PAPER A

ELASTIC CONSTANTS OF TRANSVERSELY ISOTROPIC MEDIA FROM CONSTRAINED APERTURE TRAVELTIMES

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ABSTRACT

The elastic constants that control P- and SV-wave propagation in a transversely isotropic media can be estimated by using P- and SV-wave traveltimes from either cross-well or VSP geometries. The procedure consists of two steps. First, elliptical velocity models are used to fit the traveltimes near one axis. The result is four elliptical parameters that represent direct and normal moveout velocities near the chosen axis for P- and SV-waves. Second, the elliptical parameters are used to solve a system of four equations and four unknown elastic constants. The system of equations is solved analytically yielding simple expressions for the elastic constants as a function of direct and normal moveout velocities. For *SH*-waves the estimation of the corresponding elastic constants is easier because the phase velocity is already elliptical. The procedure, introduced for homogeneous media, can be generalized to heterogeneous media by using tomographic techniques. The technique is illustrated with synthetic examples in a homogeneous medium.

INTRODUCTION

The effect of velocity anisotropy on wave propagation in homogeneous and heterogeneous media has been the subject of numerous publications. Careful forward modeling has helped interpreters to understand how velocity anisotropy manifests itself in field data. Few attempts have been made, however, to estimate the parameters that describe the complexity of velocity anisotropy, namely, the elastic constants. Estimation of the elastic constants is important because it can aid in lithologic discrimination and fracture orientation, reveal anisotropic properties of the medium not obvious in the data, and provide further imaging or full waveform inversion algorithms with background models that can be refined iteratively.

Elastic constants have been estimated primarily in the lab where the samples can be assumed to be homogeneous. When the assumption of homogeneity is not valid, the problem is more difficult because the effects of anisotropy and heterogeneity are coupled in the data. This coupling problem has been addressed by making prior assumptions about the nature of both the anisotropy and heterogeneity. For example, by assuming that the rock samples are homogeneous, Arts et al. (1991) estimate, from lab measurements, the 21 elastic constants that characterize a general triclinic system. By assuming isotropic velocities, McMechan (1983) estimates tomographically arbitrary spatial variations using 2-D and 3-D models. These two papers are examples of different trade-offs between the complexity of velocity and heterogeneity. On the one hand, the first paper uses the simplest model for heterogeneities (homogeneous) without making any assumption about the type of velocity anisotropy. On the other hand, the second paper uses the simplest model for the velocity (isotropic) without making any assumption about the nature of the heterogeneity.

A model commonly used to describe velocity anisotropy is transversely isotropic (TI). This is because transverse isotropy is the most common form of anisotropy in the subsurface. Several authors have addressed the issue of estimating elastic constants that characterize this type of anisotropy. White et al. (1983) estimate the five TI elastic constants of a homogeneous formation using a VSP geometry. Hake et al. (1984) approximate the traveltimes curves of layered models with a three-term Taylor series expansion in which the coefficient are a function of the elastic constants. Winterstein and Paulsson (1990) estimate the elastic constants from VSP and cross-well measurements in a medium with velocity linearly increasing with depth. Byun and Corrigan, (1990) propose an iterative model-based optimization scheme to invert traveltimes for the five elastic constants of a TI-layered media. More recently, Sena (1991) proposed a variant of this method in which all the calculations are done analytically, without having to go through the semblance analysis needed in Byun and Corrigan's method.

Near the axes of symmetry, a TI medium looks like an elliptically anisotropic medium. This property of transverse isotropy is used by Muir (1990a) and Dellinger et al. (1993) to approximate P- and SV-wave slowness surfaces and impulse responses with ellipses fitted near *both* horizontal and vertical axes. In a later paper, Muir (1990b) suggests the transformation of the elliptical parameters into elastic constants by using well known expressions that relate them (Levin, 1979; Levin, 1980). However, Muir implicitly assumes in this paper that traveltimes near both axes are available, which doesn't happen often.

Unfortunately, all the preceding methods fail when the data are not wide aperture or when measurements along both axes are not available. This is often the case with VSP and cross-well experiments.

I show in this paper how to obtain the elastic constants that control P- and SV-wave propagation in TI media from limited aperture traveltimes, either from VSP or from cross-well geometries. I start by fitting the traveltimes for P- and SV-waves with elliptical time-distance relations near a single axis (either vertical or horizontal). The result is four velocities: two based on the time-of-arrival and distance along a symmetry axis (the direct velocities) and two based on the differential traveltime and differential distance as the direction is perturbed (the normal moveout velocities). These four elliptical parameters are used to solve analytically a system of four equations and four unknown elastic constants. Since the procedure is based on fitting the data with elliptical velocity models, it is exact when estimating elastic constants from *SH*-wave traveltimes.

The data aperture is constrained in two different ways. First, it should not be too small to ensure that there is enough curvature to estimate the normal moveout velocities. Second, it should not be too wide to ensure that the elliptical fit remains accurate for the given wave type. I show in this paper that there is an intermediate range of ray angles that satisfy these two requirements when estimating elastic constants from either VSP or cross-well geometries.

The calculations presented here are valid for homogeneous media. When the model is heterogeneous, it can be described as a superposition of homogeneous regions, and the elliptical parameters needed at each region can be estimated tomographically, as explained by Michelena et al. (1993) and Michelena (1992a). Once the elliptical parameters at each cell are estimated, the procedure developed here for homogeneous media can be applied at each cell to obtain 2-D maps of elastic constants.

The equations I use in this paper to transform elastic constants into elliptical parameters (forward mapping) are not new. They are the same as the ones summarized by Muir (1990b), which can also be found in Levin (1979) and Levin (1980). What is new is the simultaneous solution of these equations near each axis to obtain elastic constants as a function of elliptical parameters (inverse mapping).

I start by rederiving the basic equations of the forward mapping from the expression of *P*- and *SV*-wave phase velocities in TI media. The calculations are done near both the horizontal and the vertical axes. Then, using these expressions, I solve the inverse mapping analytically. The final section illustrates the use of the technique when estimating the elastic constants of a homogeneous medium from impulse responses sampled near either the vertical axis or the horizontal axis, to simulate VSP and cross-well configurations, respectively.

FORWARD MAPPING

By doing forward modeling we create data from a given set of model parameters. When the data form a different set of model parameters, I prefer to call the process forward mapping to emphasize that the transformation is done between spaces that cannot be measured directly. In this section I explain how to do the mapping from the elastic constants of a TI media to different sets of phase velocities.

From elastic constants to phase velocities

The phase velocity expression for *P*- and *SV*-waves in TI media is (Auld, 1990)

$$
\begin{aligned}
2W_{P,SV}(\theta) &= (W_{33}+W_{44})\cos^2\theta + (W_{11}+W_{44})\sin^2\theta & \text{(1a)}\\
&\pm\sqrt{[(W_{33}-W_{44})\cos^2\theta-(W_{11}-W_{44})\sin^2\theta]^2+4(W_{13}+W_{44})^2\sin^2\theta\cos^2\theta},
\end{aligned}
$$

where $W(\theta)$ is the phase velocity squared and θ is the phase angle from the vertical. W_{ij} is the $(ij)^{th}$ elastic modulus divided by density, with units of velocity squared; I refer to the quantity W_{ij} as an "elastic constant" in the remainder of this paper. The "+" sign in front of the square root corresponds to P-waves and the "–" to SV-waves. For SH-waves, the expression for the phase velocity is (Auld, 1990)

$$W_{SH}(\theta) \;=\; W_{44}\cos^2\theta + W_{66}\sin^2\theta. \tag{1b}$$

Near the vertical and horizontal axes equation (1a) is elliptical. The velocities that describe the corresponding ellipses are called elliptical velocities. In the next two subsections, I rederive equations contained in previous papers (Levin, 1979; Levin, 1980; Muir, 1990b) that are needed to calculate these elliptical velocities from the elastic constants. The expressions that result are used later in the paper to solve the inverse mapping when the data have narrow aperture.

From elastic constants to phase velocities near the vertical axis

Expanding equation (1a) around $\theta = 0$ and neglecting terms in $\sin^4\theta$, results in:

$$\begin{aligned} 2W_{P,SV}(\theta) \;=\;& (W_{33}+W_{44})\cos^2\theta + (W_{11}+W_{44})\sin^2\theta \\ &\pm\left((W_{33}-W_{44})\cos^2\theta - (W_{11}-W_{44})\sin^2\theta + \frac{2(W_{13}+W_{44})^2}{W_{33}-W_{44}}\sin^2\theta\right). \end{aligned} \tag{2}$$

Choosing the positive root yields the P-wave phase velocity near the vertical axis, as follows:

$$W_P(\theta) \;=\; W_{P,z}\, c^2 + W_{P,xnmo}\, s^2, \tag{3}$$

where $c \;=\; \cos\theta$, $s \;=\; \sin\theta$,

$$W_{P,z} \;=\; W_{33}, \tag{4}$$

and

$$W_{P,xnmo} \;=\; W_{44} + \frac{(W_{13}+W_{44})^2}{W_{33}-W_{44}}. \tag{5}$$

$W_{P,z}$ is the vertical P-wave phase velocity squared and $W_{P,xnmo}$ is the horizontal normal moveout (*NMO*) phase velocity squared.

Choosing the negative root in equation (2) yields SV-wave phase velocities near the vertical axis, as follows:

$$W_{SV}(\theta) \;=\; W_{SV,z}\, c^2 + W_{SV,xnmo}\, s^2, \tag{6}$$

where

$$W_{SV,z} \;=\; W_{44}, \tag{7}$$

and

$$W_{SV,xnmo} \;=\; W_{11} - \frac{(W_{13}+W_{44})^2}{W_{33}-W_{44}}. \tag{8}$$

The previous expressions for the *NMO* velocities agree with the results of Thomsen (1986) and Vernik and Nur (1992).

The expression for *SH*-wave phase velocities near the vertical axis is

$$W_{SH}(\theta) \;=\; W_{SH,z}\, c^2 + W_{SH,xnmo}\, s^2, \tag{9}$$

where

$$W_{SH,z} \;=\; W_{44}, \tag{10}$$

and

$$W_{SH,xnmo} \;=\; W_{SH,x} \;=\; W_{66}. \tag{11}$$

From elastic constants to phase velocities near the horizontal axis

P- and *SV*-wave phase velocities near the horizontal axis can be obtained by interchanging W_{11} and W_{33} and c^2 and s^2 wherever they occur in equations (3) through (8). Thus, for *P*-waves the result is

$$W_P(\theta) \;=\; W_{P,x}\, s^2 + W_{P,znmo}\, c^2, \tag{12}$$

where

$$W_{P,x} \;=\; W_{11}, \tag{13}$$

and

$$W_{P,znmo} \;=\; W_{44} + \frac{(W_{13}+W_{44})^2}{W_{11}-W_{44}}. \tag{14}$$

For *SV*-waves, the expression for the phase velocity near the horizontal axis is

$$W_{SV}(\theta) \;=\; W_{SV,x}\, s^2 + W_{SV,znmo}\, c^2, \tag{15}$$

where

$$W_{SV,x} \;=\; W_{44}, \tag{16}$$

and

$$W_{SV,znmo} \;=\; W_{33} - \frac{(W_{13}+W_{44})^2}{W_{11}-W_{44}}. \tag{17}$$

Near the horizontal axis the *SH*-wave phase velocities are

$$W_{SH}(\theta) \;=\; W_{SH,x}\, s^2 + W_{SH,znmo}\, c^2, \tag{18}$$

where

$$W_{SH,x} \;=\; W_{66}, \tag{19}$$

and

$$W_{SH,znmo} \;=\; W_{SH,z} \;=\; W_{44}. \tag{20}$$

In the rest of the paper I refer to the elliptical parameters $W_{P,x}$, $W_{P,z}$, $W_{P,xnmo}$, $W_{P,znmo}$, $W_{SV,x}$, $W_{SV,z}$, $W_{SV,xnmo}$, $W_{SV,znmo}$, $W_{SH,x}$, $W_{SH,z}$, $W_{SH,xnmo}$, and $W_{SH,znmo}$ as W_*, direct or *NMO* phase velocity squared for *P*-, *SV*-, and *SH*-waves.

INVERSE MAPPING

The preceding equations that relate phase velocities and elastic constants suggest several approaches to solve the inverse mapping that depend on the recording geometry, the recording aperture, and the wave types available. From *SH*-wave phase velocities near either axis, it is always possible to estimate W_{44} and W_{66} because the *NMO* velocities are equal to corresponding direct velocities. However, from *P*- and *SV*-wave phase velocities near either axis, the estimation of the corresponding elastic constants is not straightforward. This section explains what to do in such a case.

Using *P*- and *SV*-wave full aperture phase velocities

To estimate W_{11}, W_{33}, W_{44}, and W_{13} directly from equation (1a), we need at least four phase velocities at four different angles between 0 and 90 degrees. W_{ij} is the solution of a system of nonlinear equations where the independent term is formed by these phase velocities. Along the axes, the system of equations is almost diagonal and the estimation of three elastic constants (W_{33}, W_{11}, and W_{44}) is straightforward:

$$W_{33} = W_{P,z}, \tag{21a}$$

$$W_{11} = W_{P,x}, \tag{21b}$$

$$W_{44} = W_{SV,z} = W_{SV,x}. \tag{21c}$$

The elastic constants are estimated directly from phase velocities along the axes. W_{13} can be estimated from the previous elastic constants and one phase velocity at an oblique angle, typically 45 degrees.

This approach, although simple in theory, is not applicable in many practical situations because wide aperture data are required (the angles of the observations must include 0, 90 degrees, and one intermediate measurement) in order to simplify the system of equations. This is not the case for most single-geometry data sets (either surface, or cross-well or VSP) where no rays travel along (at least) one of the axes and therefore, phase velocities along both axes cannot be estimated without having to assume a velocity symmetry (e.g., isotropic, elliptical, TI).

Using *P*- and *SV*-wave narrow aperture phase velocities

Elastic constants from phase velocities near the vertical.– When the phase angles are close to zero, it is possible to estimate elastic constants from the corresponding phase velocities by using equations (4), (5), (7), and (8), a system of four equations and four unknowns. The independent term (that I haven't explained how to obtain yet) is formed by $W_{P,z}$, $W_{P,xnmo}$, $W_{SV,z}$, and $W_{SV,xnmo}$. The solution of this system of equations is

$$W_{33} = W_{P,z}, \tag{22a}$$

$$W_{44} = W_{SV,z}, \tag{22b}$$

$$W_{13} = \sqrt{(W_{P,xnmo} - W_{SV,z})(W_{P,z} - W_{SV,z})} - W_{SV,z}, \tag{22c}$$

$$W_{11} = W_{SV,xnmo} + W_{P,xnmo} - W_{SV,z}. \tag{22d}$$

In (22d), W_{11} is a linear combination of elliptical parameters independent of the horizontal P-wave phase velocity, unlike W_{11} estimated from equation (21b). Roughly speaking, (22d) says that summing P- and SV-wave *NMO* velocities (squared) is the same as summing the elastic constants that control the horizontal wave propagation. The examples in the final section of the paper show the range of angles near the vertical for which these equations are valid.

Since this approximation simultaneously uses the elliptical parameters of two ellipses fitted near the vertical, I call it *vertical* double elliptic approximation, analogous to Muir's double elliptic approximation that uses horizontal and vertical ellipses to approximate the slowness surface and impulse response for all angles (Muir, 1990a; Dellinger et al., 1993). It is important to point out the differences between these two approximations. On the one hand, the *vertical* double elliptic approximation is used to estimate elastic constants from phase velocities near the vertical axis. Slowness surfaces and impulse responses can be calculated from these elastic constants using (1a) and the exact relationships between phase and group velocities. On the other hand, Muir's double elliptic approximation is used to estimate directly slowness surfaces and impulse responses from data near both axes without having to know the elastic constants.

Fitting P- and SV-wave phase velocities with ellipses near the vertical is not the same as using the vertical double elliptic approximation. However, the fitting is a necessary intermediate step in the estimation of elastic constants using equations (22). When the elliptical fitting is done near the horizontal axis the result is the *horizontal* double elliptic approximation, as follows.

Elastic constants from phase velocities near the horizontal.– When the phase angle is close to 90 degrees, the expressions for the elastic constants as a function of P- and SV-wave phase velocities are obtained by solving the system of equations (13), (14), (16), and (17), with the following result:

$$W_{11} = W_{P,x}, \tag{23a}$$

$$W_{44} = W_{SV,x}, \tag{23b}$$

$$W_{13} = \sqrt{(W_{P,znmo} - W_{SV,x})(W_{P,x} - W_{SV,x})} - W_{SV,x}, \tag{23c}$$

$$W_{33} = W_{SV,znmo} + W_{P,znmo} - W_{SV,x}. \tag{23d}$$

This set of equations forms the *horizontal* double elliptic approximation. It uses elliptical P- and SV-wave phase velocities near the horizontal to approximate the elastic constants. Notice that the estimation of W_{33} is independent of the P-wave phase velocity along the vertical axis. Equations (23) can be obtained from (22) by interchanging W_{11} and W_{33}, and x and z.

The estimation of W_{33} from near-horizontal phase velocities (equation (23d)) and W_{11} from near-vertical phase velocities (equation (22d)) is in both cases the sum of *NMO* velocities minus W_{44}. Michelena et al. (1993) and Michelena (1992b) show that when estimating velocities tomographically, *NMO* velocities correspond to the smallest singular values of the problem. The largest singular values correspond to velocities estimated from rays that travel along the axes. Therefore, as expected, estimating W_{33} from cross-well traveltimes alone is a harder problem than estimating W_{11} from the same data. The opposite is true when estimating W_{33} and W_{11} from VSP measurements.

Using only P-wave phase velocities near the axes

When the medium is TI and no information is available about SV-wave phase velocities, it is still possible to obtain four elastic constants from P-wave phase velocities alone near both axes. This is done by solving the system of equations (4), (5), (13), and (14), which yields:

$$W_{11} = W_{P,x}, \tag{24a}$$

$$W_{33} = W_{P,z}, \tag{24b}$$

$$W_{44} = \frac{W_{P,xnmo}W_{P,z} - W_{P,znmo}W_{P,x}}{W_{P,xnmo} + W_{P,z} - W_{P,znmo} - W_{P,x}}, \tag{24c}$$

$$W_{13} = \sqrt{(W_{P,znmo} - W_{44})(W_{P,x} - W_{44})} - W_{44}. \tag{24d}$$

This set of equations forms the P-wave double elliptic approximation of the elastic constants in a TI medium.

In this approximation, as well as in the previous ones, the assumption of transverse isotropy is crucial. When the medium is isotropic ($W_{P,x} = W_{P,z} = W_{P,xnmo} = W_{P,znmo}$) there is no way to calculate W_{44} (the shear moduli) from P-wave phase velocities alone because equation (24c) is indeterminate. When the medium is weakly anisotropic the estimation of W_{44} using this approximation may still be unreliable because both the numerator and the denominator in (24c) are close to zero. We will see later in the examples that even when the medium is moderately anisotropic this approximation breaks down quickly for phase angles not close to the axes, unlike the previous vertical and horizontal double elliptic approximations (equations (22) and (23), respectively) that have a wider range a validity.

If only SV-wave phase velocities near the axes are available, it is not possible to obtain the corresponding elastic constants because the system of equations (7), (8), (16), and (17) is underdetermined.

OBTAINING THE PHASE VELOCITIES

The expressions previously derived for the elastic constants are a function of the phase velocities. However, only in a few cases phase velocities can be easily estimated from traveltime measurements that give, instead, group velocities. The correspondence between phase and group velocities is trivial only when the model is isotropic or elliptically anisotropic or when the ray travels along an axis of symmetry.

Equations (3), (6), (12), and (15) show that the phase velocities of P- and SV-wave are elliptical near the axes of symmetry. Those of SH-waves are also elliptical [equation (1b)]. It has been shown that when the phase velocity has an elliptical shape, the corresponding impulse response is also elliptical (Levin, 1978; Byun, 1982). Therefore, the group slowness expression that corresponds to these equations has the general form

$$S^2(\phi) \;=\; S_z^2 \cos^2\phi + S_x^2 \sin^2\phi, \tag{25}$$

where ϕ is the ray angle measured from the vertical and S_* (the ray slowness) is

$$S_*^2 \;=\; \frac{1}{W_*}. \tag{26}$$

To estimate S_*, I use the expression for the traveltime of a ray that travels a distance $l = \sqrt{\Delta x^2 + \Delta z^2}$ between two points:

$$t^2 \;=\; \Delta x^2 S_x^2 + \Delta z^2 S_z^2. \tag{27}$$

This equation, which has the same form as the isotropic moveout equation, is obtained after multiplying equation (25) by l^2. Velocities estimated from the moveout near one axis using this traveltime equation are called *NMO* velocities, and velocities estimated from arrival times along the same axis are called direct velocities. Hence the different names chosen for the phase velocities W_*.

USING TRAVELTIMES TO ESTIMATE ELASTIC CONSTANTS

This section summarizes the steps needed to obtain the elastic constants of a TI medium from traveltime measurements.

Homogeneous medium

The procedure to estimate the elastic constants of a homogeneous TI medium from traveltime measurements near one axis of symmetry is the following:

1. Fit the traveltimes with elliptically anisotropic models, one model for each wave type. This gives direct and *NMO* group slownesses.

2. From the direct and *NMO* group slownesses, find the corresponding direct and *NMO* phase velocities, using equation (26).

3. From the estimated phase velocities, find the elastic constants using the equations that correspond to the given recording geometry [either (22) or (23)]. In the case of *SH* waves, the estimated phase velocities squared are the same as the corresponding elastic constants.

Heterogeneous medium

To estimate spatial variations in the elastic constants, the first step is to describe the model as a superposition of homogeneous blocks that incorporate our previous knowledge about the structure. Then, the elliptical group slownesses of the different wave types are estimated tomographically, as explained by Michelena et al. (1993). Finally, local transformations from these elliptical group velocities to elastic constants are performed at each homogeneous block by using the procedure for homogeneous media explained in the preceding section.

Estimation of elastic constants from narrow aperture traveltimes and tomographic estimation of elliptical group velocities have opposite requirements in terms of data aperture. On the one hand, the estimation of elastic constants requires traveltimes from rays that travel as close as possible to one axis [this is how equations (22) and (23) were derived.] One the other hand, the tomographic estimation of elliptical velocities requires wide ray angles to improve the conditioning of the problem, the accuracy of the *NMO* velocities and the spatial resolution of the result. Fortunately, as the examples will show, there is a common range of ray angles in which accurate results can be obtained with both procedures. However, the accuracy will depend in general on the complexity of the heterogeneity. Problems can be anticipated if the velocity varies rapidly.

Heterogeneous medium with nonvertical axes of symmetry

So far, I have assumed that the axis of symmetry is vertical. If the axis of symmetry is nonvertical we need to find its inclination first, for example, by fitting *SH*-wave traveltimes with heterogeneous elliptically anisotropic models, as explained in Michelena (1992a). Once the inclination of the axes of symmetry of the different blocks has been estimated, the elliptical group velocities of *P*- and *SV*-waves at each block are estimated using only rays that travel near the axes of symmetry. Finally, the elliptic parameters are locally transformed into elastic constants, as explained in the section "Homogeneous medium". This process assumes the axes of symmetry of the different blocks of the model are in the same plane of the survey, as explained also by Michelena (1992a).

The elastic constants estimated in this way are referred to different coordinate frames, one for each different axis of symmetry. For purposes of interpretation, having the elastic constants relative to different frames is not a problem as long as we also use the inclination of the axes of symmetry. However, for further computations (finite difference modeling, for example) it might be necessary to transform the elastic constants to a common frame. This can be done by using Bond's transformation matrices (Auld, 1990).

EXAMPLES

This section contains examples that illustrate how four elastic constants of a homogeneous TI medium can be estimated when the data are (a) P- and SV-wave traveltimes from VSP geometries with different apertures, (b) P- and SV-wave traveltimes from cross-well geometries with different apertures, and (c) P-wave traveltimes from cross-well and VSP geometries with different apertures.

The impulse response is given, and it is the same for all examples. The elastic constants that describe the medium are: $\sqrt{W_{11}} = 7400$, $\sqrt{W_{33}} = 6295$, $\sqrt{W_{13}} = 5575$, and $\sqrt{W_{44}} = 2160$, all with units of (ft/s) (Byun and Corrigan, 1990). I calculate the ellipse that fits the impulse response near the chosen axis by using a ray along the axis and a ray close to the axis. Once I have found the four needed elliptical parameters I use equations (22), (23), or (24) to convert those elliptical parameters into elastic constants.

Estimating W_{ij} from VSP P- and SV-wave traveltimes

Figure 1 shows how the vertical double elliptic approximation works for different angles around the vertical. The parameters of the ellipses that approximate the impulse response have been calculated at the angles shown by the straight lines. The left column compares the given impulse responses for P- and SV-waves (continuous lines) with the elliptical approximations around the vertical (dashed lines). With the four elliptical parameters obtained for each aperture, I calculate the elastic constants of the medium by using equation (22). From the estimated elastic constants, I calculate the corresponding impulse responses for both P- and SV-waves. The result is shown in the central column (dashed lines) simultaneously with the given impulse responses. In most cases (except at 40-degree aperture) the agreement is excellent. At 40-degree aperture the horizontal P-wave group velocity has been overestimated and the shear wave triplication is larger than expected. The right column shows the absolute value of the percentage error made in the estimation of the elastic constants. For small angles (≤ 10 degrees), the error is negligible. For angles between 10 and 30 degrees the error is smaller in W_{11} (A) than in W_{13} (F). At large angles the error in W_{11} (A) is the largest, almost 30%. Notice that up to 30 degrees, even though the error in the estimation of the elastic constants is not zero but a few percent, the differences between given and estimated impulse responses are hard to see.

In Figure 1, the elliptical approximations to the impulse responses are calculated using two ray angles at a time, one zero and one nonzero. In Figure 2, I show what happens when all these angles (or different VSP offsets) are used simultaneously to

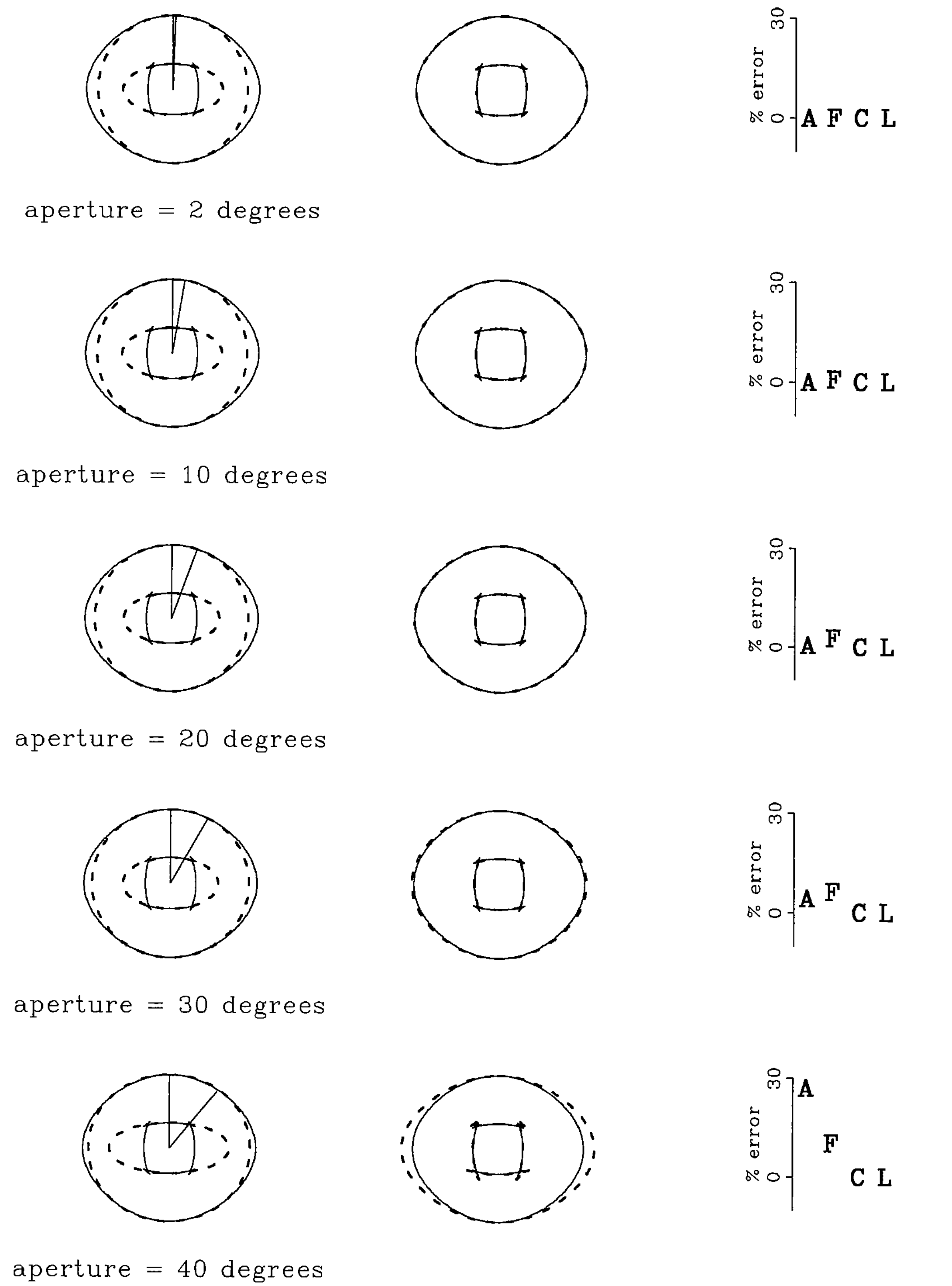

Figure 1: Left: impulse response for P- and SV-waves (continuous lines) compared with their elliptical approximations around the vertical (dashed lines). Center: given impulse responses (continuous) compared to the ones calculated from the estimated elastic constants (dashed). Right: absolute value of the error made in the estimation of the elastic constants when using the vertical double elliptic approximation. The elastic constants are A = W_{11}, F = W_{13}, C = W_{33}, and L = W_{44}. The density is assumed to be unity.

calculate the elliptical approximations and the elastic constants. The horizontal P-wave group velocity is now slightly overestimated and the shear velocities are retrieved well. The errors in the estimated elastic constants are 6% in W_{11} (A) and 4% in W_{13} (F). The errors made when using large angles only have been compensated by using also small angles.

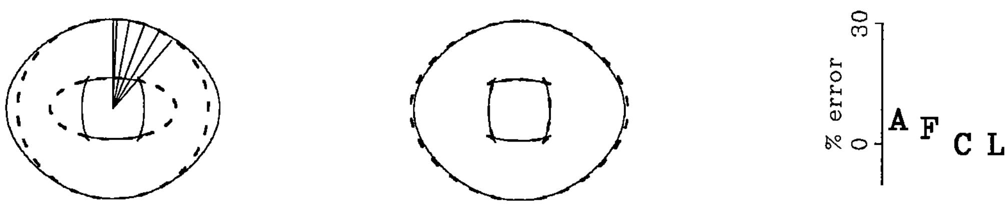

Figure 2: Left: impulse response for P- and SV-waves (continuous lines) compared with their elliptical approximations around the vertical (dashed lines). All ray angles shown are used simultaneously to calculate the elliptical approximations. Center: given impulse responses (continuous) compared to the ones calculated from the estimated elastic constants (dashed). Right: absolute value of the error made in the estimation of the elastic constants when using the vertical double elliptic approximation. The elastic constants are A = W_{11}, F = W_{13}, C = W_{33}, and L = W_{44}.

Estimating W_{ij} from cross-well P- and SV-wave traveltimes

Figure 3 shows how the horizontal double elliptic approximation works for different ray angles measured from the horizontal. When comparing this case with the VSP case (Figure 1), we see that the approximation works better for ray angles around the horizontal than around the vertical, even when the angles are large. The estimated impulse responses agree well with the given ones, and the errors in the elastic constants are negligible for angles less of fewer than 30 degrees.

The reason why the horizontal double elliptic approximation works better than the vertical double elliptic approximation is that the elliptical approximation for P-wave impulse response is more adequate at larger angles around the horizontal than around the vertical, where the group velocity is lower. Notice that the elliptic approximations around the vertical are almost circular.

Figure 4 shows that when small and large ray angles are used simultaneously to calculate the elliptical approximations around the horizontal, the result is roughly an average of the results shown in Figure 3. The agreement between given and estimated impulse responses is excellent and the error in the estimation of the elastic constants is $\approx 2\%$ for W_{13} (F) and $\approx 1\%$ for W_{33} (C).

Estimating W_{ij} from cross-well and VSP P-wave traveltimes

For TI media, the elastic constants that control the P- and SV-wave propagation can be estimated from P-wave measurements near the axes, as equation (24) shows.

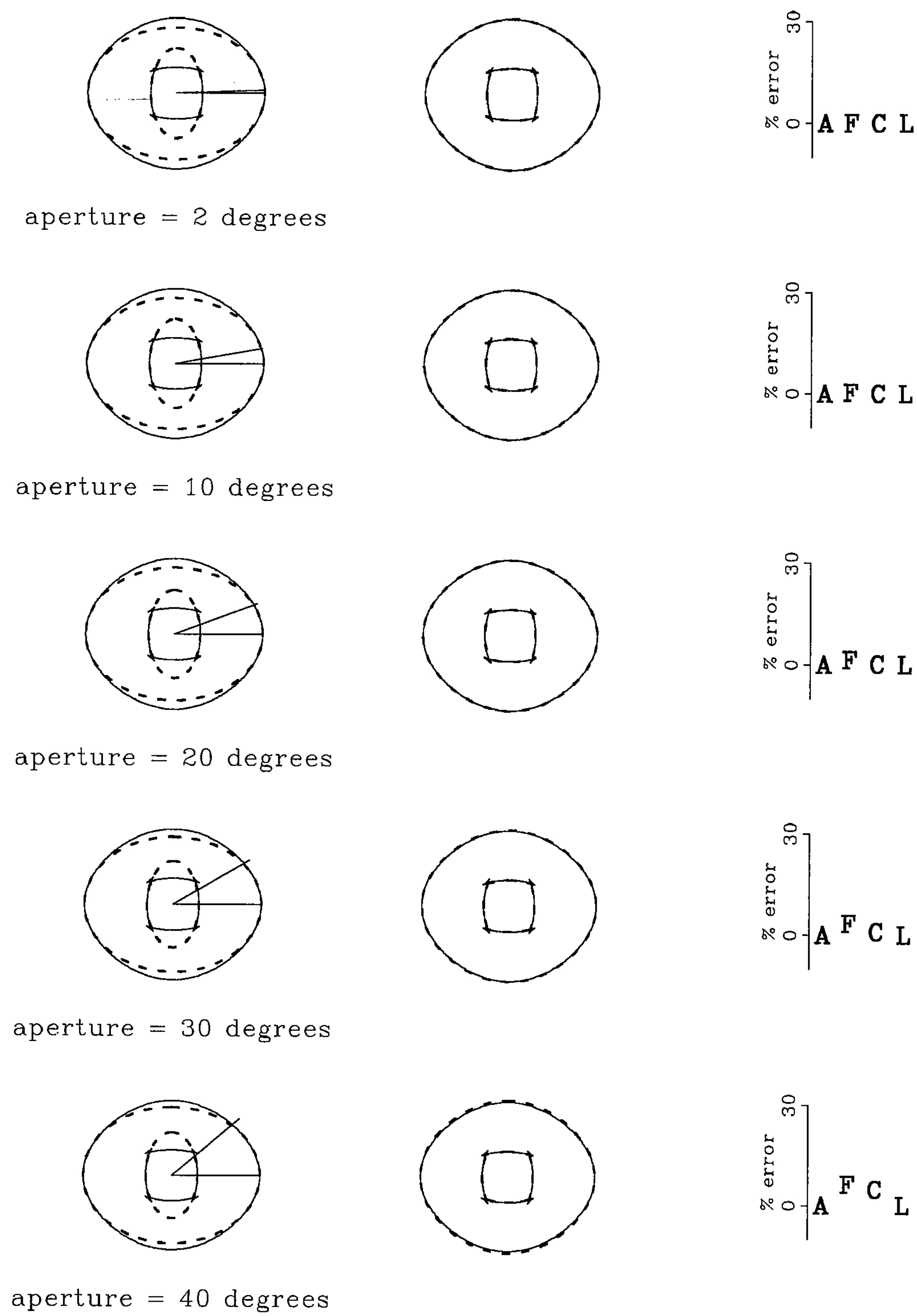

Figure 3: Left: impulse response for P- and SV-waves (continuous lines) compared with their elliptical approximations around the vertical (dashed lines). Center: given impulse responses (continuous) compared to the ones calculated from the estimated elastic constants (dashed). Right: absolute value of the error made in the estimation of the elastic constants when using the horizontal double elliptic approximation. The elastic constants are A = W_{11}, F = W_{13}, C = W_{33}, and L = W_{44}.

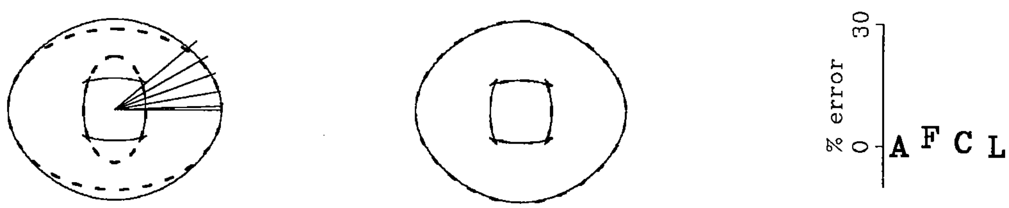

Figure 4: Left: impulse response for P- and SV-waves (continuous lines) compared with their elliptical approximations around the vertical (dashed lines). All ray angles shown are used simultaneously to calculate the elliptical approximations. Center: given impulse responses (continuous) compared to the ones calculated from the estimated elastic constants (dashed). Right: absolute value of the error made in the estimation of the elastic constants when using the horizontal double elliptic approximation. The elastic constants are A = W_{11}, F = W_{13}, C = W_{33}, and L = W_{44}.

Figure 5 shows an example. The left column shows the given P-wave impulse response (continuous line) and the approximating ellipses (dashed lines) around the horizontal and around the vertical for two different apertures. The central column shows the given and approximate impulse responses for P- and SV-waves, the approximate impulse responses calculated from the estimated elastic constants. The right column shows the absolute value of the error made in the estimation of the elastic constants. Unlike the previous approximations, the P-wave double elliptic approximation is valid only at very small angles ($\approx$ 2 degrees). At greater angles it yields highly inaccurate results, particularly in the estimation of W_{44} (L), which controls the shear wave propagation along the axes. Note that for an aperture of 10 degrees, the shear wave impulse response has been considerably underestimated.

This approximation may be hard to use in practical situations because it works only for very small angles near the axes.

CONCLUSIONS

I have shown how to estimate the elastic constants of homogeneous TI media from P-, SV-, and SH-wave traveltimes near a single axis of symmetry (either from VSP or cross-well geometries). For heterogeneous TI media, the procedure can be easily generalized by using tomographic techniques (Michelena et al. 1993; Michelena, 1992a).

The technique uses the parameters obtained by fitting traveltimes near one axis with elliptical models. For SH-wave traveltimes, the estimation of the corresponding TI elastic constants is trivial because SH-wave phase velocities are also elliptical in TI media. For P- and SV-wave traveltimes, four parameters are needed to estimate the phase velocities at all angles from measurements near one axis. These parameters are the direct and *NMO* phase velocities for P- and SV-waves. The transformation from elliptical parameters to elastic constants is simple for both VSP and cross-well geometries.

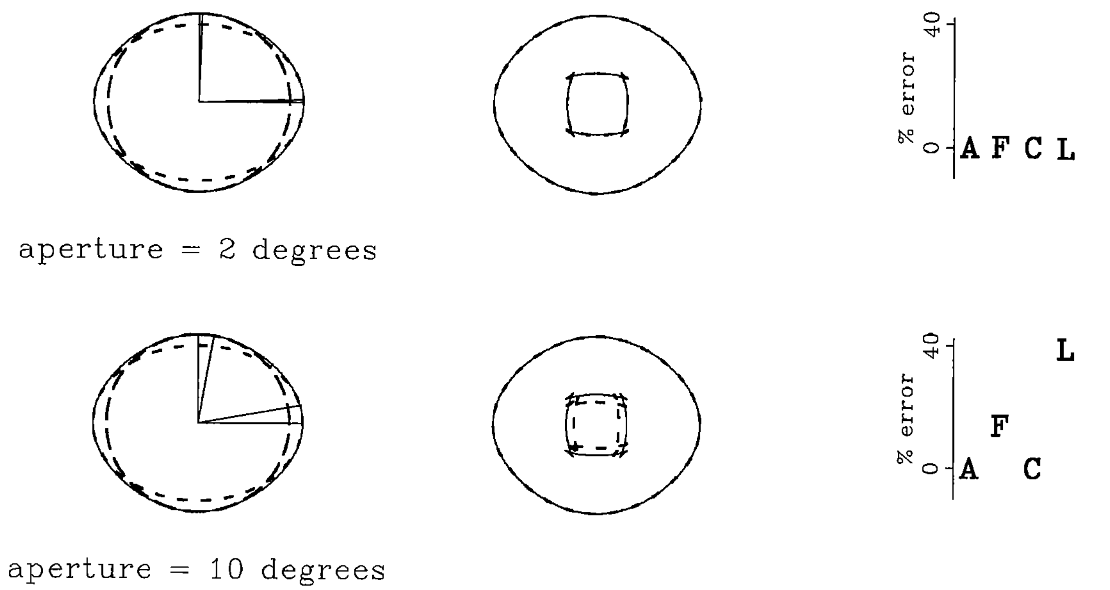

Figure 5: P-wave impulse response used to estimate the elastic constants from measurements around the axes. Left: given P-wave impulse response (continuous) and elliptical approximations around the axes (dashed). Center: given impulse response (continuous) and estimated ones (dashed) for P- and SV-waves. Right: absolute value of the error made in the estimation of the elastic constants when using the P-wave double elliptic approximation. The elastic constants are A = W_{11}, F = W_{13}, C = W_{33}, and L = W_{44}.

The accuracy of the estimation of the elastic constants from the elliptical parameters depends on the accuracy of the estimation of the *NMO* velocities. For small ray angles the approximation is excellent when using either VSP or cross-well geometries because the elliptical approximations are accurate in both cases. In practice, however, using only small ray angles may hinder an accurate estimation of *NMO* velocities. For intermediate ray angles, the estimation of vertical P-wave velocities from cross-well geometries is more accurate than the estimation of horizontal P-wave velocities from VSP. This is because the elliptical approximation for P-wave impulse response fits wider angles around the horizontal than around the vertical where the group velocity is smaller. For large ray angles the approximation doesn't work because elliptical fits are not as accurate.

When compressional and shear velocity logs are also available, they can be used either to add redundancy in the estimation of W_{11} and W_{44} or to check whether the assumption of transverse isotropy is valid or not, in particular when using cross-well measurements where vertical velocities are not well sampled.

In Michelena (1993), I show the combined application of this technique and tomographic methods to estimate spatial variations of elastic constants.

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