PAPER C

SEISMIC ATTENUATION TOMOGRAPHY BASED ON CENTROID FREQUENCY SHIFT

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ABSTRACT

We present a method for estimating seismic attenuation based on tomographic reconstruction. Tests on synthetic and field data show that the method works well for crosswell attenuation tomography with the source of broad frequency band. The data used to estimate attenuation in this method is extracted from the centroid frequency difference between incident and transmitted waves. This method is relatively insenstive to geometric spreading, reflection and transmission losses, source and receiver coupling and radiation patterns, and instrument gains.

INTRODUCTION

The recent improvement of seismic data quality makes possible attempts to estimate the heterogeneous seismic attenuation distribution. It has long been believed that attenuation is important for the characterization of rock and fluid properties, e.g., lithology, saturation, porosity, permeability and viscosity. Attempts at estimating attenuation have persisted for years. For example, Brzostowski, 1992, used seismic amplitude decay in attenuation tomography. In his method the change in amplitude is used as observed data to estimate attenuation. Amplitudes are easily contaminated by many factors such as scattering, geometric spreading, source and receiver coupling, radiation patterns and transmission/reflection losses; therefore, it is very difficult to obtain reliable attenuation estimates from the amplitude decay method. We need to seek other approaches that are more suitable for attenuation tomography. The centroid frequency shift method is one such approach.

In many natural materials, intrinsic seismic attenuation is frequency dependent, i.e., attenuation increases with frequency. The high frequency components of the seismic signal are more attenuated than the low frequency components when waves pass through fluid-filled rocks. As a result, the spectrum of incident waves experiences a frequency downshift during propagation. This phenomenon has been observed in seismic data, e.g. Hauge (1981). If we want to quantitatively determine the attenuation property base on this phenomenon, we need a relationship between the frequency shift and the attenuation parameters describing the medium. We use a method proposed by Dines and Kak (1979), and later used by Parker, et. al. (1988) for medical attenuation tomography. We also need to collect data of broad enough frequency band so that the phenomena can be clearly identified. The high frequency crosswell survey provides such data.

In this paper we present the basic theory, and discuss how the frequency shift is related to the attenuation coefficient for various input spectra. We apply the frequency shift method to crosswell attenuation tomography. Results for both synthetic and field data are presented.

THEORY

Assume that the power spectrum of an incident wave is given by *S(f)*. The received power spectrum is *R(f) = S(f)H(f)* (see Figure 1). We define the centroid frequency of the input signal *S(f)* to be

$$f_S = \frac{\int f S(f)df}{\int S(f)df}, \tag{1a}$$

and its variance to be

$$\sigma_S^2 = \frac{\int (f - f_S)^2 S(f)df}{\int S(f)df}. \tag{1b}$$

Similarly, the centroid frequency of the received signal *R(f)* is

$$f_R = \frac{\int fS(f)H(f)df}{\int S(f)H(f)df}, \tag{2}$$

where the filter *H(f)* is the response function of the medium. If *S(f)* and *H(f)* are known, we can obtain f_S, σ_S^2, f_R, and σ_R^2 by integrating according to the rules of Eqns. (1a), (1b) and (2).

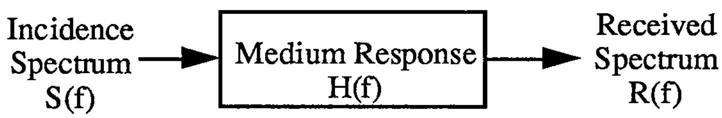


Figure 1. Relationship between incident and transmitted spectra.

Assume a constant Q model; therefore, seismic attenuation is linearly proportional to frequency. Then we can represent the attenuation response of the medium *H(f)* as

$$H(f) = \exp(-f\int \alpha_o dl),$$

where the integral is taken along the ray and $\alpha_o = \pi/(Qv)$ is attenuation coefficient. Here Q is quality factor and v is velocity. In this paper we use this model to demonstrate the frequency shift method. More complex attenuation models can be considered in the similar way. If the transmitted spectrum *S(f)* is Gaussian, i.e., given by the equation

$$S(f) = \exp[-\frac{(f-f_o)^2}{2\sigma_S^2}],$$

then from Eqns. (1) and (2),

$$f_S = f_o,$$
$$f_R = f_S - 2\sigma_S^2 \alpha_o L, \tag{3}$$

where L is the path length in the homogeneous medium of constant Q. When the medium is inhomogeneous, we replace $\alpha_o L$ by $\int \alpha_o dl$ qn. (3) can then be written as a line integral suitable for tomographic inversion:

$$\int \alpha_o dl = (f_S - f_R) / 2\sigma_S^2. \tag{4}$$

The derivation of Eqn. (4) is exact for a Gaussian input spectrum. A similar derivation (see Appendix) for non-Gaussian input spectra leads to the following special case results:

Boxcar spectra with bandwidth B:

$$\int \alpha_o dl = 12(f_S - f_R) / B^2, \qquad \alpha_o LB << 1 \tag{5}$$

Triangular spectra with bandwidth B:

$$\int \alpha_o dl = 18(f_S - f_R) / B^2, \qquad \alpha_o LB << 1 \tag{6}$$

Under the assumption of a constant-Q model we have derived tomographic Eqns. (4)-(6) for Gaussian, rectangular and triangular spectra respectively. These equations show that the attenuation coefficient for an inhomogeneous medium $\alpha_o(x,z)$ can be obtained by measuring the centroid frequency downshift $(f_S - f_R)$ of the incident and transmitted waves. The integrated attenuation equals to the downshift multiplied by a scaling factor. From Eqn. (3), we see that a broader input bandwidth σ_S^2 leads to a larger frequency change. Therefore, a broad input frequency band may be important for a robust estimation of $\alpha_o(x,z)$. Crosswell seismic survey with a high frequency downhole source provides a good opportunity to perform attenuation tomography using frequency shift method.

Since only Eqn (4) obtained from Gaussian spectrum is an exact formula, we should choose signals with the spectrum of Gaussian shape as source if it is possible.

Though Eqns (4)-(6) are derived from spectra of much different shapes, they are somewhat similar. This similarity may imply the robustness of this method, that is, it is not sensitive to the small change in spectrum shapes.

TESTS ON SYNTHETIC AND FIELD DATA

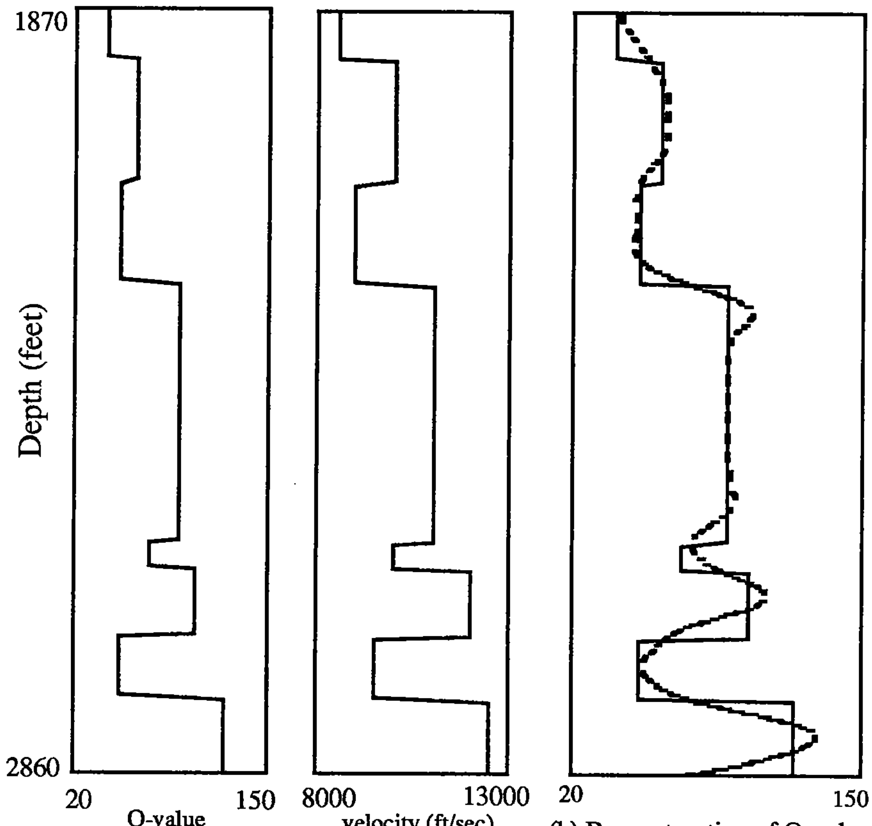
Before applying Eqn. (4) to field data, let us first examine its validity by considering a synthetic example for the VSP geometry. In this case, we rewrite Eqn. (4) as

$$\alpha_{oi} = \frac{1}{2\sigma_i^2} \frac{\Delta f_i}{\Delta z_i}, \tag{7}$$

where $\Delta f_i = f_i - f_{i+1}$ is the centroid frequency difference between i^{th} and $(i+1)^{th}$ VSP level, Δz_i the distance between i^{th} and $(i+1)^{th}$ level, α_{oi} is the attenuation coefficient between i^{th} and $(i+1)^{th}$ level, and σ_i the bandwidth at i^{th} receiver determined from Eqn. (1b).

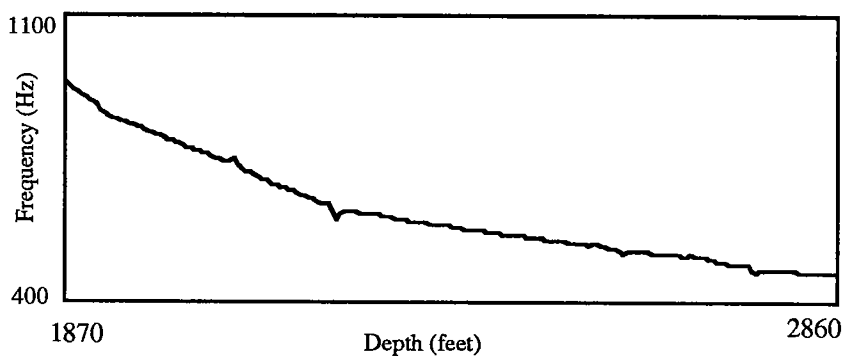
A viscoelastic wave equation (Apsel, 1979) is used for the seismic modeling. The attenuation in this model is introduced by using a complex velocity. A zero offset VSP with 220 receivers is calculated using VESPA. Figure 2a shows the model used. The frequency band of the source is 10 Hz - 2010 Hz. This frequency band is much broader than the real VSP, but it is in the frequency range of field crosswell data.

We select a time window that includes the first arrival. Figure 2c is a plot of the centroid frequencies picked from the data, which down shifts from 1000 Hz to 460Hz over a range of nearly 1000 feet. After smoothing the curve and calculating the downshift data, we obtain the attenuation coefficient α_o. Then using the definition $Q = \pi / (\alpha_o v)$, we get the estimated Q-values shown in Figure 2b, where the dashed line represents the reconstructed Q-value that fits the original synthetic model (solid line) pretty well. The thicker the layer is the better the inversion result. But even for a relatively thin layer, for example, the layer around the depth of 2600 ft, we also get a satisfactory result.



(a) Model

(b) Reconstruction of Q-value (dashed line).



(c) Center frequency picks from the synthetic VSP data.

Figure 2. A test of the frequency shift method using VSP. Model shown in (a) is used to calculate a VSP. (c) is the center frequencies obtained from the VSP, which are used to get the reconstructed Q-values shown in (b).

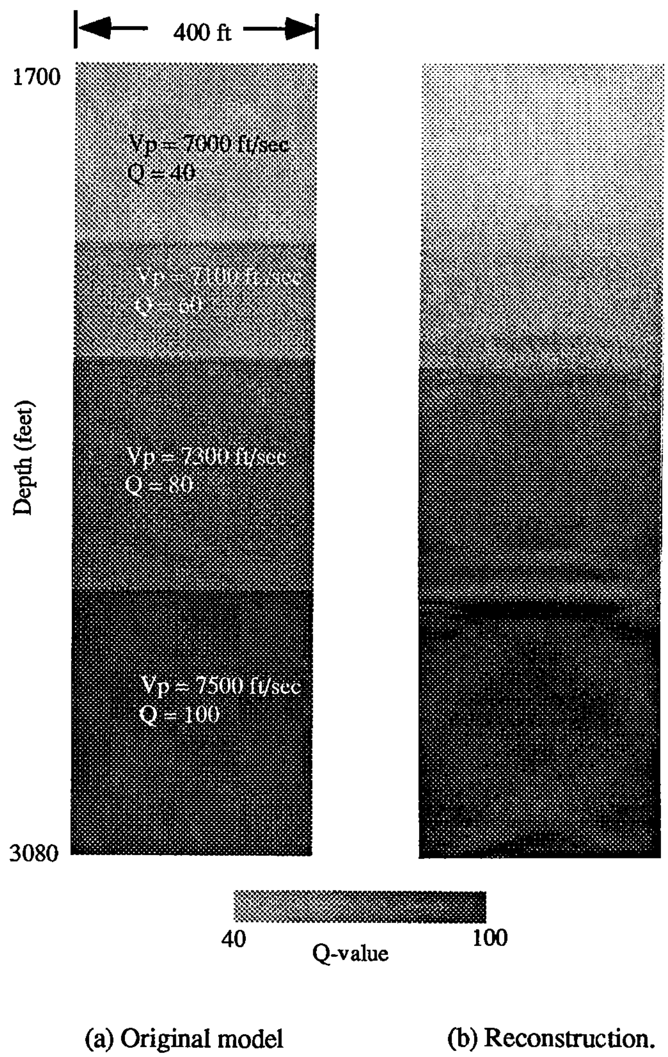


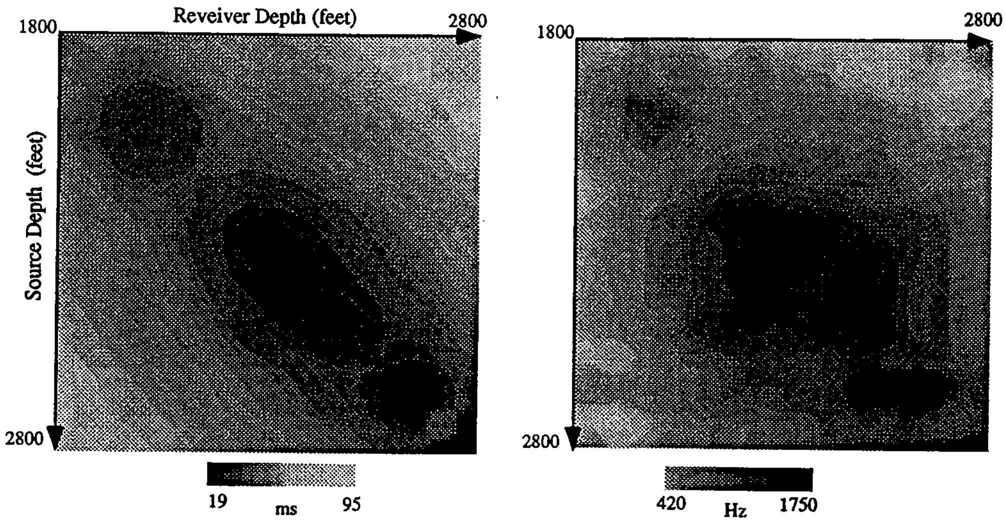
Figure 3. Computer simulation of attenuation tomography using the frequency shift method.

For crosswell tomographic simulation we use the model shown in Figure 3a. We placed 70 sources in one well and 70 receivers in the other with a well offset of 400 ft. Figure 3b is the reconstruction of Q-values using Eqn (4). Thus we see that tomography, based on inversion of the line integral of Eqn. (4) is capable of imaging the Q field.

For field data, we choose a data set collected at BP's Devine test site. A linear sweep from 200Hz to 2000Hz was used. Figure 4b shows the centroid frequency picks f_R. For comparison we also show the travel time picks from the same data set in Figure 4a. Figure 5d gives the Q-value reconstruction from the frequency downshift data. Compare this Q-value image with the velocity reconstruction shown in Figure 5b from traveltimes, The geological structure and a well log are also shown in Figure 5. The correlation with the lithology is good. For Eqn (4), we need to know the source centroid frequency f_S and bandwidth σ_S, and the received centroid frequency f_R. We directly measure f_R, but have no direct measurement of the source input spectrum. In this field data test we choose $f_S = \max\{f_R\} = 1750\text{Hz}$ and $\sigma_S^2 =$ average of σ_R^2, i.e. the average of the variance at the receivers. These are not the best estimations. We are investigating other methods of estimating these parameters, including incorporating them as unknowns into the inversion problem.

CONCLUSIONS

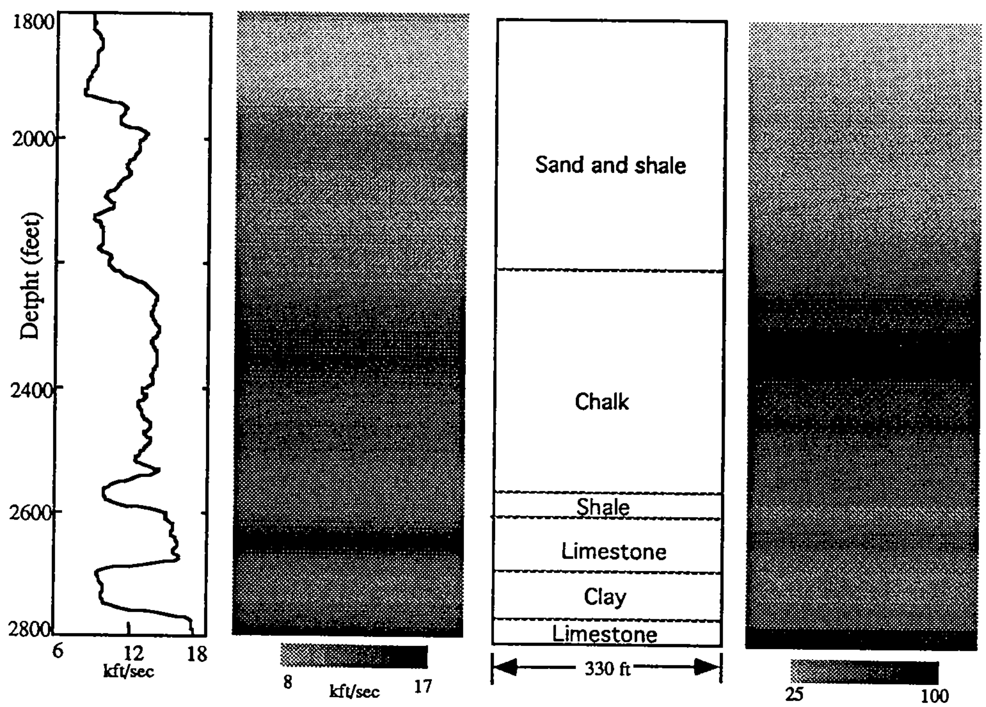
Frequency-dependent attenuation causes a change in wave spectra. For a constant Q-model and a Gaussian spectrum this change is simple: the difference in centroid frequency between the incident (input) and transmitted (received) waves is proportional to the integrated attenuation times a scaling factor which is determined from the input spectrum. This fact results in a simple formula that can be used for attenuation tomography. Although the method is sensitive, it is best on data with a broad frequency band. The crosswell geometry with a high frequency downhole source provides a good opportunity to use this method. A field data test shows that the attenuation tomogram has a good correlation with lithology and the velocity tomogram.



(a) A chart of travel time picks

(b) A chart of center frequency picks

Figure 4. Center frequency picks and travel time picks from Devine data. A horizontal line in the chart represents a common source gather,and a vertical line represents a common receiver gather. These two charts exhibit a good correlation: high center frequencies corresponding to slow travel times.



(a) P velocity log

(b) Velocity imaging

(c) Lithology

(d) Q-value imaging

Figure 5. Attenuation and velocity tomograms of Devine data.

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APPENDIX

If *S(f)* is rectangular shape with a width of B, then

$$f_S = \int_0^B f df \; / \; \int_0^B df = B/2,$$

$$f_R = \frac{\int_0^B f\, e^{-fa_oL} df}{\int_0^B e^{-fa_oL}\, df} = \frac{a_oL}{1-e^{-a_oLB}}\left[\frac{1}{(a_oL)^2} - \frac{Be^{-a_oLB}}{a_oL}\left(\frac{1}{a_oLB}+1\right)\right].$$

For Q =100, v =15000 ft/s, B = 500Hz, L =100ft, we get $\alpha_oLB = (\pi/Qv)LB = 0.1 <<$ 1. In the case of $\alpha_oLB << 1$,

$$\exp(-\alpha_o LB) \approx 1 - \alpha_o LB + \frac{1}{2}(\alpha_o LB)^2 - \frac{1}{6}(\alpha_o LB)^2$$

and

$$f_R \approx \frac{B}{2} - \frac{B^2}{12}\alpha_o L = f_S - \frac{B^2}{12}\alpha_o L,$$

or write it as tomographic equation

$$\int a_o dl \approx 12(f_S - f_R) / B^2.$$

Similarly, if *S(f)* is a right triangle with a side of B, we get

$$f_S = \frac{1}{3}B,$$

$$f_R \approx \frac{B}{3} - \frac{B^2}{18}\alpha_o L = f_S - \frac{1}{18}\alpha_o L$$

and

$$\int a_o dl \approx 18(f_S - f_R) / B^2.$$