PAPER I

RESOLUTION OF CROSSWELL TOMOGRAPHY WITH TRANSMISSION AND REFLECTION TRAVELTIMES

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ABSTRACT

Transmission traveltimes alone in crosswell tomography are insufficient to determine the slowness field between the wells accurately. Horizontal smearing is common. In many crosswell data sets, traveltimes from reflections can also be picked. We study here the effect that including these reflected traveltimes has on the resolution of both the slowness field and the reflector depth. When reflected traveltimes are used, more unknown parameters need to be introduced to solve for the position of the reflectors. We compute the diagonal elements of the resolution matrix for two realistic crosswell geometries based upon the acquisition parameters used in the McElroy crosswell field experiment. We compare the resolution when no reflections are used, when upgoing reflections only are used, and when both upgoing and downgoing reflections are used. Our results indicate that the reflector positions are very well determined, so these new unknowns can be determined from the new traveltime data. Including reflection traveltimes improves the resolution of the slowness field somewhat, particularly near the reflectors.

INTRODUCTION

In crosswell seismic tomography, transmission traveltimes from shot locations in one well to receiver locations in another well are measured, and the goal is to determine the slowness field (reciprocal of wavespeed or velocity) between the wells. Unfortunately, transmission traveltimes alone are insufficient to determine the slowness field accurately (Menke, 1984; Bregman, 1986; Bregman *et al.*, 1989; Calnan and Schuster, 1989, 1990; Langan and Bube, 1995). The most poorly determined features in crosswell transmission tomography are horizontally zero-mean, vertically homogeneous perturbations of a slowness field ("stripes"); these slowness perturbations cause little or no perturbation in the traveltimes, so they are difficult to recover from traveltime data alone. Stated less formally, in general the horizontal location of features in the slowness model is poorly determined by the traveltime data.

A number of approaches have been used to augment the transmission traveltimes with other data in order to obtain an improved estimate of the slowness field. Wavefield inversion has been coupled to traveltime inversion (Luo and Schuster, 1991). There has been much recent interest in using reflections in crosswell seismic data to estimate the slowness field, both in wavefield imaging and for augmenting transmission traveltimes with reflection traveltimes (Calnan and Schuster, 1989, 1990; Stewart and Marchisio, 1991; Lazaratos, 1993; Lazaratos *et al.*, 1992, 1993). We focus here on the effect of including reflection traveltimes together with transmission traveltimes in crosswell tomography.

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Calnan and Schuster (1989, 1990) proposed combining transmission traveltimes with reflection traveltimes in tomography to obtain some improvement in the resolution of the slowness field. In their analyses, it appears that they fixed the reflector depths while solving for the slowness field, rather than solving for both the reflector depths and the slowness field simultaneously. It should be noted that they did this work before the first high quality reflection image was produced by Lazaratos *et al.* (1992) and it was not even known at the time whether one could "pick" reflection events in prestack or post-stack data.

The purpose of our study is to analyze the effect that adding reflection traveltimes has on the resolution of slownesses *and* reflector depths in crosswell tomography. A key issue is whether the addition of more unknowns (the reflector positions) damages the potential gain of adding more information (the reflection traveltimes). We examine the diagonal elements of the resolution matrix for a selected series of linearized problems in order to address this question.

THE RESOLUTION MATRIX

One way to study the ability of the traveltimes to resolve features of the model is to study the derivative matrix $\underline{\mathbf{A}}$ of traveltimes with respect to model parameters at a particular background model. This matrix tells us how perturbations in the background model change the traveltimes. We choose a background model with constant slowness and a single flat reflector. Even though the background model is constant, the slowness field (a function of x and z) and the reflector depth (a function of x) have complete freedom (up to the level of discretization) in the possible perturbations.

The choice of a constant background model forces symmetry on the problem which we hope brings out worst-case behavior for conditions where we have good raypath coverage over most of the region. On can easily envision complicated models whereby the ray coverage is either locally sparse or mostly parallel, such that the resolution problems are quite different from the ones posed here.

The resolution matrix of $\underline{\mathbf{A}}$ can be described in terms of its singular value decomposition, $\underline{\mathbf{A}} = \underline{\mathbf{U}}\, \underline{\Sigma}\, \underline{\mathbf{V}}^{\mathrm{T}}$, where $\underline{\mathbf{U}}$ and $\underline{\mathbf{V}}$ are orthogonal matrices and $\underline{\Sigma}$ is a diagonal matrix whose diagonal elements are the singular values of $\underline{\mathbf{A}}$: $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0$. The columns of $\underline{\mathbf{V}}$ are the singular vectors in the model space. For a given noise level, and a choice of a minimum signal-to-noise ratio (e.g., 1), a cutoff l in the index of the singular values can be chosen so that whenever $j > l$, the expected signal-to-noise ratio for the coefficient of the j^{th} column of $\underline{\mathbf{V}}$ (in the solution of the linearized problem) is less than the chosen minimum. Let $\underline{\mathbf{V}}_l$ be the matrix consisting of the first l columns of $\underline{\mathbf{V}}$. The resolution matrix (for this noise level) is

$$\underline{\mathbf{R}} = \underline{\mathbf{V}}_l\, \underline{\mathbf{V}}_l^{\mathrm{T}}.$$

The diagonal elements of $\underline{\mathbf{R}}$ can be shown to satisfy $0 \leq r_{ii} \leq 1$. If $l = n$ (so no columns are dropped), $\underline{\mathbf{R}} = \underline{\mathbf{I}}$. In general, the closer r_{ii} is to 1, the better the resolution of the i^{th} model parameter.

RESOLUTION STUDIES

We formulated two sets of synthetic problems for study based upon two crosswell experiments in the McElroy Field, one set based upon a 180-ft spacing between wells, and the other based on a 600-ft spacing between wells. Both synthetic studies used source and receiver geometries similar to the actual field experiments.

Tables 1 and 2 summarize the parameters of the problems we investigated. We traced rays through a constant slowness model between all appropriate source and receiver locations. A reflector was placed at a depth of 2900 ft, approximately the midline of the 100-ft thick reservoir. We calculated the traveltime derivatives from ray tracing, and then computed the derivative matrix $\underline{\mathbf{A}}$ and the resolution matrix $\underline{\mathbf{R}}$ (for a given noise level).

Note that the reflector position is independent of the slowness structure. We have found that this decoupling is essential in order to obtain the best resolution possible in studies where surface seismic data are used to resolve reflector depths with traveltime tomography (Bube *et al.*, 1995). Another observation we have made with surface seismic data is that one must be careful not to *underparametrize* the slowness field as well in order to resolve reflector depths accurately--if a proposed simplification of slowness structure in the model is not justified by what is present in the real world, errors associated with an oversimplification of the slowness field are forced to create errors in the reflector depth. Even though one may not be able to resolve accurately the complicated slowness structure, this freedom must be present in the slowness model. The interesting result is that one has to avoid both overparametrizing *and* underparametrizing the slowness model!

We computed three raypath coverages for each well spacing; *transmitted* rays only, *transmitted* rays plus *upwardly reflected* rays, and *transmitted* plus *upwardly reflected* rays plus *downwardly reflected* rays. We based the singular value cutoff for the computation of resolution diagonals upon an assumed noise level. For the 185-ft well spacing a 1% noise level was chosen, which is equivalent to about 0.15 msec error (slightly greater than the 0.1 msec sample interval). For the 600-ft well spacing a 2% noise level was chosen, equivalent to about 0.9 msec error. Too fine a grid spacing can strongly affect this analysis due to overparametrization. We chose a grid spacing of 10 ft for the narrow profile and 20 ft for the wide profile. In both profiles this corresponds to four times the source and receiver spacing, which is normally great enough to prevent spatial aliasing of the slowness field (Bube and Resnick, 1984; Bube *et al.*, 1985).

The contour plots of the resolution diagonals for slowness are shown if Figures 1 and 2, which correspond to the 180-ft profile and the 600-ft profile, respectively. The magnitude of the contour values indicates what fraction of a perturbation to the constant background will be recovered at each grid point location. A value of 0.8 is generally considered good. Lower values usually mean there will be considerable smearing of the image.

Figure 1A shows that for a 10-ft grid spacing and a 1% noise level, and using the transmitted data only, we should expect good slowness resolution on the 180-ft profile through the central half of the acquisition area. Note that there will be some degradation just below the reservoir interval of 2850 ft to 2950 ft. The addition of a reflector and upgoing reflected events (Figure 1B) substantially improves the slowness resolution for a zone about 75-ft thick just above the reflector. There is also a slight improvement in slowness resolution just below the central part of the reflector. The further addition of downgoing reflections (Figure 1C) improves the slowness resolution below the central partof the reflector in a zone about 50 ft thick vertically. Note that the raypath coverage for the downgoing reflections is not as extensive as the upgoing raypath coverage (Table 1).

The resolution diagonals for the reflector depth parameters all exceed 0.993 for the 180-ft profile. After scaling slowness and reflector depth appropriately (because their units are different), an estimate for the errors predicted for the reflector depths yields a value of only a few inches. Therefore, the reflector depths are well determined in this example.

Figure 2A shows that for a 20-ft grid spacing and 2% noise level, we will get slightly poorer slowness resolution on the 600-ft profile. In particular, the slowness resolution is considerably poorer in the lower portion of the reservoir. The addition of upgoing reflections from a reflector at 2900 ft (Figure 2B) improves the resolution substantially. The addition of downgoing reflections (Figure 2C) only slightly improves the slowness resolution below the reflector. Note that there are fewer than 4000 downgoing reflected rays out of total of about 60,000 (Table 2). This explains in part the minimal impact of the downgoing reflections.

The resolution diagonals for the reflector depths all exceed 0.975 for the 600-ft profiles, leading to an estimate of a maximum error in the reflector depth parameters on the order of 0.5 ft to 2.5 ft. One can expect good resolution of the reflector depths, in spite of the somewhat mediocre slowness resolution.

CONCLUSIONS

We have performed a resolution analysis for two realistic crosswell geometries by computing the diagonals of the resolution matrix. The addition of reflection traveltimes to the transmission traveltimes adds more parameters to be determined (the reflector depth parameters). Fortunately, even when reflections from only one side of the reflector are used, the reflector depths are very well determined. Thus, crosswell transmission and reflection tomography should put the reflectors in the right place, *even if there are still errors in the slowness field.* As we noted, decoupling the reflector position from the slowness structure is probably critical for this conclusion to be valid, as is the degree of parametrization of the slowness field. Including reflection traveltimes improves the resolution of the slowness field, particularly near the reflector (on the same side as the reflections). Including reflection traveltimes from several reflectors should further improve the slowness resolution.

The potential application of transmission plus reflection traveltime tomography would be in situations where one wants to know the precise structure of selected reflection events in a reflection image. Although a reflection image obtained by migration or VSP-CDP mapping may be well focused, the actual geometric structure of the reflections may be subtly distorted due to errors in the velocity model used in the time-to-depth conversion. Traveltime tomography, when applied to both transmitted and reflected events, should provide superior depth resolution of these selected reflection events. A potential side benefit would be a more accurate velocity model.

A second potential application is in trying to obtain improved slowness images for larger well spacings when they raypath coverage is less than ideal. The improved angular coverage that the reflections provide will improve the slowness resolution somewhat, particularly near the selected reflectors.

Van Schaack (this volume) has implemented a transmission plus reflection traveltime tomography method and tested it on both synthetic data and field data. His preliminary results support the conclusions of our study.

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Table 1. Parameters Governing The Analysis For The 180-ft Well Spacing

Ray Pattern for Inversion	Sources	Receivers	Rays	Grid Spacing	Unknowns	Noise Level	S.V. Cut*	Figure
Transmitted Rays Only	240 @ 2.5 ft	240 @ 2.5 ft	57,600	10 ft	1,159	1%	337	1A
Trans. + Upgoing Reflections	240 @ 2.5 ft	240 @ 2.5 ft	77,200	10 ft	1,178	1%	325	1B
Trans. + Upgoing + Downgoing	240 @ 2.5 ft	240 @ 2.5 ft	87,200	10 ft	1,178	1%	323	1C

*Singular Vectors Cut Off

Table 2. Parameters Governing The Analysis For The 600-ft Well Spacing

Ray Pattern for Inversion	Sources	Receivers	Rays	Grid Spacing	Unknowns	Noise Level	S.V. Cut*	Figure
Transmitted Rays Only	200 @ 5.0 ft	200 @ 5.0 ft	40,000	20 ft	1,581	2%	816	2A
Trans. + Upgoing Reflections	200 @ 5.0 ft	200 @ 5.0 ft	59,600	20 ft	1,611	2%	776	2B
Trans. + Upgoing + Downgoing	200 @ 5.0 ft	200 @ 5.0 ft	63,200	20 ft	1,611	2%	766	2C

*Singular Vectors Cut Off

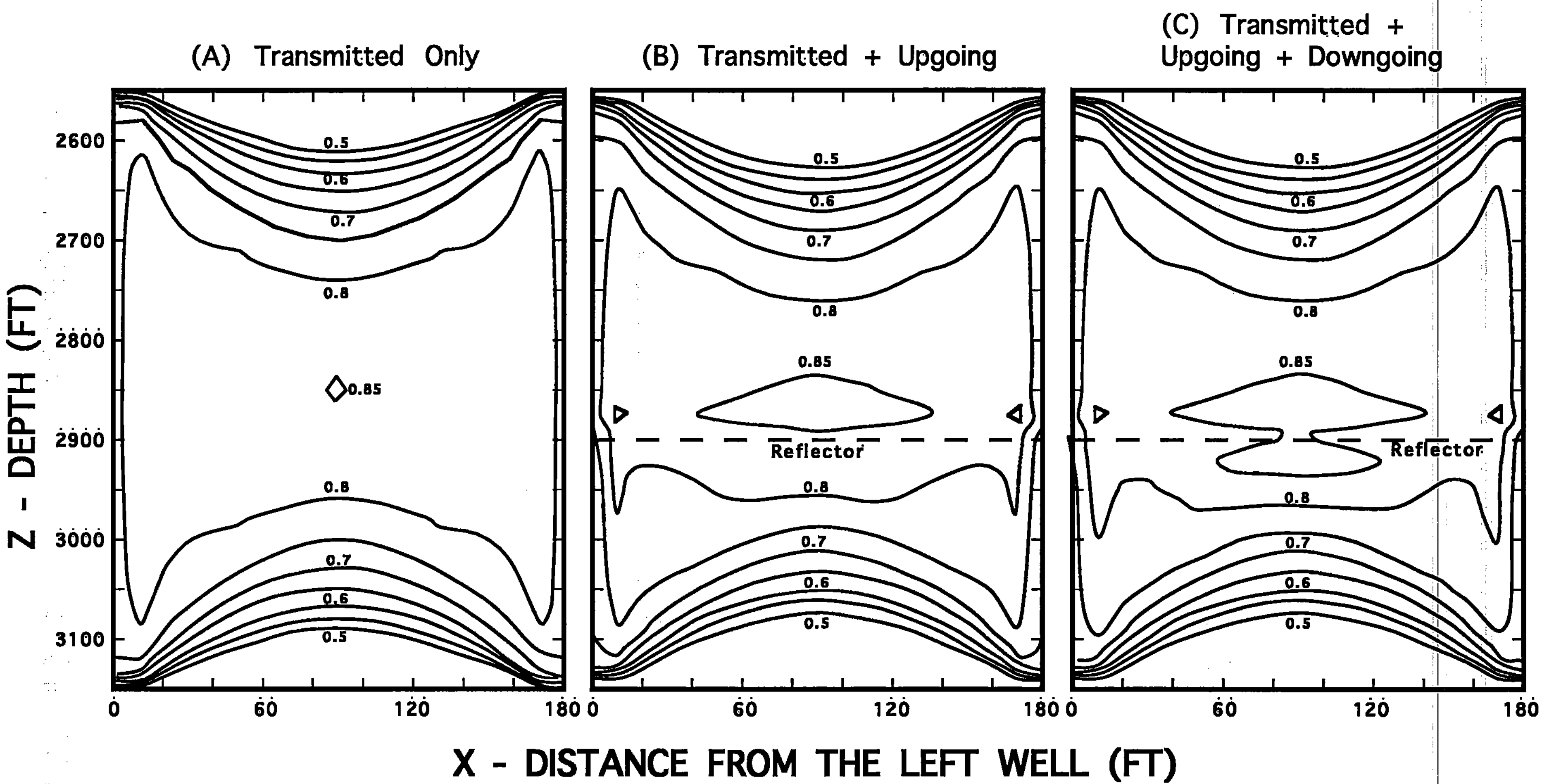

Figure 1. Contoured resolution diagonals for a 180-ft crosswell profile. Three cases are shown: (a) transmitted raypaths only, (b) transmitted plus upwardly reflected raypaths, and (c) transmitted plus upwardly plus downwardly reflected raypaths. These resolution diagonals indicate how good the slowness resolution will be between the wells for a given acquisition geometry, model parametrization, and noise level. A value of 1.0 would indicate perfect recovery of the slowness information at a given point. Lesser values indicate poorer recovery and subsequent smearing (usually) of the image about the neighboring grid points.

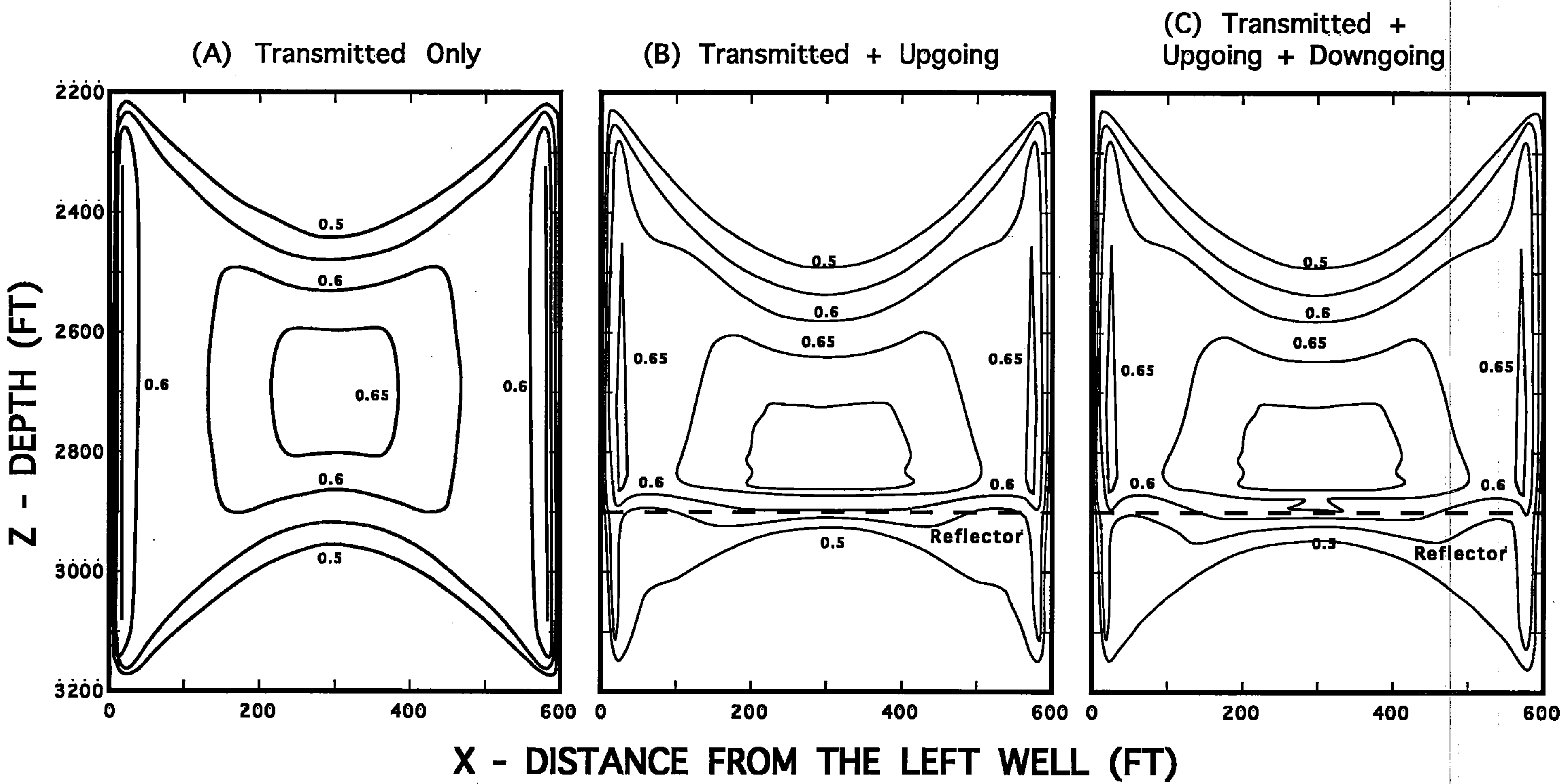

Figure 2. Contoured resolution diagonals for a 600-ft crosswell profile. Three cases are shown: (a) transmitted raypaths only, (b) transmitted plus upwardly reflected raypaths, and (c) transmitted plus upwardly plus downwardly reflected raypaths. These resolution diagonals indicate how good the slowness resolution will be between the wells for a given acquisition geometry, model parametrization, and noise level. A value of 1.0 would indicate perfect recovery of the slowness information at a given point. Lesser values indicate poorer recovery and subsequent smearing (usually) of the image about the neighboring grid points.