PAPER C

CALCULATING FRESNEL ZONES FOR CROSSWELL TOMOGRAPHY

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ABSTRACT

Ray-theoretic tomography relies on the high frequency approximation of ray theory. Using ray theory, the traveltime from a source to a receiver is simply the line integral of slowness along the raypath. In practical applications the high frequency assumption of ray theory is often incorrect. In these cases the traveltime should be calculated as a function of the slowness within a wavepath.

The first step in wavepath tomography is defining the wavepath. This can be a difficult task in a general 2-dimensional medium. I define the Fresnel volume wavepaths numerically using an energetic-arrival finite-differences eikonal solver. Traveltime maps calculated in a 2-dimensional medium for the source and receiver are added together and any traveltime on this combined map falling within $1/2T$ of the energetic arrival defines a point lying within the Fresnel volume. Here, T is the period of the source wavelet. The advantage of using the energetic arrival solver is that critically refracted energy is ignored in the wavepath and traveltime calculations. This is a useful feature because forward-modeling critical refractions is inherently inaccurate in the non-linear traveltime tomography inversion. The use of the energetic arrival eikonal solver provides a fast, robust technique to calculate Fresnel wavepaths.

I use these wavepaths in a simple tomographic application. In this application the wavepaths are used as the basis for backprojecting over an area rather than a line. This builds a smoothing criterion into the tomographic inversion.

INTRODUCTION

The technique most commonly used to process crosswell seismic data is traveltime tomography. Seismic traveltime tomography uses the source-to-receiver propagation times for multiple source and receiver positions to estimate the velocity structure of a surveyed zone. The general approach to solving this inversion problem is to perturb an assumed velocity model to minimize the difference between the traveltimes calculated using the assumed model and the observed traveltimes. There are two particularly important parts in this inversion. First is the forward modeling step, e.g., calculating traveltimes through the assumed model. Second is the backprojection technique. The backprojection technique is the formulation used to update the velocity model to minimize the difference between the calculated and observed traveltimes.

The most popular method used for calculating traveltimes through the velocity model is "ray-tracing". The use of ray-tracing implies a fundamental assumption of high frequency and invokes Snell's law along the path to calculate a ray trajectory. To satisfy the infinite frequency assumption, the spatial variation of velocity must be small compared with the wavelength of the seismic source. The "mathematical ray" resulting from the infinite frequency assumption of ray theory is a trajectory with zero volume, and the source-to-receiver traveltime is the line integral of slowness along this trajectory.

Often the high frequency assumption is not valid for a particular experiment. A typical approach in this situation is to apply a smoothing criterion to solve for only the part of the velocity field that does satisfy the high frequency assumption. The smoothing can be applied to the velocity model between iterations (Nemeth et al., 1993), or it can be built into the solution of the linear set of equations with techniques such as "convolutional quelling" (Meyerholtz et al., 1989) and damped least squares (Menke, 1984). A different approach is to apply the smoothing in the parameterization of the inverse problem. In this case the model is parameterized in terms of smooth basis functions (Michelena and Harris, 1991).

Woodward (1989) and Harlan (1990) have followed the idea of *wavepaths* based on Fresnel regions. Woodward, following work done by Hagedoorn (1954), replaces raypaths used in tomography with wavepaths calculated using the Rytov approximation. The calculation of these wavepaths is not always straightforward since it requires solutions for Green's functions in a 2-dimensional medium. Harlan estimates wavepaths by solving for all paths within a model for a given source-receiver geometry which do not exceed the minimum Fermat time by more than half a temporal wavelength. This technique has the advantage that it is easy to conceptualize and implement.

In this paper I describe a technique similar to Harlan's for determining wavepaths. The advantage of this technique is that it uses a finite-differences eikonal solver designed to calculate the traveltimes of *energetic* arrivals (Mo, 1994). Calculating only the energetic arrivals avoids problems which can result when the traveltimes of critical refractions, or head waves, are used. To illustrate one potential application, I use of the wavepaths as a smoothing function in a crosswell traveltime tomographic inversion.

DEFINING FRESNEL WAVEPATHS

The principle of Huygens-Fresnel states that any path, from source to receiver, that has a traveltime within half the period of the source wavelet of the minimum path time, contributes to the first arrival. This principle is just another way of expressing the idea that a wave traveling along the trajectory of a raypath is influenced not only by the velocities along that path, but also by the velocities in the near vicinity. I will use this idea as a basis to define wavepaths for use in backprojection in the crosswell tomographic inversion.

I introduce one difference into the wavepath definition: the minimum path time used is for the energetic travel path. Typically the minimum, or Fermat, traveltime is used in tomography. This is done because it is much easier to uniformly identify first arrivals in a crosswell data set than direct arrivals. Unfortunately, this philosophy can lead to several problems when high velocity contrasts lead to first arrival traveltimes from critical refractions.

Critical refractions typically result from raypaths which are at some point defined by an interface of fast and slow media. Crosswell transmission traveltime tomography, in general, lacks the resolution required to define these interfaces. For this reason the forward-modeled raypath will seldom be correct. Another related problem results from the inherent non-linearity of the traveltime inversion problem. The raypath is required to estimate the traveltime through the model and to define the path of backprojection. Unfortunately, this raypath is a function of the velocity model that is being calculated. As with many non-linear problems, the system is linearized by using an estimate of the velocity model to determine the raypaths and calculated traveltimes. This estimated model is improved and used in the next iteration as the new velocity estimate. Hopefully, the velocity model eventually converges to an accurate solution. However, until this velocity structure is found with a fair degree of accuracy, the conditions required to trace the refracted path will not be present. Another ambiguity lies in whether any particular critically refracted event comes from above or below the receiver.

For all these reasons I focus on using only the direct arrival traveltimes. One drawback of this philosophy is that it requires interpreting the data while picking traveltimes to avoid picking the traveltimes of critically refracted events. With the aid of local geologic knowledge, model based pick estimates, and personal expertise (acquired through practice), this is not an impossible problem. In my implementation, the Fresnel wavepath is the region defined by a set of point meeting the following criterion: a point falls within the Fresnel wavepath if the traveltime of a wave traveling from the source to that point, plus the traveltime from that point to the receiver, is less than or equal to the minimum direct arrival traveltime plus one half the period of the source wavelet.

CALCULATING FRESNEL WAVEPATHS

The key to efficiently computing the Fresnel wavepaths is the energetic-arrival finite-differences eikonal solver (Mo, 1994). This solver is similar to the one developed by Vidale (1988) except that it computes the arrival times of the direct arriving energy and ignores critically refracted waves. Details of how this algorithm works can be found in Paper K, this volume.

Operationally, the eikonal solver runs using a 2-D gridded slowness model and a starting location, or source point. The solver then produces a traveltime map with the same dimensions as the original slowness map except that each node of the map represents the traveltime from the starting point to that node instead of the local slowness.

Fresnel Wavepath Calculation

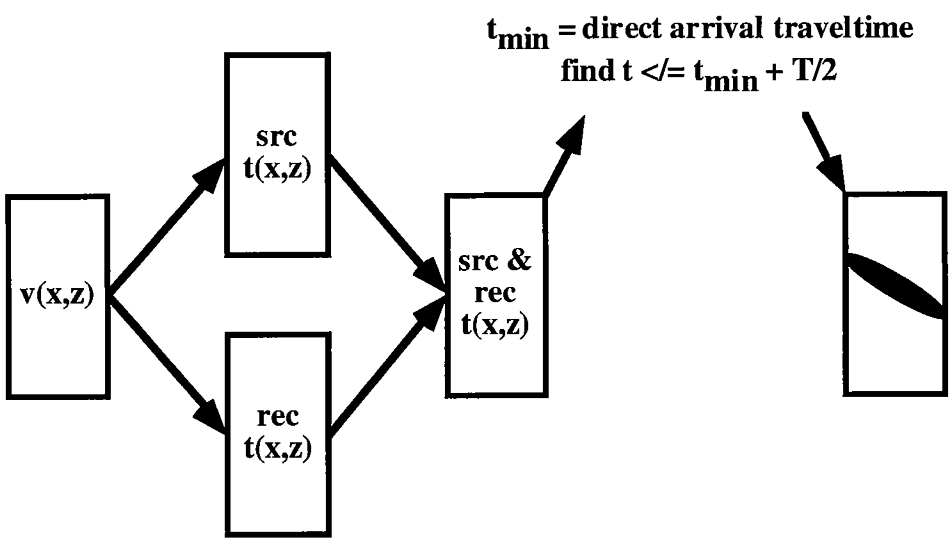

Figure 1: A schematic of the Fresnel wavepath calculation. Traveltime maps are calculated for the source and receiver positions from an input velocity model. The traveltime maps are calculated with a energetic-arrival finite-differences eikonal solver. These velocity maps are then added. A filter is applied to the combined map identifying all traveltimes falling within the direct arrival time and that time plus T/2, 1/2 the period of the source signal. The result of this filter is the Fresnel volume wavepath.

The procedure used to calculate the Fresnel wavepath is shown in Figure 1. To better illustrate this procedure I will go through the steps using the velocity model shown in Figure 2. The source location is at zero offset and at adepth of 175 ft. The receiver is located at an offset of 150 ft and a depth of 375 ft. The first step in calculating the Fresnel

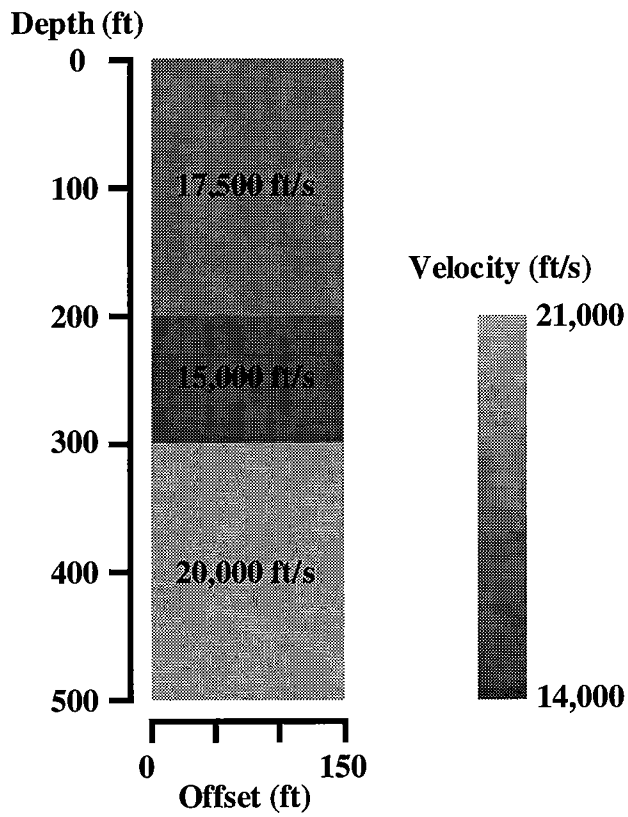

Figure 2: A simple 3 layer velocity model used to illustrate the technique used to calculate the Fresnel wavepath.

wavepath is to run the eikonal solver to create two traveltime maps, one where the source location is the starting point and another where the receiver location is the starting point. The results of these calculations are shown in Figure 3. The images in Figure 3 are displayed with a "random" colortable to accentuate the isochrons. Notice in the near vicinity of the starting point in each traveltime map that the isochrons are essentially circular in shape. This is expected where the velocity is constant. The isochrons deform at the interfaces due to the refraction of the transmitted energy.

The next step is to add the traveltime images of Figure 3. The result of this step is shown in Figure 4. This figure is also plotted using a "random" colortable. This combined traveltime map exhibits several interesting features. First, the isochrons represent trajectories where the traveltime from the source, to the isochron, to the receiver is constant. These isochrons are equivalent to Kirchoff migration ellipses. Second, the central dark-gray area, which includes the source and receiver locations, is representative of the Fresnel wavepath. Although it is not within the resolution of this colortable, the central path, connecting the source to the receiver, is the "mathematical" raypath. All points falling on this path will have the same value, equivalent to the source-to-receiver traveltime.

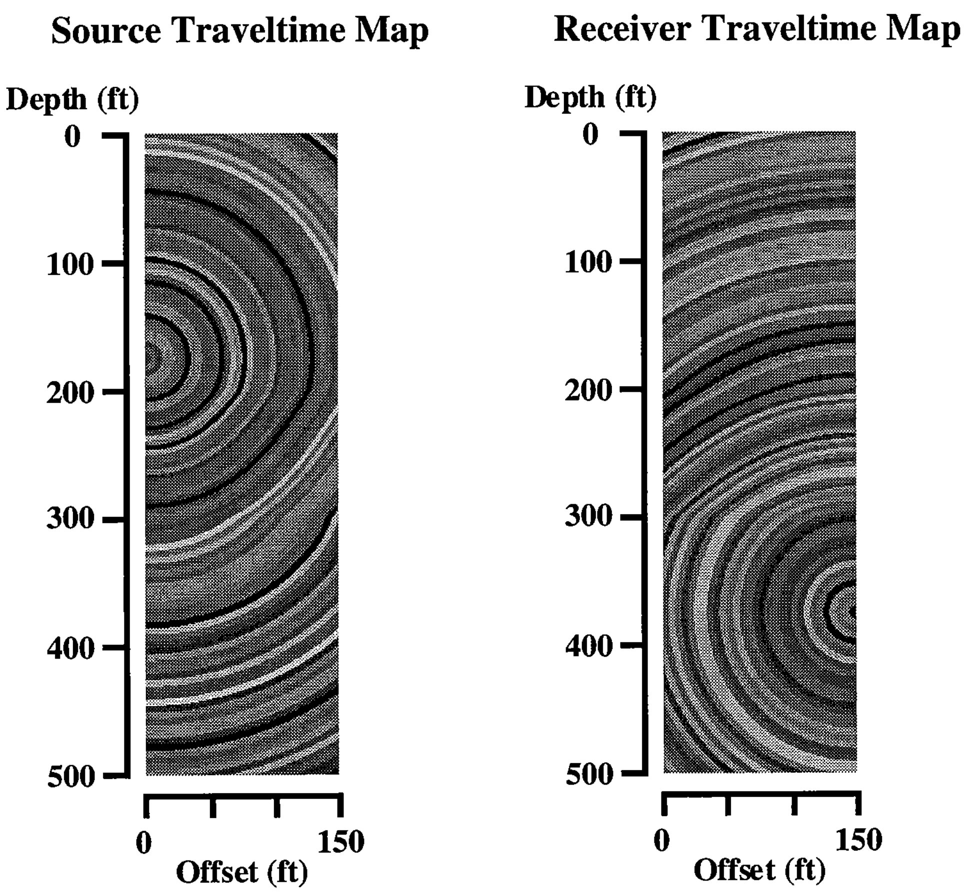

Figure 3: The above illustrations are eikonal traveltime maps for the source (left) and receiver (right). The traveltime maps are displayed with a "random" colortable to accentuate the isochrons of the traveltime maps.

Another potential application of this map is the determination of reflection points. The point at which the minimum traveltime is found along a line (e.g. reflector) drawn arbitrarily through this map, is the reflection point for that line. That minimum traveltime is also the reflection traveltime. If there is no possible reflection point along the line, the minimum time will be found at one of the two points where the line goes off the traveltime map.

These reflection points and times are useful in the calculation of XSP-CDP mapping trajectories.

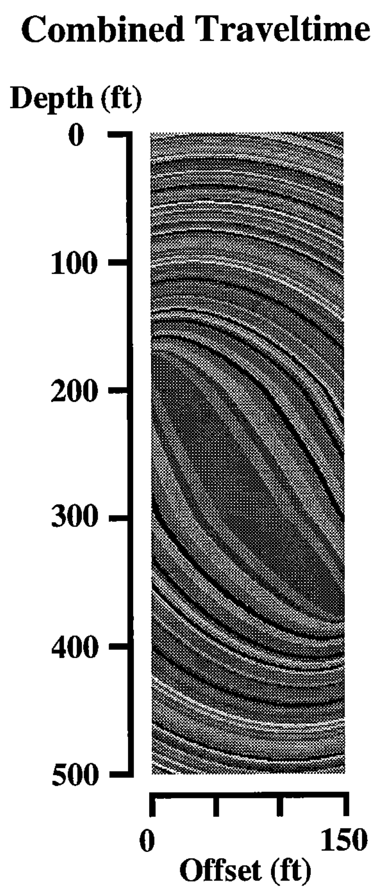

Figure 4: The above figure is the result of adding the source and receiver traveltime maps shown in Figure 3. A "random" colortable is used to highlight the traveltime contours. Notice the equi-colored contours are essentially equivalent to Kirchoff migration ellipses. Although the central "wavepath" is representative of the shape of the Fresnel wavepath the actual wavepath is determined by traveltime values falling between the direct arrival time, and the direct time + 1/2T. (one half the period of the source wavelet).

Finally, to determine the Fresnel wavepath, all times in Figure 4 are found which fall between the direct arrival traveltime, to this time plus one half the period of the source wavelet (inclusive). These points constitute the Fresnel wavepath. For this example I assume a source wavelet with a period of 1 ms which corresponds to a center frequency of 1000 Hertz. Therefore, the window used in determining the Fresnel wavepath is from 14.7 ms (the direct arrival traveltime) to 15.2 ms (14.7 ms + 1.0/2.0 ms). The Fresnel wavepath for this example is shown in Figure 5. The Fresnel wavepath in a homogeneous medium is defined by an ellipse. The wavepath shown in Figure 5 is deformed due to the velocity contrasts in the model and is clipped where the wavepath extends outside the model.

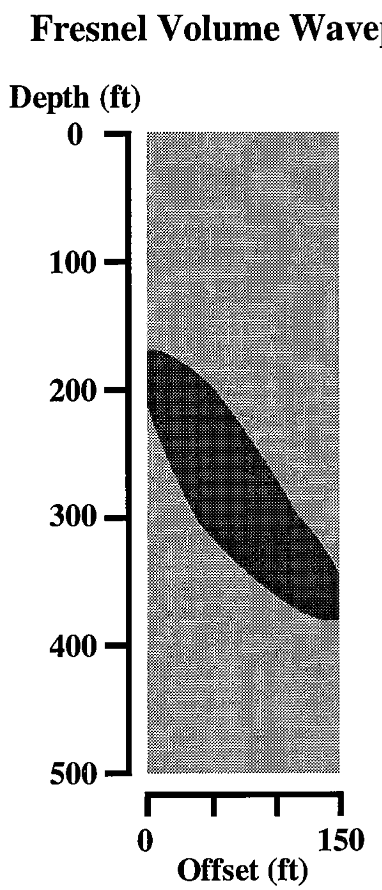

Figure 5: The Fresnel wavepath for the example shown in Figure 2. This wavepath includes all paths which have traveltimes within 0.5 ms of the direct arrival traveltime . The 0.5 ms window represents the appropriate value for a source wavelet with a period of 1 ms.

USING FRESNEL WAVEPATHS IN TRAVELTIME TOMOGRAPHY

Backprojection using volumeless rays

Using the high frequency ray approximation, the traveltime along path l, in a medium where the slowness is defined by $S(x,z)$, can be written as

$$t = \int_l S(x,z)dl \qquad (1)$$

Typically, the raypath is a function of the slowness field, $S(x,z)$, and a traveltime inversion must be performed in a sequence of linearized steps. Linearization is accomplished by using a slowness estimate, $S'(x,z)$, as the starting point of the inversion. This estimate defines the raypath, l', and is updated in the inversion to minimize the difference between t, the measured traveltime, and t', the traveltime calculated through the estimate $S'(x,z)$. One of the approximations made in the linearization is that the l' equals l. Equation (1) in terms of these new parameters is

$$t = \int_{l'} S'(x,z)dl' + R \tag{2}$$

where R, the traveltime residual, is defined by

$$R = t - t' \tag{3}$$

and t' is the traveltime measured through slowness field $S'(x,z)$:

$$t' = \int_{l'} S(x,z)dl' \tag{4}$$

To include the residual term inside the integral, multiply and divide R by $\int_{l'} dl'$ and rewrite the equation,

$$t = \int_{l'} (S'(x,z) + \frac{R}{l'})dl' \tag{5}$$

Equation (5) shows a method to use the traveltime residual to modify the continuous slowness field S' along the raypath so that the calculated traveltime matches the measured traveltime. This process is referred to as "backprojection". A single linear step in a tomographic inversion is called an "iteration". In an iteration the raypaths are defined by the estimated velocity model $S'(x,z)$ and do not change. The backprojection step is performed for all the raypaths and the final correction to $S'(x,z)$ is the average of the residual corrections. This process is the simultaneous iterative reconstruction technique (SIRT) (Dines and Lytle, 1979).

Backprojection using Fresnel wavepaths

Backprojection across Fresnel wavepaths is similar to backprojection along raypaths. In the case of wavepaths, the backprojection is done over an area instead of along a line. In this application I will use the Fresnel wavepaths to define an area to be used for backprojection. This formulation is still ray-theoretic in nature since the calculated traveltime used to calculate the residual is the energetic arrival eikonal time. To fully utilize the wavepath in a wave-theoretic traveltime inversion a formulation must be developed which uses all the information within the wavepath. In this formulation all the velocities within the wavepath contribute to the final calculated traveltime and perturbing any of these velocities would result in perturbing the calculated traveltime. Although a wave-based formulation would be superior to a ray-based formulation, the research on calculating wave-based traveltimes is still in progress.

The simple technique I describe utilizes the wavepath as a smoother. In this case the residual calculated using the ray-theoretic traveltime is backprojected over the entire wavepath. The advantage offered by this simple application is that the smoothing typically required in any ray-based inversion is defined honoring the wave nature of the data rather than in an ad hoc manner.

The wavepath backprojection formulation

Similar to equation (1) above, the Fresnel volume traveltime can be written as

$$t = G\iint_a w(x,z)S(x,z)da \qquad (6)$$

In this equation $w(x,z)$ is a weighting factor and G is a geometric factor used to scale the contributions to obtain a traveltime measurement. I determine G empirically by forward-modeling. The first step in computing G is to rewrite the equation replacing the weighted slowness terms by the average of the weighted slowness terms. Equation (6) becomes

$$t = G\iint_a S_{avg}da \qquad (7)$$

where S_{avg} is defined as

$$S_{avg} = \frac{\iint_a w(x,z)S(x,z)da}{\iint_a w(x,z)da} \tag{8}$$

These equations are now used to define *G*. This is done using the direct arrival eikonal time through the estimated slowness field *S'(x,z)*. This traveltime, *t'*, was calculated when the wavepath was defined. Rewriting equation (7) using the estimated slowness field and the calculated traveltime,

$$t' = G\iint_a S'_{avg}\, da \tag{9}$$

In equation (9), *G* is the only unknown. Solving for *G*

$$G = \frac{t'}{S'_{avg}} \tag{10}$$

Now a backprojection formulation can be written starting with equation (7) using the same philosophy as applied in equations (2)–(5). First, rewrite equation (7) in terms of the observed traveltime, the geometric factor, the estimated slowness model, and the traveltime residual:

$$t = G\iint_a S'_{avg}\, da + R \tag{11}$$

The residual term, *R*, is defined by equation (3). Now include *R* inside the integral. To do this expand S_{avg} and multiply and divide *R* by an integral form to provide a common denominator. This expression is

$$t = G\iint_a \frac{\iint_a w(x,z)S'(x,z)da'}{\iint_a w(x,z)da'}da + R\frac{\dfrac{G\iint_a da'}{\iint_a w(x,z)da'}}{\dfrac{G\iint_a da'}{\iint_a w(x,z)da'}} \tag{12}$$

Rearranging terms and putting *R* inside the integral,

$$t = \frac{G}{\iint_a w(x,z)da'}\iint_a\left(\iint_a w(x,z)S'(x,z) + \frac{R\iint_a w(x,z)da'}{G\iint_a da'}da'\right)da \tag{13}$$

Implementation

Although equation (13) appears complicated it is fundamentally the same as equation (5). To implement equation (13) in an inversion algorithm the slowness model is dicretized as a finely meshed grid. In this parameterization the areas of integration become a summation over the nodes. Equation (13) can now be written in its discretized form,

$$t = \frac{G}{\sum\limits_{i,j} w_{i,j}} \sum_{i',j'} \left(w_{i',j'} S_{i',j'} + \frac{R \sum\limits_{i,j} w_{i,j}}{GN} \right) \tag{14}$$

If the weighting function is assumed to be unitary within the Fresnel volume then the summation of the weighting function is equal to the number of grid points within the Fresnel volume, *N*. This further simplifies the equation,

$$t = \frac{G}{N} \sum_{i',j'} \left(S_{i',j'} + \frac{R}{G} \right) \tag{15}$$

FRESNEL VOLUME BACKPROJECTION EXAMPLE

I have run a traveltime inversion using the Fresnel volume backprojection formulation on a synthetic data set. The simple model used is seen in Figure 2. The data set includes 101 sources by 101 receivers evenly spaced along the sides of the model every 5 ft. In my inversion I implemented equation (14) to utilize the weighting function. I designed a simple function that linearly ramps the wavepath's weight from a value of one on the axis of the Fresnel volume to zero at the edge. Starting with a homogeneous model, four iterations were run using a SIRT-type algorithm. Each iteration was run for 20 backprojections.

The results of the inversion are shown in Figure 6. The inversion has converged reasonably well towards the solution. Some artifacts are seen near the interfaces and at the top and bottom of the image. Although the Fresnel wavepaths might provide additional stability, the inversion still contains artifacts typical of inversions which use the high frequency ray assumption. The forward modeled traveltime used in my inversion is calculated using the eikonal solver with its ray-theoretic assumptions. The wavepath is used simply to provide a smoothing criterion. The results of the wavepath inversion suggest that developing a formulation where the traveltime is calculated from the wavepath might provide improved results.

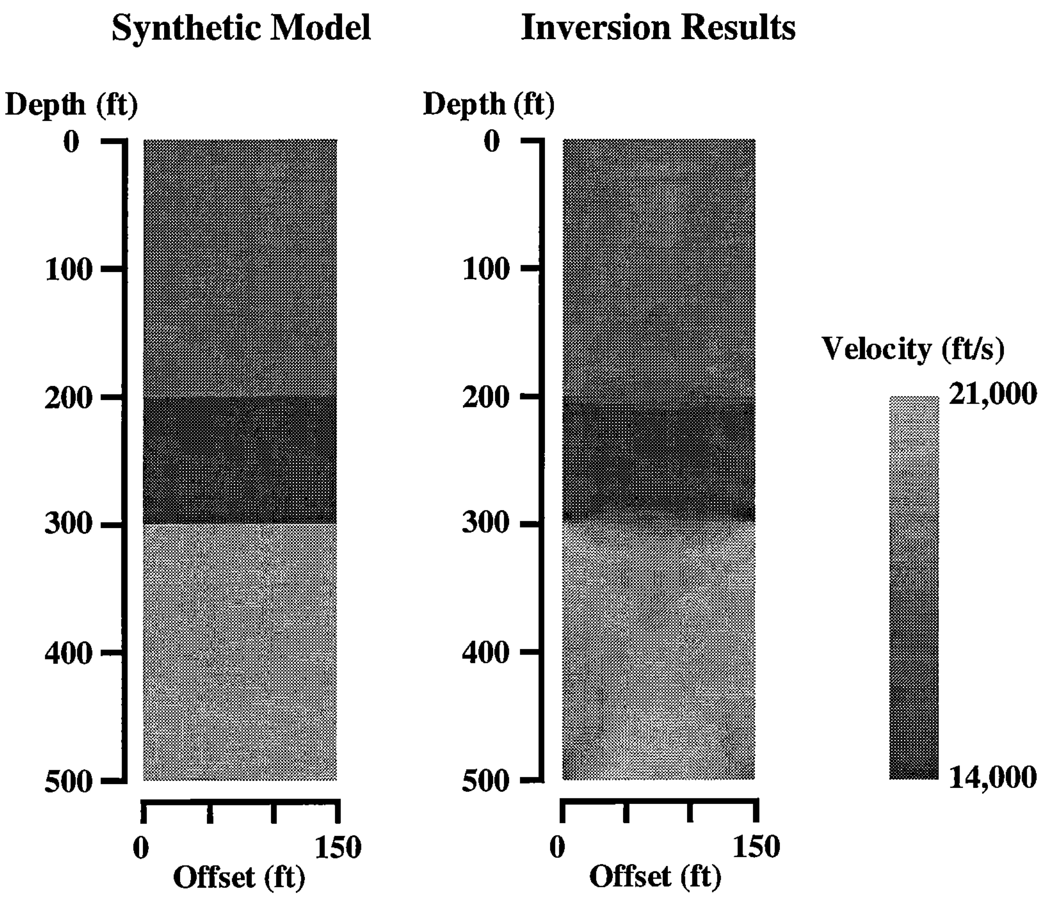

Figure 6: The results of the Fresnel wavepath inversion. In this inversion the Fresnel wavepaths were used to provide a basis for smoothing the inversion. This smoothing is accomplished during backprojection. This inversion consisted of 4 iterations of 20 backprojections each. The starting model was homogeneous.

DISCUSSION

I have described a technique by which energetic arrival Fresnel volumes can be easily calculated. A simple application was designed to use these wavepaths as a smoothing criterion in a crosswell traveltime inversion. The results of this inversion do not show much improvement in speed or accuracy compared with more standard smoothing techniques. The ease with which these wavepaths are calculated might be more fully utilized if the wavepaths are applied in a fullwave inversion formulation. They might also be used as a basis set in a "fat ray" formulation such as that described by Michelena and Harris (1991).

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