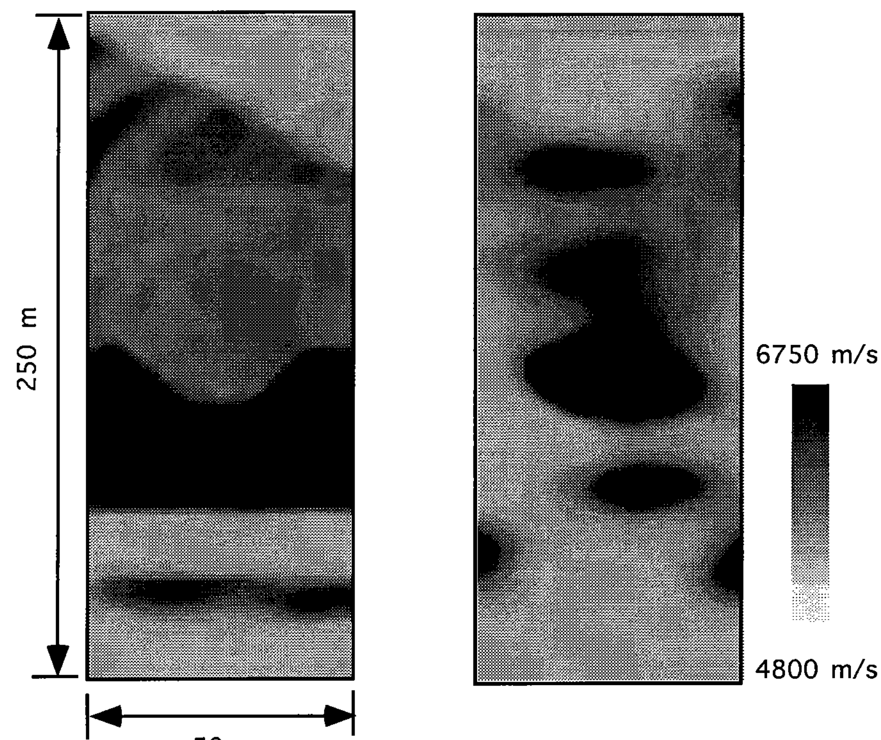

(e) (f)