PAPER K

TRACE INTERPOLATION IN THE F-X DOMAIN

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ABSTRACT

In this paper we present a method for trace interpolation in the f-x domain. The method relies on the fact that linear events in t-x are predictable in f-x. For the complex Wiener prediction we use an extension of the Burg technique, which allows us to forego the usual assumption that the data is zero at places where it is unavailable, in this case not only the end but also the unknown values in the middle. This approach for interpolation should be very useful in the crosswell geometry, because as previously shown in the surface seismics data, the prediction permits the data to have a few jumps. Other methods will smear those jumps that are critical for traveltime tomography.

INTRODUCTION

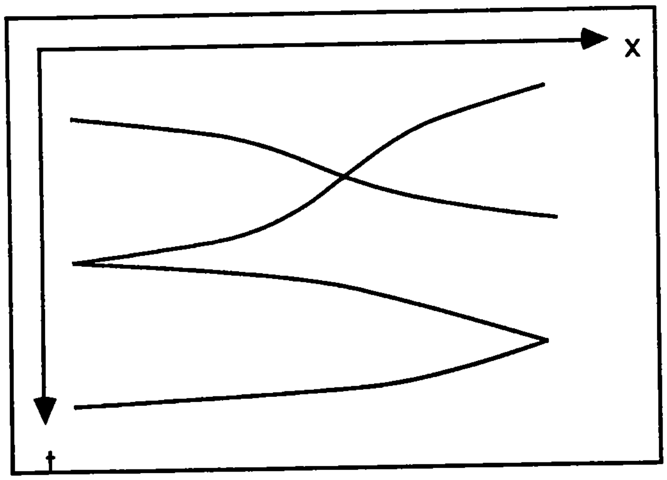
Prediction in the f-x domain is a very successful method for removing random noise from seismic data (Canales, 1984). In that domain, linear events are perfectly predictable with a Wiener prediction filter. The prediction is made at each frequency, after a Fourier transform in time. The complex form of the prediction is needed, since the transformed data is complex.

Once you have the proper prediction filter, it can be used to interpolate missing data, as long as there are not very large gaps. The main problem is to estimate the autocorrelation function from incomplete data.

Here we will use an extension of the Burg technique that handles the missing data in the same well known way as it handles the missing end points.

F-X PREDICTION

The method assumes that the traces are composed of delayed impulses as shown in the next figure:



The events are defined by:

T-X Domain

$$u(t,x) = \sum_k a_k \, \delta(\, t - g_k(x)\,)$$

F-X Domain

$$u(\omega,x) = \sum_k a_k \, e^{-i\omega \, g_k(x)}$$

where: a_k represent the strength of the impulses

and: $g_k(x)$ are delay functions that define the shape of the events.

After assuming the events are linear and allowing the events to have an arbitrary wavelet, the frequency domain form of the model is (Canales, 1984) :

$$u(\omega,x) = \sum_k v_k(\omega) \, e^{-i\omega \, \beta_k \, x}$$

This suggests that the model is composed of purely sinusoidal complex functions, and thus is perfectly predictable with a complex Wiener filter (Canales, 1984).

It turns out that some small amount of curvature is handled correctly, since such a curved event can be represented with a combination of a few linear ones.

THE BURG TECHNIQUE

The needed statistical information is contained in the prediction filter, the autocorrelation, the power spectrum and the reflection coefficients. Once you know one function, you can estimate the other ones. The standard technique starts with an estimate of the autocorrelation function.

The Burg technique estimates the reflection coefficients directly from the data, without explicitly obtaining the autocorrelation, (Burg,1975). The technique is particularly good for short time series because of its treatment of the end effects (Claerbout, 1972).

We will use the complex form of the Burg algorithm, but for simplicity we here start with the Burg algorithm for real data. Its extension to complex data is very simple (Claerbout, 1976).

Start with :

$$f_0(z) = b_0(z) = x(z)$$

Update as:

$$f_{n+1}(z) = f_n(z) - c_{n+1} z^{n+1} b_n(z)$$

$$b_{n+1}(z) = b_n(z) - c_{n+1} z^{-n-1} f_n(z)$$

Where $f_n(z)$ is the z-transform of the forward prediction error (using a filter of order n) and $b_n(z)$ is the backward prediction error one and $x(z)$ is the data. Note that z is the delay operator so that the relation indicates shifting before adding.

Burg's formula for the (n+1)-th reflection coefficient c_{n+1} is:

$$c_{n+1} = \frac{2 * \mathbf{f}_n \cdot \mathbf{b}_n}{\mathbf{f}_n \cdot \mathbf{f}_n + \mathbf{b}_n \cdot \mathbf{b}_n}$$

Where $\mathbf{f}_n$ is the vector representation of the forward prediction error and $\mathbf{b}_n$ is the backward prediction error. The c_n are of magnitude less than one as required by the minimum-phase assumption.

The scalar c_{n+1} is the solution to the following Least Squares Problem (Claerbout,1976).

$$\begin{bmatrix} \mathbf{f}_{n+1} \\ \mathbf{b}_{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_n \\ \mathbf{b}_n \end{bmatrix} - \begin{bmatrix} \mathbf{b}_n \\ \mathbf{f}_n \end{bmatrix} c_{n+1}$$

The updating and the computation of the reflection coefficients use the Burg treatment of the end effects. That is, at each iteration we eliminate the end points from the calculation.

The previous one least squares problem with one unknown parameter relates pairs data from the forward and backward prediction errors. The Burg technique simply eliminates the pairs at the end where the data is missing. We can do this and still have an acceptable value for the reflection coefficient, since its bounding depends entirely on the Schwartz inequality for vectors.

INTERPOLATION WITH THE BURG TECHNIQUE

It is very natural to use the Burg technique in the f-x prediction method, because during the calculation of the reflection coefficients we always update the prediction error. This error is subtracted from the original data to give the predicted signal.

Another advantage is that the Burg technique is good for short time series, because it is not necessary to assume zero values at the ends. In the f-x prediction case it means we do not have to assume zero traces at the ends.

For the interpolation case it is even more important, since we can extend the treatment of the end effects to missing data. We simply eliminate pairs that have a missing data point, not only at the ends but also in the middle.

One important fact is that for the missing data at the end, once a data point is eliminated it is never used in subsequent lags. On the other hand, a data point in the middle may pair with a missing data point at one lag and with a known point at another lag. In on case the data point will not be used and in the other case it will be.

RESULTS

In figure 1 we show a synthetic section. It has continuous events with some curvature and a large amount of added random noise. Traces 19, 20, 22, 23, 45, 46 and 47 have been zeroed (counting from the right).

Figure 2 shows the section from figure 1, after f-x Prediction. The missing traces have been predicted with the one-step ahead prediction filter. Note that in general a great deal of random noise has been attenuated, this is the standard benefit of f-x prediction.

Figure 3 show the section after using the new treatment of missing data. The only difference in the method is the estimation of the reflection coefficients since the prediction is done in exactly the same way. Note that the amplitudes in figure 2 are biased towards zero. This is of course because the estimation assumed those traces to be zero for the standard f-x Prediction. On the other hand the new treatment of missing data has predicted the missing traces correctly.

Note that the results are different from linear interpolation between traces. The noise has not been interpolated.

CONCLUSIONS

We have presented a method for interpolation of missing traces based on f-x prediction via the Burg technique. By extending the Burg treatment of the end effects for missing data, we estimate the reflection coefficients and thus the prediction filter without assuming zeroes for the missing data.

The technique predicts traces that satisfy a very simple interpretational model, and it can handle events with some curvature.

The method is very valuable in two aspects. First it improves the standard random noise attenuation when traces are missing and secondly it provides us with the interpolated traces.

For larger gaps, an average from forward and backward prediction should be used, as the prediction deteriorates with prediction distance.

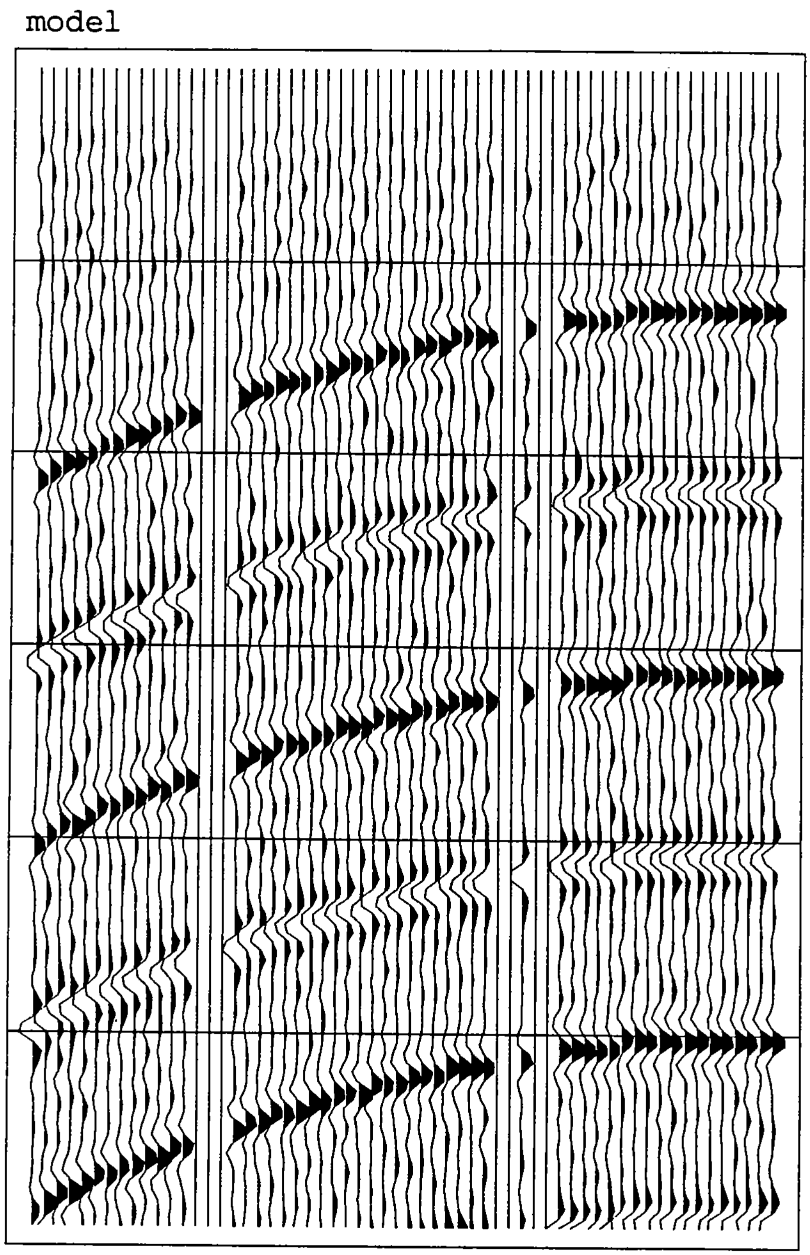


Figure 1
Synthetic input model.

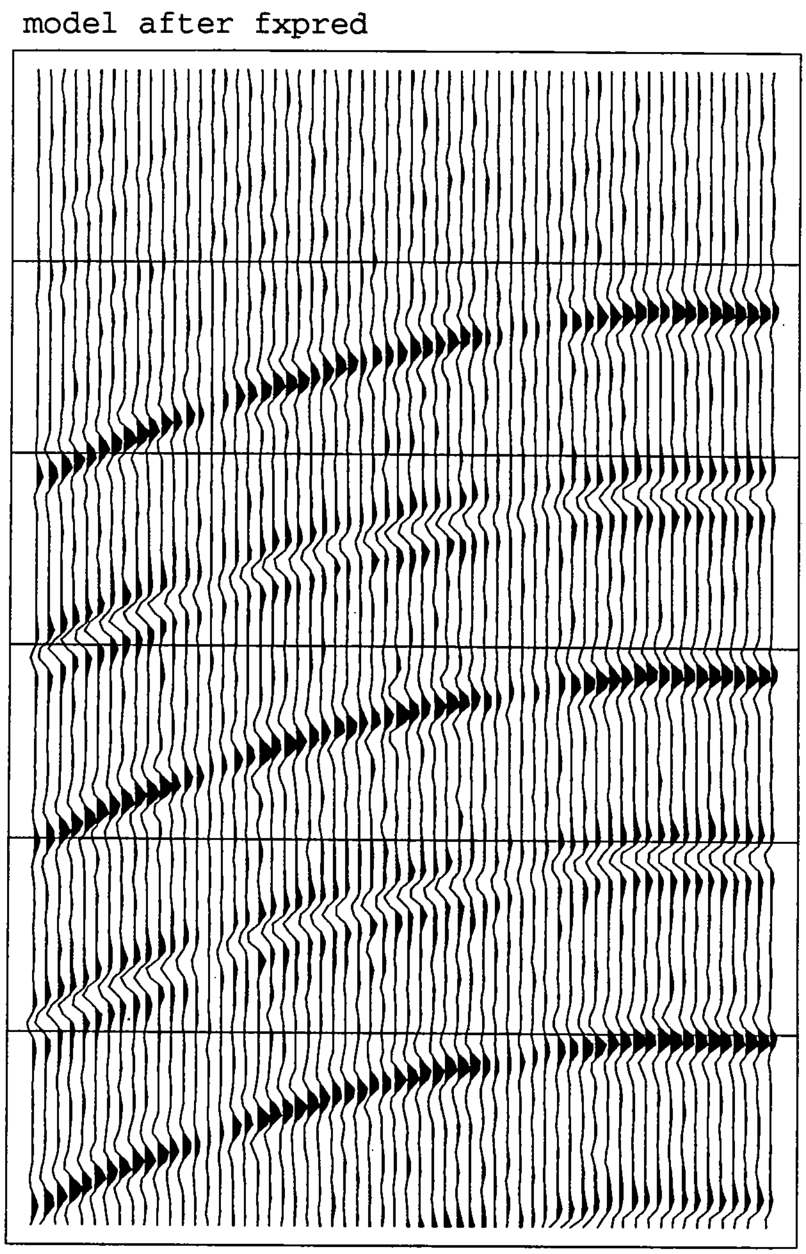


Figure 2
Model after Standard f-x Prediction.

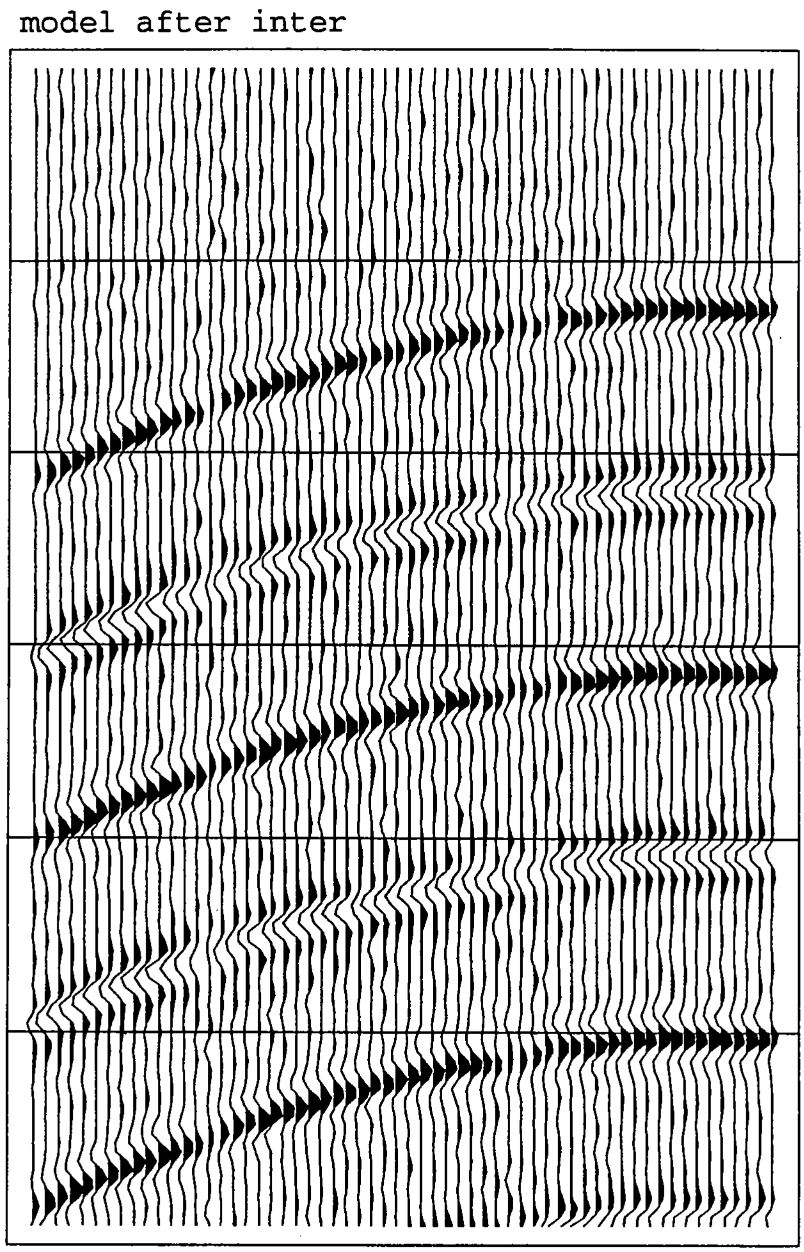


Figure 3
Model after f-x Prediction with the new method

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