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Viscoplastic deformation in unconsolidated reservoir sands (Part 1): Laboratory observations and time-dependent end cap models

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ABSTRACT: Laboratory studies of deforming unconsolidated reservoir sands from the Wilmington Field, CA and the Gulf of Mexico indicate that a significant portion of the deformation is both time-dependent and permanent. Furthermore, a threshold viscous compaction pressure has been identified in these sands, marking the transition from elastic to viscoplastic behavior, and which in general can be approximated by the maximum in situ effective pressure experienced by the sand at depth. Because the viscous component of deformation is significant, a standard elastic-plastic end cap model is not sufficient, and a model that includes viscoplasticity must be used. An appropriate model for unconsolidated sands can be developed by incorporating Perzyna viscoplasticity theory into the modified Cambridge clay cap model. Perzyna viscoplasticity theory simply states that pressure (and the location of the end cap) should follow a power law function of strain rate when a material is deforming viscoplastically. Hydrostatic compression tests were conducted at volumetric strain rates of 10^{-6}, 10^{-5}, and 10^{-4} per second in order to find values for the required model parameters, namely the threshold viscous compaction pressure as a function of strain rate. As a result, by using an end cap model and Perzyna viscoplasticity theory, changes in porosity in both the elastic and viscoplastic regimes can be predicted as a function of both stress path and strain rate.

1. INTRODUCTION

Inelastic porosity loss and its associated compaction and subsidence is commonly observed in unconsolidated sand and shale and weakly consolidated chalk reservoirs during production. A classic example of this is the Ekofisk field, where both field evidence and laboratory studies showed that production-induced compaction was permanent, and that the observations could be modeled with an elastic-plastic cap-type constitutive equation [1]. More recently, Chan and Zoback [2] used the modified Cambridge clay cap model to describe the deformation of unconsolidated sands from the Gulf of Mexico, and developed the DARS (Deformation of Reservoir Space) method of transferring model parameters from laboratory boundary conditions to reservoir boundary conditions in order to predict changes in porosity associated with production. Fossum and Fredrich [3] derived a unique and continuous end cap model by analyzing laboratory data from a variety of unconsolidated earth materials, and built the resulting constitutive equations into a large 3-D finite element code capable of meter-scale deformation analysis of reservoirs and aquifers. Incorporating cap-type constitutive laws into finite element models is not unique to reservoir analysis; such models are commonly used in civil engineering and soil mechanics at both the field scale [4,5] and the laboratory scale [6]. End cap elastic-plastic constitutive laws have proven to be robust and reliable predictors of the deformation of a variety of unconsolidated materials over several orders of magnitude in scale.

There are several advantages to choosing an end cap constitutive law for describing elastic-plastic materials. The main advantage for geomechanical applications is that the model provides a means of quantitatively predicting changes in porosity as a function of stress under both shearing and compaction. In addition, most end cap models require solving for only a few parameters in order to be fully defined. End cap models provide a means for mapping out the elastic-plastic transition in materials under variety of boundary conditions including shearing, uniaxial stress, and hydrostatic compression. Finally, end cap models have been used to successfully describe a wide range of materials, from unconsolidated soils and clays to cemented sandstones [7].

However, end cap models can be simplistic when used to describe certain materials. For example, materials which fail to deform according to the assumptions of associated plasticity (in associated plasticity the direction of maximum strain is parallel to the direction of maximum stress) cannot be accurately described. Also, since instantaneous time-independent deformation is assumed, end cap models fail to describe time-dependent materials. Because unconsolidated reservoir sands and shales exhibit significant time-dependent deformation [8,9,10,11], existing end cap models must be modified if they are to be used to describe the constitutive behavior of such materials. This has been done previously, by Adachi and Oka for example [12], who succeeded in describing laboratory data on clays by expanding the original Cambridge clay model to include rate-dependent viscoplastic deformation.

Given that end cap models are generally successful in predicting changes in porosity associated with plastic deformation, but breakdown when applied to materials such as unconsolidated sands that exhibit time-dependent deformation, it seems logical to try to expand end cap models such that porosity changes can be accurately predicted even in time-dependent materials. Since porosity changes are directly linked to changes in fluid flow properties in reservoirs, and production associated porosity loss in unconsolidated sand reservoirs can be large (10-15% at Wilmington), accurate prediction of porosity change is crucial to reservoir management. In this paper we expand the modified Cambridge clay model used by Chan and Zoback [2] to include time-dependent deformation by incorporating Perzyna viscoplasticity theory. When assuming that the Cam clay parameter 'M' is constant, the new time-dependent model predicts that the end cap expands with increasing strain-rate. Specifically, the elastic-viscoplastic transition pressure is predicted to follow a power law function of volumetric strain-rate. Laboratory tests conducted under constant strain-rate boundary conditions on unconsolidated reservoir sands will be used to verify the model and constrain the necessary parameters.

2. THEORY PART 1–MODIFIED CAMBRIDGE CLAY MODEL

The modified Cambridge clay model can be thought of as a simplified case in the general class of "end cap" models based on the concept of critical state. Critical state, or critical void-ratio, of an unconsolidated material is defined as a deformed state in which the individual particles are arranged such that no volume change takes place during shearing [13,14]. For example, when a loose soil sample is sheared, it deforms until finally reaching a state of collapse, at which point (in terms of stress) the sample is said to have reached the plastic yield surface. If loaded beyond this initial yield surface, plastic deformation continues to occur until the soil reaches the critical void ratio (or state), at which point the void ratio remains constant, and all additional deformation occurs as shear. In contrast, when a dense (or cemented) soil or rock sample is sheared, stress increases to some peak value, then decreases to some residual level, at which point it remains basically constant with continuing shear. Initially the dense material compacts, until the peak stress is reached, and then it dilates until the volumetric strain remains constant (at the residual stress), at which point the material is said to have reached the critical state, and shear can continue indefinitely without further volume changes.

While end cap models are firmly mathematically grounded in plasticity theory, they were initially developed to describe laboratory observations of yielding soils. Hvorslev [15] observed that the constant void ratio contours on the triaxial plane (σ_1 vs. $\sqrt{2}\sigma_3 = \sqrt{2}\sigma_2$) were identical during both undrained and drained conditions. From this observation, Roscoe and the Cambridge group proposed models for the yielding of soils and clays based on the balance of internal and external work.

For the original Cam clay model, it was assumed that only shear strain contributed to dissipated work, and therefore the only component of strain that contributed to the plastic deformation. However, for the modified Cam clay model, both the volumetric and shear strains contribute to the total plastic strain and dissipate work. Other assumptions built into the Cam clay model include the shape of the yield surface, associated plastic flow, time-independent deformation, and constant values for model parameters (independent of pressure and strain) [4,14,16].

In terms of a conventional axisymmetric triaxial ($\sigma_2 = \sigma_3$) laboratory apparatus, the work done on a right cylindrical sample is,

$$dW = \sigma_1 d\varepsilon_1 + 2\sigma_3 d\varepsilon_3$$
$$dW = \left(\frac{\sigma_1 + 2\sigma_3}{3}\right)\left(d\varepsilon_1 + 2d\varepsilon_3\right) + \left(\sigma_1 - \sigma_3\right)\tfrac{2}{3}\left(d\varepsilon_1 - d\varepsilon_3\right) \quad (1)$$

with stresses and strains defined so as to be easily measured from the lab apparatus,

$$p = \frac{\sigma_1 + 2\sigma_3}{3} = \frac{J_1}{3}$$
$$q = \sigma_1 - \sigma_3 = \sqrt{3J_{2D}} \quad (2)$$
$$d\varepsilon_v = d\varepsilon_1 + 2d\varepsilon_3$$
$$d\varepsilon_s = \tfrac{2}{3}\left(d\varepsilon_1 - d\varepsilon_3\right),$$

where p is the mean pressure (this can also be written as the mean effective pressure, p'), J_1 is the first invariant of the stress tensor, q is the differential stress, J_{2D} is the second invariant of the deviatoric stress tensor, ε_v is the volumetric strain, and ε_s is the deviatoric strain. After defining the stresses and strains above, the work exerted on the sample can be written as,

$$dW_{ex} = pd\varepsilon_v + qd\varepsilon_s. \quad (3)$$

Now, according to the modified Cam clay model, the dissipation of work due to sample deformation (internal work) is assumed to be,

$$dW_{in} = p\sqrt{\left(d\varepsilon_v^p\right)^2 + M^2\left(d\varepsilon_s^p\right)^2}, \quad (4)$$

where $d\varepsilon^p$ refers to a plastic strain (elastic strain should not dissipate work), and M is the slope of the critical state line, defined on p-q axes as a particular ratio of q/p, which comes from laboratory measurements.

The yield surface can be derived by setting the 2 work equations equal and separating the stress and strain terms. This gives the strain ratio in terms of stress ratio. Then, by assuming associated plasticity, the strain path is defined as perpendicular to the stress path, or,

$$\frac{d\varepsilon_s^p}{d\varepsilon_v^p} = -\frac{dp}{dq}. \quad (5)$$

This results in two equivalent equations for strain ratio, which can be combined and integrated to give the yield surface. The yield surface is functionally dependent on strain ratio, which in turn is dependent on both the stress ratio (associated plasticity) and the arbitrary functional form of dW_{in}.

For the modified Cam clay, following the procedure in the previous paragraph results in an elliptical yield surface with 2 free parameters,

$$M^2p^2 - M^2pp_o + q^2 = 0, \quad (6)$$

where p_o is the location of the yield surface on the p-axis, and M and p_o are variables which need to be experimentally determined. Note that p_o also serves as a hardening parameter, which moves along the p-axis as deformation continues through the elastic-plastic transition, and is tied to a particular volumetric strain or porosity. In other words, as p_o increases, porosity decreases. Also note that if M is assumed to be constant, then the yield surface simply scales with p_o. For a full derivation of the original and modified Cam clay models, please refer to Desai and Siriwardane [4].

Figure 1 shows a schematic version of the Cam clay end cap model. The first thing to note is the critical state line, with slope M, and that the yield surface intersects this line at its highest point. For unconsolidated or loosely consolidated materials, yield surfaces only exist between the critical state line and the p-axis, with the intersection of the yield surface and the p-axis given by p_o, or P^*. Note that the projection of the critical state line on the p-q plot is similar to the conventional idea of fixed failure envelopes such as the coefficient of static friction line in the linear Mohr-Coulomb model. However, in contrast to conventional plasticity models (such as Mohr-Coulomb), the critical state surface is not fixed, but allowed to evolve as the sample yields and continues to deform. Compaction results in an expansion of the yield surface; dilation actually causes contraction of the yield surface. As the sample deforms in shear, it evolves toward the critical state line until it reaches the critical state, at which point any additional plastic flow occurs with constant volume, as required by the associated flow rule (strain is normal to the tangent of the yield surface and therefore vertical – i.e. shear strain only).

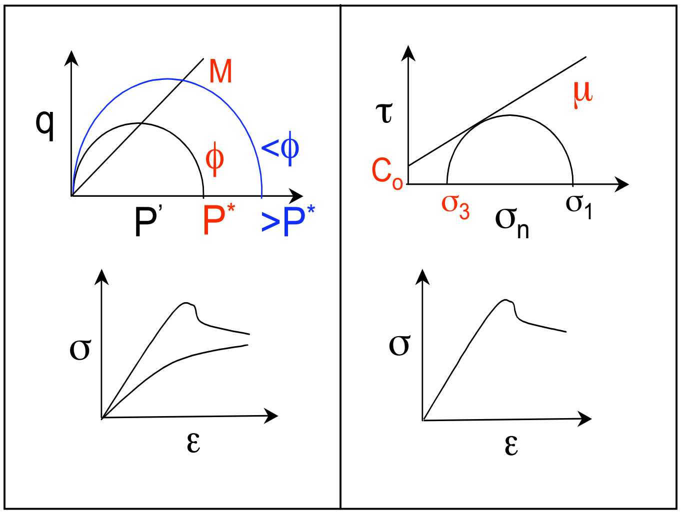

Figure 1: Schematic depiction of a typical end cap model and the Mohr-Coulomb friction model. The left half of the figure shows the end cap model, which allows for plastic failure due to compaction, while the Mohr-Coulomb model (right half) only provides for plastic failure due to shear. This difference is depicted in the stress-strain curves; in the left side figure, both the shear failure (top curve) and compaction failure (bottom curve) approach the critical state line with increasing strain. The critical state line is shown in the q-p plot as the line with slope M.

3. THEORY PART 2 – PERZYNA VISCOPLASTICITY

One of the limitations of traditional end cap models such as the modified Cam clay described in the previous section is that the models only describe materials with a static, time-independent yield surface. For materials with significant time-dependent deformation, such models will generally not suffice without modification. In this section, we describe one way of modifying traditional end cap models to include time-dependent plastic deformation, namely by incorporating Perzyna viscoplasticity.

The basic concept behind the inclusion of viscoplasticity into end cap models is relatively simple; the single static time-independent yield surface is simply replaced with a dynamic yield surface whose position in stress space is dependent on time or rate in addition to porosity or state. In other words, the elastic-plastic constitutive law which defines the static yield surface becomes an elastic-viscoplastic constitutive law. The inclusion of viscoplasticity only adds a moderate increase in model complexity – later in this section we show that the viscoplasticity can be described simply by adding a scaling term to the traditional static end cap. See Figure 2 for a schematic comparison of the static and dynamic end cap models.

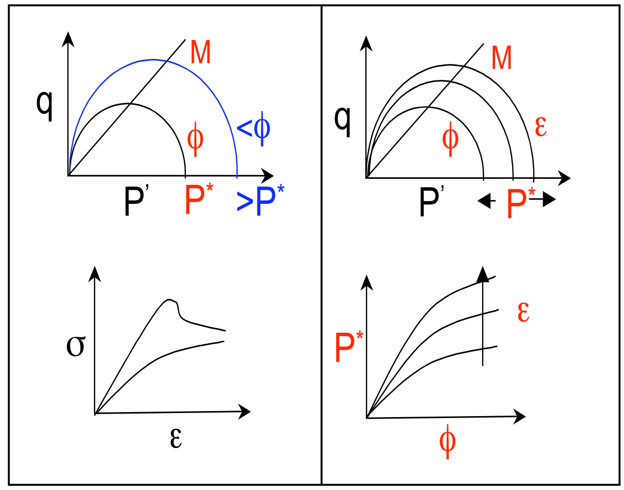

Figure 2: Applying viscoplasticity means that the end cap becomes dynamic rather than static. This means that the size of the end cap at a particular porosity varies as a function of strain rate. Higher strain rates typically correspond to higher P* values. The left side of the figure shows the traditional elastic-plastic end cap, while the right half of the figure shows the elastic-viscoplastic end cap.

Specifically, the theory of viscoplasticity described by Perzyna [17] can be thought of as a yield stress that depends on some function of strain and follows a power law function of strain rate. We have selected Perzyna viscoplasticity as the time-dependent modifier for the modified Cam clay model because Adachi and Oka [12] used it successfully in conjunction with the original Cam clay model to describe the deformation of clays and soils, and because the unconsolidated reservoir sands we have tested [11] generally exhibit power law creep. In addition, Leroueil et al. [18] conducted a series of laboratory tests on clays with the purpose of identifying which parameters were necessary to define a constitutive law, and concluded that deformation was a function of stress, strain, and strain-rate, but not stress-rate.

The Perzyna viscoplasticity model can be derived in five steps, following Adachi and Oka [12]. First, the total strain-rate is divided into elastic and viscoplastic components,

$$\dot{\varepsilon}_T = \dot{\varepsilon}_e + \dot{\varepsilon}_{vp.} \tag{7}$$

It follows that the corresponding stress rate is a function of the elastic strain rate,

$$\dot{\sigma} = C\!:\!\left(\dot{\varepsilon}_T - \dot{\varepsilon}_{vp}\right), \tag{8}$$

where C is an elastic modulus. Third, the viscoplastic strain-rate is assumed to be a power law function of the overstress,

$$\dot{\varepsilon}_{vp} = \sigma\left[\frac{\langle f \rangle}{\overline{\sigma}_o}\right]^N \frac{\partial f}{\partial \sigma}, \qquad (9)$$

where σ is stress, N is a variable, f is the overstress, and σ_o-bar is the initial yield stress.

The overstress is defined as the amount that the measured stress exceeds the yield stress. As assumed in the Cam-clay models, the yield stress is not fixed but allowed to evolve, and Perzyna [17] assumed that the yield stress increased linearly with equivalent plastic strain,

$$\overline{\sigma} = \langle \overline{\sigma}_o + hk \rangle, \qquad (10)$$

where h is a variable and k is the equivalent viscoplastic strain. Finally, it needs to be stated that the yield stress (f) and the viscoplastic strain rate are both zero when the measured stress is less than the yield stress,

$$\langle f \rangle = \begin{cases} f & f > 0 \\ 0 & f \le 0 \end{cases}. \qquad (11)$$

In other words, when stress is less than the yield stress, the material behaves elastically.

Now, for the case of the modified Cam clay model, the Perzyna yield stress f is replaced by the end cap yield surface,

$$f = M^2 p^2 - M^2 p p_o + q^2, \qquad (12)$$

and it becomes apparent that the entire end cap should scale with strain-rate. Since the end cap also scales with p_o, the rate-dependence can be conveniently placed on the p_o term, and the necessary Perzyna parameters can be easily measured in the laboratory by simply measuring hydrostatic pressure as a function of volumetric strain rate.

4. LABORATORY STUDIES AND MODEL VERIFICATION

To verify that Perzyna (power law) viscoplasticity is an appropriate choice for modeling the rate-dependent yielding of unconsolidated sands, hydrostatic compression tests were conducted at volume strain rates between 10^{-7} and 10^{-4} per second. All tests were conducted on cleaned and dried, right cylindrical samples of unconsolidated reservoir sand from the Wilmington field in California. One-inch diameter, two-inch long sample plugs were obtained from four-inch diameter core that was collected in the Upper Terminal zone of the Wilmington field at a depth of approximately 3000 feet. The samples were outfitted with two half-inch stroke linear potentiometers to measure axial displacement, an LVDT-based chain gauge to measure radial displacement, and top and bottom pore lines to enable the sample to drain to ambient atmospheric pressure. All tests were conducted using an NER Autolab 2000 conventional triaxial press, with command signal feedback configured such that the confining pressure was controlled by the volumetric strain. For more details on the experimental setup, see the Appendix in [11].

By assuming that Wilmington sand can be described with a modified Cam clay end cap, then both the static end cap and Perzyna viscoplastic parameters can be solved for simultaneously simply by conducting hydrostatic compression tests at varying volumetric strain rates. There are four static end cap parameters to solve for – the elastic bulk modulus K, the incremental volumetric plastic strain parameter λ, the position of the yield surface on the p-axis (p*), and the position of the critical state line M, which is assumed to be time-independent and constant with a value of 1.2 (corresponding to a 0.6 coefficient of friction – see [2] for details). In addition, the offset parameter γ and the power parameter N need to be found to establish a power law relationship between viscoplastic strain rate and p*. It should be noted that it is virtually impossible to determine the actual static location of p* for a viscoplastic material since p* scales with strain rate. In this case, the actual location of the static p* does not matter, since it only serves as a scaling factor in Equation 9, and should not affect the measurement of N.

The pressure-volumetric strain curves shown in Figure 3 show the results from tests conducted on unconsolidated Wilmington sand at volumetric strain rates between 10^{-7} and 10^{-5} per second. In this figure it is obvious that the overall deformation exhibits rate-dependence, and that a positive relationship exists between pressure and strain rate. Second, based on the standard method of interpreting consolidation plots [4], the location of the yield surface (p*) can be seen to be consistently located at approximately 4% volumetric strain, i.e.

at the point of maximum change in slope. Failure is associated with a strain in this case, rather than a stress, because an end cap represents a locus of failure stresses at a particular value of porosity, or strain.

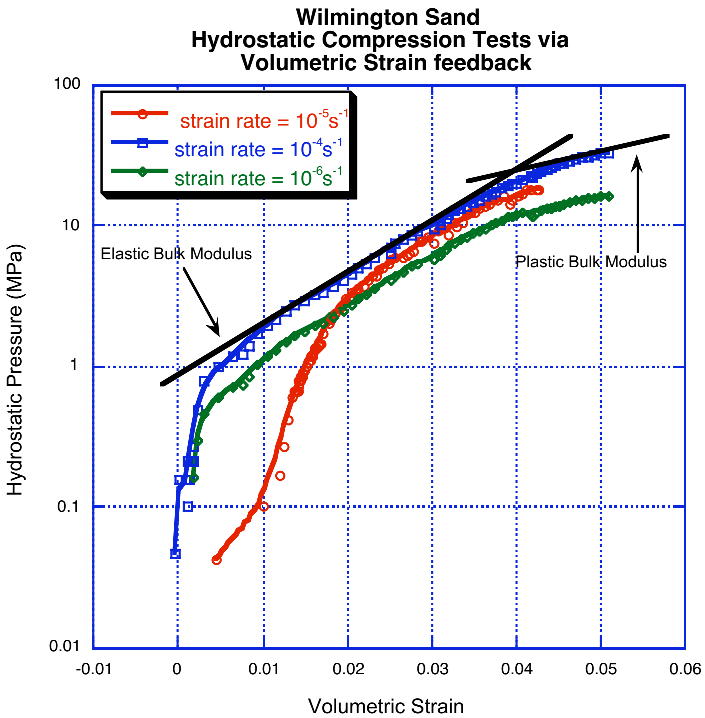

Figure 3: Hydrostatic compression tests controlled by volumetric strain rate. These tests were conducted to measure the plastic limit (P*) as a function of strain rate. The lines overlaying the data show how the elastic and plastic bulk modulus were determined. P* was interpreted to correspond to 4% volumetric strain, as shown by the crossover from elastic to plastic deformation. According to the end cap model, failure occurs at a given strain, rather than a given stress. Refer to the text for details.

The figure also shows that the elastic deformation is homogeneous and relatively independent of strain rate. The bulk modulus quickly evolves with pressure to be nearly constant and independent of rate, with a value of approximately 400 MPa. The volumetric plastic strain parameter is more difficult to determine due to the limited amount of data, but appears to increase slightly with strain rate. The values for λ are given in the table below.

Table 1: Elastic Modulus and Elastic-Plastic Transition for Wilmington Sand

Strain Rate	λ Elastic Modulus (MPa)	P^* (MPa)
10^{-4}/s	400	21
10^{-5}/s	470	16
10^{-6}/s	440	10

The results in Figure 4 verify that Wilmington sand deforms in accordance with Perzyna viscoplasticity. Plotting volumetric strain rate against the confining pressure measured at 4% volumetric strain (the elastic-viscoplastic transition) results in data which follow a power law, as predicted. Note that the p* pressure is in the range of the maximum in situ pressure experienced by the samples in the field, as previously found in Chapter 2. For unconsolidated Wilmington sand, the power law that describes the data is

$$\dot{\varepsilon}_{vp} = 2.36x10^{-13} p^{6.58}. \qquad (13)$$

with N equal to approximately 7. This equation comes from Equation 9, modified to reflect the fact that the truly static value of the yield pressure is unknown. A complete elastic-viscoplastic constitutive law for hydrostatic compression can now be constructed for Wilmington sand. Since each sample tested was interpreted to undergo the elastic-viscoplastic transition at 4% volumetric strain, the elastic and plastic components can be largely decoupled without introducing large errors. In the elastic domain, volumetric strain rate is related to pressure rate through the bulk modulus. In the plastic domain, the rate of pressure buildup depends on the volumetric strain rate in accordance with Perzyna viscoplasticity and is given by Equation 13.

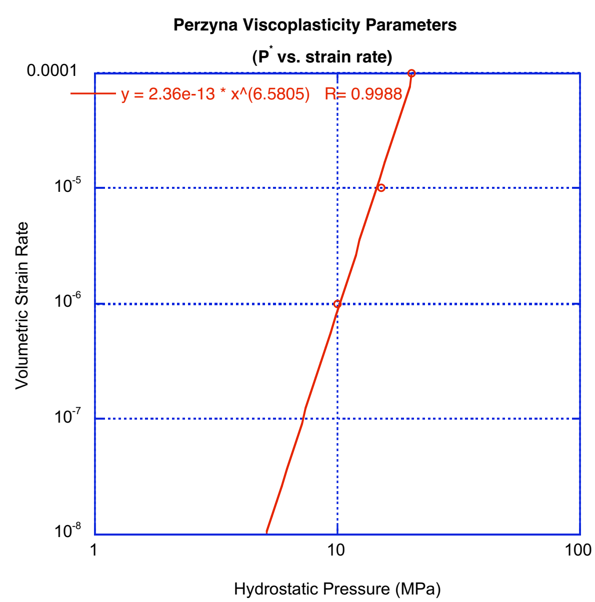

Figure 5: As proposed by Perzyna, the P* values for unconsolidated Wilmington sand are related to strain rate through a power law. This figure can be used to estimate the P* in a reservoir if the strain rate is known, or vice versa.

5. DISCUSSION

It is important to know the magnitude of rate-dependent viscoplastic effects. For Wilmington sand, the effects are large – increasing the pressure by 50% results in an order of magnitude increase in strain rate. Observations and appropriate modeling of rate-dependent viscoplastic behavior can have significant implications for reservoir geomechanics and the production of hydrocarbons and water. First, the accuracy of estimating in situ porosity from laboratory core samples improves because rate-dependence can be accounted for. Second, incorporating viscoplasticity into end cap models as done here will result in more accurate predictions of reservoir porosity change with production. Improved reservoir porosity predictions can be made because a means exists for converting changes in pressure and porosity measured at lab strain rates (10^{-6}) to field strain rate (10^{-9}). It should also be noted that the choice of end cap model in the case of simple hydrostatic compression is less important than the observation that an end cap exists, since the full end cap can easily be mapped in the laboratory by running tests with different stress paths.

In addition, these observations should prove helpful to reservoir engineers working in fields with unconsolidated sands. These sands appear to fail at stresses just larger than the maximum in situ pressure. This means that most fields begin to deform inelastic ally as soon as production begins. Knowledge of the magnitude and functional form of viscoplastic deformation in the field can then be used as a tool to control reservoir performance. Please note that care needs to be taken when using this model in fields where the stress path is not hydrostatic. Since the only stress path used here was hydrostatic, only hydrostatic conditions can be modeled in the field with any degree of certainty.

For example, in fields where the reduction of permeability with porosity and pressure is small, the compaction drive provided by viscoplastic deformation can be used to increase production rate. On the other hand, in fields such as Wilmington, where subsidence is a problem, pressure drawdown should be minimized and pressure maintenance should be started immediately in order to slow the rate at which subsidence occurs.

6. CONCLUSIONS

Laboratory observations of the deformation of unconsolidated sands from the Wilmington field, California, show that there is significant viscous deformation once pressure is increased beyond a certain threshold. The existence of a threshold viscous compaction pressure, coupled with observations that much of the deformation is permanent, suggests an end cap elastic-plastic constitutive model such as the modified Cambridge clay model. However, the viscous component of deformation requires that the time and rate independent Cambridge clay model be appropriately modified to include the viscous effects. Perzyna power law viscoplasticity is selected as the modifier for the static end cap model because it satisfies observations of power law creep strain and because it can be easily incorporated into the end cap constitutive equations while only adding two additional free parameters. Integrating Perzyna viscoplasticity into the end cap model results in an elastic-viscoplastic rather than an elastic-plastic constitutive law, and successfully describes our data. Hydrostatic compression tests conducted as a function of volumetric strain rate provide the data required for fitting the Perzyna viscoplasticity equation. Our results for unconsolidated sand suggest that a 50% increase in effective pressure causes an order of magnitude increase in volumetric strain rate in the viscoplastic domain.

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