ACOUSTIC ATTENUATION LOGGING USING THE CENTROID FREQUENCY SHIFT AND AMPLITUDE RATIO METHODS: A NUMERICAL STUDY

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ABSTRACT

The centroid frequency shift method is proposed to estimate seismic attenuation from full waveform acoustic logs. This approach along with the amplitude ratio method is applied to investigate the attenuation properties of the *P* head wave in fluid-filled boreholes. The generalized reflection and transmission coefficients method is used to perform forward modeling. We suggest an empirical formula to describe the frequency-dependent geometrical spreading of the *P*-wave in a borehole. We simulate a more realistic borehole by including a mudcake and an invaded zone which are modeled by a large number of radially symmetric thin layers. The numerical tests show that the invaded zone exhibits a very strong influence on the attenuation measurement.

INTRODUCTION

Many efforts have been made to estimate the seismic attenuation from sonic logs. Existing methods include amplitude ratio, spectral ratio, partition coefficients and full waveform inversion (Paillet and Cheng 1991). We propose another method which is based on centroid frequency shift. The centroid frequency shift method has been applied to crosswell seismic attenuation (Quan & Harris, 1993). In crosswell profiling this method measures the spectral centroid difference between incident and transmitted waves and uses this difference to estimate the attenuation (1/Q). For the full waveform logging we take the advantage of multiple receivers and measure the spectral centroid difference of P-waves between two or more receivers for attenuation estimation. In acoustic logs the first arrival is a P head wave refracted from the borehole wall. In order to understand the

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attenuation property of the head wave in boreholes and provide a guide to in situ measurement, we use the generalized reflection and transmission coefficients method (Chen, et al., 1994) to model waves in boreholes, We then apply the centroid frequency shift method along with the amplitude ratio method for inversion. This approach is tested with numerical simulations.

FORWARD MODELING

In the sonic logging problem, we measure stress waves in a fluid-filled borehole where the normal stress $p^{(1)}$ is given by calculating a k-ω integral (Chen et al., 1994):

$$p^{(1)}(r,z,t) = \frac{\rho^{(1)}}{4\pi} \int_{-\infty}^{+\infty} \omega^2 F(\omega) e^{-i\omega t} \{ \int_{-\infty}^{+\infty} i \frac{J_o(v_\alpha^{(1)} r) e^{i v_\alpha^{(1)} r^{(1)}} \hat{R}_{+-}^{(1)}}{1 - \hat{R}_{+-}^{(1)} e^{i v_\alpha^{(1)} r^{(1)}}} e^{ikz} dk + \frac{1}{R} e^{i k_\alpha^{(1)} R} \} d\omega. \quad (1)$$

where, $R = \sqrt{r^2 + z^2}$, z = source-receiver offset, r = the distance of receiver from borehole center, $\rho^{(1)}$ = fluid density, $\hat{R}_{+-}^{(1)}$ = the generalized reflection coefficient calculated from Eqn (16) in (Chen et al., 1994), $F(\omega)$ = source spectrum, and $v_\alpha^{(1)} = \sqrt{(\omega / \alpha^{(1)})^2 - k^2}$ with $\alpha^{(1)}$ = P-wave velocity in the fluid. The superscript is the layer number, and the first layer is fluid-filled borehole. The discrete wave-number technique and FFT are used to evaluate Eqn (1). In our simulation the spectrum of source function $F(\omega)$ corresponding to displacement is described by (Tsang and Rader, 1979)

$$F(\omega) = \frac{1}{\omega^2} \frac{8\alpha\omega_o(\alpha_s - i\omega)}{[(\alpha_s - i\omega)^2 + \omega_o^2]^2}$$

with $\omega_o = 2\pi \times 14000$ Hz and $\alpha_s = 0.5\ \omega_o/\pi$. The attenuation is introduced through complex velocity defined by (Aki and Richards, 1980)

$$v(\omega) = v(\omega_o)[1 + \frac{1}{\pi Q} \log(\frac{\omega}{\omega_o}) - \frac{i}{2Q}], \quad (2)$$

where Q is the quality factor for either P-wave or S-wave, and v is either P-wave velocity or S-wave velocity.

ATTENUATION ESTIMATION FORM SONIC LOGGING

The first arrival in full waveform sonic logs is a P head wave. We separate it from later arrivals using a time window. We use this first arrival to estimate P-wave attenuation (1/Q). The magnitude of recorded wave spectrum $R_{i+1}(f)$ at r= 0 and z=z_{i+1} is written as (see Figure 1)

$$R_{i+1}(f) = \frac{R_i(f)}{G_i} G_{i+1} \exp[-\alpha_{oi} f (z_{i+1} - z_i)], \tag{3}$$

where G represents geometrical spreading, and

$$\alpha_{oi} = \pi/(v_i Q_i) \tag{4}$$

is the intrinsic seismic attenuation coefficient. In general, the velocity v_i and attenuation coefficient α_{oi} vary with depth z_i. In our model, we consider radial variation but no vertical variation. The subscript "i" indicates the i^{th} receiver in a receiver array. As noticed by some authors (e.g., Page 144 in Paillet and Cheng, 1991), the geometrical spreading of this refracted head wave is highly frequency dependent. But how the geometrical spreading depends on frequency is not clear. In this study we assume that

$$G_{i+1} = G(f, z_{i+1}) \sim \frac{\exp(-\alpha_g f z_{i+1})}{z_{i+1}^p} \tag{5}$$

where α_g is an attenuation coefficient related to frequency-dependent geometrical spreading, and p is the power of "true" geometric spreading factor. Combining Eqns (3) and (5) we have

$$R_{i+1}(f) = (\frac{z_i}{z_{i+1}})^p R_i(f) \exp[-f(\alpha_g + \alpha_{oi})(z_{i+1} - z_i)] \tag{6}$$

We will empirically determine α_g and p from numerical simulation for a given borehole structure. We use two techniques to estimate the intrinsic attenuation coefficient α_o. The first one is frequency shift method (Quan and Harris, 1993). In this method we define the spectral centroid of $R_i(f)$ to be

$$f_i = \frac{\int f R_i(f) df}{\int R_i(f) df}, \tag{7}$$

and its variance to be

$$\sigma_i^2 = \frac{\int (f - f_i)^2 R_i(f) df}{\int R_i(f) df}. \tag{8}$$

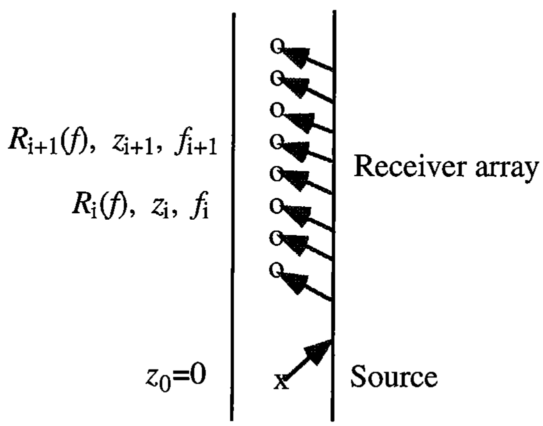


Figure 1. Sonic log geometry. $R_i(f)$ is the signal spectrum at the i^{th} reciver, z_i is the receiver depth and f_i is the centroid frequency of $R_i(f)$ defined in Eqn (7).

Then, the intrinsic attenuation coefficient α_{oi} at depth z_i can be obtained from following equation

$$\alpha_{oi} = \frac{1}{\sigma_i^2} \frac{\Delta f_i}{\Delta z_i} - \alpha_g. \tag{9}$$

where $\Delta f_i = f_i - f_{i+1}$ is the frequency down-shifting between two receivers, and $\Delta z_i = z_{i+1} - z_i$. Another method is based on the amplitude ratio. Rearranging Eqn (6) and taking the logarithm we obtain

$$\alpha_{oi} = \frac{1}{f \Delta z_i} \log[\frac{R_i(f) z_i^p}{R_{i+1}(f) z_{i+1}^p}] - \alpha_g. \tag{10}$$

In the next section we use Eqns (9) and (10) to estimate the attenuation coefficient α_{oi} from synthetic seismograms obtained for the sonic log geometry.

NUMERICAL SIMULATIONS

Simple boreholes

Let us first consider a simple open borehole whose model parameters are shown in Figure 2. Using Eqn (1) we obtain the seismograms in Figure 3. Then we perform a Fourier transform to the first arrival (*P*-wave) that is just covered by a short time window and use Eqn (7) to calculate the spectral centroid (Curve A in Figure 4). Changing the Q_P value in Figure 2 to Q_P = 80 and Q_P =120 we obtain Curves B and C, respectively. We then use Eqn (9) (frequency shift method) and Eqn (10) (amplitude ratio method) to estimate Q_P values. The slope of the centroid frequency curves carries information about Q_P.

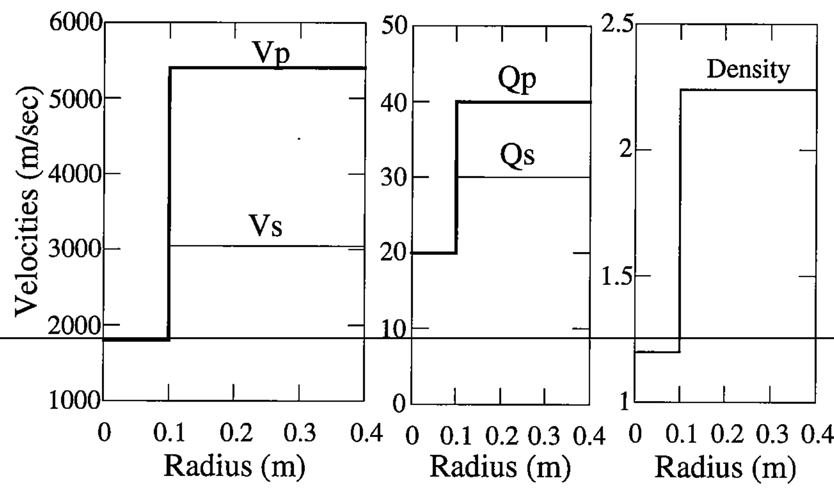


Figure 2. A simple fluid-filled open borehole.
The radius of the borehole is 0.1 m.

If we ignore the effect of frequency-dependent spreading ($\alpha_g = 0$), then from Eqns (9) (frequency shift method only) we estimate $\tilde{Q}_P$= 37, 69, and 95 which correspond to the given values Q_P = 40, 80, and 120, respectively. Here, Eqn (4) is used to convert α_o into Q. Obviously, the estimated $\tilde{Q}_P$'*s* are systematically smaller than the given Q_P's. If we do not ignore the effect of frequency-dependent spreading and find α_g from a measurement then use it as correction for all the measurements, we obtain the *corrected*

$\tilde{Q}_p$= 40, 82, and 120. The correction $\alpha_g = \pi/(vQ_g)$ with $Q_g = 450$ is found by solving Eqn (9) from a single measurement.

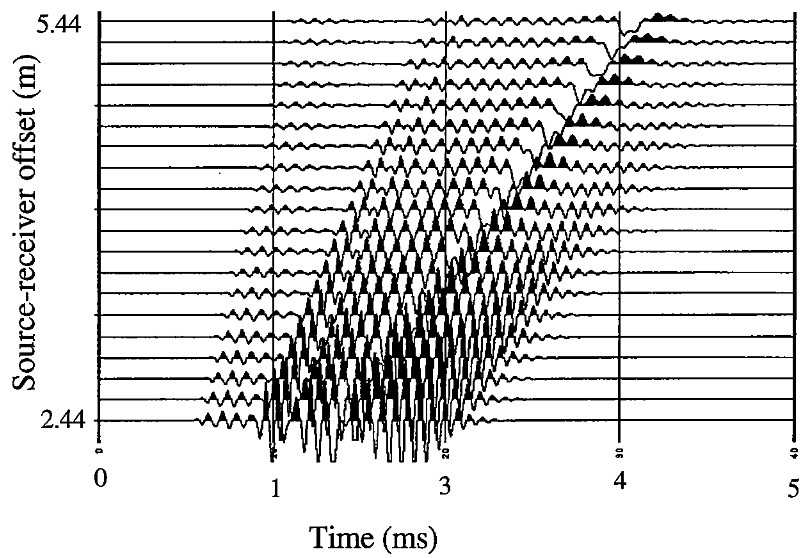


Figure 3. Synthetic seismograms in a simple borehole shown in Figure 1.

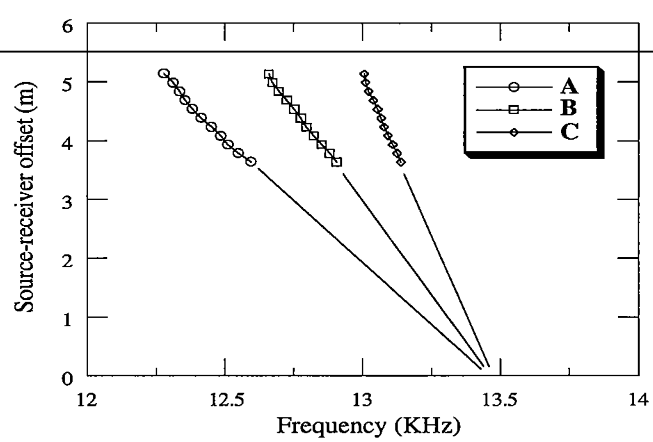


Figure 4. Centroid frequency picks corresponding to Q_p=40 (Curve A), Q_p=80 (Curve B) and Q_p=120 (Curve C).

Let us now consider the same synthetic data using the amplitude ratio method. Choosing the same value of α_g obtained above and setting power p in Eqn (10) to be 0.87, we obtain the estimated $\tilde{Q}_p$= 40, 80 and 119, respectively. If we set $\alpha_g = 0$ and $p =$ 1, equivalent to the conventional amplitude ratio method (e.g., Cheng et al, 1982), we obtain $\tilde{Q}_p$= 43, 93, and 148, respectively.

Boreholes with Mudcake and Invaded Zone

The borehole model shown in Figure 2 is idealized. We design a more realistic model which consists of a fluid-filled borehole, a mudcake and an invaded zone. Since the modeling algorithm can efficiently handle an arbitrary number of radial layers, we use ten thin layers to model the invaded zone (Figure 5). The Q_p of the invaded zone varies from 35 to 40. Figure 6 shows the seismograms calculated using this model. For a closer comparison we pick out the first traces (nearest source-receiver offset) from Figures 3 and 6 and plot them together in Figure 7. The mudcake and invaded zone exhibit a significant effect on the wave trains. The amplitude of the first arrival (P-wave) becomes relatively higher when the invaded zone is present. The attenuation study in the following paragraph shows more detailed effects of the invaded zone.

We also change the Q_p of the invaded zone in Figure 6 to be 75-80 and calculate another data set. Using Eqn (9) (frequency shift method) with $\alpha_g = 0$, we obtain the estimated $\tilde{Q}_p$= 16.5 and 16.7, which correspond to the given values of Q_p = 35-40 and 75-80, respectively. If we use Eqn (10) (amplitude ratio method) with $\alpha_g = 0$ and $p = 1$, then the estimated $\tilde{Q}_p$= 78 and 747, which again correspond to the given values of 35-40 and 75-80, respectively. However, if we choose $\alpha_g = 0$ and $p = 0.6$ in Eqn (10), we obtain the estimated $\tilde{Q}_p$= 41 and 78 which are very close to the predicted values. These experiments show that the geometrical spreading in a borehole with an invaded zone is more complex than in a simple borehole. We can not simply use $p = 1$ in Eqn (5) to describe the geometrical spreading effects in a complex borehole. The frequency-dependent term $\exp(-\alpha_g f z_{i+1})$ in Eqn (5) may be too simple, since we could not find a single correction α_g to recover all the estimated $\tilde{Q}_p$'s to the predicted values.

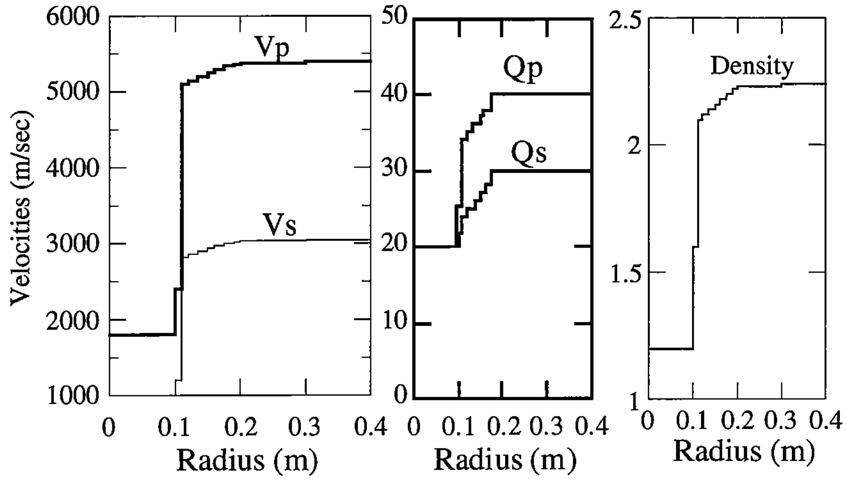


Figure 5. A borehole with mudcake and invaded zone.

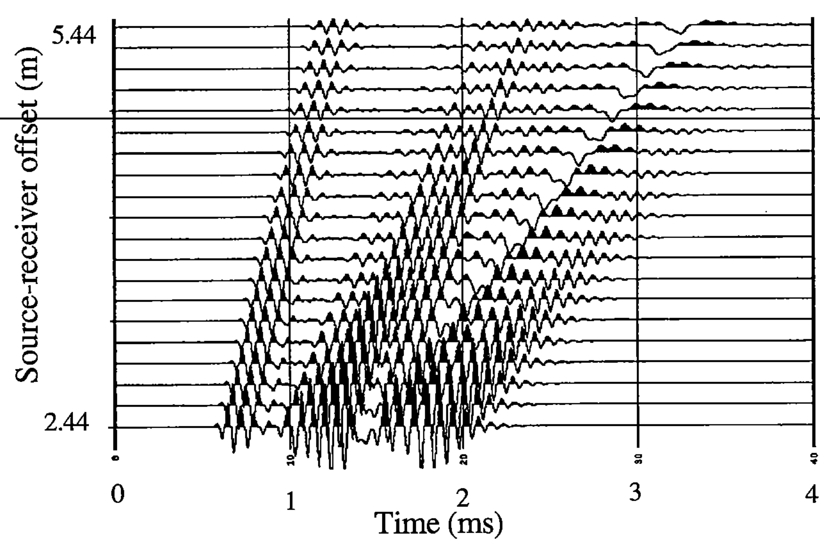
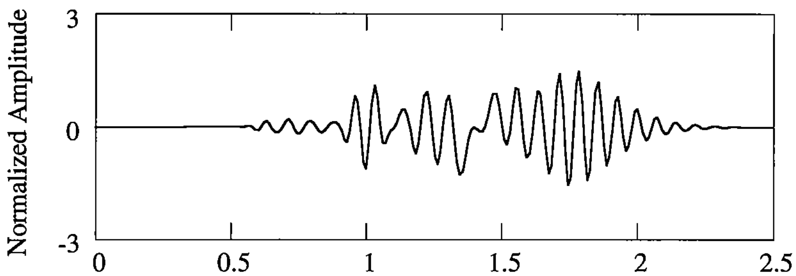
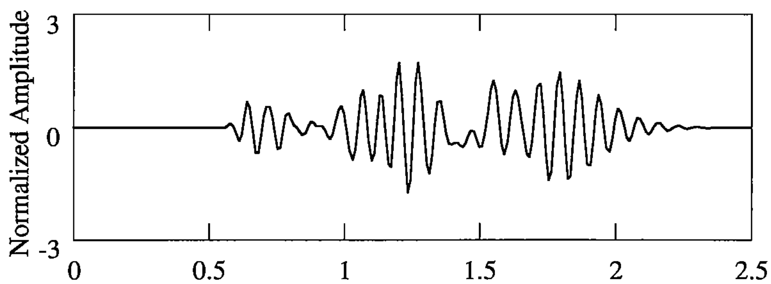


Figure 6. Synthetic seismograms in a complicated open borehole



(a) A trace from a simple open borehole.



(b) A trace from an invaded open borehole.

Figure 7. Comparison of seismograms calculated from simple and complicated boreholes.

DISCUSSIONS

The examples in the previous section indicate the attenuation property of P-waves. The estimated results, more or less, are affected by window length and window types used to calculate attenuation. For the purpose of reducing the effect of window length and the contamination of later wave trains we only use the seismograms for source to receiver separation greater than 3.5 m in Figures (3) and (6).

The frequency shift method is strongly affected by α_g, the attenuation coefficient related to the frequency-dependent geometrical spreading (see Eqn (5) for definition). It is independent of the "true' geometrical spreading factor $1/z^p$.

The test on the borehole with an invaded zone shows that if we set $\alpha_g = 0$ and $p = 1$, the Q_p value estimated from centroid frequency shift method (Eqn 9) is too small, and the Q_p value estimated from amplitude ratio method (Eqn 10) is too large. The amount of the difference between them may carry information about borehole structures, for example, the depth of the invaded zone.

CONCLUSIONS

The generalized reflection/transmission coefficients method is applied to numerically investigate seismic wave attenuation in simple boreholes and complicated boreholes with invaded zones. In order to measure the intrinsic attenuation we need to remove the geometrical spreading effect which is complex and highly frequency-dependent. If we use the frequency shift method for a simple borehole, we can introduce a simple geometrical spreading factor to do the correction. For a complicated borehole we need more work to figure out the correction term. For the amplitude ratio method the power p in geometrical spreading factor $1/z^p$ can be different from 1.

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