PAPER D

ANISOTROPY FROM HEAD WAVES IN CROSSWELL DATA PART 1 : THEORY

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ABSTRACT

A method for using head waves observed in crosswell data to analyze anisotropic properties of sedimentary rocks was developed. Head waves are critical refractions generated at the interface of low and high velocity formations. Anisotropic properties of the low velocity formation significantly affect the velocity of the head waves because the head waves propagate obliquely through the formation. Unlike surface seismic or VSP survey, the crosswell recording geometry makes it easy to observe such head waves. This paper focuses on the theory of estimating anisotropic properties from head waves in crosswell seismic and well logs using a model of a horizontally layered transverse isotropic medium. Our theoretical study has led to a simple relationship between traveltimes of head waves observed on a wellbore and the velocity of seismic waves in the anisotropic medium. The relationship easily yields the phase velocity and the incident angle of the head waves, which are usually missing information in surface seismic or VSP survey. The information is essential to estimating anisotropic properties of sedimentary rocks.

INTRODUCTION

The model of a transverse isotropy is widely accepted for describing elastic anisotropy of sedimentary rocks. Horizontal layering of different isotropic (or anisotropic) media yields an equivalent elastically anisotropic property, if the wave length of the wave is much larger than the typical layer thickness. This type of anisotropy is called transverse isotropy. In general, elastic anisotropy can be directly associated with elastic constants of the materials. Elastic materials have twenty-one elastic constants. Usually, symmetrical properties reduce the number of independent constants. In the case of a transverse isotropy, the anisotropic behavior is governed by five independent constants (Backus, 1962). Berryman (1979) led to a relation between the phase velocity and the group velocity of seismic waves generated by a point source located in the transversely isotropic medium.

The measurement of elastic constants is usually performed in a laboratory using core samples (Tosaya, 1982; Jones and Wang, 1981). In contrast, in-situ estimation of elastic constants is rarely performed because of the limitation of resolution of in-situ measurements and the difficulties in obtaining sufficient data for determining elastic constants.

In recent years, crosswell seismic tomography has been developed in order to determine the detailed velocity structure between two wells. While surface seismic and VSP surveys observe nearly vertical velocities, this new crosswell geometry makes it possible to measure an accurate horizontal velocity. As a result, the integration of these techniques enables the detailed analysis of elastic properties of sedimentary rocks. Winterstein and Paulsson (1990) discussed anisotropic properties of a shale using both crosswell and VSP data, and determined in-situ five in-situ elastic constants according to a model of a transverse isotropy.

Head waves are critical refraction generated at the interface of fast and slow velocity formations. These waves can be easily observed in the crosswell geometry. Unlike direct body waves between two wells, the incident angle of the head waves depends only on the velocity contrasts at the interface. A large velocity contrast at the interface enhances the detection of head waves in crosswell data and makes it easy to recognize the effect of the anisotropy of a low velocity formation, regardless of the formation thickness.

In this paper, we present a method for estimating anisotropic parameters and elastic constants by combining head waves in crosswell geometry with sonic logs. The method is based on the theory of an acoustic wave propagation in a transverse isotropic medium (Thomsen, 1986). We first verify a simple relation between the apparent velocity of head waves at receiver arrays and a true phase velocity of head waves propagating through a low velocity formation. This relation leads to a practical method for deducing elastic constants and anisotropic parameters from in-situ measurements.

SEISMIC WAVE PROPAGATION

Horizontal layering of isotropic or anisotropic materials gives rise to a behavior of elastic aniostropy if the wave length of a wave is much larger than the typical thickness of each layer (Backus, 1962). This type of anisotropy is called transverse isotropy (see Figure 1). The elastic constants consists of five independent components (Thomsen, 1986). The elastic constants can be expressed in the following two-rank tensor notation:

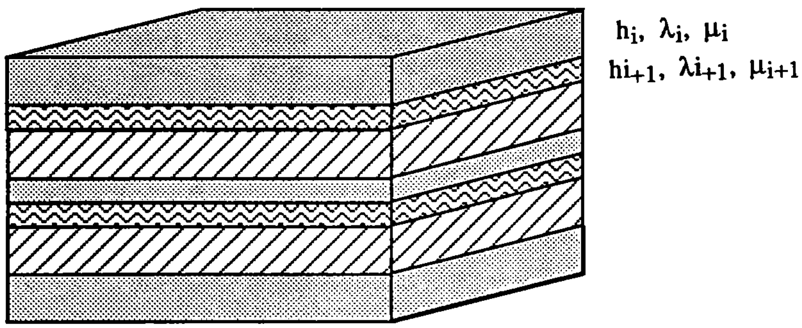

Figure 1: The equivalent model of a transverse isotropic medium. The horizontal layering of thin isotropic beds shows anisotropic property if the wave length of a seismic wave is much greater than the typical thickness of each layer.

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad (1)$$

$C_{12} = C_{11} - 2C_{66}.$

Backus (1962) clarified that these five constants are deducable from the following averaging of Lamb's constant λ_i and shear modulus μ_i of each thin isotropic layer :

$$C_{11} = \frac{1}{\Sigma h_i} \Sigma \frac{4\mu_i(\lambda_i+\mu_i)h_i}{\lambda_i+2\mu_i} + \left(\frac{1}{\Sigma h_i}\Sigma \frac{h_i}{\lambda_i+2\mu_i}\right)^{-1} \left(\frac{1}{\Sigma h_i} \Sigma \frac{\lambda_i h_i}{\lambda_i+2\mu_i}\right)^2 \quad (2)$$

$$C_{33} = \left(\frac{1}{\Sigma h_i} \Sigma \frac{h_i}{\lambda_i+2\mu_i}\right)^{-1} \quad (3)$$

$$C_{13} = \left(\Sigma \frac{h_i}{\lambda_i+2\mu_i}\right)^{-1} \Sigma \frac{\lambda_i h_i}{\lambda_i+2\mu_i} \quad (4)$$

$$C_{44} = \left(\frac{1}{\Sigma h_i} \Sigma \frac{h_i}{\mu_i}\right)^{-1} \quad (5)$$

$$C_{66} = \frac{1}{\Sigma h_i} \Sigma h_i \mu_i \quad (6)$$

where h is the thickness of each layer. Hook's law provides a relation between stresses and strains. Using the above expression, the relationship is given by

$$T_i = \Sigma C_{ij} S_j \qquad (i, j = 1, 2, 3). \quad (7)$$

Ti and Sj are abbreviated expressions for components of stress and strain tensors :

$$T_1 = \sigma_{xx},\ T_2 = \sigma_{yy},\ T_3 = \sigma_{zz},\ T_4 = \sigma_{yz},\ T_5 = \sigma_{xz},\ T_6 = \sigma_{xy}$$

$$S_1 = \frac{du_x}{dx},\ S_2 = \frac{du_y}{dy},\ S_3 = \frac{du_z}{dz},$$

$$S_4 = \frac{du_z}{dy} + \frac{du_y}{dz},\ S_5 = \frac{du_z}{dx} + \frac{du_x}{dz},\ S_6 = \frac{du_y}{dx} + \frac{du_x}{dy}$$

On the other hand, the equations of motion are given by

$$\frac{d\sigma_{ij}}{dx_j} = \rho \frac{d^2u_i}{dt^2} \qquad (i = 1, 2, 3 \quad X_1 = x, X_2 = y, X_3 = z) \tag{8}$$

Suppose that a plane wave is propagating in X-Z plane, and the symmetry axis is Z direction. Then, the displacements vector U is expressed as

$$U = \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} e^{i\{wt-k(lxX+lzZ)\}}. \tag{9}$$

where

$$l_x = \sin\theta \qquad l_z = \cos\theta \tag{10}$$

θ is an angle between the direction of the plane wave and the symmetry axis. Using Eqns. 2 and 3, the displacements Ux, Uy, Uz must satisfy the following christoffel equations :

$$k^2 \begin{bmatrix} C_{11}l_x^2+C_{44}l_z^2 & 0 & (C_{13}+C_{44})l_xl_z \\ 0 & C_{66}l_x^2+C_{44}l_z^2 & 0 \\ (C_{13}+C_{44})l_xl_z & 0 & C_{44}l_x^2+C_{33}l_z^2 \end{bmatrix} \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} = \rho\omega^2 \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix}. \tag{11}$$

The dispersion relations are given by

$$\begin{vmatrix} C_{11}l_x^2+C_{44}l_z^2-\rho\omega^2/k^2 & 0 & (C_{13}+C_{44})l_xlz \\ 0 & C_{66}l_x^2+C_{44}l_z^2-\rho\omega^2/k^2 & 0 \\ (C_{13}+C_{44})l_xl_z & 0 & C_{44}l_x^2+C_{33}l_z^2-\rho\omega^2/k^2 \end{vmatrix} = 0 \tag{12}$$

This equation leads three seismic modes propagating in a transverse isotropic medium. The following three quadratic forms were denoted by Thomsen (1986):

$$\rho V_{SH}^2(\theta) = C_{66}\sin^2\theta + C_{44}\cos^2\theta \tag{13}$$

$$\rho V_P^2(\theta) = \frac{1}{2}\left[C_{33}+C_{44}+(C_{11}-C_{33})\sin^2\theta+D(\theta)\right] \tag{14}$$

$$\rho V_{SV}^2(\theta) = \frac{1}{2}\left[C_{33}+C_{44}+(C_{11}-C_{33})\sin^2\theta-D(\theta)\right] \tag{15}$$

$$D(\theta) = \{(C_{33}-C_{44})^2+2[2(C_{13}+C_{44})^2-(C_{33}-C_{44})(C_{11}+C_{33}-2C_{44})]\sin^2\theta$$
$$+[(C_{11}+C_{33}-2C_{44})^2-4(C_{13}+C_{44})^2]\sin^4\theta\}^{1/2}.$$

These modes correspond to SH, quasi-P, and quasi-SV waves propagating in a transverse isotropic medium.

The behaviors of the seismic waves are somewhat different from those in isotropic media. First, the velocity of seismic waves depends on the direction of the incident angle (angular dispersion). This angular dispersion is directly associated with the fact that the direction of the seismic ray is not parallel to the wave normal (see Figure 2). The seismic energy propagates with a group velocity. In anisotropic media, the group velocity is different from the phase velocity which corresponds to the speed of a wave normal. Second, as for quasi-P and quasi-SV modes, the direction of displacements U_x and U_z are no longer parallel or perpendicular to the wave normal.

Berryman 1979) induced a simple relation between the phase velocity and the group velocity for seismic waves generated by a point source in a transversely isotropic medium :

$$\tan\phi = \frac{V(\theta)\sin\theta + \frac{dV}{d\theta}\cos\theta}{V(\theta)\cos\theta - \frac{dV}{d\theta}\sin\theta} \qquad (16)$$

$$V_g(\phi) = \sqrt{V(\theta)^2 + \left(\frac{dV}{d\theta}\right)^2} \qquad (17)$$

where $V(\theta)$ and $V_g(\phi)$ are phase and group velocities, respectively. In-situ measurements of seismic waves in a crosswell geometry is based on the group velocity and angle (see Figure 3).

Eqn. 17 shows that in the case of an angular dispersion, a group velocity is always equal to or greater than the corresponding phase velocity. If the group and phase velocities are same, then the phase angle and the group angle is also same.

It should be noted that explicit expressions of $V_P(\phi)$ and $V_{sv}(\phi)$ are very complicated. This suggests that even if we can determine the $V(\phi)$ and ϕ from observed direct waves, it is difficult to directly induce elastic constants from such results. However, as we discuss later, if we use head waves, we can directly obtain a phase velocity $V_P(\theta)$.

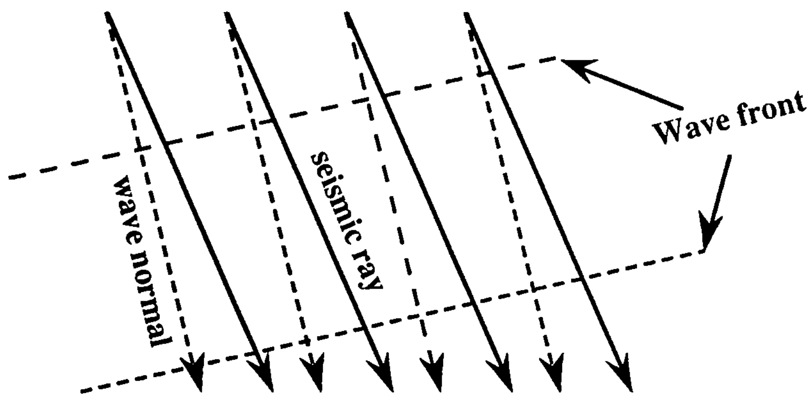

Figure 2: Difference between the wave normals and the seismic rays for incident plane waves in an anisotropic medium. The seismic rays are oblique to the wave front.

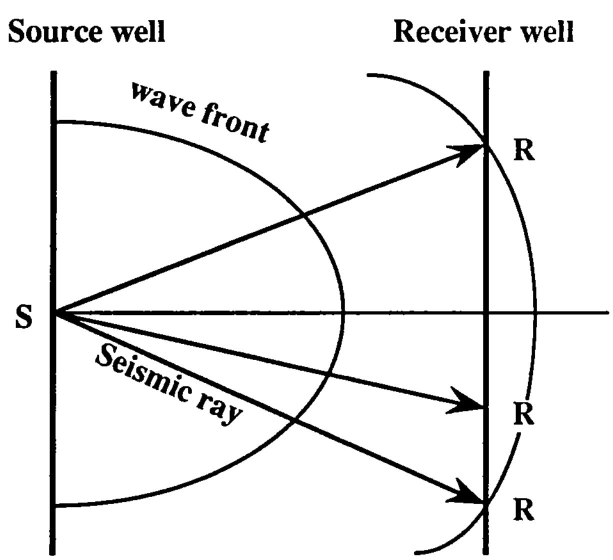

Figure 3: Detection of seismic waves in a crosswell geometry. In a homogeneous medium, the travel time is simply expressed as $T = L/Vg(\phi)$ where L is a distance between the source and the receiver.

GENERALIZED SNELL'S LAW

Seismic waves refract at the interface of two elastic media. Regardless of isotropic or anisotropic, the component of a slowness surface (reciprocal of a phase velocity) parallel to the interface has to remain constant (Henneke,1971). That is expressed as

$$\frac{K_{x1}}{\omega} = \frac{K_{x2}}{\omega} = P(\text{constant}) \qquad (18)$$

where the X-axis is parallel to the interface, and K_x expresses an X-component of the wave number K (see Figure4). This is defined as the generalized Snell's law in this paper. Suppose that a symmetry axis of a transverse iostropic medium coincides with Z direction. Then, K is a function of θ, the angle between the Z-axis and the incident wave.

$$K_x = K(\theta)\sin\theta, \quad K_z = K(\theta)\cos\theta \qquad (19)$$

$$V(\theta) = \frac{\omega}{K(\theta)} \qquad (20)$$

The generalized Snell's law can be simply expressed as

$$\frac{\sin\theta}{V(\theta)} = P_\theta(\text{constant}). \qquad (21)$$

It should be noted that the velocity used in these equation is a phase velocity. As already discussed, we can define a group velocity in anisotropic media. The expression of the Snell's law using the group velocity is also significant because in-situ measurements of seismic waves are usually based on the group angle and velocity.

Fermat's principle requires that the path of a seismic ray between two points minimize the traveltime along the pass. According to the procedure shown in Appendix A, we can obtain

$$\frac{\sin\phi}{V_g(\phi)} - \frac{dV_g}{d\phi}\frac{\cos\phi}{V(\phi)^2} = P_\phi(\text{constant}) \qquad (22)$$

along the seismic ray. Using Eqn. 11 and 12, we can show that Eqn.17 is equivalent to Eqn. 16, that is $P_\theta = P_\phi$ (see Appendix B).

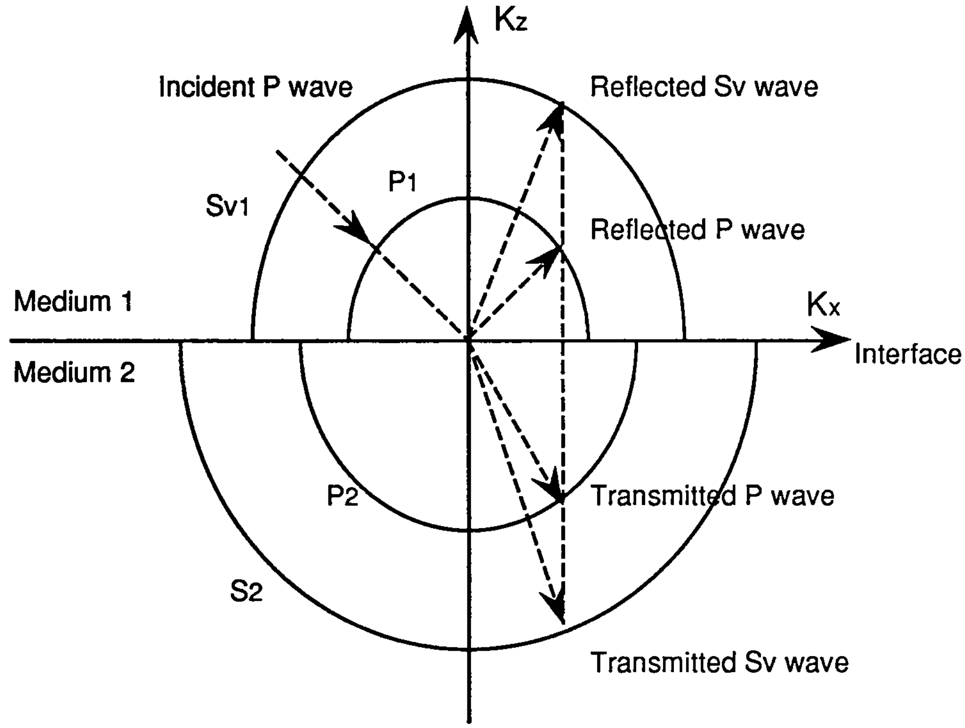

Figure 4: Slowness field in anisotropic media. An incident P wave refracts and reflects at the interface between the two media. The generalized Snell's law requires that the horizontal component of the slowness of each mode is equal to that of the incident wave.

Thus, the generalized Snell's law in anisotropic media is represented by

$$\frac{\sin\theta}{V(\theta)} = \frac{\sin\phi}{V_g(\phi)} - \frac{dV_g}{d\phi}\frac{\cos\phi}{V_g(\phi)^2} = P \text{ (constant)} \tag{23}$$

HEAD WAVES IN CROSSWELL SURVEYS

A head wave is critical refraction at the interface of two different media. This head wave can be easily detected in the crosswell geometry. The following situations are required for the detection.

- Both an energy source and a receiver are located in a low velocity medium.
- The velocity contrast at the interface is large enough to create head waves between two wells.
- The head waves arrive at receivers earlier than direct waves because of the small amplitude of the head waves..

Figure 5 shows the concept of the detection of head waves and direct wave in the crosswell geometry.

Anisotropic property of the low velocity formation significantly affects the traveltimes of head waves observed in the crosswell geometry. The incident angle depends only on the velocity contrasts between the two media, regardless of the formation thickness. The detection of head waves itself implies a large velocity contrast at the interface.

The incident angle of a seismic ray at the critical refraction is defined as an angle such that the seismic ray becomes horizontal after the refraction. Figure 6 illustrates the ray path of head waves. The traveltime is divided into the three parts.

T_s : Traveltime between the source and the interface
T_2 : Traveltime along the interface
T_r : Traveltime between the interface and the receiver

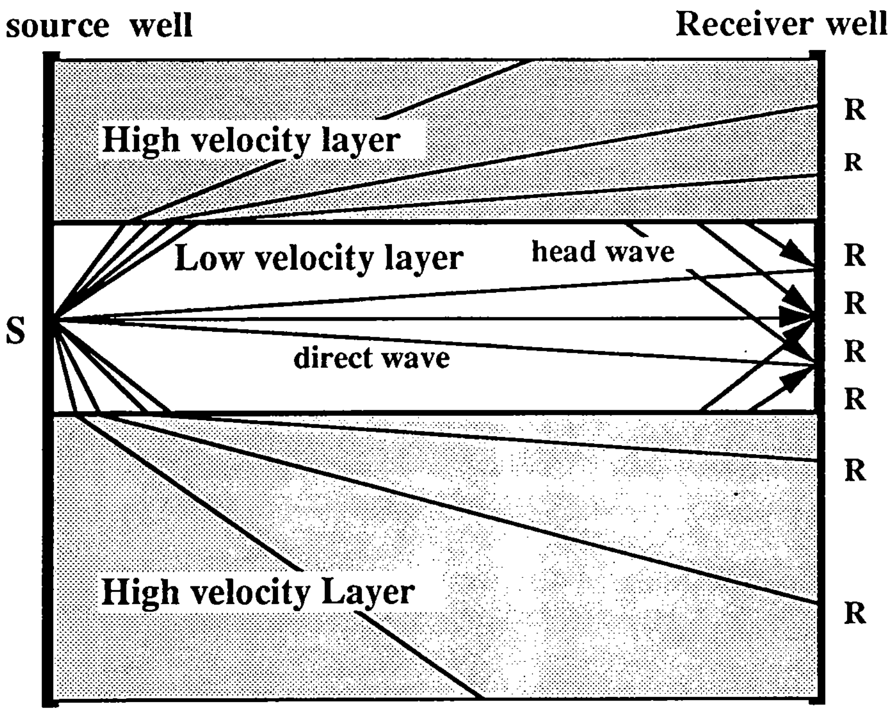

Figure 5: Observation of head waves in a crosswell survey. If both a source and a receiver are located in a low velocity layer, head waves are detectable.

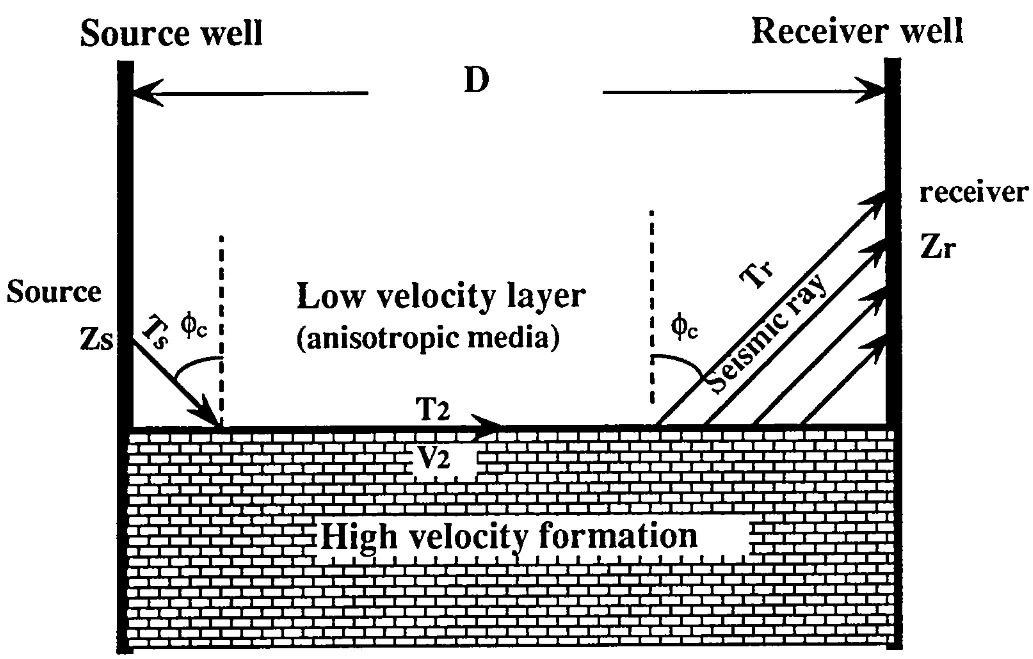

Figure 6: Seismic ray path of head waves observed in a crosswell geometry. After the critical refraction, the seismic ray propagates along the interface, turning to the low velocity formation.

T_s, T_2, and T_r are simply given by

$$T_s = \frac{\Delta Z_s}{V_g(\phi_c)\cos\phi_c} \tag{24}$$

$$T_2 = \frac{D - \tan\phi_c(\Delta Z_s + \Delta Z_r)}{V_2} \tag{25}$$

$$T_r = \frac{\Delta Z_r}{V(\phi_c)\cos\phi_c} \tag{26}$$

The total traveltime is

$$T = \frac{D}{V_2} + \left(\frac{1}{V(\phi_c)\cos\phi_c} - \frac{\tan\phi_c}{V_2}\right)(\Delta Z_s + \Delta Z_r) \tag{27}$$

where

D	:	Horizontal distance between the two well
ϕ_c	:	Incident angle (group angle) at the interface
$V_g(\phi_c)$	:	Group velocity of head waves in a low velocity formation
V_2	:	Horizontal velocity of a high velocity formation
ΔZ_s	:	Vertical offset distance of a source from the interface
ΔZ_r	:	Vertical offset distance of a receiver from the interface

V_2 is equal to the horizontal phase velocity of the high velocity formation under the assumption of transversely isotropic media with a vertical symmetry axis. The generalized Snell's law requires that the head wave satisfy the following equation :

$$\frac{\sin\phi_c}{V_g(\phi_c)} - \frac{dV_g}{d\phi c}\frac{\cos\phi_c}{V_g(\phi_c)^2} = \frac{\sin\theta_c}{V(\theta_c)} = \frac{1}{V_2} \tag{28}$$

where θ_c is the corresponding phase angle. Using equations (16), (17) and (25),

$$\frac{1}{V_g(\phi_c)\cos\phi_c} - \frac{\tan\phi_c}{V_2} = \frac{\cos\theta_c}{V(\theta_c)} \tag{29}$$

$$\cos\theta_c = \sqrt{1 - \left(\frac{V(\theta_c)}{V_2}\right)^2} \tag{30}$$

$$\frac{1}{V_g(\phi_c)\cos\phi_c} - \frac{\tan\phi_c}{V_2} = \sqrt{\frac{1}{V(\theta_c)^2} - \frac{1}{V_2^2}}. \tag{31}$$

Putting this to Eqn. 27, the traveltime is given by

$$T = \frac{D}{V_2} + \sqrt{\frac{1}{V(\theta_c)^2} - \frac{1}{V_2^2}}\,(\Delta Z_s + \Delta Z_r). \tag{32}$$

If the low velocity formation is weakly inhomogeneous in lateral, the second term of the right -hand side can be divided into two parts.

$$T = \frac{D}{V_2} + \sqrt{\frac{1}{V_s(\theta_c)^2} - \frac{1}{V_2^2}}\,\Delta Z_s + \sqrt{\frac{1}{V_r(\theta_c)^2} - \frac{1}{V_2^2}}\,\Delta Z_r \tag{33}$$

where suffixes s and r represent the source and the receiver, respectively.

Travel time T has a linear relation with ΔZ_s and ΔZ_r. This means that the head waves behave like plane waves around the receiver array in a wellbore. According to the calculations shown in Appendix B, we can show that θ_c is the angle between the interface and the wave front while ϕ_c corresponds to the angle between the seismic rays and the normal direction to the interface. In a transverse isotropic medium, the wave front is not perpendicular to the direction of the seismic rays. Figure 7 illustrates the behavior of head waves around a receiver array.

An apparent head wave velocity V_{hd} is defined as a reciprocal of the gradient of travel times along the subsequent receivers :

$$\frac{\Delta T}{\Delta Z} = \frac{1}{V_{hd}} \tag{34}$$

From Eqns. 33 and 34, V_{hd} and $V(\theta_c)$ have a simple relation :

$$\frac{1}{V_{hd}} = \sqrt{\frac{1}{V(\theta_c)^2} - \frac{1}{V_2^2}}. \tag{35}$$

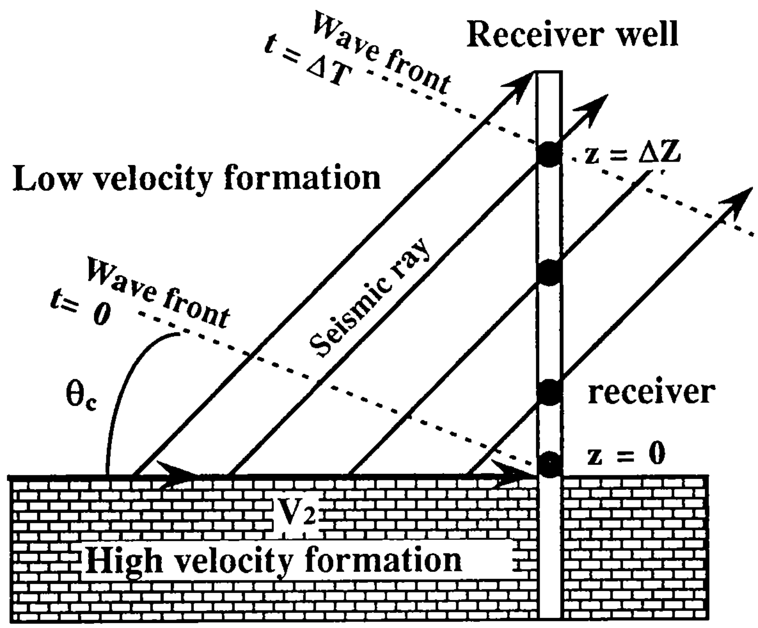

Figure 7: Behavior of head waves around receivers. θc corresponds to the angle between the wave front and the interface.

Thus, the traveltime T can be simply expressed by apparent head wave velocities observed at a source array and a receiver array :

$$T = \frac{D}{V_2} + \frac{\Delta Z_s}{V_{hds}} + \frac{\Delta Z_r}{V_{hdr}} \tag{36}$$

Analysis of Head Waves

As shown in Eqn. 36, traveltimes of head waves have a linear relation with ΔZ_s and ΔZ_r. The coefficients of ΔZ_s and ΔZ_r are reciprocals of V_{hd} around source and receiver wells. Eqn. 36 can be rewritten as

$$T = \alpha D + \beta \Delta Z_s + \gamma \Delta Z_r \tag{37}$$

where

$$\alpha = \frac{T_0}{D}, \quad \beta = \frac{1}{V_{hds}}, \quad \gamma = \frac{1}{V_{hdr}}. \tag{38}$$

All of α, β and γ have a dimension of slowness. .The least square method can easily determine the coefficients, α, β and γ from observed head waves. It should be noted that β and γ are not significantly affected by the constant bias on ΔZ_s and ΔZ_r. For the determination of these coefficients, Z_s and Z_r may be relative vertical distance from a arbitrarily fixed origin . That is

$$\Delta Z_s = Z_s - Z_0 \qquad \Delta Z_r = Z_r - Z_0 \tag{39}$$

where Z_0 is the depth of a local origin. (This is one of the advantages of the head wave analysis because the accurate depth of the interface is not required. Sometimes, it is difficult to determine the accurate location of the boundary.) In contrast, the value of α is sensitive to the bias for ΔZ_s and ΔZ_r. This suggests that we should avoid determining V_2 from α. Fortunately, V_2 can be easily derived from the direct waves propagating along the interface.

The least square method leads to the following simultaneous equation :

$$Ax = y \tag{40}$$

$$A = \begin{bmatrix} Dn & \sum_{i=1}^{n} Z_{si} & \sum_{i=1}^{n} Z_{ri} \\ D\sum_{i=1}^{n} Z_{si} & \sum_{i=1}^{n} Z_{si}^2 & \sum_{i=1}^{n} Z_{si}Z_{ri} \\ D\sum_{i=1}^{n} Z_{ri} & \sum_{i=1}^{n} Z_{si}Z_{ri} & \sum_{i=1}^{n} Z_{ri}^2 \end{bmatrix} \quad X = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \quad Y = \begin{bmatrix} \sum_{i=1}^{n} T_i \\ \sum_{i=1}^{n} Z_{si}T_i \\ \sum_{i=1}^{n} Z_{ri}T_i \end{bmatrix}.$$

$V_s(\theta_c)$ and $V_r(\theta_c)$ are simply induced from the solution of the above equation:

$$V_s(\theta_c) = \frac{1}{\sqrt{\beta^2 + \frac{1}{V_2^2}}} \qquad V_r(\theta_c) = \frac{1}{\sqrt{\gamma^2 + \frac{1}{V_2^2}}}. \tag{41}$$

The generalized Snell's law gives the incident angle (critical phase angle)

$$\theta_c = \frac{V(\theta_c)}{V_2}. \tag{42}$$

Consequently, the head wave analysis allows us to simultaneously obtain $V(\theta_c)$ and θ_c.

ESTIMATION OF ELASTIC CONSTANTS

The behaviors of quasi-P and quasi-SV waves can be describe by four independent elastic constants C_{11}, C_{33}, C_{44}, and C_{13}. Except for C_{13}, each constant is directly associated with an in-situ measurement :

$$C_{11} = \rho V_h^2 \qquad C_{33} = \rho V_v^2 \qquad C_{44} = \rho V_{sv}^2 \tag{43}$$

A crosswell survey can measure the horizontal velocity V_h from a direct wave between two wells. V_v and V_{sv} are vertically propagating P and S waves, which can be provided by an acoustic logging such as a full wave sonic log. In contrast, C_{13} cannot be estimated from the above velocities because C_{13} only affects the velocity of a obliquely propagating seismic wave.

A more convenient representation is provided in terms of the parameters s, e and δ^* for describing $V(\theta)$, the phase velocity of a quasi-P wave (Thomsen, 1986):

$$V^2(\theta) = V_v^2[\ 1 + \varepsilon\sin^2\theta + E(\theta)\ ] \tag{44}$$

$$E(\theta) = \frac{1-\sigma^2}{2}\left[\sqrt{1+\frac{4\delta^*\sin^2\theta\cos^2\theta+4(1-\sigma^2+\varepsilon)\varepsilon\sin^4\theta}{(1-\sigma^2)^2}} - 1\right] \tag{43}$$

$$\sigma = \frac{V_{sv}}{V_h} = \sqrt{\frac{C_{44}}{C_{11}}} \tag{45}$$

$$\varepsilon = \frac{1}{2}\left(\frac{V_h^2}{V_v^2} - 1\right) = \frac{1}{2}\left(\frac{C_{11}}{C_{33}} - 1\right). \tag{46}$$

Fortunately, σ and ε does not include the term of C_{13}. These two parameters can be easily determined by the in-situ measurements. However, δ^* does depend on C_{13} :

$$\delta^* = \frac{1}{2C_{33}^2}[\ 2(C_{13}+C_{44})^2 - (C_{33} - C_{44})(C_{11} + C_{33} - 2C_{44})] \tag{47}$$

This implies that an additional oblique velocity is required for determining δ^*. Using $V(\theta_c)$, σ, and ε, we can calculate the value of δ^*:

$$\delta^* = -(1-\sigma^2+\varepsilon)\varepsilon\tan^2\theta_c + \frac{\left(\frac{V^2(\theta_c)}{V_v^2} - 1 - \varepsilon\sin^2\theta_c\right)\left(\frac{V^2(\theta_c)}{V_v^2} - \sigma^2 - \varepsilon\sin^2\theta_c\right)}{\sin^2\theta_c\cos^2\theta_c}. \tag{47}$$

Thus, the analysis of head waves contributes directly to obtaining the value of δ^*.

Based on the model, we would like to propose a practical method for estimating δ^* and the associated C_{13}. The method consists of the following steps.

1. Determine V_{hd} from travel times of head waves.
2. Derive $V(\theta_c)$ and θ_c.
3. Estimate σ and ε from sonic logs and crosswell data.
4. Calculate δ^* and C_{13} using the above information.

CONCLUSION

Our theoretical study demonstrated that the analysis of head waves has several advantages for estimating anisotropic parameters and the associated elastic constants:

- The analysis of head waves enables the derivation of an incident angle and a velocity of a head wave in a phase domain. This makes it easy to estimate δ^* and C_{13}.
- The incident angle of head waves depends only on the velocity contrast between low and high velocity formation. The detection of head waves in the crosswell geometry itself suggests a large velocity contrast. Anisotropic property of the low velocity formation significantly affects the velocity of head waves, regardless of the formation thickness..
- The exact location of the interface that create head waves is not required because an apparent head wave velocity can be determined by the gradient of travel times along a receiver array in a wellbore

Based on the above advantages, we proposed a practical method for estimating δ^* and C_{13} by combining head waves with other in-situ measurements.

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APPENDIX A: SNELL'S LAW IN A GROUP DOMAIN

According to the geometry illustrated in Figure A-2, the travel time between S and R can be written as

$$T = \frac{h_1}{V_{g1}(\phi_1)\cos\phi_1} + \frac{h_2}{V_{g2}(\phi_2)\cos\phi_2}. \tag{A-1}$$

The incident and refracted angles are given by

$$\tan\phi_1 = \frac{x}{h_1} \qquad \text{and} \qquad \tan\phi_2 = \frac{L-x}{h_2}. \tag{A-2}$$

Fermat's principle requires that the position of P make the traveltime minimum. That is,

$$\frac{dT}{dx} = 0. \tag{A-3}$$

Then,

$$\frac{dT}{dx} = \frac{d\phi_1}{dx}\frac{d}{d\phi_1}\left(\frac{h_1}{V_g(\phi_1)\cos\phi_1}\right) + \frac{d\phi_2}{dx}\frac{d}{d\phi_2}\left(\frac{h_2}{V_g(\phi_2)\cos\phi_2}\right). \tag{A-4}$$

From Eqn. A-2,

$$\frac{d\phi_1}{dx} = \frac{\cos^2\phi_1}{h_1} \qquad \frac{d\phi_2}{dx} = -\frac{\cos^2\phi_2}{h_2}. \tag{A-5}$$

Putting these relations into equation (53), we can derive

$$\frac{\sin\phi 1}{V_g(\phi_1)} - \frac{dV_g}{d\phi_1}\frac{\cos\phi 1}{V_g(\phi_1)^2} = \frac{\sin\phi_2}{V_g(\phi_2)} - \frac{dV_g}{d\phi_2}\frac{\cos\phi_2}{V_g(\phi_2)^2}. \tag{A-6}$$

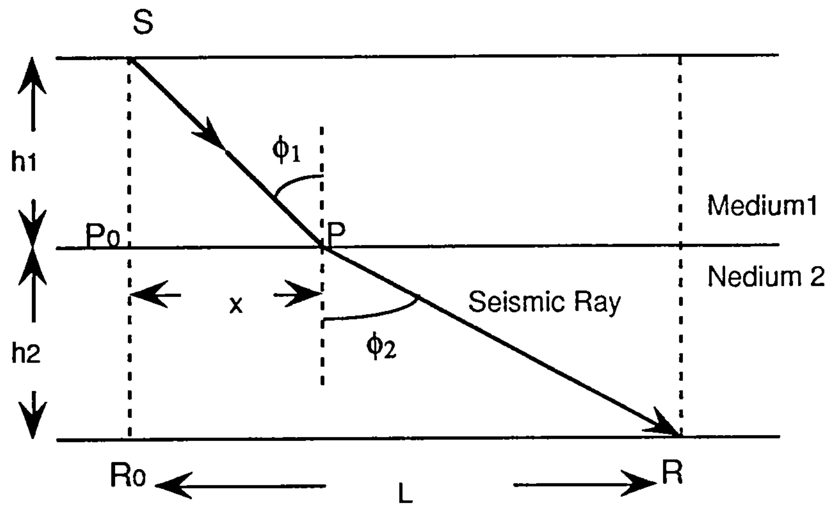

Figure A-1: Refraction of seismic rays at the interface.

APPENDIX B: WAVE FRONT OF HEAD WAVES

Let's define α, L_1, L_2 and d as illustrated in Figure B-1. α corresponds to the angle between the wave front and the interface. The angle between the seismic rays and the normal direction to the interface is given by a group angle ϕ_c. The travel time from P to O along the interface has to be equal to that from P to Q in a medium 1. This requires that

$$\frac{L_1}{V(\phi_c)\sin\phi_c} = \frac{L_1+L_2}{V_2}. \tag{B-1}$$

The relation among L_1, L_2, d, α and ϕ_c are given by

$$\frac{d}{L_1} = \cot\phi_c \qquad\qquad \frac{d}{L_2} = \tan\alpha \tag{B-2}$$

From the above two equations, we can derive

$$\cot\alpha = \frac{1}{\cos\phi_c}\left(\frac{V_2}{V(\phi_c)} - \sin\phi_c\right). \tag{B-3}$$

According to the generalized Snell's law for a critical refraction,

$$V_2 = \frac{V(\theta_c)}{\sin\theta_c}. \tag{B-4}$$

In addition, the relation between the phase and group velocities described in Eqns. 16 and 17 gives

$$\sin\phi_c = \frac{V(\theta_c)\sin\theta_c + \frac{dV}{d\theta}\cos\theta_c}{V_g(\phi_c)} \tag{B-5}$$

$$\cos\phi_c = \frac{V(\theta_c)\cos\theta_c - \frac{dV}{d\theta}\sin\theta_c}{V_g(\phi_c)}. \tag{B-6}$$

Then,

$$\frac{V_2}{V(\phi_c)} - \sin\phi_c = \frac{\cot\theta_c(V(\theta_c) - \frac{dV}{d\theta}\sin\theta_c)}{V_g(\theta_c)} = \cot\theta_c\cos\phi_c. \tag{B-7}$$

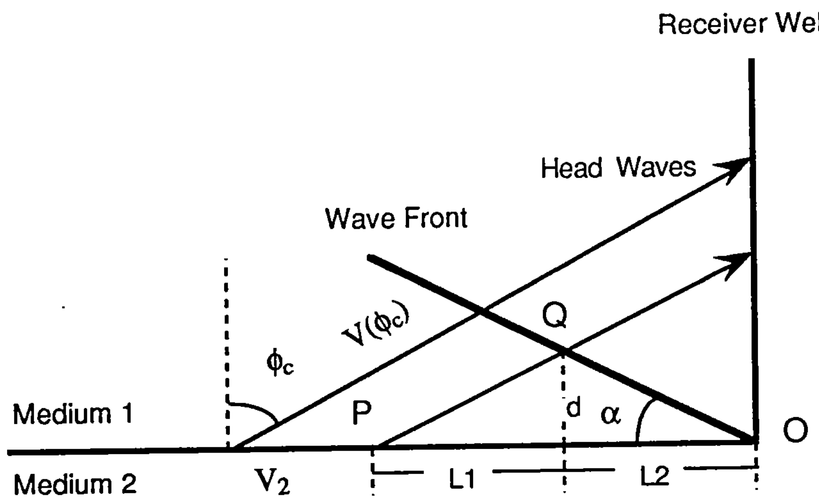

Figure B-1: Wave front of head waves around the receivers. $V(\phi_c)$ and V_2 represent the critical group velocity of head waves and the horizontal velocity in a medium 2, respectively.

Finally, we derive

$$\cot\alpha = \cot\theta_c \qquad \alpha = \theta_c. \tag{B-8}$$

This result immediately concludes that the angle between the wave front and the interface is equal to the critical phase angle.

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