PAPER B

SEISMIC ATTENUATION TOMOGRAPHY USING THE FREQUENCY SHIFT METHOD: PRACTICAL CONSIDERATIONS AND APPLICATIONS

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ABSTRACT

This paper focuses on the implementation and application of seismic attenuation tomography based on the central frequency shift method. The frequency shift method uses the central frequency difference between incident and transmitted waves as the data to calculate the attenuation coefficient. The central frequency of transmitted waves can be measured from recorded seismograms, but the central frequency of incident waves may not be directly obtained. We suggest an approach which includes this frequency as an unknown in the inversion problem. This method is applied to 1-D geological structure (Devine data) and 2-D geological structure (King Mountain data).

INTRODUCTION

We have introduced the central frequency shift method to seismic wave attenuation tomography. Quan and Harris (1993) presented the basic theory with verification tests. In this paper we discuss more problems related to implementation of the method, and give more crosswell real data examples of 1-D and 2-D cases. Integrated geological interpretation with attenuation tomograms is in process.

BRIEF OF THE THEORY

We use a Gaussian spectrum as an example to briefly review the basic idea of the central frequency shift method. We assume that an incident wave has a spectrum of Gaussian distribution:

$$|S(f)| = \exp[-\frac{(f-f_s)^2}{2\sigma_s^2}], \qquad (1)$$

and the attenuation response of the medium is given by

$$|H(f)| = \exp[-f \int_{ray} \alpha_o df] , \qquad (2)$$

where α_o is attenuation coefficient. Then the wave spectrum recorded at a receiver still has a Gaussian shape which is represented as

$$|R(f)| = |S(f)||H(f)| = A \exp[-\frac{(f - f_R)^2}{2\sigma_s^2}], \qquad (3)$$

where,

$$A = \exp[-\frac{f_S^2 - f_R^2}{2\sigma_s^2}]$$

and

$$f_R = f_S - \sigma_s^2 \int_{ray} \alpha_o df . \qquad (4)$$

Equation (4) can be rewritten as a tomographic inversion equation

$$\int_{ray} \alpha_o df = \frac{(f_S - f_R)}{\sigma_s^2} . \qquad (5)$$

where $(f_S - f_R)$ is the central frequency difference between incident and transmitted waves, and $1/\sigma_s^2$ acts as a scaling factor. (Note that Eqn (4) in Quan & Harris (1993) had a typing error).

PRACTICAL CONSIDERATIONS

Static Correction Of Source Frequency f_S

Eqn (5) is the basic equation used for attenuation tomography which can be written in a discrete form as

$$\sum_j \alpha_{oj}^i l_j^i = \frac{f_S - f_R^i}{\sigma_s^2} . \qquad (6)$$

Here index i represents the i[th] ray and j is for the j[th] pixel of the medium, l_j^i is the ray length within the jth pixel. In practice we can measure f_R from recorded seismograms, but may not directly obtain the source central frequency f_S and variance σ_s^2. From Eqns (1) and (3) we understand that the source spectrum $|S(f)|$ and receiver spectrum $|R(f)|$ exhibit the same variance σ_s^2, under the conditions given in Eqns (1) and (2). Therefore, we can choose the average of variances σ_R^2 at receivers as σ_s^2. We include the source spectral frequency f_S as an unknown in addition to α_j^i. By solving simultaneous equations we obtain attenuation coefficients α_j^i as well as f_S. Let

$$f_S = \bar{f}_S + \Delta f, \tag{7}$$

where $\bar{f}_S = \max\{f_R^i\}$ is an initial estimation of f_S, and Δf is static correction. Then

$$\frac{f_S - f_R^i}{\sigma_s^2} = \frac{\bar{f}_S + \Delta f - f_R^i}{\sigma_s^2} = \frac{\bar{f}_S - f_R^i}{\sigma_s^2} + \frac{\Delta f}{\sigma_s^2}. \tag{8}$$

Eqn (6) can be written as

$$\sum_j \alpha_j^i l_j^i - \frac{\Delta f}{\sigma_s^2} = \frac{\bar{f}_S - f_R^i}{\sigma_s^2}, \tag{9}$$

where α_j^i and Δf are unknowns to solve. We need to properly scale the coefficients of these simultaneous equations to make the numerical calculation stable, since coefficients l_j^i and $1/\sigma_s^2$ have different dimensions.

<u>Data Processing</u>

We first pick and align the direct wave. Then we mix traces to reduce scattering interference, and perform FFT to the direct wave which is covered by a short time window. The central frequency f_R and variance σ_s^2 are calculated by the following formulas:

$$f_R = \frac{\int f|R(f)|df}{\int |R(f)|df},$$

$$\sigma_s^2 = \frac{\int (f - f_R)^2 |R(f)| df}{\int |R(f)| df}.$$

If we treat $(\bar{f}_S - f_R^i)/\sigma_S^2$ as "travel time", α_j^i as "slowness", and add one more term $-\Delta f / \sigma_s^2$ into the system of equations, then we can slightly modify the techniques and programs for travel time tomography to do the attenuation tomography.

APPLICATIONS

Devine Data Survey 2

Quan and Harris (1993) took a test on the Devine crosswell data set. We here apply the modified inversion method and program based on Eqn (9) to the same data again. To run the program, we only need to input travel time picks, central frequency picks and spectral variances. The program selects the average variance, the initial model and the source frequency, and then calculates the velocity tomogram and attenuation tomogram with source frequency static correction. Figure 1 shows data picks and inversion results for this data set. Figure 1a displays the central frequency picks which are used to inverse the P-wave attenuation coefficient α_0. We use straight rays and a 1-D model for the inversion. The model in vertical direction is divided into 300 pixels. Figure 1c gives the calculated α_0 which is displayed in the form of $1/\alpha_0$. Figure 1b shows travel time picks which are used to inverse P-wave velocity displayed in Figure 1f. In Figure 1d we convert attenuation α_o and velocity v to Q-values by the definition

$$Q = \frac{\pi}{v\alpha_o}. \tag{10}$$

The geological structure and a well log are shown in Figures 1e and 1g which exhibit an excellent agrement with the inversion results. structure.

King Mountain Data

The geological structure in the King Mountain survey is complicated. We use a 2-D model for this crosswell data set. The model is divided into 30 (horizontal) by 60 (vertical). pixels. The central frequency picks of this data range from 600 - 1000 Hz. Figure 2 shows the 2-D P-wave velocity and attenuation tomograms which exhibit good correlation.

CONCLUSIONS

The frequency shift method can be used to estimate seismic attenuation, even in a complicated medium. The source frequency static correction this method makes the attenuation estimation relatively unique and stable.

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REFERENCE

Quan, Y. and Harris, J. M., 1993, Seismic attenuation tomography based on centroid frequency shift: Expanded Abstract of the 63rd SEG annual meeting (See also STP Annual Report, Vol. 4, No. 1, 1993