# PAPER L

# *SEISMOGRAM SYNTHESIS FOR RADIALLY MULTI-LAYERED MEDIA USING THE GENERALIZED R/T COEFFICIENTS METHOD*

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## *ABSTRACT*

A new method based on generalized reflection and transmission coefficients is proposed to calculate the waves in radially multi-layered media. The method is used to simulate full waveform sonic logs in cemented and cased boreholes and crosswell seismic profiles in situations where we need to consider borehole effects. Our formulations are renormalized; therefore, they are more stable and efficient than previous methods for numerical computation. The new formulations are tested by comparing our numerical results with available previous work, showing excellent agreement.

## *INTRODUCTION*

Sonic logging in cased boreholes is useful for evaluating the quality of the cement job. Also, effort is being made to measure the formation properties from full waveform sonic logs run in cased boreholes. A cased borehole and any near borehole alteration can be modeled as a radially layered medium. Study of waves in a radially layered medium is useful for understanding and interpreting full waveform sonic logs. Tubman et al. (1984) used the Thomson-Haskell method for modeling the cased borehole. We propose a new approach using generalized reflection and transmission (R/T) coefficients to solve this problem. The generalized R/T coefficients method is widely used in modeling the elastic waves in vertically layered media because of its computational stability and efficiency over the Thomson-Haskell method, especially for high frequency problems (see, e.g., Luco & Apsel, 1983; Kennett, 1983; Chen, 1993). Yao & Zheng (1985) derived a set of formulas for computing synthetic seismograms in radially layered borehole environments using the generalized R/T coefficient method, but they did not conduct numerical tests to check their

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formulation. We derive a set of alternative formulations which are more stable and efficient than the previous methods for numerical computation. First, We introduce the concepts of modified and generalized reflection and transmission (R/T) matrices for the radially layered media and derive a recursive scheme to calculate them. Then, we determine the wave fields using the R/T matrices. To check the formulation, we compare our calculated results with those of Cheng and Toksoz (1981) for a two-layer open borehole model, of Tubman (1984) for a four-layer cased hole model. The comparison shows a good agreement. Quan et al. (1994) used this method to investigate acoustic attenuation logging and geometrical spreading of the P head wave in boreholes with mudcake and invaded zone.

## *GOVERNING EQUATIONS AND THEIR GENERAL SOLUTIONS*

The radially multi-layered model considered in this study is shown in Figure 1. The first layer ( $r < r^{(1)}$) is fluid. For this axially symmetrical problem, there only exist *P* and *SV* waves, i.e., the displacement can be written as

$$\mathbf{u} = \nabla\phi + \nabla \times (\mathbf{e}_\theta \psi). \tag{1}$$

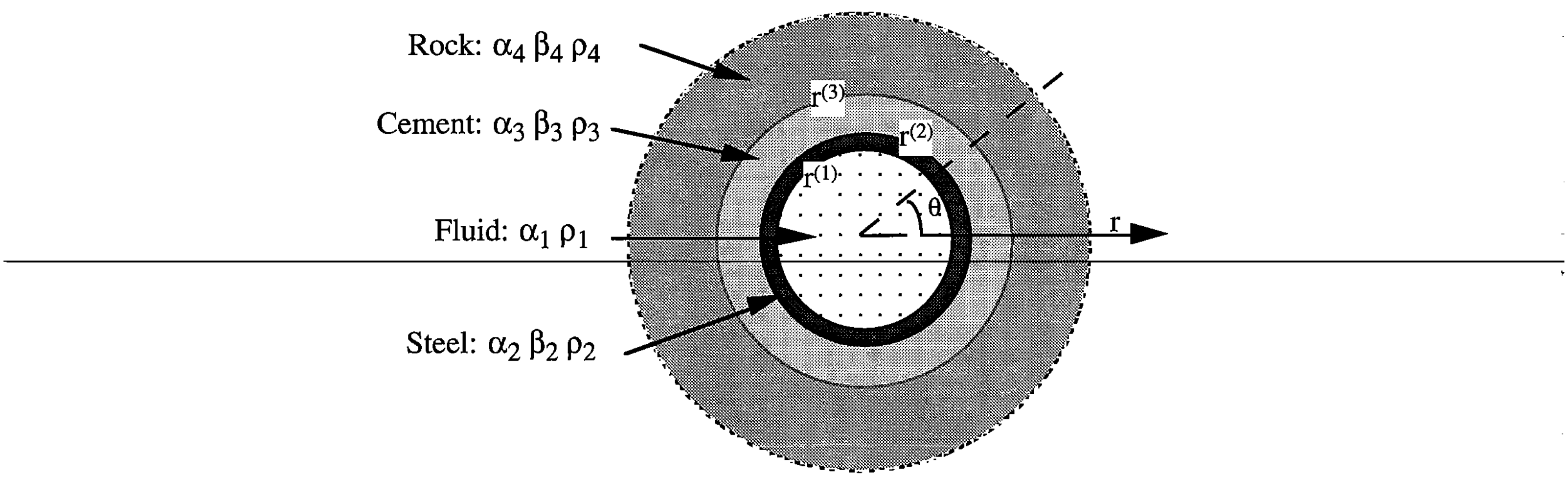


Figure 1. A radially multi-layered model.

Substituting Eqn (1) into the elastodynamic equation in the frequency domain we obtain

$$\frac{\partial^2\phi^{(j)}}{\partial r^2} + \frac{1}{r}\frac{\partial\ \phi^{(j)}}{\partial r} + \frac{\partial^2\phi^{(j)}}{\partial z^2} + k_\alpha^{(j)2}\phi^{(j)} = F(\omega)\frac{\delta(r)\delta(z)}{2\pi r}\delta_{j1}, \tag{2a}$$

$$\frac{\partial^2\psi^{(j)}}{\partial r^2}+\frac{1}{r}\frac{\partial\psi^{(j)}}{\partial r}-\frac{\psi^{(j)}}{r^2}+\frac{\partial^2\psi^{(j)}}{\partial z^2}+k_\beta^{(j)^2}\psi^{(j)}=0, \tag{2b}$$

for $r^{(j-1)}<r<r^{(j)}$, $j=1,2,...,N$, (define $r^{(0)}=0$). In Equation (1), $k_\alpha^{(j)}=\omega/\alpha^{(j)}$, $k_\beta^{(j)}=\omega/\beta^{(j)}$, $F(\omega)$ is the source spectrum, α and β are *P*-wave and S-wave velocities, respectively. In the $k-\omega$ domain, the solutions of Eqns (2a) and (2b) are

$$\tilde{\phi}_k^{(j)}(r)=c_{p_-}^{(j)}e^{iv_\alpha^{(j)}(r^{(j)}-r)}\overline{H}_o^{(2)}(v_\alpha^{(j)}r)+c_{p_+}^{(j)}e^{iv_\alpha^{(j)}(r-r^{(j-1)})}\overline{H}_o^{(1)}(v_\alpha^{(j)}r), \tag{3}$$

$$\tilde{\psi}_k^{(j)}(r)=c_{s_-}^{(j)}e^{iv_\beta^{(j)}(r^{(j)}-r)}\overline{H}_1^{(2)}(v_\beta^{(j)}r)+c_{s_+}^{(j)}e^{iv_\beta^{(j)}(r-r^{(j-1)})}\overline{H}_1^{(1)}(v_\beta^{(j)}r), \tag{4}$$

where $v_\alpha^{(j)}=\sqrt{k_\alpha^{(j)2}-k^2}$, $\mathrm{Im}\{v_\alpha^{(j)}\}>0$, $v_\beta^{(j)}=\sqrt{(k_\beta^{(j)})^2-k^2}$, $\mathrm{Im}\{v_\beta^{(j)}\}>0$, and $j$ =2,3, ..., N, N+1. $\overline{H}_n^{(1)}(x)=e^{-ix}H_n^{(1)}(x)$ and $\overline{H}_n^{(2)}(x)=e^{ix}H_n^{(2)}(x)$ are the first and second kind *renormalized* Hankel functions of $n^{\text{th}}$ order, respectively. $c_{p_-}^{(j)},c_{p_+}^{(j)},c_{s_-}^{(j)}$ and $c_{s_+}^{(j)}$ are unknowns, where "+" refers to outgoing waves and "-" refers to incoming waves. Instead of the usual Hankel functions, we used the renormalized Hankel functions whose asymptotic behavior for *large argument* is very gentle for expressing our solutions. For $j$ = 1, in the fluid-filled borehole, we have

$$\begin{aligned}\tilde{\phi}_k^{(1)}(r)&=cJ_o(v_\alpha^{(1)}r)-\frac{i}{8\pi}F(\omega)H_o^{(1)}(v_\alpha^{(1)}r)\\&=c_-^{(1)}e^{iv_\alpha^{(1)}(r^{(1)}-r)}\overline{H}_o^{(2)}(v_\alpha^{(1)}r)+(c_+^{(1)}+s_+)e^{iv_\alpha^{(1)}r}\overline{H}_o^{(1)}(v_\alpha^{(1)}r)\end{aligned} \tag{5}$$

where

$$c_+^{(1)}=c_-^{(1)}e^{iv_\alpha^{(1)}r^{(1)}} \text{ and } s_+=-\frac{i}{8\pi}F(\omega). \tag{6}$$

Using above potential solutions we obtain the displacements and stresses in the cylindrical layers ($j$ > 1) as

$$\begin{bmatrix}\tilde{u}_k^{(j)}(r)\\\tilde{v}_k^{(j)}(r)\\\tilde{\sigma}_k^{(j)}(r)\\\tilde{\tau}_k^{(j)}(r)\end{bmatrix}=\begin{bmatrix}e_{11}^{(j)}(r)&e_{12}^{(j)}(r)&e_{13}^{(j)}(r)&e_{14}^{(j)}(r)\\e_{21}^{(j)}(r)&e_{22}^{(j)}(r)&e_{23}^{(j)}(r)&e_{24}^{(j)}(r)\\e_{31}^{(j)}(r)&e_{32}^{(j)}(r)&e_{33}^{(j)}(r)&e_{34}^{(j)}(r)\\e_{41}^{(j)}(r)&e_{42}^{(j)}(r)&e_{43}^{(j)}(r)&e_{44}^{(j)}(r)\end{bmatrix}\begin{bmatrix}c_{p_-}^{(j)}\\c_{s_-}^{(j)}\\c_{p_+}^{(j)}\\c_{s_+}^{(j)}\end{bmatrix}, \tag{7}$$

where

$$e_{11}^{(j)} = -v_\alpha^{(j)} \overline{H}_1^{(2)}(v_\alpha^{(j)} r) e^{iv_\alpha^{(j)}(r^{(j)}-r)}, \qquad e_{12}^{(j)} = -ik\overline{H}_1^{(2)}(v_\beta^{(j)} r) e^{iv_\beta^{(j)}(r^{(j)}-r)},$$

$$e_{13}^{(j)} = -v_\alpha^{(j)} \overline{H}_1^{(1)}(v_\alpha^{(j)} r) e^{iv_\alpha^{(j)}(r-r^{(j-1)})}, \qquad e_{14}^{(j)} = -ik\overline{H}_1^{(1)}(v_\beta^{(j)} r) e^{iv_\beta^{(j)}(r-r^{(j-1)})},$$

$$e_{21}^{(j)} = ik\overline{H}_o^{(2)}(v_\alpha^{(j)} r) e^{iv_\alpha^{(j)}(r^{(j)}-r)}, \qquad e_{22}^{(j)} = v_\beta^{(j)} \overline{H}_o^{(2)}(v_\beta^{(j)} r) e^{iv_\beta^{(j)}(r^{(j)}-r)},$$

$$e_{23}^{(j)} = ik\overline{H}_o^{(1)}(v_\alpha^{(j)} r) e^{iv_\alpha^{(j)}(r-r^{(j-1)})}, \qquad e_{24}^{(j)} = v_\beta^{(j)} \overline{H}_o^{(1)}(v_\beta^{(j)} r) e^{iv_\beta^{(j)}(r-r^{(j-1)})},$$

$$e_{31}^{(j)} = 2\mu^{(j)}[\Omega^{(j)} \overline{H}_o^{(2)}(v_\alpha^{(j)} r) + v_\alpha^{(j)} \overline{H}_1^{(2)}(v_\alpha^{(j)} r)/r] e^{iv_\alpha^{(j)}(r^{(j)}-r)},$$

$$e_{32}^{(j)} = -ik\mu^{(j)} v_\beta^{(j)} [\overline{H}_o^{(2)}(v_\beta^{(j)} r) - \overline{H}_2^{(2)}(v_\beta^{(j)} r)] e^{iv_\beta^{(j)}(r^{(j)}-r)},$$

$$e_{33}^{(j)} = 2\mu^{(j)}[\Omega^{(j)} \overline{H}_o^{(1)}(v_\alpha^{(j)} r) + v_\alpha^{(j)} \overline{H}_1^{(1)}(v_\alpha^{(j)} r)/r] e^{iv_\alpha^{(j)}(r-r^{(j-1)})},$$

$$e_{34}^{(j)} = -ik\mu^{(j)} v_\beta^{(j)} [\overline{H}_o^{(1)}(v_\beta^{(j)} r) - \overline{H}_2^{(1)}(v_\beta^{(j)} r)] e^{iv_\beta^{(j)}(r-r^{(j-1)})},$$

$$e_{41}^{(j)} = -2ik\mu^{(j)} v_\alpha^{(j)} \overline{H}_1^{(2)}(v_\alpha^{(j)} r) e^{iv_\alpha^{(j)}(r^{(j)}-r)},$$

$$e_{42}^{(j)} = 2\mu^{(j)} \Omega^{(j)} \overline{H}_1^{(2)}(v_\beta^{(j)} r) e^{iv_\beta^{(j)}(r^{(j)}-r)},$$

$$e_{43}^{(j)} = -2ik\mu^{(j)} v_\alpha^{(j)} \overline{H}_1^{(1)}(v_\alpha^{(j)} r) e^{iv_\alpha^{(j)}(r-r^{(j-1)})},$$

$$e_{44}^{(j)} = 2\mu^{(j)} \Omega^{(j)} \overline{H}_1^{(1)}(v_\beta^{(j)} r) e^{iv_\beta^{(j)}(r-r^{(j-1)})},$$

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and, $\Omega^{(j)} = k^2 - \frac{1}{2} k_\beta^{(j)2}$ and $r^{(j-1)} < r < r^{(j)}$. For the first layer, i.e., the liquid layer, we have

$$\begin{bmatrix} \tilde{u}_k^{(1)}(r) \\ \tilde{\sigma}_k^{(1)}(r) \end{bmatrix} = \begin{bmatrix} e_{11}^{(1)}(r) & e_{12}^{(1)}(r) \\ e_{21}^{(1)}(r) & e_{22}^{(1)}(r) \end{bmatrix} \begin{bmatrix} c_{p-}^{(1)} \\ c_{p+}^{(1)} \end{bmatrix}, \tag{8}$$

where

$$e_{11}^{(1)} = -v_\alpha^{(1)} \overline{H}_1^{(2)}(v_\alpha^{(1)} r) e^{iv_\alpha^{(1)}(r^{(1)}-r)}, \qquad e_{12}^{(1)} = -v_\alpha^{(1)} \overline{H}_1^{(1)}(v_\alpha^{(1)} r) e^{iv_\alpha^{(1)} r},$$

$$e_{21}^{(1)} = -\lambda^{(1)} k_\alpha^{(1)^2} \overline{H}_0^{(2)}(v_\alpha^{(1)} r) e^{iv_\alpha^{(1)}(r^{(1)}-r)}, \qquad e_{22}^{(1)} = -\lambda^{(1)} k_\alpha^{(1)^2} \overline{H}_0^{(1)}(v_\alpha^{(1)} r) e^{iv_\alpha^{(1)} r}$$

and $c_{p-}^{(1)} = c_-^{(1)}$, $c_{p+}^{(1)} = c_+^{(1)} + s_+$.

## *R/T MATRICES AND SOLUTION SYNTHESIS*

In the preceding section, we obtained the general solutions with unknowns, $c_{p_-}^{(j)}, c_{p_+}^{(j)}, c_{s_-}^{(j)}$ and $c_{s_+}^{(j)}$ for $j$ = 2, 3, ..., N+1, and $c_-^{(1)}$. These unknowns can be determined by imposing boundary conditions at each interface. For the first interface (liquid-solid boundary), the boundary conditions are

$$\begin{bmatrix} \tilde{u}_k^{(1)}(r^{(1)}) \\ \tilde{\sigma}_k^{(1)}(r^{(1)}) \end{bmatrix} = \begin{bmatrix} \tilde{u}_k^{(2)}(r^{(1)}) \\ \tilde{\sigma}_k^{(2)}(r^{(1)}) \end{bmatrix} \tag{9}$$

and

$$0 = \tilde{\tau}_k^{(2)}(r^{(1)}). \tag{10}$$

For the $j^{\text{th}}$ interface ($j$ = 2, 3,..., N) which is solid-solid boundary, we have

$$\begin{bmatrix} \tilde{u}_k^{(j)}(r^{(j)}) \\ \tilde{v}_k^{(j)}(r^{(j)}) \\ \tilde{\sigma}_k^{(j)}(r^{(j)}) \\ \tilde{\tau}_k^{(j)}(r^{(j)}) \end{bmatrix} = \begin{bmatrix} \tilde{u}_k^{(j+1)}(r^{(j)}) \\ \tilde{v}_k^{(j+1)}(r^{(j)}) \\ \tilde{\sigma}_k^{(j+1)}(r^{(j)}) \\ \tilde{\tau}_k^{(j+1)}(r^{(j)}) \end{bmatrix}. \tag{11}$$

To effectively determine the unknowns for each layer, we introduce the generalized reflection and transmission (R/T) matrices and derive their explicit expressions by using the above boundary conditions.

<u>Modified R/T matrices</u>

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The modified R/T matrices for solid-solid interfaces are defined in the relations

$$\begin{aligned} \mathbf{c}_-^{(j)} &= \mathbf{R}_{+-}^{(j)}(\mathbf{c}_+^{(j)} + \delta_{1j}s_+) + \mathbf{T}_-^{(j)}\mathbf{c}_-^{(j+1)} \\ \mathbf{c}_+^{(j+1)} &= \mathbf{T}_+^{(j)}(\mathbf{c}_+^{(j)} + \delta_{1j}s_+) + \mathbf{R}_{-+}^{(j)}\mathbf{c}_-^{(j+1)}, \end{aligned} \quad \text{for } j = 1, 2, ...., \text{N}, \tag{12}$$

where $\mathbf{c}_\pm^{(j)} = [c_{p\pm}^{(j)}, c_{s\pm}^{(j)}]^{\text{T}}$ and $\mathbf{R}_{+-}^{(j)}$, $\mathbf{R}_{-+}^{(j)}$, $\mathbf{T}_+^{(j)}$ and $\mathbf{T}_-^{(j)}$ are the modified R/T matrices for the $j$-th interface. Substituting Eqn (7) into Eqn (11), then comparing with Eqn (12) we obtain

$$\begin{bmatrix} \mathbf{R}_{+-}^{(j)} & \mathbf{T}_{-}^{(j)} \\ \mathbf{T}_{+}^{(j)} & \mathbf{R}_{-+}^{(j)} \end{bmatrix} = \begin{bmatrix} e_{11}^{(j)}(r^{(j)}) & e_{12}^{(j)}(r^{(j)}) & -e_{13}^{(j+1)}(r^{(j)}) & -e_{14}^{(j+1)}(r^{(j)}) \\ e_{21}^{(j)}(r^{(j)}) & e_{22}^{(j)}(r^{(j)}) & -e_{23}^{(j+1)}(r^{(j)}) & -e_{24}^{(j+1)}(r^{(j)}) \\ e_{31}^{(j)}(r^{(j)}) & e_{32}^{(j)}(r^{(j)}) & -e_{33}^{(j+1)}(r^{(j)}) & -e_{34}^{(j+1)}(r^{(j)}) \\ e_{41}^{(j)}(r^{(j)}) & e_{42}^{(j)}(r^{(j)}) & -e_{43}^{(j+1)}(r^{(j)}) & -e_{44}^{(j+1)}(r^{(j)}) \end{bmatrix}^{-1}$$

$$\times \begin{bmatrix} -e_{13}^{(j)}(r^{(j)}) & -e_{14}^{(j)}(r^{(j)}) & e_{11}^{(j+1)}(r^{(j)}) & e_{12}^{(j+1)}(r^{(j)}) \\ -e_{23}^{(j)}(r^{(j)}) & -e_{24}^{(j)}(r^{(j)}) & e_{21}^{(j+1)}(r^{(j)}) & e_{22}^{(j+1)}(r^{(j)}) \\ -e_{33}^{(j)}(r^{(j)}) & -e_{34}^{(j)}(r^{(j)}) & e_{31}^{(j+1)}(r^{(j)}) & e_{32}^{(j+1)}(r^{(j)}) \\ -e_{43}^{(j)}(r^{(j)}) & -e_{44}^{(j)}(r^{(j)}) & e_{41}^{(j+1)}(r^{(j)}) & e_{42}^{(j+1)}(r^{(j)}) \end{bmatrix}, \qquad (13)$$

for $j = 2, 3, \ldots, N$. Similarly, from Eqns (8), (9) and (10), we find

$$\begin{bmatrix} \mathbf{R}_{+-}^{(1)} & \mathbf{T}_{-}^{(1)} \\ \mathbf{T}_{+}^{(1)} & \mathbf{R}_{-+}^{(1)} \end{bmatrix} = \begin{bmatrix} e_{11}^{(1)}(r^{(1)}) & -e_{13}^{(2)}(r^{(1)}) & -e_{14}^{(2)}(r^{(1)}) \\ e_{21}^{(1)}(r^{(1)}) & -e_{33}^{(2)}(r^{(1)}) & -e_{34}^{(2)}(r^{(1)}) \\ 0 & -e_{43}^{(2)}(r^{(1)}) & -e_{44}^{(2)}(r^{(1)}) \end{bmatrix}^{-1}$$

$$\times \begin{bmatrix} -e_{12}^{(1)}(r^{(1)}) & e_{11}^{(2)}(r^{(1)}) & e_{12}^{(2)}(r^{(1)}) \\ -e_{22}^{(1)}(r^{(1)}) & e_{31}^{(2)}(r^{(1)}) & e_{32}^{(2)}(r^{(1)}) \\ 0 & e_{41}^{(2)}(r^{(1)}) & e_{42}^{(2)}(r^{(1)}) \end{bmatrix}. \qquad (14)$$

Generalized R/T matrices

The generalized R/T matrices, $\hat{\mathbf{R}}_{+-}^{(j)}$ and $\hat{\mathbf{T}}_{+}^{(j)}$, are defined via following relations:

$$\begin{cases} \mathbf{c}_{+}^{(j+1)} = \hat{\mathbf{T}}_{+}^{(j)}(\mathbf{c}_{+}^{(j)} + \delta_{j1}s_{+}) \\ \mathbf{c}_{-}^{(j)} = \hat{\mathbf{R}}_{+-}^{(j)}(\mathbf{c}_{+}^{(j)} + \delta_{j1}s_{+}) \end{cases} \quad \text{for } j = 1, 2, \ldots\ldots, N. \qquad (15)$$

Substituting Eqn (16) into Eqns (12) and (13) and rearranging them, we obtain a recursive relation

$$\begin{cases} \hat{\mathbf{T}}_{+}^{(j)} = [\mathbf{I} - \mathbf{R}_{-+}^{(j)}\hat{\mathbf{R}}_{+-}^{(j+1)}]^{-1}\mathbf{T}_{+}^{(j)} \\ \hat{\mathbf{R}}_{+-}^{(j)} = \mathbf{R}_{+-}^{(j)} + \mathbf{T}_{-}^{(j)}\hat{\mathbf{R}}_{+-}^{(j+1)}\hat{\mathbf{T}}_{+}^{(j)} \end{cases} \quad \text{for } j = \text{N, N-1, ..., 2,1}, \qquad (16)$$

where **I** is the unit matrix. Eqn (16) provides an efficient recursive scheme to calculate the generalized R/T matrices from the modified R/T matrices. Our formulations are numerically more stable than the previous methods (e.g., Tubman et al, 1984 and Yao & Zheng, 1985) because of the use of the renormalized Hankel functions and the renormalization factors $e^{iv_{\alpha,\beta}^{(j)}r^{(j)}}$ and $e^{-iv_{\alpha,\beta}^{(j)}r^{(j-1)}}$.

Solution synthesis

Having the generalized R/T matrices, we can compute the unknowns $\mathbf{c}_{\pm}^{(j)}$ for any layer. Thus, we can compute the displacements and stresses for any layer. In a sonic logging problem, we are interested in the stress waves in the fluid-filled borehole where the normal stress is

$$\tilde{\sigma}_k^{(1)}(r) = e_{21}^{(1)}(r)c_-^{(1)} + e_{22}^{(1)}(r)(c_+^{(1)} + s_+)$$
$$= \lambda^{(1)}k_\alpha^{(1)2}\left[\frac{2J_o(v_\alpha^{(1)}r)e^{iv_\alpha^{(1)}r^{(1)}}\hat{R}_{+-}^{(1)}}{\hat{R}_{+-}^{(1)}e^{iv_\alpha^{(1)}r^{(1)}}-1} - H_o^{(1)}(v_\alpha^{(1)}r)\right]s_+. \quad (17)$$

Taking inverse Fourier transforms over $k$ and ω, we obtain the solution in spatial and time domains as

$$\sigma^{(1)}(r,z,t) = \frac{\rho^{(1)}}{4\pi}\int_{-\infty}^{+\infty}\omega^2 F(\omega)e^{-i\omega t}\{\int_{-\infty}^{+\infty} i\frac{J_o(v_\alpha^{(1)}r)e^{iv_\alpha^{(1)}r^{(1)}}\hat{R}_{+-}^{(1)}}{1-\hat{R}_{+-}^{(1)}e^{iv_\alpha^{(1)}r^{(1)}}}e^{ikz}dk$$
$$+\frac{1}{R}e^{ik_\alpha^{(1)}R}\}d\omega. \quad (18)$$

where, $R = \sqrt{r^2 + z^2}$.

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## *NUMERICAL IMPLEMENTATION*

The discrete wave-number technique (Bouchon & Aki, 1977) and FFT are used to numerically evaluate the $k-\omega$ integral in Eqn (18). A complete program package is written to implement our new algorithm. To check the validity of our formulation and program, we test a two-layer open hole model chosen from Cheng and Toksoz (1981) and a four-layer cased borehole model chosen from Tubman et al. (1984). The spectrum of source function is described by

$$F(\omega) = \frac{1}{\omega^2}\frac{8\alpha\omega_o(\alpha - i\omega)}{[(\alpha - i\omega)^2 + \omega_o^2]^2},$$

and attenuation is introduced through complex velocity defined by

$$v(\omega) = v(\omega_{ref})[1 + \frac{1}{\pi Q}\log(\frac{\omega}{\omega_{ref}}) - \frac{i}{2Q}],$$

where $Q$ is the quality factor for either P-wave or S-wave, and $v$ is either P-wave velocity or S-wave velocity.

The seismograms computed by the generalized R/T matrix method are shown in Figures 2 and 3. Figures 4 and 5 are taken from the papers mentioned above, respectively. Comparisons of our result with theirs show very good agreement. The model parameters used in Figures 2 and 4 are shown in Table 1. The parameters of the source function are: $\omega_o=2\pi$ x15000Hz and $\alpha= 0.5\omega_o/\pi$. The source-receiver separation is 2.44 m. For Figures 3 and 5, the model is referred to Figure 1 whose parameters are given in Table 2. The source-receiver separation is 3.048m. The parameters of the source function are: $\omega_o=2\pi$ x 13000Hz and $\alpha=0.5\omega_o/\pi$.

| Layer | $r$ (cm) | $\alpha$ (km/s) | $\beta$ (km/s) | $\rho$ (g/cm$^3$) | $Q_p/Q_s$ |
|---|---|---|---|---|---|
| Fluid | 10.2 | 1.83 | – | 1.2 | $\infty$ |
| formation | $\infty$ | 5.94 | 3.05 | 2.3 | $\infty$ |

Table 1. Model parameters used for Figures 2 and 4.

| Layer | $r$ (cm) | $\alpha$ (km/s) | $\beta$ (km/s) | $\rho$ (g/cm$^3$) | $Qp$ | $Qs$ |
|---|---|---|---|---|---|---|
| Fluid | 4.7 | 1.68 | – | 1.2 | 20 | – |
| Casing | 5.72 | 6.1 | 3.35 | 7.5 | 1000 | 1000 |
| Cement | 10.2 | 2.82 | 1.73 | 1.92 | 40 | 30 |
| Formation | $\infty$ | 4.88 | 2.6 | 2.16 | 60 | 60 |

Table 2. Model parameters used for Figures 3 and 5.

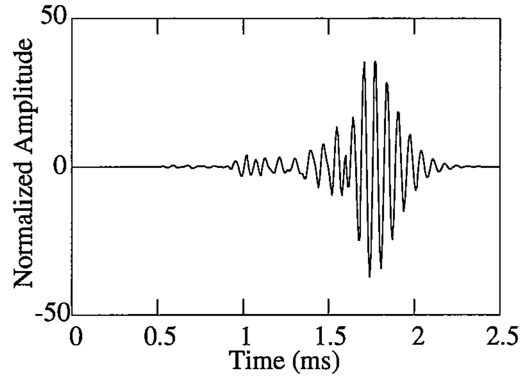


Figure 2. Seismogram calculated by the generalized R/T coefficients method using a *two-layer* open hole model.

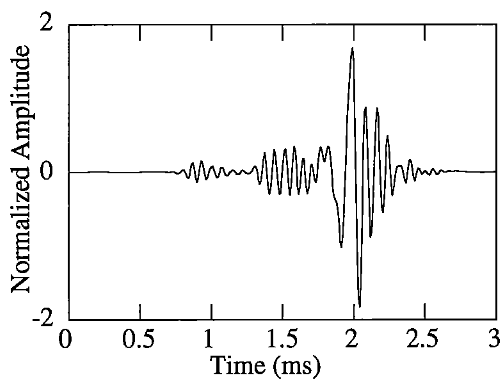


Figure 3. Seismogram calculated by the generalized R/T coefficients method using a *four-layer* open hole model.

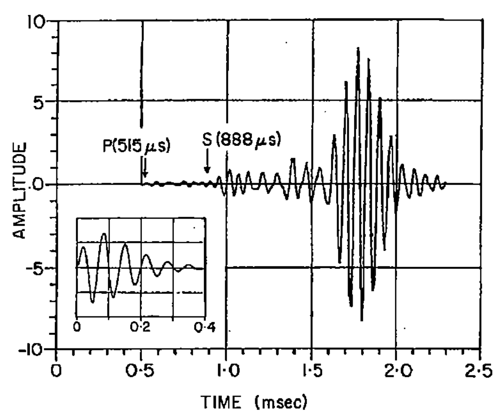


Figure 4. Seismogram taken from Page 1047 in Cheng & Toksoz (1981) for comparison with Figure 2.

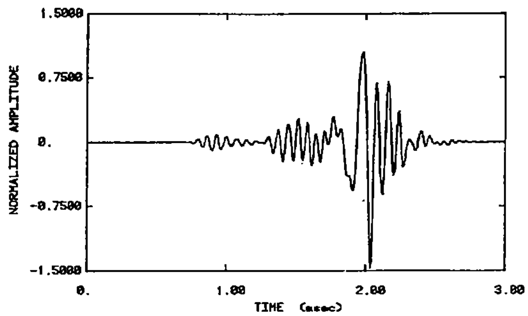


Figure 4. Seismogram taken from Page 1055 in Tubman (1984) for comparison with Figure 3.

## CONCLUSIONS

An approach based on generalized reflection and transmission coefficients is developed to calculate waves in radially layered media. The renormalized Hankel functions and the renormalization factors are introduced to make the numerical procedure more stable. Aside from the examples presented in the previous section, we have also successfully simulated crosswell seismic profiles and the open hole with invaded zone using this method.

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