PAPER E

LATTICE PARAMETERIZATION FOR TOMOGRAPHY

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ABSTRACT

Problems of two-dimensional tomographic inversion are commonly formulated with orthogonal basis functions such as rectangular homogeneous pixels. In order to represent the so-called "high resolution" tomogram, a large number of small pixels are often used. In order to stabilize the inversion algorithm or to reduce unsightly discontinuities, the pixel model is often smoothed after inversion. My contention is that 2-D transmission tomography should be used only for "low resolution" imaging, that is, resolution of greater than 1 wavelength. When a high resolution result is required, e.g., < 1 wavelength, one should look to including more than transmission data, for example reflection traveltimes, reflection amplitudes and diffractions. The low resolution tomogram should be smoothly heterogeneous and described by as few parameters as possible, thus insuring a stable and robust inversion. In this paper, I propose using a lattice parameterization for slowness field. The use of lattices is not new to forward models but are rarely used in the tomography. In my implementation, slowness values between nodes of the lattice are found by bi-linear interpolation. I show that certain simplifying approximations reduce the inversion based on the lattice to string algorithm.

INTRODUCTION

For consistently, parameterization of the inverse model should match that of your forward model. If square orthogonal pixels are used for the inverse model, then square orthogonal pixels should be used in the forward model. The advantages of this model was emphasized by Michelena and Harris [1991]. A common mistake found in many tomography algorithms is to define the forward model on a grid and the inverse model with homogeneous pixels. This inconsistency affects the rate of convergence, accuracy, and stability of the tomography algorithm. This subtle but importance difference in the two models is illustrated in Figure 1. Let's review the models.

The pixel model: Slowness is defined everywhere within homogeneous blocks of finite size. The block homogeneous pixel result in a discontinuous representation of the field. Pixel values are often smoothed to reduce the unsightly roughness. Semi-continuous variations over a large area require many small pixels.

The lattice model: Slowness is defined at regularly spaced nodes. Between nodes, the slowness is obtained using bi-linear interpolation. The result is a smoothly varying slowness field. Continuous variations can be easily represented by a few nodes, even for a large region. Nodes may be irregularly spaced, non-uniform in either dimension, or adapted to the geometry of a structural feature.

FORWARD MODELING

For crosswell, the lattice model is especially useful. Variations between wells may be modeled with just a few interwell nodes, thus yielding a largely over-determined system of linear equations. See (7) below. Smooth variations in regions of the size of several wavelengths are easily implemented, thereby asking tomography to provide only the low resolution image but to do so with high confidence. The one-dimensional continuous model illustrated in Figure 2 is described by only 2 nodes.

As discussed by Harris [1992], ray theory is most often used to model traveltimes for crosswell tomography. A general ray equation for heterogeneous, isotropic, linear, elastic media can be derived from the wave equation or from Fermat's Principle. The ray equation can be written compactly as

$$\frac{d}{d\ell}\left[\mathrm{u}\left(\frac{d\mathbf{r}}{d\ell}\right)\right] = \nabla \mathrm{S} \tag{1}$$

where $\mathrm{S}(\mathbf{r}) = \mathrm{V}^{-1}(\mathbf{r})$ is the slowness, i.e., reciprocal of the velocity, $\nabla \mathrm{S}$ is gradient of the slowness, ℓ is the increment of length along the ray path. Once a ray path connecting points a and b is found by solving (1), a traveltime is obtained by integrating the slowness along the path:

$$\mathrm{t} = \int_{\mathrm{a}}^{\mathrm{b}} \mathrm{S}(\mathbf{r})\, d\ell \tag{2}$$

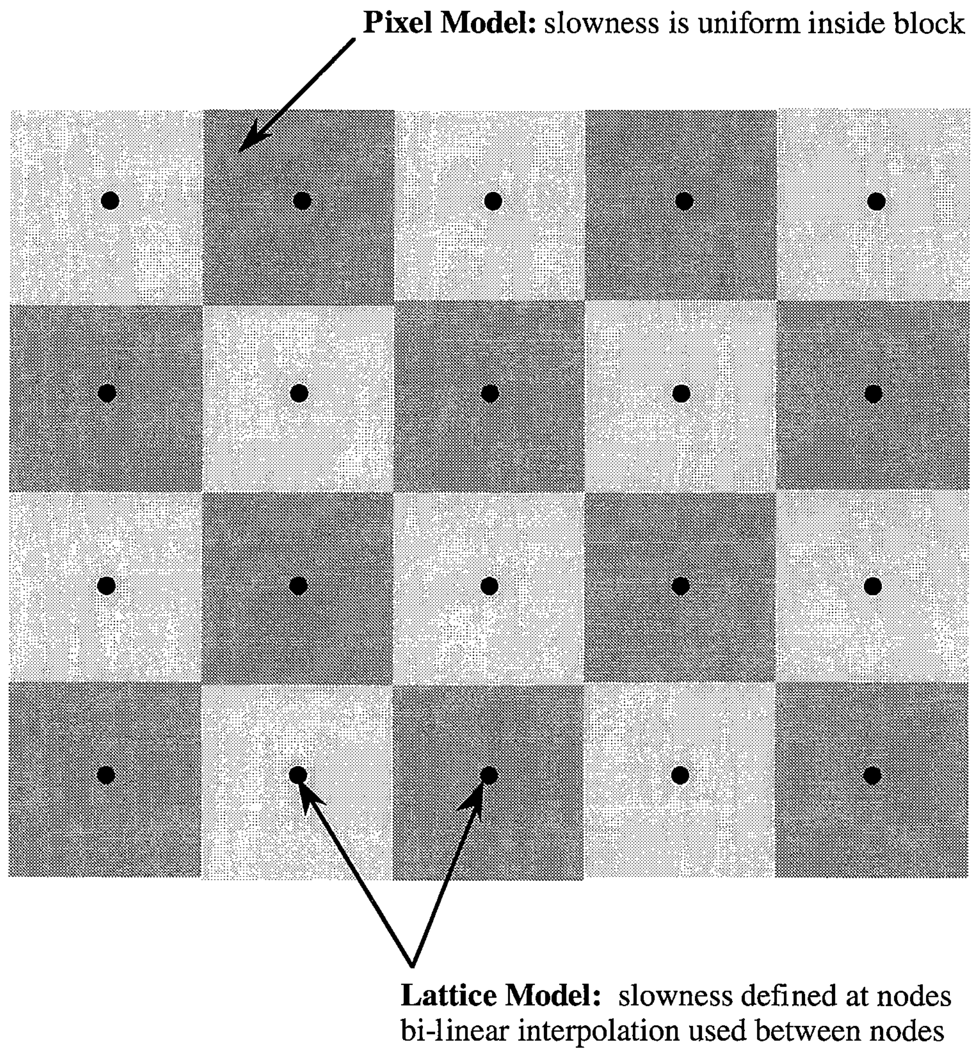

Figure 1. Lattice and pixel parameterization for forward and inverse models.

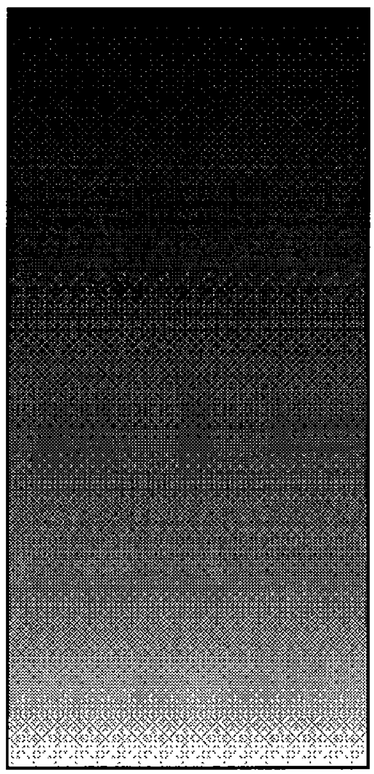

Figure 2. This one-dimensional vertical gradient is defined by two nodes: one at the top of 10 kft/s and one at the bottom of 20/kft/s. Values in between are calculated with a bi-linear interpolator.

In practice, the forward model traveltime is obtained by discretizing (2) in the following way:

$$t_j = \sum_{m=1}^{M_j} \hat{S}_{jm} \Delta\ell \tag{3}$$

where $\hat{S}_{jm}$ are a set of "interpolated" slowness values along the jth ray path and M_j is the number of equi-spaced steps of length $\Delta\ell$ along the ray path. $\hat{S}_{jm}$ is interpolated from the lattice using N-term interpolation:

$$\hat{S}_{jm} = \sum_{i=1}^{N} d_{ijm} S_i \tag{4}$$

where the S_i's are the values of slowness at the N nodes of the lattice and the d_{ijm}'s are the interpolation coefficients. In practice, I use bi-linear interpolation from the four nearest nodes. Substituting (4) into (3) gives the model traveltime equation suitable for inversion:

$$t_j = \sum_{i=1}^{N} W_{ij} S_i \tag{5}$$

The elements W_{ij} comprise the projection matrix made up from the bi-linear coefficients:

$$W_{ij} = \Delta\ell \sum_{m=1}^{M_j} d_{ijm} \tag{6}$$

INVERSE MODELING

Now, because the forward model (5) is expressly written in terms of the lattice nodes, the inverse model should be parameterized in terms of unknown coefficients for the lattice nodes. Many authors have used this nodal representation in the forward model, but then turn around and use rectangular pixels for the inverse model [e.g., McMechan]. The inverse problem is the find the set of slownesses $\{S_i\}$ satisfying (5) when a set of traveltimes $\{t_j\}$ are known and the elements of the projection matrix W_{ij} are computed:

$$t_j = \sum_{i=1}^{N} W_{ij} S_i \qquad j=1,2, \ldots, M \tag{7}$$

where M is the number of traveltimes and N the number of unknowns. When bi-linear interpolation is used, $[W_{ij}]$ is sparse. Typically, M may be very large as 60,000 and N as large as 10,000 or more; therefore the matrix $[W_{ij}]$ is large and sparse. In such cases, the system of equations (5) may be solved using Kaczmarz method of projections [Tanabe, 1971]. This method is implemented with the following algebraic solver:

$$S_i^{(j)} = S_i^{(j-1)} + \frac{t_j - \tau_j}{\sum_{k=1}^{M_j} W_{ik}^2} W_{ij} \tag{8}$$

where t_j is the observed traveltime and τ_j is the calculated traveltime in the (j-1) model. According to (8), updates to the model are made one ray at a time, i.e., ART. In practice, we update the model only once for all rays (SIRT). The SIRT solver is implemented in the TIMS program called LATTICE.

The problem of calculating the elements is illustrated in Figure 3. The coefficients d_{ijm} must be found for each point along the ray. It would appear that the calculation of the elements of the projection matrix would be an expensive part of the algorithm. While it is not prohibitively expensive in comparison to ray tracing, there is nevertheless a cost. In computing the elements, it is important that the accuracy be such that the $\sum_{i=1}^{M_j} W_{ij} = L_j$.

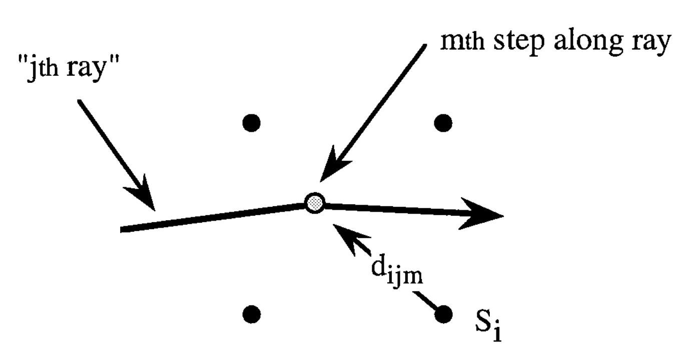

Figure 3. Slowness along the ray path is calculated by interpolation of values from the four neighboring nodes. When node spacing is small, a nearest neighbor approximation can be used and the lattice model reduces to the string model.

Under certain conditions, e.g., when the lattice spacing is small, a reasonable approximation is to use the slowness from the nearest neighbor. In such situations, all the interpolation coefficients are zero except for one and the matrix elements $W_{ij} = \Delta\ell$ for the ART solver (8) becomes

$$S_i^{(j)} - S_i^{(j-1)} = \frac{t_j - \tau_j}{M_j(\Delta\ell)^2}\Delta\ell = \frac{t_j - \tau_j}{L_j} \tag{9}$$

Eqn. (9) is precisely the solver implemented in strings [Harris, 1991]; therefore, strings is simply a special case that is appropriate for small lattice spacing.

SUMMARY

A lattice parameterization is proposed for traveltime tomography. Coarse lattices offer the advantage of generating smooth continuously varying models that are parameterized by a few coefficients. Inversion for lattice parameters may result in a significantly over-determined systems of equations, thus leading to robust and stable algorithms. Lattices, with bi-linear interpolation between nodes, are often used in forward models and should be used for inverse models as well. Under certain special conditions, reconstruction techniques based on the lattice model reduces exactly to the strings algorithm. The inversion code LATTICE has been implemented and is undergoing test on synthetic and field data. I have discussed the uniform lattice with regularly spaced nodes. Work is underway to generate a non-uniform lattice, perhaps adaptively adjusted for iterative tomography.

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