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# A statistical evaluation of intact rock failure criteria constrained by polyaxial test data for five different rocks

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**Abstract**

In this study we examine seven different failure criteria by comparing them to published polyaxial test data ($\sigma_1 > \sigma_2 > \sigma_3$) for five different rock types at a variety of stress states. We employed a grid search algorithm to find the best set of parameters that describe failure for each criterion and the associated misfits. Overall, we found that the polyaxial criteria Modified Wiebols and Cook and Modified Lade achieved a good fit to most of the test data. This is especially true for rocks with a highly $\sigma_2$-dependent failure behavior (e.g. Dunham dolomite, Solenhofen limestone). However, for some rock types (e.g. Shirahama Sandstone, Yuubari shale), the intermediate stress hardly affects failure and the Mohr–Coulomb and Hoek and Brown criteria fit these test data equally well, or even better, than the more complicated polyaxial criteria. The values of $C_0$ yielded by the Inscribed and the Circumscribed Drucker–Prager criteria bounded the $C_0$ value obtained using the Mohr–Coulomb criterion as expected. In general, the Drucker–Prager failure criterion did not accurately indicate the value of $\sigma_1$ at failure. The value of the misfits achieved with the empirical 1967 and 1971 Mogi criteria were generally in between those obtained using the triaxial and the polyaxial criteria. The disadvantage of these failure criteria is that they cannot be related to strength parameters such as $C_0$. We also found that if only data from triaxial tests are available, it is possible to incorporate the influence of $\sigma_2$ on failure by using a polyaxial failure criterion. The results for two out of three rocks that could be analyzed in this way were encouraging. 

*Keywords:* Rock strength; Rock failure criteria; Intermediate principal stress; Polyaxial test data

## 1. Introduction

A number of different criteria have been proposed to describe brittle rock failure. In this study we aim to find which failure criterion, and which parameters, best describes the behavior of each rock type by minimizing the mean standard deviation misfit between the predicted failure stress and the experimental data. With this approach we can benchmark the different criteria against a variety of rock strength data for a variety of lithologies. This work also allowed us to investigate the influence of the intermediate stress on rock failure. We tested two conventional "triaxial" criteria (the Mohr–Coulomb and the Hoek and Brown criteria), which ignore the influence of the intermediate principal stress and are thus applicable to conventional triaxial test data ($\sigma_1 > \sigma_2 = \sigma_3$), three true triaxial, or polyaxial criteria (Modified Wiebols and Cook, Modified Lade, and Drucker–Prager), which consider the influence of the intermediate principal stress in polyaxial strength tests ($\sigma_1 > \sigma_2 > \sigma_3$) and two empirical criteria (Mogi 1967 and Mogi 1971). It is very important to mention that we did not investigate the behavior of the conventional "triaxial" criteria in their 3D versions (taking into account $\sigma_2$), as they have been widely used in their standard 2D version, especially when studying wellbore stability. The five rock types investigated were: amphibolite from the KTB site, Dunham dolomite, Solenhofen limestone, Shirahama sandstone and Yuubari shale.

In the sections below, we first define the various failure criteria we are evaluating and the rock types tested. We then define the statistical procedure we developed for evaluating the various strength criteria for each rock type. After presenting the results of our statistical analysis and evaluating the fit of each criterion for each rock type, we briefly examine the question of whether rock strength parameters obtained with triaxial tests ($C_0$, $\mu_i$) can be utilized in polyaxial failure criteria.

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**Nomenclature**

| | |
|---|---|
| $\sigma_{ij}$ | effective stress with $i$ and $j = 1,2,3$ (Eq. (1)) |
| $S_{ij}$ | total stress with $i$ and $j = 1,2,3$ (Eq. (1)) |
| $P_0$ | pore pressure (Eq. (1)) |
| $\tau$ | shear stress (Eq. (2)) |
| $S_0$ | shear strength or cohesion of the material (Eq. (2)) |
| $\mu_i$ | coefficient of internal friction (Eq. (2)) |
| $\sigma_n$ | normal stress (Eq. (2)) |
| $\sigma_1$ | major principal effective stress at failure (Eq. (3)) |
| $\sigma_3$ | least principal effective stress at failure (Eq. (3)) |
| $C_0$ | uniaxial compressive strength (Eq. (3)) |
| $m$ | constant that depends on rock type (Eq. (5)) |
| $s$ | constant that depends on the quality of the rock mass (Eq. (5)) |
| $I_1$ | first stress invariant (Eqs. (6) and (7)) |
| $I_3$ | third stress invariant (Eqs. (6) and (7)) |
| $P_a$ | atmospheric pressure (Eq. (6)) |
| $M'$, $\eta_1$ | material constants (Eq. (6)) |
| $\kappa_1$ | constant that depends on the density of the soil |
| $I_1'$ | modified first stress invariant (Eqs. (9) and (10)) |
| $I_3'$ | modified third stress invariant (Eqs. (9) and (11)) |
| $S$ | parameter related to the cohesion of the rock (Eqs. (9) and (12)) |
| $\eta$ | parameter representing the internal friction of the rock (Eqs. (9) and (13)) |
| $\phi$ | angle of internal friction (Eq. (4)) |
| $J_1$ | mean effective confining stress (Eq. (15)) |
| $J_2^{1/2}$ | $(3/2)^{1/2}\tau_{oct}$ (Eq. (16)) |
| $\tau_{oct}$ | Octahedral shear stress (Eq. (17)) |
| $A$ | parameter related to $C_0$ and $\mu_i$ (Eqs. (14) and (20)) |
| $B$ | parameter related to $C_0$ and $\mu_i$ (Eqs. (14), (19) and (20)) |
| $C$ | parameter related to $C_0$ and $\mu_i$ (Eqs. (14), (18), (19) and (20)) |
| $C_1$ | parameter related to $C_0$ and $\mu_i$ (Eq. (18)) |
| $\beta$ | may represent the contribution of $\sigma_2$ to the normal stress on the fault plane (Eq. (21)) |
| $\sigma_{m,2}$ | effective mean pressure on faulting $((\sigma_1 + \sigma_3)/2)$ |
| $K$ | empirical constant (Eqs. (23) and (24)) |
| $\alpha$ | material constant (Eq. (24)) |
| $X$, $Y$ | variables (Eq. (29)) |
| Corr[$X$, $Y$] | correlation of two variables $X$ and $Y$ (Eq. (29)) |
| Cov[$X$, $Y$] | covariance of two variables $X$ and $Y$ (Eq. (29)) |
| $s_X$ | standard deviation of $X$ (Eq. (29)) |
| $s_Y$ | standard deviation of $Y$ (Eq. (29)) |

In this paper $\sigma_{ij}$ is defined as the effective stress and is given by

$$\sigma_{ij} = S_{ij} - P_0, \quad (1)$$

where $S_{ij}$ is total stress and $P_o$ is pore pressure.

### 1.1. Mohr–Coulomb criterion

Mohr proposed that when shear failure takes place across a plane, the normal stress $\sigma_n$ and the shear stress $\tau$ across this plane are related by a functional relation characteristic of the material

$$|\tau| = S_0 + \mu_i\sigma_n, \quad (2)$$

where $S_0$ is the shear strength or cohesion of the material and $\mu_i$ is the coefficient of internal friction of the material.

Since the sign of $\tau$ only affects the sliding direction, only the magnitude of $\tau$ matters. The linearized form of the Mohr failure criterion may also be written as

$$\sigma_1 = C_0 + q\sigma_3, \quad (3)$$

where

$$q = [(\mu_i^2 + 1)^{1/2} + \mu_i]^2 = \tan^2(\pi/4 + \phi/2), \quad (4)$$

where $\sigma_1$ is the major principal effective stress at failure, $\sigma_3$ is the least principal effective stress at failure, $C_0$ is the uniaxial compressive strength and $\phi$ is the angle of internal friction equivalent to atan($\mu_i$). This failure criterion assumes that the intermediate principal stress has no influence on failure.

The yield surface of this criterion is a right hexagonal pyramid equally inclined to the principal-stress axes. The intersection of this yield surface with the $\pi$-plane is a hexagon. The $\pi$-plane (or deviatoric plane) is the plane which is perpendicular to the straight-line $\sigma_1 = \sigma_2 = \sigma_3$. Fig. 1 shows the yield surface of the Mohr–Coulomb criterion and Fig. 2a shows the representation of this criterion in $\sigma_1 - \sigma_2$ space for a $C_0 = 60$ MPa and $\mu_i = 0.6$.

### 1.2. Hoek and Brown criterion

This empirical criterion uses the uniaxial compressive strength of the intact rock material as a scaling parameter, and introduces two dimensionless strength parameters, $m$ and $s$. After studying a wide range of experimental data, Hoek and Brown [1] stated that the relationship between the maximum and minimum stress

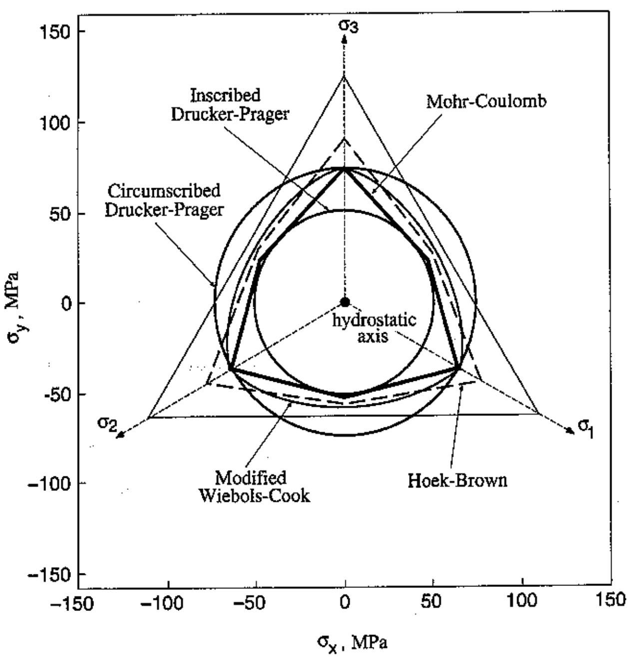



Fig. 1. Yield envelopes projected in the $\pi$-plane for the Mohr–Coulomb criterion, the Hoek and Brown criterion, the Modified Wiebols and Cook criterion and the Circumscribed and Inscribed Drucker–Prager criterion.

is given by

$$\sigma_1 = \sigma_3 + C_0\sqrt{m\frac{\sigma_3}{C_0} + s}, \tag{5}$$

where $m$ and $s$ are constants that depend on the properties of the rock and on the extent to which it had been broken before being subjected to the failure stresses $\sigma_1$ and $\sigma_3$.

The Hoek and Brown failure criterion was originally developed for estimating the strength of rock masses for application to excavation design.

According to Hoek and Brown [1,2], $m$ depends on rock type and $s$ depends on the characteristics of the rock mass. Below we list ranges for $m$-values, given some characteristic rock types:

(a) $5<m<8$ Carbonate rocks with well developed crystal cleavage (dolomite, limestone, marble).
(b) $4<m<10$ Lithified argillaceous rocks (mudstone, siltstone, shale, slate).
(c) $15<m<24$ Arenaceous rocks with strong crystals and poorly developed crystal cleavage (sandstone, quartzite).
(d) $16<m<19$ Fine-grained polymineralllic igneous crystalline rocks (andesite, dolerite, diabase, rhyolite).
(e) $22<m<33$ Coarse-grained polymineralllic igneous and metamorphic rocks (amphibolite, gabbro, gneiss, granite, norite, quartz-diorite).

While these values of $m$ obtained from lab tests on intact rock are intended to represent a good estimate when laboratory tests are not available, we will compare them with the values obtained for the five rocks studied. For intact rock materials, $s=1$. For a completely granulated specimen or a rock aggregate, $s=0$.

Fig. 1 shows that the intersection of the Hoek and Brown yield surface with the $\pi$-plane is approximately a hexagon. The sides of the Hoek and Brown pyramid are not linear planes, as it is the case for the Mohr–Coulomb criterion, but second-order planes (giving parabola in the normal stress-shear stress plane). In our example, the curvature is so small that the sides look like straight lines. In Fig. 2b it is possible to see the behavior of this criterion in $\sigma_1-\sigma_2$ space for $C_0=60$ MPa, $m=16$ and $s=1$. Hoek and Brown is represented by straight lines like Mohr–Coulomb.

### 1.3. Modified Lade criterion

The Lade criterion is a three-dimensional failure criterion for frictional materials without effective cohesion. It was developed for soils with curved failure envelopes [3]. This criterion is given by

$$((I_1^3/I_3) - 27)(I_1/p_a)^{m'} = \eta_1, \tag{6}$$

where

$$I_1 = S_1 + S_2 + S_3, \tag{7}$$

$$I_3 = S_1S_2S_3, \tag{8}$$

where $p_a$ is the atmospheric pressure expressed in the same units as the stresses, and $m'$ and $\eta_1$ are material constants.

In the modified Lade criterion developed by Ewy [4], $m'$ was set equal to zero in order to obtain a criterion, which is able to predict a linear shear strength increase with increasing $I_1$. In this way the criterion is similar to that proposed by Lade and Duncan [5] in which $(I_1^3/I_3) = \kappa_1$ where $\kappa_1$ is a constant whose value depends on the density of the soil. For considering materials with cohesion, Ewy [4] introduced the parameter $S$ and also included the pore pressure as a necessary parameter.

Doing all the modifications and defining appropriate stress invariants the following failure criterion was obtained by Ewy [4]

$$(I_1')^3/I_3' = 27 + \eta, \tag{9}$$

where

$$I_1' = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S) \tag{10}$$

and

$$I_3' = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S), \tag{11}$$

where $S$ and $\eta$ are material constants. The parameter $S$ is related to the cohesion of the rock, while the parameter $\eta$ represents the internal friction. These

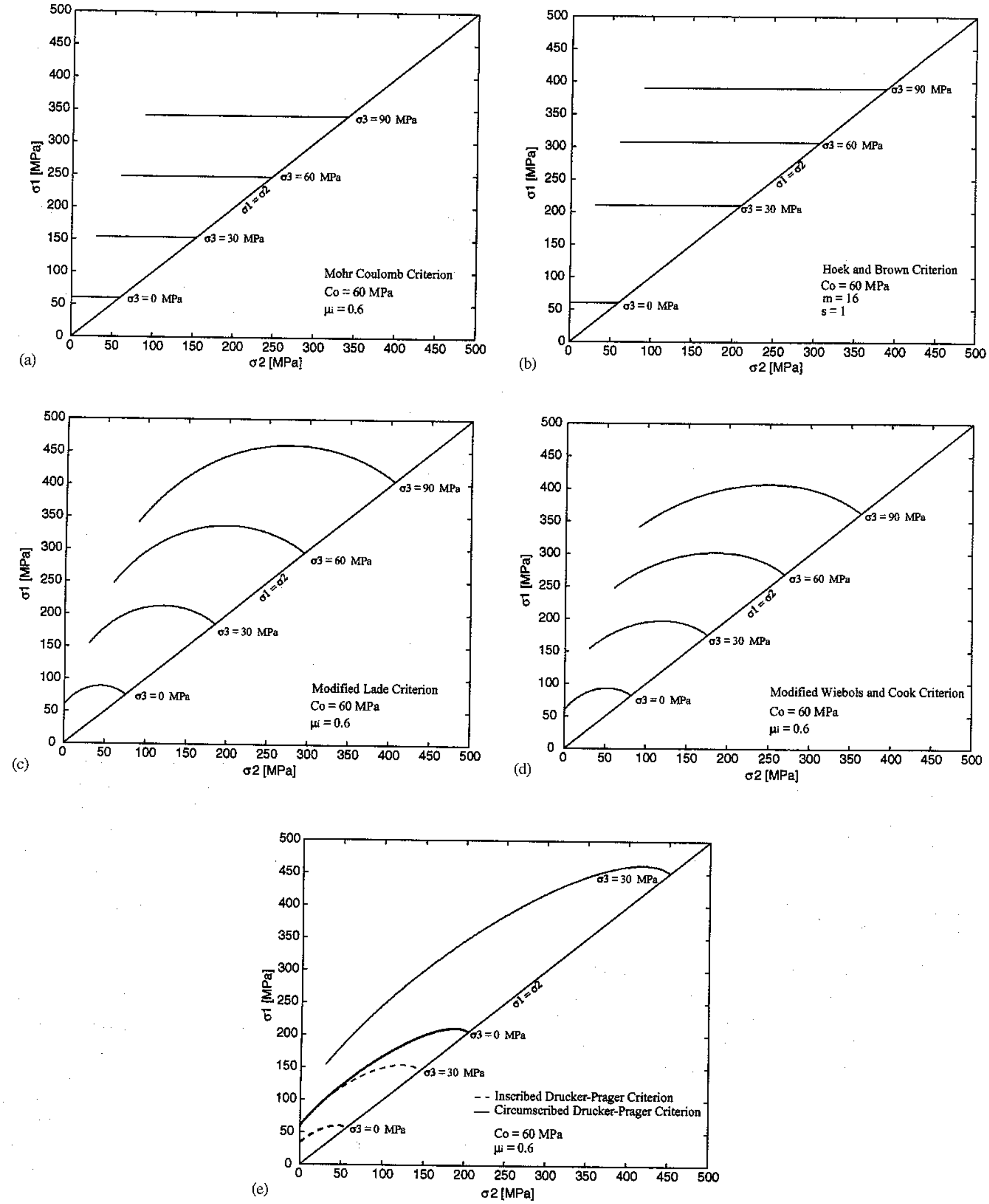



Fig. 2. In order to observe how some of the failure criteria analyzed in this study behave in the $\sigma_1 - \sigma_2$ space, we plotted the curves corresponding to $\sigma_3 = 0$, 30, 60 and 90 MPa using $C_0 = 60$ MPa and $\mu_i = 0.6$. For the Hoek and Brown criterion we used $C_0 = 60$ MPa, $m = 16$ and $s = 1$. (a) Mohr–Coulomb criterion. (b) Hoek and Brown criterion. (c) Modified Lade criterion. (d) Modified Wiebols and Cook criterion. (e) Inscribed and Circumscribed Drucker–Prager criterion only for $\sigma_3 = 0$ and 30 MPa.

parameters can be derived directly from the Mohr–Coulomb cohesion $S_0$ and internal friction angle $\phi$ by

$$S = S_0/\tan\phi, \tag{12}$$

$$\eta = 4(\tan\phi)^2(9 - 7\sin\phi)/(1 - \sin\phi), \tag{13}$$

where $\tan\phi = \mu_i$ and $S_0 = C_0/(2q^{1/2})$ with $q$ as defined in Eq. (4).

The modified Lade criterion first predicts a strengthening effect with increasing intermediate principal stress $\sigma_2$ followed by a slight reduction in strength once $\sigma_2$ becomes "too high" [4]. This typical behavior of the Modified Lade criterion can be observed in Fig. 2c where it has been plotted in $\sigma_1 - \sigma_2$ space for $C_0 = 60$ MPa and $\mu_i = 0.6$.

### 1.4. Modified Wiebols and Cook criterion

Zhou [6] presented a failure criterion, which is an extension of the Circumscribed Drucker–Prager criterion (described later) with features similar to the effective strain energy criterion of Wiebols and Cook [7].

The failure criterion described by Zhou predicts that a rock fails if

$$J_2^{1/2} = A + BJ_1 + CJ_1^2, \tag{14}$$

where

$$J_1 = (1/3) \times (\sigma_1 + \sigma_2 + \sigma_3) \tag{15}$$

and

$$J_2^{1/2} = \sqrt{\frac{1}{6}((\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2)}, \tag{16}$$

where $J_1$ is the mean effective confining stress and $J_2^{1/2} = (3/2)^{1/2}\tau_{oct}$, where $\tau_{oct}$ is the octahedral shear stress

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_2 - \sigma_1)^2}. \tag{17}$$

The parameters $A$, $B$, and $C$ are determined such that Eq. (14) is constrained by rock strengths under triaxial ($\sigma_2 = \sigma_3$) and biaxial ($\sigma_1 = \sigma_2$) conditions. By substituting the given conditions plus the uniaxial rock strength ($\sigma_1 = C_0$, $\sigma_2 = \sigma_3 = 0$) into Eq. (14), it is determined that

$$C = \frac{\sqrt{27}}{2C_1 + (q-1)\sigma_3 - C_0} \times \left(\frac{C_1 + (q-1)\sigma_3 - C_0}{2C_1 + (2q+1)\sigma_3 - C_0} - \frac{q-1}{q+2}\right) \tag{18}$$

with $C_1 = (1 + 0.6\mu_i)C_0$.

$$B = \frac{\sqrt{3}(q-1)}{q+2} - \frac{C}{3}(2C_0 + (q+2)\sigma_3) \tag{19}$$

and

$$A = \frac{C_0}{\sqrt{3}} - \frac{C_0}{3}B - \frac{C_0^2}{9}C. \tag{20}$$

The rock strength predictions produced using Eq. (14) are similar to that of Wiebols and Cook [7] and thus the model described by Eq. (14) represents a modified strain energy criterion, which we call Modified Wiebols and Cook. For polyaxial states of stress, the predictions made by this criterion are greater than that of the Mohr–Coulomb criterion. This can be seen in Fig. 1 because the failure envelope of the Modified Wiebols and Cook criterion just coincides with the outer apices of the Mohr–Coulomb hexagon. This criterion is also plotted in $\sigma_1 - \sigma_2$ space in Fig. 2d.

### 1.5. Mogi 1967 empirical criterion

Mogi studied the influence of the intermediate stress on failure by performing confined compression tests ($\sigma_1 > \sigma_2 = \sigma_3$), confined extension tests ($\sigma_1 = \sigma_2 > \sigma_3$) and biaxial tests ($\sigma_1 > \sigma_2 > \sigma_3 = 0$) on different rocks. He recognized that the influence of the intermediate principal stress on failure is non-zero, but considerably smaller than the effect of the minimum principal stress [8]. When he plotted the maximum shear stress $(\sigma_1 - \sigma_3)/2$ as a function of $(\sigma_1 + \sigma_3)/2$ for failure of Westerly Granite, he observed that the extension curve lied slightly above the compression curve and the opposite happened when he plotted the octahedral shear stress $\tau_{oct}$ as a function of the mean normal stress $(\sigma_1 + \sigma_2 + \sigma_3)/3$ for failure of the same rock. Therefore, if $(\sigma_1 + \beta\sigma_2 + \sigma_3)$ is taken as the abscissa (instead of $(\sigma_1 + \sigma_3)$ or $(\sigma_1 + \sigma_2 + \sigma_3)$), the compression and the extension curves become coincidental at a suitable value of $\beta$. Mogi argued that this $\beta$ value is nearly the same for all brittle rocks but we will test this assertion. The empirical criterion has the following formula

$$(\sigma_1 - \sigma_3)/2 = f_1[(\sigma_1 + \beta\sigma_2 + \sigma_3)/2], \tag{21}$$

where $\beta$ is a constant smaller than 1. The form of the function $f_1$ in Eq. (21) is dependent on rock type and it should be a monotonically increasing function. This criterion postulates that failure takes place when the distortional energy increases to a limiting value, which increases monotonically with the mean normal pressure on the fault plane. The term $\beta\sigma_2$ may correspond to the contribution of $\sigma_2$ to the normal stress on the fault plane because the fault surface, being irregular, is not exactly parallel to $\sigma_2$ and it would be deviated approximately by $\arcsin(\beta)$.

### 1.6. Mogi 1971 empirical criterion

This empirical fracture criterion was obtained by generalization of the von Mises's theory. It is formulated by

$$\tau_{oct} = f_1(\sigma_1 + \sigma_3), \tag{22}$$

where $f_1$ is a monotonically increasing function. According to Mogi [9] the data points tend to align in a single curve for each rock, although they slightly scatter in some silicate rocks. The octahedral stress is not always constant but increases monotonically with $(\sigma_1 + \sigma_3)$. Failure will occur when the distortional strain energy reaches a critical value that increases monotonically with the effective mean pressure on the slip planes parallel to the $\sigma_2$ direction. The effective mean pressure on faulting is $(\sigma_1 + \sigma_3)/2$ or $\sigma_{m,2}$; therefore, $\tau_{oct}$ at fracture is plotted against $\sigma_{m,2}$. Mogi applied this failure criterion to different kinds of rocks and it always gave satisfactory results.

For both Mogi criteria, as $f_1$ has to be a monotonically increasing function, we fit the data using three kind of functions: power law, linear and second-order polynomial, in order to find the best-fitting curve, that is, the one with the least standard deviation mean misfit.

### 1.7. Drucker–Prager criterion

The von Mises criterion may be written in the following way

$$J_2 = k^2, \tag{23}$$

where $k$ is an empirical constant. The extended von Mises yield criterion or Drucker–Prager criterion was originally developed for soil mechanics [10].

The yield surface of the modified von Mises criterion in principal stress space is a right circular cone equally inclined to the principal-stress axes. The intersection of the $\pi$-plane with this yield surface is a circle. The yield function used by Drucker and Prager to describe the cone in applying the limit theorems to perfectly plastic soils has the form

$$J_2^{1/2} = k + \alpha J_1, \tag{24}$$

where $\alpha$ and $k$ are material constants. The material parameters $\alpha$ and $k$ can be determined from the slope and the intercept of the failure envelope plotted in the $J_1$ and $(J_2)^{1/2}$ space. $\alpha$ is related to the internal friction of the material and $k$ is related to the cohesion of the material, in this way, the Drucker–Prager criterion can be compared to the Mohr–Coulomb criterion. When $\alpha = 0$, Eq. (24) reduces to the Von Mises criterion.

The Drucker–Prager criterion can be divided into an outer bound criterion (or Circumscribed Drucker–Prager) and an inner bound criterion (or Inscribed Drucker–Prager). These two versions of the Drucker–Prager criterion come from comparing the Drucker–Prager criterion with the Mohr–Coulomb criterion. In Fig. 1 the two Drucker–Prager options are plotted together with the Mohr–Coulomb criterion in the $\pi$-plane. The inner Drucker–Prager circle only touches the inside of the Mohr–Coulomb criterion and the outer Drucker–Prager circle coincides with the outer apices of the Mohr–Coulomb hexagon.

The Inscribed Drucker–Prager criterion is obtained when [11,12]

$$\alpha = \frac{3 \sin \phi}{\sqrt{9 + 3 \sin^2 \phi}} \tag{25}$$

and

$$k = \frac{3 C_0 \cos \phi}{2\sqrt{q}\sqrt{9 + 3 \sin^2 \phi}}, \tag{26}$$

where $\phi$ is the angle of internal friction, that is, $\phi = \tan^{-1} \mu_i$.

The Circumscribed Drucker–Prager criterion is obtained when [11,6]

$$\alpha = \frac{6 \sin \phi}{\sqrt{3}(3 - \sin \phi)} \tag{27}$$

and

$$k = \frac{\sqrt{3} C_0 \cos \phi}{\sqrt{q}\,(3 - \sin \phi)}. \tag{28}$$

As Eqs. (25) and (27) show, $\alpha$ only depends on $\phi$, which means that $\alpha$ has an upper bound for both cases. When $\phi = 90°$, $\mu_i = \infty$ as $\tan(90) = \infty$, so the value of $\alpha$ converges to 0.866 in the Inscribed Drucker–Prager case and to 1.732 in the Circumscribed Drucker–Prager case. Fig. 3 shows the behavior of $\alpha$ with respect to $\mu_i$. The asymptotic values are represented by a thick dashed line for each case. As $\alpha$ is obtained from the slope of the failure envelope in $J_1 - (J_2)^{1/2}$ space, according to its value we are able to discern whether the Inscribed or the Circumscribed Drucker–Prager criteria can be applied to the data. If the value of $\alpha$ for a specific rock is greater than the upper bound (asymptotic value), the values of $C_0$ and $\mu_i$ cannot be obtained, which means that the Drucker–Prager criteria cannot be compared to Mohr–Coulomb. If it is not necessary to find the values of $C_0$ and $\mu_i$ then the Drucker–Prager failure criterion can always be applied.

In Fig. 2e we present the behavior of both Drucker–Prager criteria for $C_0 = 60$ MPa and $\mu_i = 0.6$ in comparison with other failure criteria studied here. As it is shown in Fig. 2e, for the same values of $C_0$ and $\mu_i$, the Inscribed Drucker–Prager criterion predicts failure at lower stresses than the Circumscribed Drucker–Prager criterion.

## 2. Strength data

The five rock types investigated were amphibolite from the KTB site, Dunham dolomite, Solenhofen

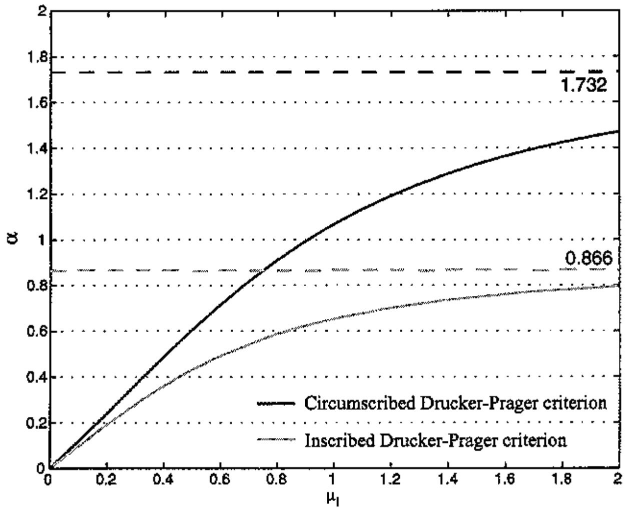



Fig. 3. Parameter $\alpha$ from the Drucker–Prager criterion versus $\mu_i$. The asymptotic value of $\alpha$ is represented by a thick dash line. For the Inscribed Drucker–Prager (Eq. (25)) the asymptotic value of $\alpha$ is 0.866 and for the Circumscribed Drucker–Prager (Eq. (27)) the asymptotic value of $\alpha$ is 1.732.

limestone, Shirahama sandstone and Yuubari shale. The polyaxial data of these rocks were obtained from published works as follows: the data of the amphibolite from the KTB site was kindly provided by Chang and Haimson from their work on the KTB amphibolite [13], the data for the Dunham dolomite and Solenhofen limestone from Mogi [9], and the data for the Shirahama sandstone and Yuubari Shale from Takahashi & Koide [14]. Tables 15–19 show the polyaxial test data for each rock. It is important to mention that we are not assessing the quality of the data in this study. Instead, our goal is to statistically find the best-fitting parameters with different failure criteria by utilizing the experimental data in a statistically comprehensive manner.

In order to quantify the influence of $\sigma_2$ on failure, we calculated the correlation coefficient between $\sigma_1$ and $\sigma_2$ for each $\sigma_3$ for every rock.

The correlation coefficient is the correlation of two variables, defined by [15]

$$\mathrm{Corr}[X,Y] = \frac{\mathrm{Cov}[X,Y]}{s_X s_Y}, \qquad (29)$$

where $s_X$ and $s_Y$ are the standard deviations of $X$ and $Y$, respectively. The correlation function lies between $-1$ and $+1$. If the value assumed is negative, $X$ and $Y$ are said to be negatively correlated, if it is positive they are said to be positively correlated and if it is zero they are said to be uncorrelated. If $\sigma_1$ increases with $\sigma_2$, the correlation coefficient also increases. If $\sigma_1$ does not change with $\sigma_2$, then the correlation coefficient would be near zero.

Fig. 4 shows the correlation coefficient of $\sigma_1$ to $\sigma_2$ to illustrate the influence of $\sigma_2$ on strength for the different rocks as a function of $\sigma_3$. The rocks with the highest influence of $\sigma_2$ on failure are Dunham dolomite, Solenhofen limestone and the amphibolite from the KTB site. The Yuubari shale shows an intermediate influence of $\sigma_2$ on failure and the Shirahama sandstone presents an unusual behavior as the influence of $\sigma_2$ on failure markedly varies with $\sigma_3$. The strong $\sigma_2$ dependence of strength of most of the rocks tested suggest that, in general, polyaxial strength criteria would be expected to work best. Although the behavior of Shirahama sandstone is so variable that it is difficult to assess which kind of failure criterion would work best.

## 3. Results

To consider the applicability of four of the failure criteria to the experimental data, we performed a grid search allowing $C_0$ and $\mu_i$ to vary over a specific range. We chose the best-fitting combination of $C_0$ and $\mu_i$ for a specific rock by minimizing the mean standard deviation misfit to the test data. The failure criteria analyzed using this approach were the Mohr–Coulomb criterion, the Hoek and Brown criterion, the Modified Lade criterion and the Modified Wiebols and Cook criterion. As the Hoek and Brown criterion does not depend on $\mu_i$, but on $m$ and $s$, the grid search was made varying $C_0$, $m$ and $s$. Fig. 5a shows the misfit contours for the Modified Wiebols and Cook criterion to the Shirahama sandstone data. A minimum is very well defined allowing us to accurately determine the $C_0$ and $\mu_i$ that describe the failure of this rock in terms of this criterion. In Fig. 5b, the fit of this criterion with the best-fitting parameters is shown. By doing a grid search, in addition to obtaining the best-fitting parameters $C_0$ and $\mu_i$, it enables us to observe the sensitivity of the failure criterion when the parameters are changed—this can be observed in Appendix B. That is, a grid search allows us to look at the whole solution space at once.

Fig. 6 presents all the results for the Mohr–Coulomb criterion with the best-fitting parameters for each rock type. As the Mohr–Coulomb does not take into account the influence of $\sigma_2$, the fit is a horizontal straight line. Therefore, the best fit would be one that goes through the middle of the data set for each $\sigma_3$. The smallest misfits associated with utilizing the Mohr–Coulomb criterion were obtained for the Shirahama sandstone and the Yuubari shale. The largest misfits were for Dunham dolomite, Solenhofen limestone and KTB amphibolite, which are rocks presenting the greatest influence of the intermediate principal stress on failure (Fig. 4). The mean misfit obtained using the Mohr–Coulomb criterion is consistently larger than that obtained using the polyaxial failure criteria for rocks presenting a large influence of $\sigma_2$ on failure like Dunham

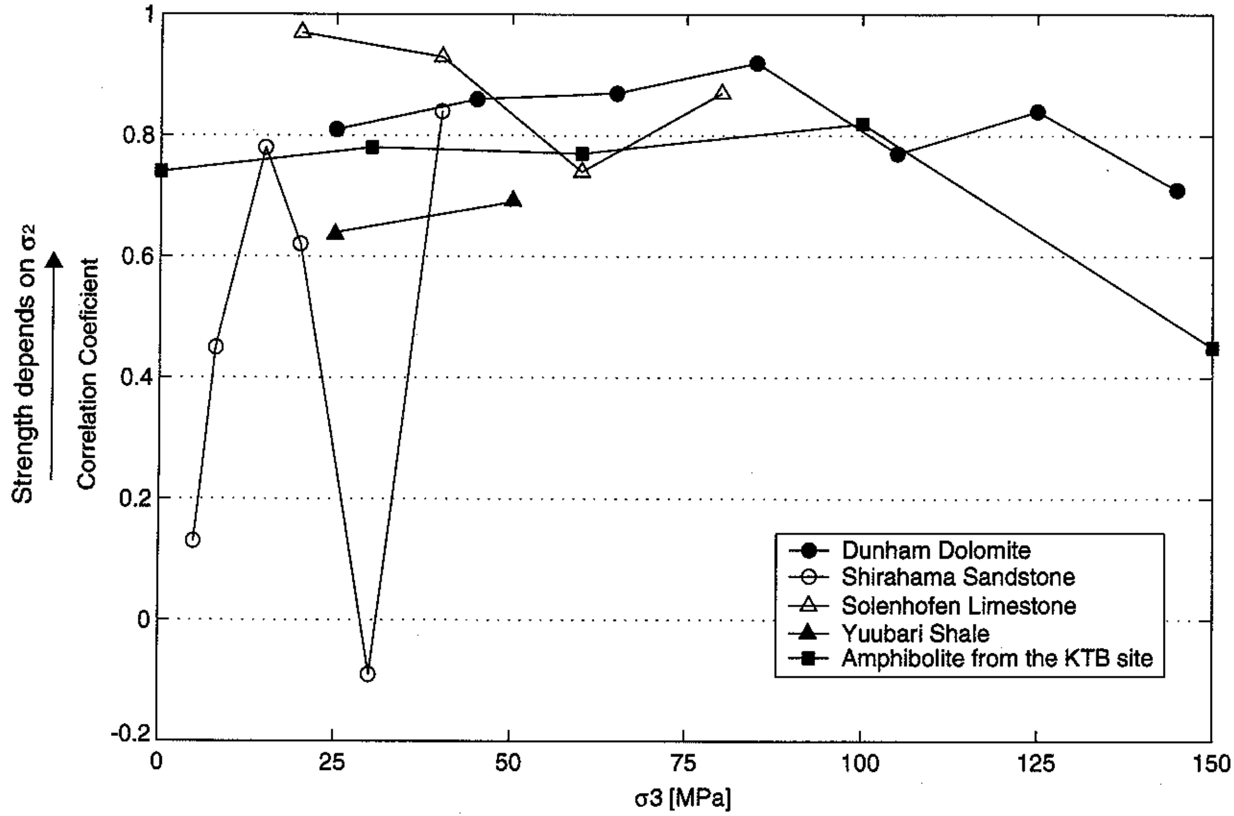



Fig. 4. Correlation coefficient versus $\sigma_3$ for all the rocks studied in this work. When the correlation coefficient approximates to 1, that means that $\sigma_1$ increases with $\sigma_2$, which also means that failure occur at higher stresses than if $\sigma_1$ does not depend of $\sigma_2$.

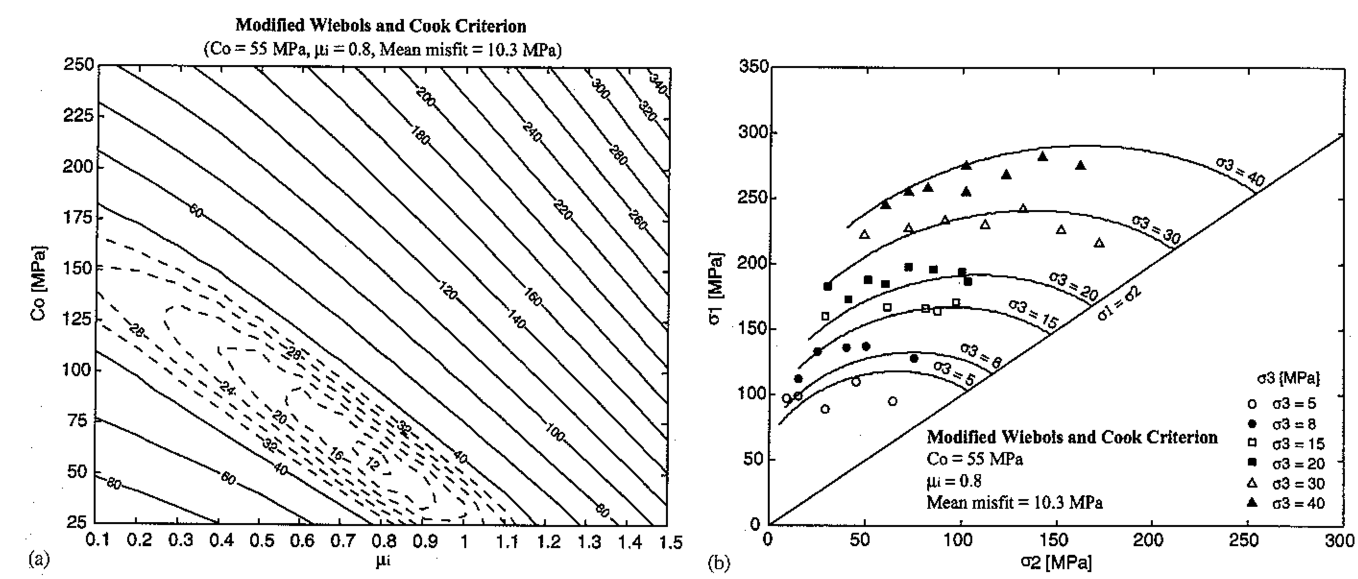



Fig. 5. Modified Wiebols and Cook criterion for the Shirahama sandstone. (a) Best-fitting solution compared to the actual data. (b) Contour plot of the misfit to the experimental data for various combinations of $C_0$ and $\mu_i$.

dolomite and Solenhofen limestone. It is important to realize that the Mohr–Coulomb criterion tends to overestimate the value of $C_0$ when applied to polyaxial data. The misfit data shown in Fig. 20 indicates that the Mohr–Coulomb criterion is always very well constrained with respect to $C_0$ and $\mu_i$.

As can be seen in Fig. 7, the Hoek and Brown criterion fit the experimental data well, especially for the

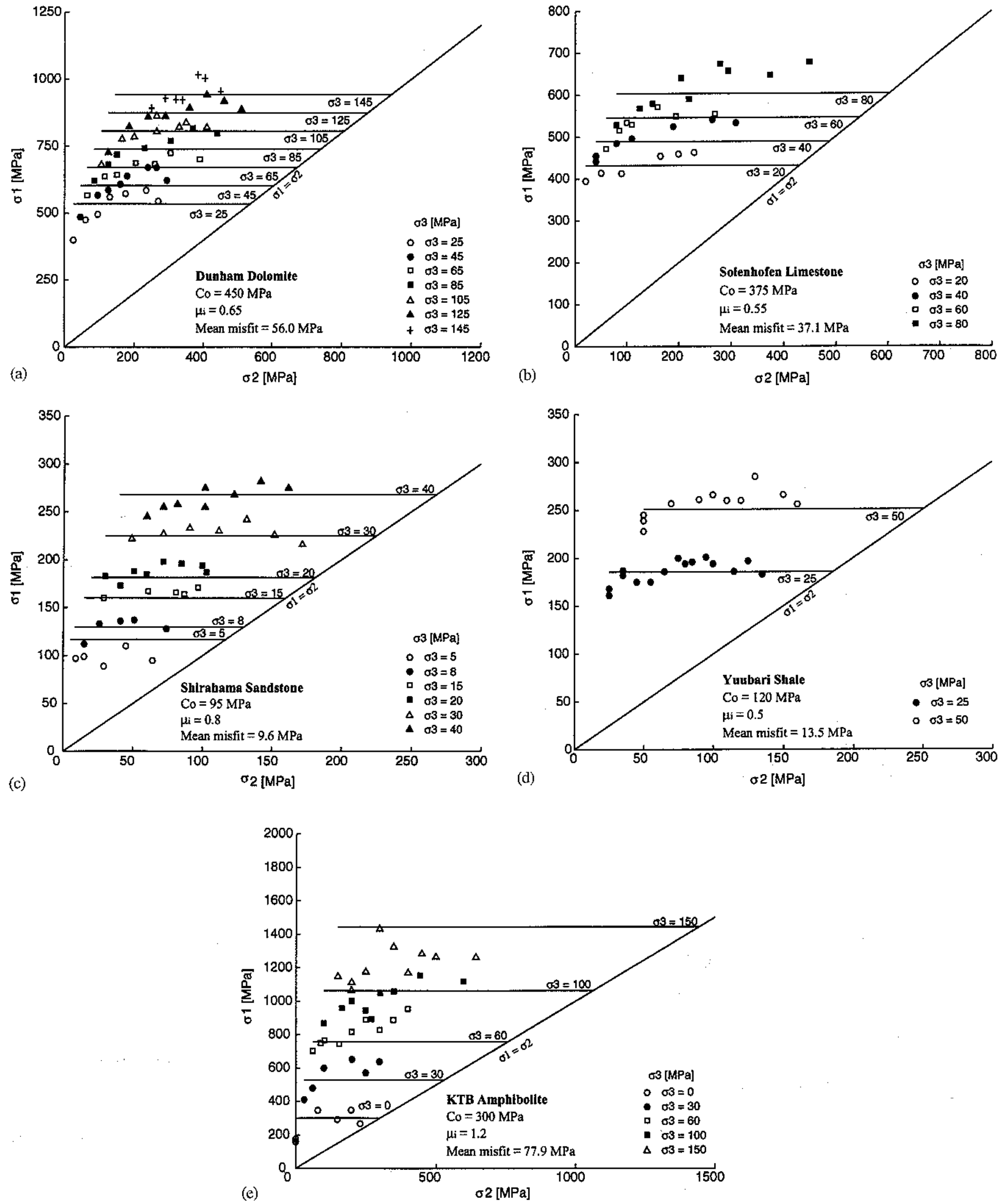



Fig. 6. Best-fitting solution for all the rocks using the Mohr–Coulomb criterion (Eq. (3)). (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

Yuubari shale and the Shirahama sandstone. The values of $m$ found in this study coincide with those reported by Hoek and Brown, for the same kinds of rocks except for the Solenhofen limestone, for which we obtained a value of $m = 4.6$ and the values of $m$ reported are in the range of 5–8. However, Fig. 21b shows that for $m = 5$, the

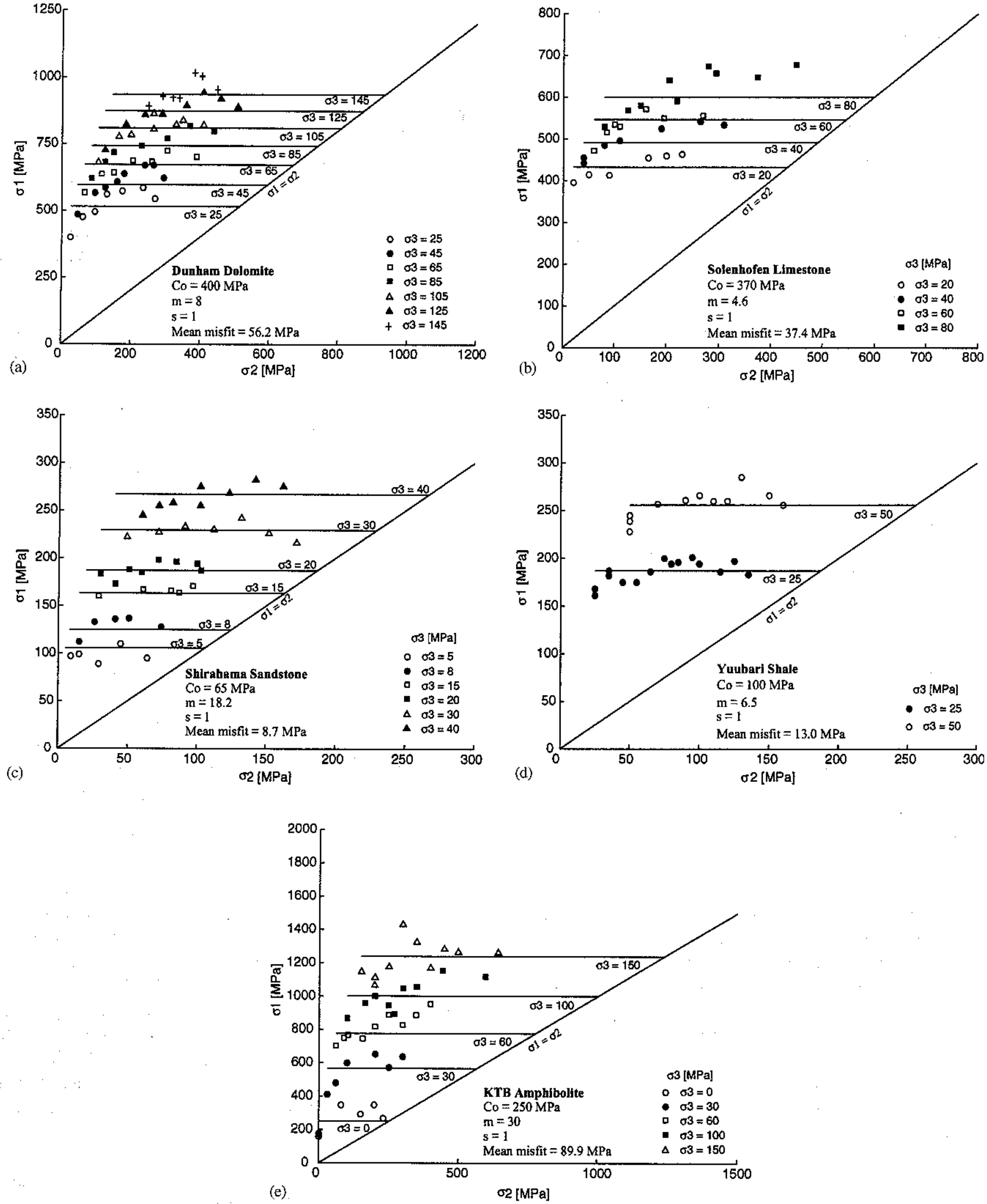



Fig. 7. Best-fitting solution for all the rocks using the Hoek and Brown criterion (Eq. (5)). (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

misfit is essentially the same ($\pm 3$ MPa) as for $m = 4.6$. The value of $s$ was 1 for every rock studied, as expected for intact rocks. The compressive strength ($C_0$) found using the Hoek and Brown criterion was consistently lower than that found using the Mohr–Coulomb criterion, but (as shown below) was greater than those found using the polyaxial criteria. The only exception is the KTB amphibolite for which the Hoek and Brown criterion, the Modified Lade criterion and the Modified Wiebols and Cook criterion yield the same values for $C_0$. As can be seen in Fig. 21, the contour misfits for the Hoek and Brown failure criterion allows us to constrain $C_0$ but not $m$; i.e., for the same misfit, there is a wide range of values of $m$ capable of reproducing approximately that same misfit.

The Modified Lade criterion (Fig. 8) works very well for the rocks with a high $\sigma_2$-dependance on failure, that is, Dunham dolomite and Solenhofen limestone. For the KTB amphibolite, this criterion reasonably reproduces the trend of the experimental data but not as well as for the Dunham dolomite. We obtained a similar result for the Yuubari shale, which was expected as this rock presents an intermediate $\sigma_2$-dependance on failure. The fit to the Shirahama sandstone data do not reproduce the trends very well. This is due to the varying $\sigma_2$-dependance for different $\sigma_3$, which makes the approximations more difficult. Fig. 22 shows that the Modified Lade criterion yields well constrained rock strength parameters.

Similar to the Modified Lade criterion, the Modified Wiebols and Cook criterion also works best for rocks with a strong influence of $\sigma_2$ on failure. The results obtained for this criterion are shown in Fig. 9. The Modified Wiebols and Cook criterion and the Modified Lade criterion, both achieve very good fits. For rocks with a high $\sigma_2$-dependance, the Modified Wiebols and Cook criterion works very well, as was the case for the Dunham dolomite, KTB amphibolite and Solenhofen limestone. For the Yuubari shale, with an intermediate $\sigma_2$-dependance, the criterion reproduces the trend of the data equally well. For rocks presenting a variable $\sigma_2$-dependance, the fitting can be more complicated. Some sets of $\sigma_3$ are very well matched while others in the same rock present a poor fit. This is the case for the Shirahama Sandstone, where the Modified Wiebols and Cook criterion does not reproduce the trend of the data very well due to the varying $\sigma_2$-dependance of failure for different $\sigma_3$. As shown in Fig. 23, both $C_0$ and $\mu_i$ are very well constrained for this failure criterion.

Fig. 10 shows the results obtained for each rock using the Mogi 1967 empirical criterion. The maximum shear stress $(\sigma_1 - \sigma_3)/2$ was plotted against the appropriate normal stress $(\sigma_1 + \beta\sigma_2 + \sigma_3)/2$. The different symbols show different $\sigma_3$ values and they form a single relation for each rock. The values of $\beta$ are reported in Table 1. As Fig. 10 shows, the strength data points can be fit by a power law approximation for every rock. While the Mogi 1967 criterion works well and gives insight into the influence of $\sigma_2$ on failure, it does not provide directly the strength parameter $C_0$.

We found that the value of $\beta$ for Dunham dolomite was 0.5, which is markedly different than the value of 0.1 reported by Mogi [8], which means that the fracture plane is deviated by 30° from the $\sigma_2$-direction and not by 5.7°. In addition, the value of $\beta$ for Solenhofen limestone was not nearly zero as reported by Mogi [9] but 0.45 which is equivalent to a deviation angle of $\sim$27°. The differences between the results we found in this study and the ones carried out by Mogi for this failure criterion, are due to the difference in data taken into account, that is, Mogi worked with triaxial (compression and extension) test data and biaxial test data and we worked with polyaxial test data.

Chang and Haimson [13] reported that the amphibolite from the KTB site failed in brittle fashion along a fracture plane striking sub-parallel to the direction of $\sigma_2$. According to our findings, the fracture plane should be deviated approximately $\sim$8° from the $\sigma_2$-direction, which agrees with the observations of Chang and Haimson who made an extensive study of the polyaxial mechanical behavior of the KTB amphibolite. The Shirahama sandstone presented the lowest value of $\beta = 0.06$, which means that the fracture plane is almost parallel ($\sim$3°) to the $\sigma_2$-direction. The value of $\beta$ for the Yuubari shale was 0.25 equivalent to $\sim$14°.

Fig. 11 shows the results obtained for the Mogi 1967 empirical criterion in $\sigma_1 - \sigma_2$ space. It can be seen that this failure criterion is represented by a quasi-rectilinear function. In Tables 7–11 the mean misfits in $\sigma_1 - \sigma_2$ space are reported. It can be seen that for the Dunham dolomite and the Solenhofen limestone (i.e., the rocks with higher $\sigma_2$-dependence), the mean misfit achieved by this criterion is between the values of the misfit achieved by the triaxial failure criteria and the other two polyaxial failure criteria (Modified Lade and Modified Wiebols and Cook). For the KTB amphibolite and the Shirahama sandstone, the mean misfit is greater than those obtained by the same triaxial and polyaxial criteria mentioned before. For the Yuubari shale, the Mogi 1967 failure criterion achieved the least mean misfit; however, the mean misfit yielded by the Modified Wiebols and Cook criterion was only 20% larger than the one obtained using the Mogi 1967 criterion. As the latter does not provide information about $C_0$, it might be better, in general, to use the Modified Wiebols and Cook criterion, which does provide information about $C_0$ and $\mu_i$.

Fig. 12 shows the results obtained for the Mogi 1971 empirical criterion. $\tau_{oct}$ at fracture is plotted against $(\sigma_1 + \sigma_3)/2$ or $\sigma_{m,2}$. The different symbols show different $\sigma_3$ values and they form a single curve for each rock. We fit the data using three kinds of functions: power law, linear and second-order polynomial, in order to find the best-fitting curve, that is, the one with the least standard deviation mean misfit. Tables 2–6 show the mean misfits associated to each function for each rock. We show only

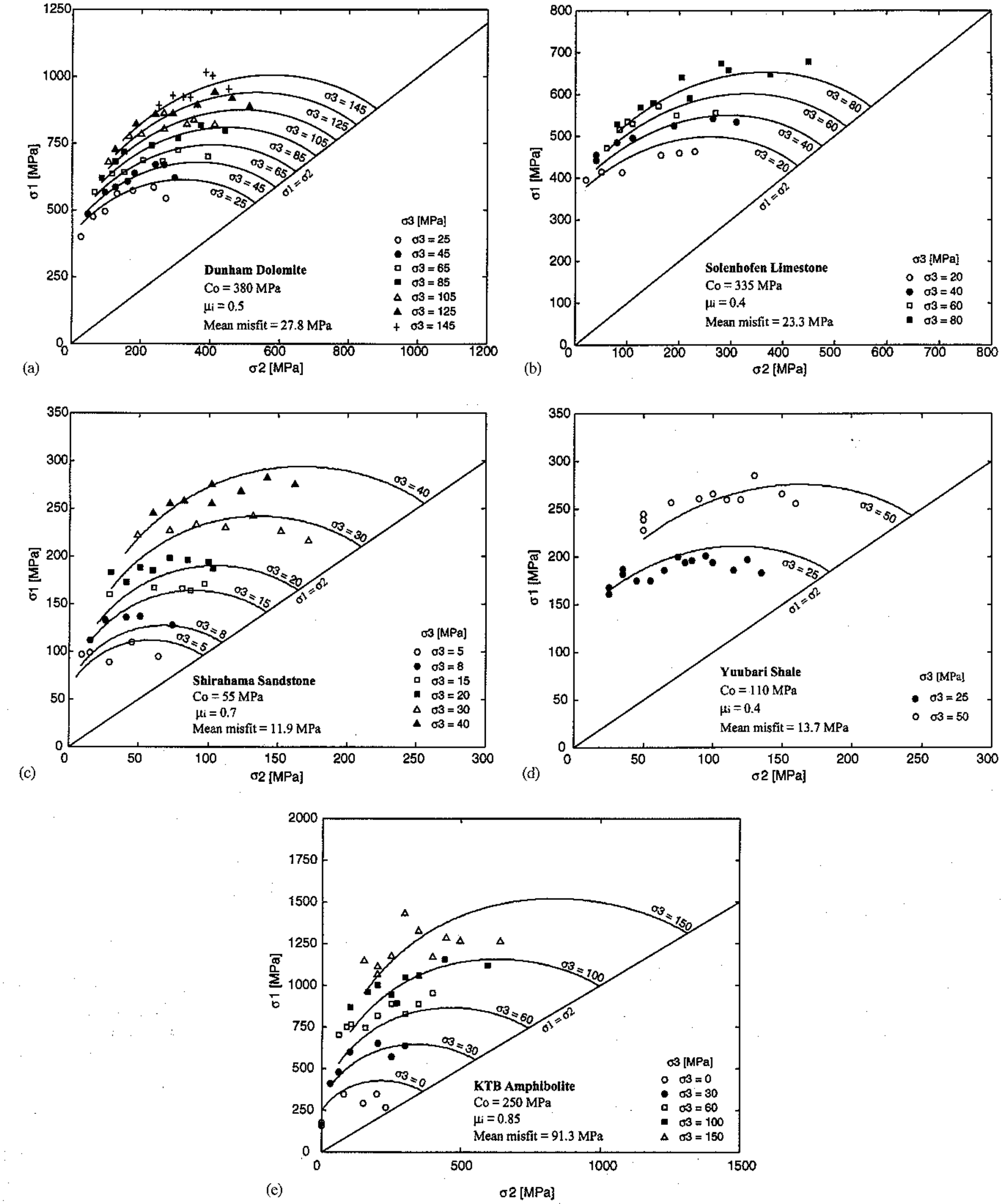



Fig. 8. Best-fitting solution for all the rocks using the Modified Lade criterion (Eq. (9)). (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

the best-fitting functions in Fig. 12. For the amphibolite of the KTB site, we used the power law failure criterion reported by Chang and Haimson [13]. We also analyzed the second-order polynomial and linear fittings for the same rock, but these functions did not fit the data as well as the power law function.

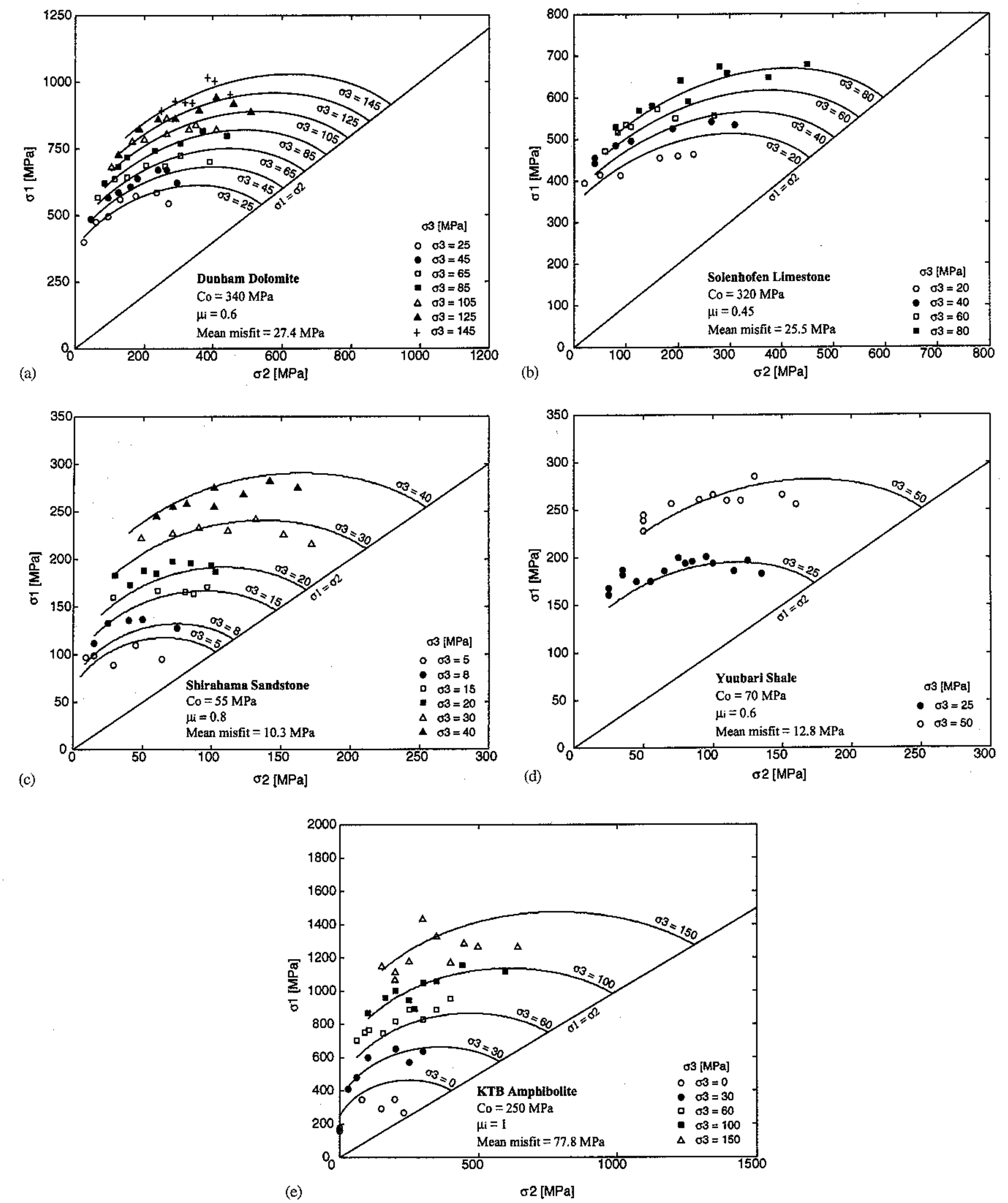



Fig. 9. Best-fitting solution for all the rocks using the Modified Wiebols and Cook criterion (Eq. (14)). (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

In summary, the Mogi 1971 empirical failure criterion is able to reproduce all the failure stresses for the rocks in the $\tau_{oct} - \sigma_{m,2}$ space using a monotonically increasing function. In most cases, a power law fit works best. However, in the $\sigma_1 - \sigma_2$ space, as can be seen in Fig. 13, this failure criterion yields (in some cases), physically

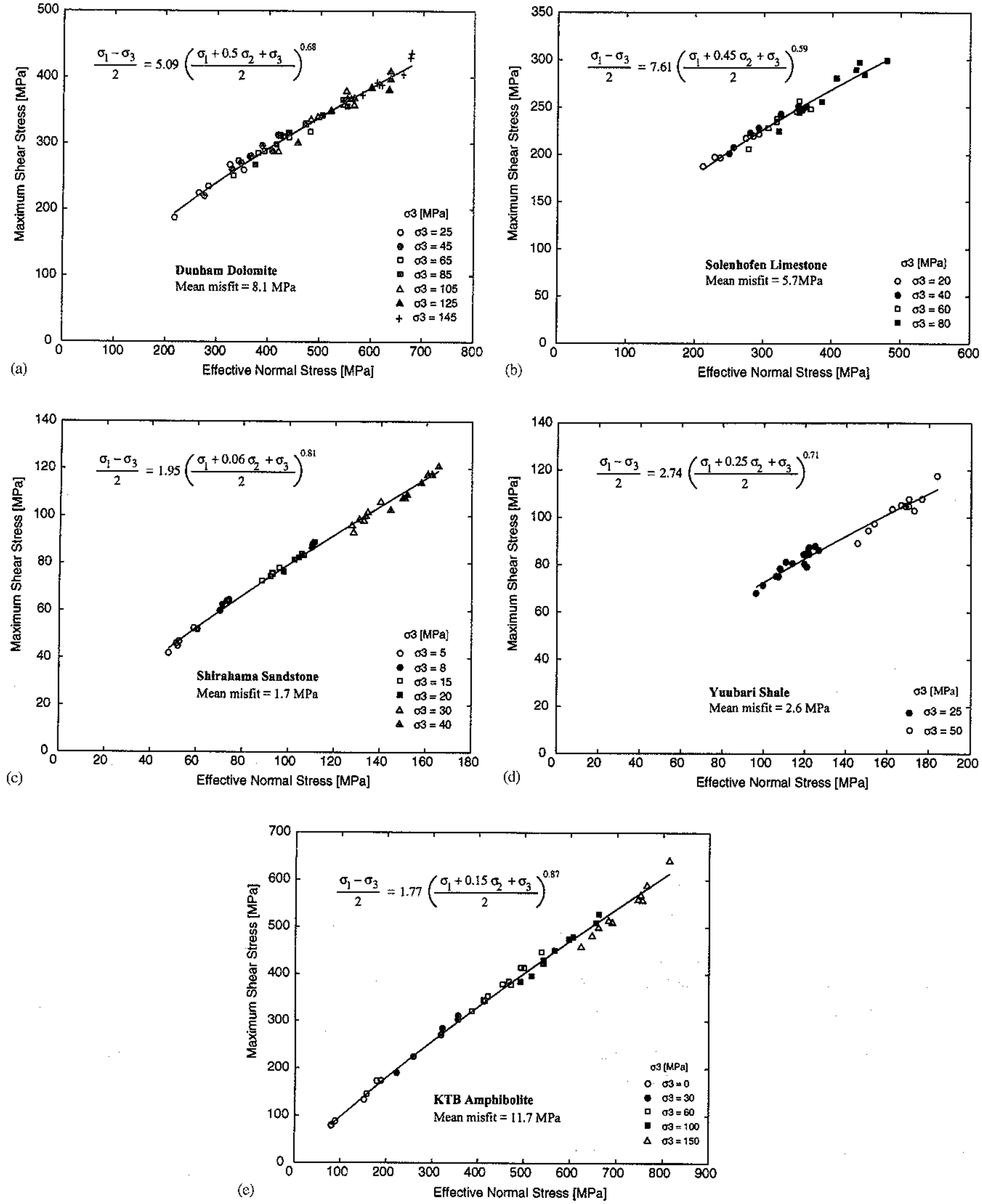



Fig. 10. Best-fitting solution for all the rocks using the Mogi 1967 criterion plotted in $(\sigma_1 - \sigma_3)/2 - (\sigma_1 + \beta\sigma_2 + \sigma_3)/2$ space (Eq. (21)). (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

unreasonable solutions. It first predicts a strengthening effect with increasing intermediate principal stress $\sigma_2$ followed by a considerable 40–60% reduction in strength once $\sigma_2$ becomes too high. Also, for Shirahama sandstone and KTB amphibolite there are some values of $\sigma_2$ having two values of $\sigma_1$ at failure, which is

Table 1
Best-fitting parameters and mean misfits for the Mogi 1967 failure criterion in $(\sigma_1-\sigma_3)/2$–$(\sigma_1+\beta\sigma_2+\sigma_3)/2$ space

| Type of rock | Failure criterion | Mean misfit (MPa) |
|---|---|---|
| Dunham dolomite | $\frac{\sigma_1-\sigma_3}{2}=5.09\left[\frac{\sigma_1+0.5\sigma_2+\sigma_3}{2}\right]^{0.68}$ | 8.1 |
| Solenhofen limestone | $\frac{\sigma_1-\sigma_3}{2}=7.61\left[\frac{\sigma_1+0.45\sigma_2+\sigma_3}{2}\right]^{0.59}$ | 5.7 |
| Shirahama sandstone | $\frac{\sigma_1-\sigma_3}{2}=1.95\left[\frac{\sigma_1+0.06\sigma_2+\sigma_3}{2}\right]^{0.81}$ | 1.7 |
| Yuubari shale | $\frac{\sigma_1-\sigma_3}{2}=2.74\left[\frac{\sigma_1+0.25\sigma_2+\sigma_3}{2}\right]^{0.71}$ | 2.6 |
| KTB amphibolite | $\frac{\sigma_1-\sigma_3}{2}=1.77\left[\frac{\sigma_1+0.15\sigma_2+\sigma_3}{2}\right]^{0.87}$ | 11.7 |

physically impossible. This is not an artifact of the graphic representation but of the mathematical definition. The reason why this failure criterion fits the data so well in the $\tau_{oct}-\sigma_{m,2}$ space is because it takes advantage of this dual solution to actually fit the data in that space. For the Shirahama sandstone the best-fitting failure criterion is the Modified Wiebols and Cook as well as for the KTB amphibolite, the latter contradicting the results of Chang and Haimson [13], who reported the Mogi 1971 failure criterion as the best-fitting failure criterion for the KTB amphibolite. However, this failure criterion does a very good job fitting the data of the Dunham dolomite, the Solenhofen limestone and the Yuubari shale in both spaces. In Tables 7–11, the mean misfits for the $\sigma_1-\sigma_2$ space, associated to each rock, are reported.

To analyze the rock strength data with the Drucker–Prager criterion, we obtained the relationship between $J_1$ and $(J_2)^{1/2}$ using minimum least squares and finding the standard deviation mean misfit directly, without a grid search. We were able to determine which criteria are applicable for which rocks, based on the range of values that $\alpha$ (Eqs. (25) and (27)) is allowed to have.

As it was shown in Fig. 3, the parameter $\alpha$ ranges between 0 and 0.866 for the Inscribed Drucker–Prager criterion and between 0 and 1.732 for the Circumscribed Drucker–Prager criterion. If the value of $\alpha$ obtained using the linear fit falls within these values, it is possible to find the respective $C_0$ and $\mu_i$ for a given rock. This is the case for Dunham dolomite, Solenhofen limestone, Shirahama sandstone and Yuubari shale. The values of $C_0$ obtained for each rock using the Inscribed and the Circumscribed Drucker–Prager criterion give a range within which the value of $C_0$ obtained using Mohr–Coulomb is contained, as was expected. However, for the KTB amphibolite, the value of $\alpha$ was within the range for the Circumscribed Drucker–Prager criterion but outside the range for the Inscribed Drucker–Prager criterion, therefore we were only able to find the parameters for $C_0$ and $\mu_i$ using the relationships from the Circumscribed Drucker–Prager criterion. All best-fitting strength parameters are summarized in Tables 7–11.

Fig. 14 presents the fits of the rock strength data and the respective coefficients in the $J_1$ and $(J_2)^{1/2}$ space, in which the Drucker–Prager criterion was developed. The parameters $C_0$ and $\mu_i$ are summarized in the table presented in the same figure. Fig. 15 shows the data in $\sigma_1-\sigma_2$ space. At low values of $\sigma_2$ ($\sigma_2 < 100$ MPa), the Drucker–Prager criterion is able to reproduce the trend of the data for the Dunham dolomite, the Solenhofen limestone and the Yuubari shale (for $\sigma_3 = 25$ MPa), but for the other rocks, the curves do not even reproduce the trend of the data. That is, the Drucker–Prager failure criterion does not accurately indicate the value of $\sigma_1$ at failure.

## 4. Behavior of the different failure criteria in relation to each rock

As summarized in Tables 7–11, the mean misfits obtained using the two triaxial failure criteria are about within ~10% of each other and the mean misfits using the two polyaxial failure criteria are also within ~10% of each other. However, the mean misfits for the

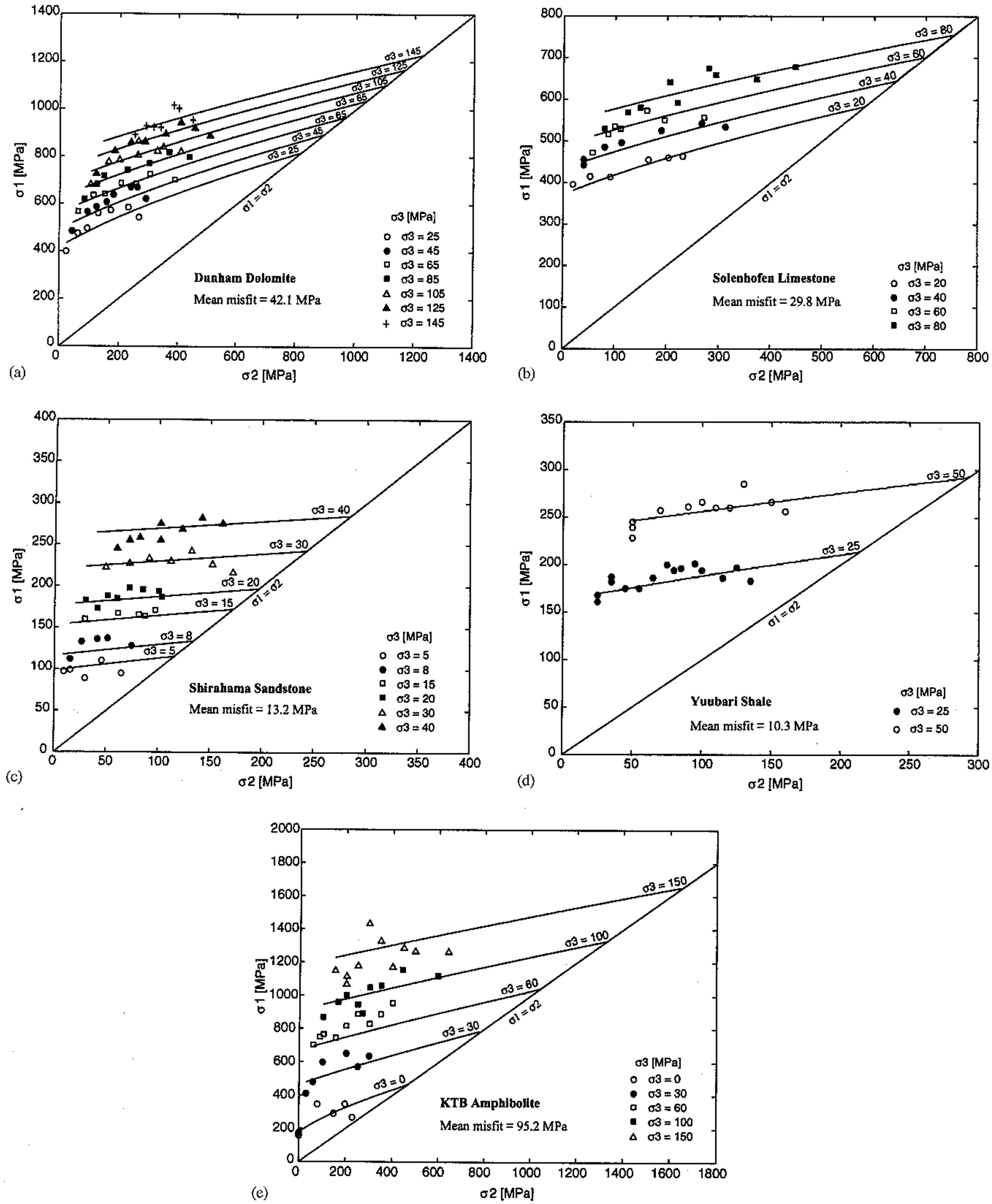



Fig. 11. Best-fitting solution for all the rocks using the Mogi 1967 criterion plotted in $\sigma_1 - \sigma_2$ space. (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

polyaxial failure criteria, are ~40–50% less than for the triaxial failure criteria. The Mogi 1967 empirical criteria yielded the lowest mean misfit for the Yuubari shale but it was only 20% less than the mean misfit yielded by the Modified Wiebols and Cook for the same rock. The Mogi 1971 empirical criterion yielded the lowest mean

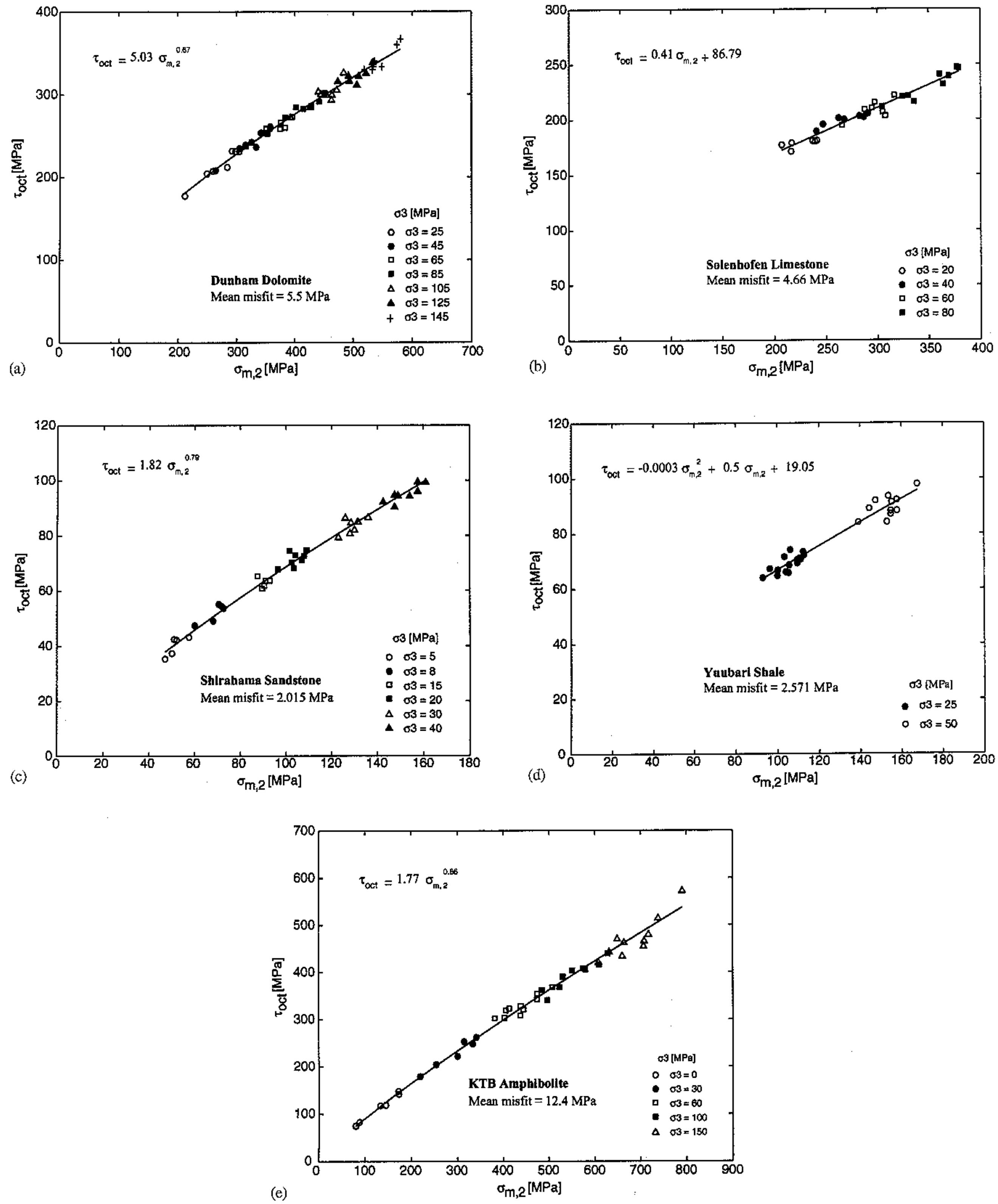



Fig. 12. Best-fitting solution for all the rocks using the Mogi 1971 criterion plotted in $\tau_{oct}-\sigma_{m,2}$ space (Eq. (22)). (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

Table 2
Best-fitting parameters and mean misfits for the Mogi 1971 criterion for the Dunham dolomite in $\tau_{oct}-\sigma_{m,2}$ space

| Type of function | Failure criterion | Mean misfit (MPa) |
|---|---|---|
| Power law | $\tau_{oct} = 5.503\sigma_{m,2}^{0.67}$ | 5.5 |
| Second-order polynomial | $\tau_{oct} = -0.0001\sigma_{m,2}^2 + 0.58\sigma_{m,2} + 66.53$ | 10.4 |
| Linear | $\tau_{oct} = 0.46\sigma_{m,2} + 89.41$ | 5.7 |

Table 3
Best-fitting parameters and mean misfits for the Mogi 1971 criterion for the Solenhofen limestone in $\tau_{oct}-\sigma_{m,2}$ space

| Type of function | Failure criterion | Mean misfit (MPa) |
|---|---|---|
| Power law | $\tau_{oct} = 8.12\sigma_{m,2}^{0.57}$ | 4.8 |
| Second-order polynomial | $\tau_{oct} = 0.0003\sigma_{m,2}^2 + 0.2\sigma_{m,2} + 111.4$ | 4.7 |
| Linear | $\tau_{oct} = 0.41\sigma_{m,2} + 86.79$ | 4.66 |

Table 4
Best-fitting parameters and mean misfits for the Mogi 1971 criterion for the Shirahama sandstone in $\tau_{oct}-\sigma_{m,2}$ space

| Type of function | Failure criterion | Mean misfit (MPa) |
|---|---|---|
| Power law | $\tau_{oct} = 1.82\sigma_{m,2}^{0.79}$ | 2.015 |
| Second-order polynomial | $\tau_{oct} = -0.0009\sigma_{m,2}^2 + 0.7\sigma_{m,2} + 5.5$ | 2.023 |
| Linear | $\tau_{oct} = 0.54\sigma_{m,2} + 14.48$ | 2.2 |

Table 5
Best-fitting parameters and mean misfits for the Mogi 1971 criterion for the Yuubari shale in $\tau_{oct}-\sigma_{m,2}$ space

| Type of function | failure criterion | Mean misfit (MPa) |
|---|---|---|
| Power law | $\tau_{oct} = 2.75\sigma_{m,2}^{0.69}$ | 2.573 |
| Second-order polynomial | $\tau_{oct} = -0.0003\sigma_{m,2}^2 + 0.5\sigma_{m,2} + 19.05$ | 2.571 |
| Linear | $\tau_{oct} = 0.43\sigma_{m,2} + 23.93$ | 2.572 |

Table 6
Best-fitting parameters and mean misfits for the Mogi 1971 criterion for the KTB amphibolite in $\tau_{oct}-\sigma_{m,2}$ space

| Type of function | Failure criterion | Mean misfit (MPa) |
|---|---|---|
| Power law [13] | $\tau_{oct} = 1.77\sigma_{m,2}^{0.86}$ | 12.4 |
| Second-order polynomial | $\tau_{oct} = -0.0001\sigma_{m,2}^2 + 0.7\sigma_{m,2} + 8.28$ | 16.3 |
| Linear | $\tau_{oct} = 0.64\sigma_{m,2} + 36.37$ | 13.6 |

misfit for the Solenhofen limestone but it was only 17% less than the misfit yielded by the Modified Lade criterion. The mean misfits for the Drucker–Prager failure criterion were within the 10% of the triaxial failure criteria misfits for the Dunham dolomite and the Solenhofen limestone. However, for the other rocks, the misfits using the Drucker–Prager criterion were 2–3 times larger than the misfits using the simpler triaxial failure criteria.

In Fig. 16 we present a summary of the best-fitting curves for all the rocks in this study, for all the failure criteria at the minimum and maximum values of $\sigma_3$ used in the lab tests. It demonstrates that obtaining data under nearly biaxial conditions ($\sigma_2 \sim \sigma_1$) would be helpful in characterizing rock failure.

The parameters giving the best fit for each criterion are summarized in Table 7 for the Dunham dolomite. Failure of Dunham dolomite depends strongly on the intermediate principal stress. The triaxial Hoek and Brown and Mohr–Coulomb criteria misfits are essentially the same and the $C_0$ values determined with the two criteria differ by ~10% (~50 MPa). The obtained value of $s$ is 1 (as for an intact rock) and $m = 8$, which is in the range of values reported by Hoek and Brown [1,2] for carbonate rocks. As shown in Fig. 21a, the $m$ values that range between 7 and 8 fit the data almost equally well, as for $m = 5$, the misfits would be twice as large as the misfit for $m = 8$. The Modified Lade criterion is able to fit almost all the data points. Note that the misfit with this polyaxial criterion is less than half of that from the triaxial criteria. The Modified Wiebols and Cook criterion has the same misfit as the Modified Lade criterion. $C_0$ only differs by ~12% (~40 MPa) and $\mu_i$ by 20% (0.1). The Modified Wiebols and Cook yielded the least mean misfit for this rock. The Mogi empirical failure criteria fit the data very well as it can be seen in Figs. 11 and 13. The misfit associated to the Mogi 1967 criterion is 1.5 times larger than the one associated to the best-fitting failure criterion for this rock. The misfit yielded by the Mogi 1971 failure criterion is the same as the Modified Lade and Modified Wiebols and Cook criteria. The values of $C_0$ corresponding to the Inscribed and the Circumscribed Drucker–Prager criteria (Fig. 12) bound the value of $C_0$ for the Mohr–Coulomb criterion as expected. Fig. 16a shows that the best-fitting failure criteria for this rock are the Modified Lade and the Modified Wiebols and Cook.

The results for the Solenhofen limestone were qualitatively similar for the triaxial and the polyaxial (Modified Lade and Modified Wiebols and Cook) failure criteria (Table 8). The Mohr–Coulomb and the Hoek and Brown criteria fit the data equally well and represent an average fit of the data as it can be seen in Figs. 6b and 7b. The value of $m$ was 4.6, which is 10% lower than the lower bound of the range of $m$ corresponding to carbonate rocks ($5 < m < 8$). However,

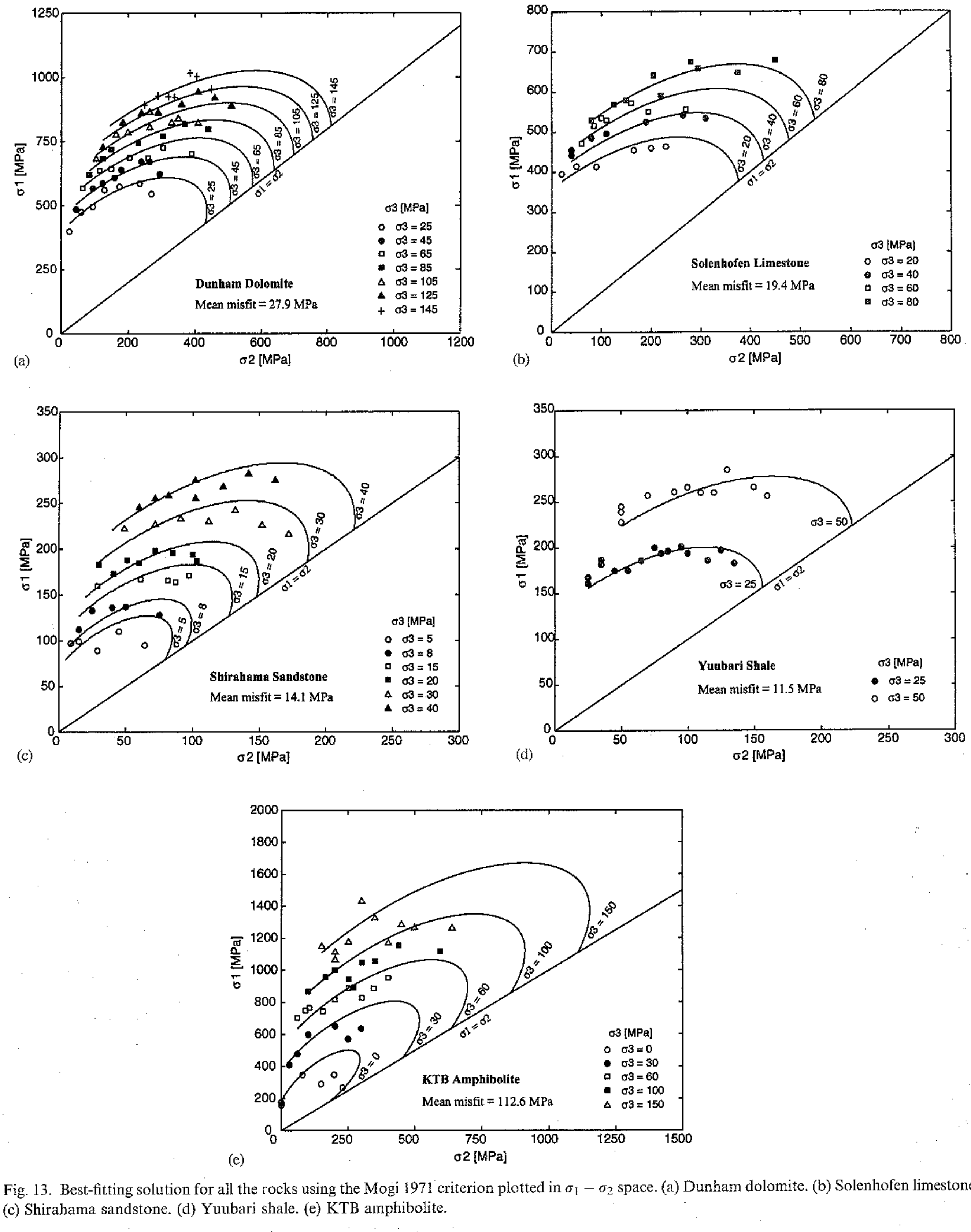



Fig. 13. Best-fitting solution for all the rocks using the Mogi 1971 criterion plotted in $\sigma_1 - \sigma_2$ space. (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

Table 7
Best-fitting parameters and mean misfits (in $\sigma_1 - \sigma_2$ space) for Dunham dolomite

| Failure criterion | $C_0$ (MPa) | $\mu_i$ | $m$ | $s$ | Mean misfit (MPa) |
|---|---|---|---|---|---|
| Mohr–Coulomb | 450 | 0.65 | — | — | 56.0 |
| Hoek–Brown | 400 | — | 8 | 1 | 56.2 |
| Modified Wiebols and Cook | 340 | 0.6 | — | — | 27.4 |
| Modified Lade | 380 | 0.5 | — | — | 27.8 |
| Mogi 1967 | $\frac{\sigma_1 - \sigma_3}{2} = 5.09\left[\frac{\sigma_1 + 0.5\sigma_2 + \sigma_3}{2}\right]^{0.68}$ | | | | 42.1 |
| Mogi 1971 | $\tau_{oct} = 5.03\sigma_{m,2}^{0.67}$ | | | | 27.9 |
| Drucker–Prager | $J_2^{1/2} = 0.5J_1 + 159.1$ | | | | 51.6 |
| Inscribed Drucker–Prager | 723 | 0.64 | — | — | — |
| Circumscribed Drucker–Prager | 393 | 0.42 | — | — | — |

Table 8
Best-fitting parameters and mean misfits (in $\sigma_1 - \sigma_2$ space) for Solenhofen limestone

| Failure criterion | $C_0$ (MPa) | $\mu_i$ | $m$ | $s$ | Mean misfit (MPa) |
|---|---|---|---|---|---|
| Mohr–Coulomb | 375 | 0.55 | — | — | 37.1 |
| Hoek–Brown | 370 | — | 4.6 | 1 | 37.4 |
| Modified Wiebols and Cook | 320 | 0.45 | — | — | 25.5 |
| Modified Lade | 335 | 0.4 | — | — | 23.3 |
| Mogi 1967 | $\frac{\sigma_1 - \sigma_3}{2} = 7.61\left[\frac{\sigma_1 + 0.45\sigma_2 + \sigma_3}{2}\right]^{0.59}$ | | | | 29.8 |
| Mogi 1971 | $\tau_{oct} = 0.41\sigma_{m,2} + 86.79$ | | | | 19.4 |
| Drucker–Prager | $J_2^{1/2} = 0.3J_1 + 167.2$ | | | | 35.9 |
| Inscribed Drucker–Prager | 574.5 | 0.37 | — | — | — |
| Circumscribed Drucker–Prager | 371 | 0.28 | — | — | — |

Table 9
Best-fitting parameters and mean misfits (in $\sigma_1 - \sigma_2$ space) for Shirahama sandstone

| Failure criterion | $C_0$ (MPa) | $\mu_i$ | $m$ | $s$ | Mean misfit (MPa) |
|---|---|---|---|---|---|
| Mohr–Coulomb | 95 | 0.8 | — | — | 9.6 |
| Hoek–Brown | 65 | — | 18.2 | 1 | 8.7 |
| Modified Wiebols and Cook | 55 | 0.8 | — | — | 10.3 |
| Modified Lade | 55 | 0.7 | — | — | 11.9 |
| Mogi 1967 | $\frac{\sigma_1 - \sigma_3}{2} = 1.95\left[\frac{\sigma_1 + 0.06\sigma_2 + \sigma_3}{2}\right]^{0.81}$ | | | | 13.2 |
| Mogi 1971 | $\tau_{oct} = 1.82\sigma_{m,2}^{0.79}$ | | | | 14.1 |
| Drucker–Prager | $J_2^{1/2} = 0.6J_1 + 27.7$ | | | | 28.3 |
| Inscribed Drucker–Prager | 175.7 | 0.88 | — | — | — |
| Circumscribed Drucker–Prager | 74.7 | 0.51 | — | — | — |

Fig. 21b shows that for $m = 5$, the misfit is essentially the same ($\pm 3$ MPa) than for $m = 4.6$. The Modified Wiebols and Cook and the Modified Lade criterion yielded very similar values of $C_0$ and $\mu_i$ and their misfits are very similar. Figs. 8b and 9b show that for both criteria, the fitting curves corresponding to all the $\sigma_3$

Table 10
Best-fitting parameters and mean misfits (in $\sigma_1 - \sigma_2$ space) for Yuubari shale

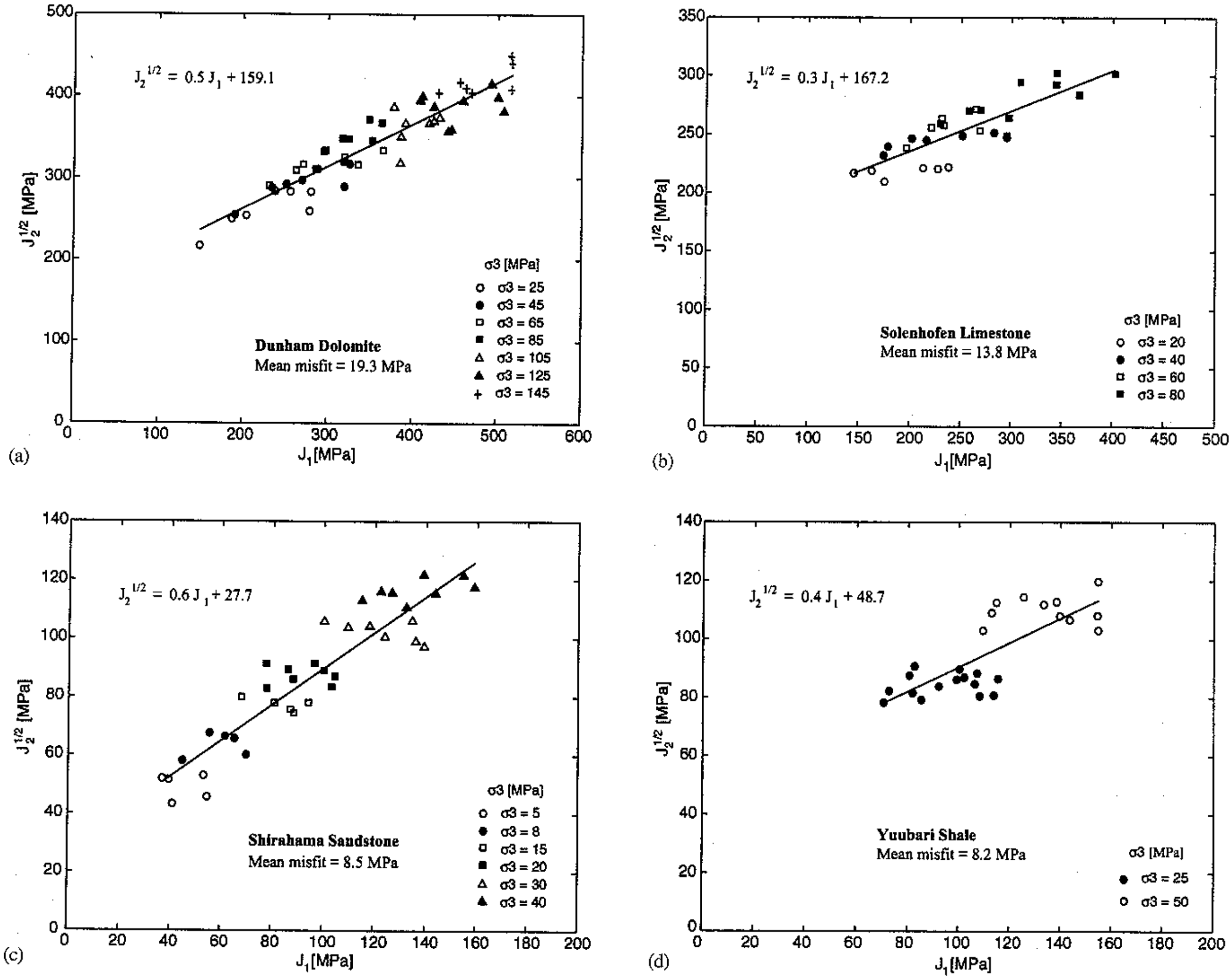
| Failure criterion | $C_0$ (MPa) | $\mu_i$ | $m$ | $s$ | Mean misfit (MPa) |
|---|---|---|---|---|---|
| Mohr–Coulomb | 120 | 0.50 | — | — | 13.5 |
| Hoek–Brown | 100 | — | 6.5 | 1 | 13.0 |
| Modified Wiebols–Cook | 70 | 0.6 | — | — | 12.8 |
| Modified Lade | 110 | 0.4 | — | — | 13.7 |
| Mogi 1967 | $\frac{\sigma_1 - \sigma_3}{2} = 2.74\left[\frac{\sigma_1 + 0.25\sigma_2 + \sigma_3}{2}\right]^{0.71}$ | | | | 10.3 |
| Mogi 1971 | $\tau_{oct} = -0.0003\sigma_{m,2}^2 + 0.5\sigma_{m,2} + 19.05$ | | | | 11.5 |
| Drucker–Prager | $J_2^{1/2} = 0.4J_1 + 48.7$ | | | | 21.0 |
| Inscribed Drucker–Prager | 176.8 | 0.48 | — | — | — |
| Circumscribed Drucker–Prager | 111 | 0.34 | — | — | — |

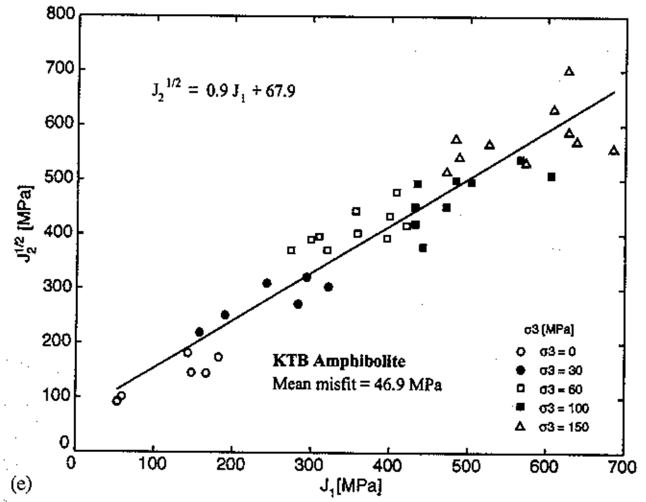
Table 11
Best-fitting parameters and mean misfits (in $\sigma_1 - \sigma_2$ space) for the KTB amphibolite

| Failure criterion | $C_0$ (MPa) | $\mu_i$ | $m$ | $s$ | Mean misfit (MPa) |
|---|---|---|---|---|---|
| Mohr–Coulomb | 300 | 1.2 | — | — | 77.9 |
| Hoek–Brown | 250 | — | 30 | 1 | 89.9 |
| Modified Wiebols–Cook | 250 | 1 | — | — | 77.8 |
| Modified Lade | 250 | 0.85 | — | — | 91.3 |
| Mogi 1967 | $\frac{\sigma_1 - \sigma_3}{2} = 1.77\left[\frac{\sigma_1 + 0.15\sigma_2 + \sigma_3}{2}\right]^{0.87}$ | | | | 95.2 |
| Mogi 1971 [13] | $\tau_{oct} = 1.77\sigma_{m,2}^{0.86}$ | | | | 112.6 |
| Drucker–Prager | $J_2^{1/2} = 0.9J_1 + 67.9$ | | | | 161.5 |
| Inscribed Drucker–Prager | — | — | — | — | — |
| Circumscribed Drucker–Prager | 236.5 | 0.75 | — | — | — |

values are very good except for $\sigma_3 = 20$ MPa. The Mogi 1967 empirical failure criterion (Fig. 11b) reproduces the trend of the data very well, indicating that the value of $\beta$ indeed corresponds to the contribution of $\sigma_2$ on failure. The value of $\beta = 0.45$ implies that the fracture plane is about 26.7° deviated from the direction of $\sigma_2$. The misfit achieved by this failure criterion was 35% larger than the misfit achieved by the Mogi 1971 criterion, which yielded the least mean misfit for this rock. Fig. 13b shows that this criterion fits the data very well, however, as it does not provide information about the compressive strength or the coefficient of internal friction then it would be more practical to use the values of these parameters given by the Modified Lade criterion which yielded a mean misfit only 16% larger than the mean misfit obtained using the Mogi 1971 criterion. The Drucker–Prager criterion (Fig. 15b) only slightly reproduces the trend of the data and its misfit is approximately 1.5 times larger than the one obtained with the Modified Wiebols and Cook criterion. Fig. 16b shows that the best-fitting failure criteria for this rock are the Mogi 1967, the Modified Lade and the Modified Wiebols and Cook criteria. It also shows that the fit of the data by the triaxial failure criteria is equivalent. The Drucker–Prager criterion gives the worst prediction of $\sigma_1$ at failure.

According to the misfits given by each criterion for the Shirahama sandstone (Table 9), it would be logical to think that both triaxial criteria (Mohr–Coulomb and Hoek and Brown), the Modified Lade criterion and the Modified Wiebols and Cook criterion fit the data well, as their respective misfits are nearly the same. However, the way they approximate the data is different and as the Shirahama sandstone presents an unusual $\sigma_2$-dependence, the approximations are not completely satisfactory. The Mohr–Coulomb criterion fits the data very well for some values of $\sigma_3$ but not for the entire data set because of the unusual $\sigma_2$-dependence. The Hoek and Brown criterion (Figs. 6c and 7c) achieves better fit than the Mohr–Coulomb criterion, probably because it has an additional degree of freedom. We found that the value of $m$ for this rock is 18.2, which is within the range of $m$ values for arenaceous rocks. As shown in Fig. 21c the $m$ values that range between 16 and 20 fit the data similarly well, as for $m = 15$ (lower limit) and $m = 24$ (upper limit), the misfit would be approximately twice as large as the misfit for $m = 18.2$. The Hoek and Brown criterion yielded the least mean misfit for this rock. Both




| Rocks \ Failure criteria | IDP Co [MPa] | IDP μi | CDP Co [MPa] | CDP μi |
|---|---|---|---|---|
| Dunham dolomite | 723 | 0.6 | 393 | 0.4 |
| Solenhofen limestone | 574.5 | 0.4 | 371 | 0.3 |
| Shirahama sandstone | 175.7 | 0.9 | 74.7 | 0.5 |
| Yuubari shale | 176.8 | 0.5 | 111 | 0.3 |
| KTB amphibolite | - | - | 236.5 | 0.8 |

IDP: Inscribed Drucker-Prager criterion
CDP: Circumscribed Drucker-Prager criterion

Fig. 14. Best-fitting solution for all the rocks using the Drucker–Prager criterion plotted in $J_1-(J_2)^{1/2}$ space (Eq. (24)). (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite. The parameters $C_0$ and $\mu_i$ are summarized in the table for each rock for the Inscribed (IDP) and Circumscribed (CDP) Drucker–Prager criterion.

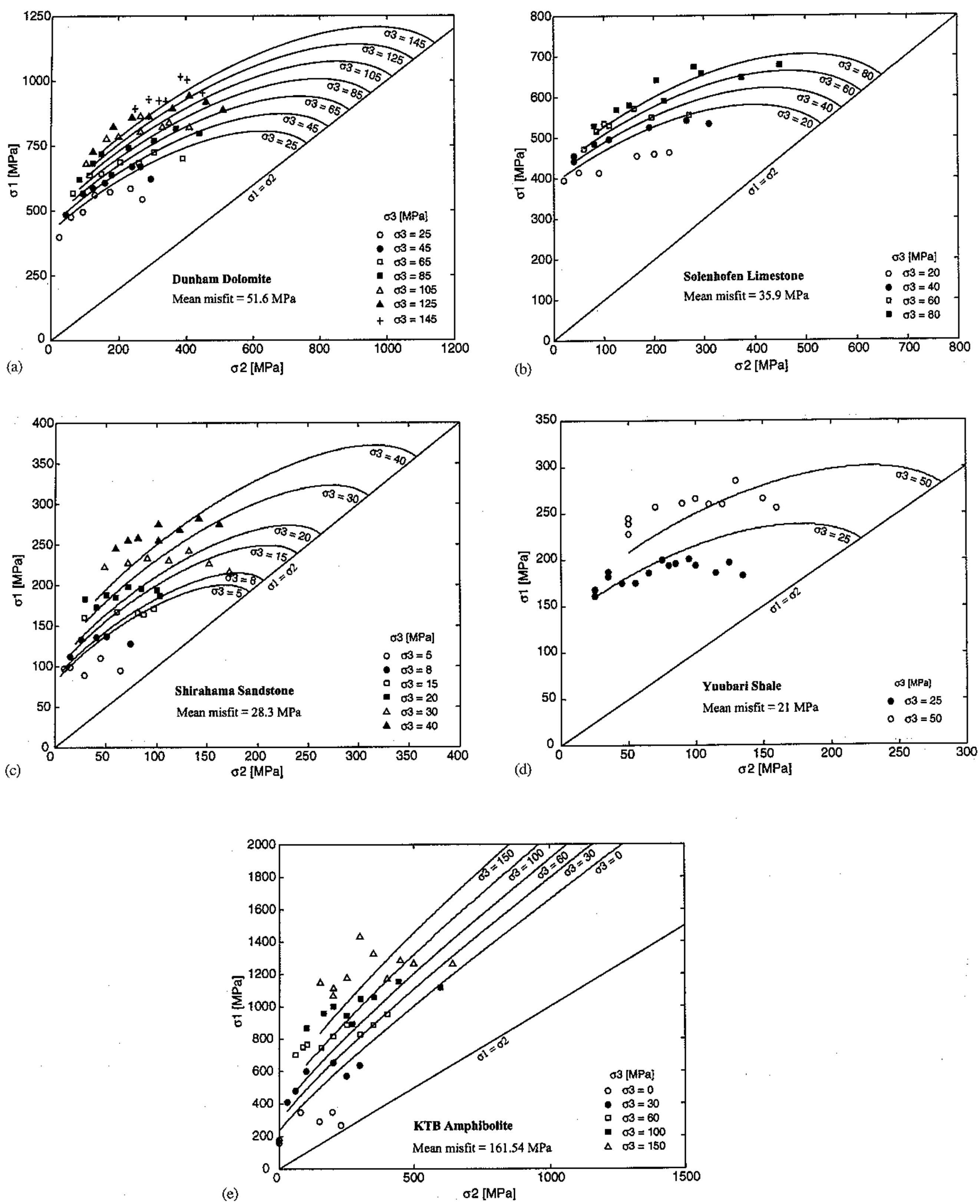



Fig. 15. Best-fitting solution using the Drucker–Prager criterion plotted in $\sigma_1 - \sigma_2$ space. (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

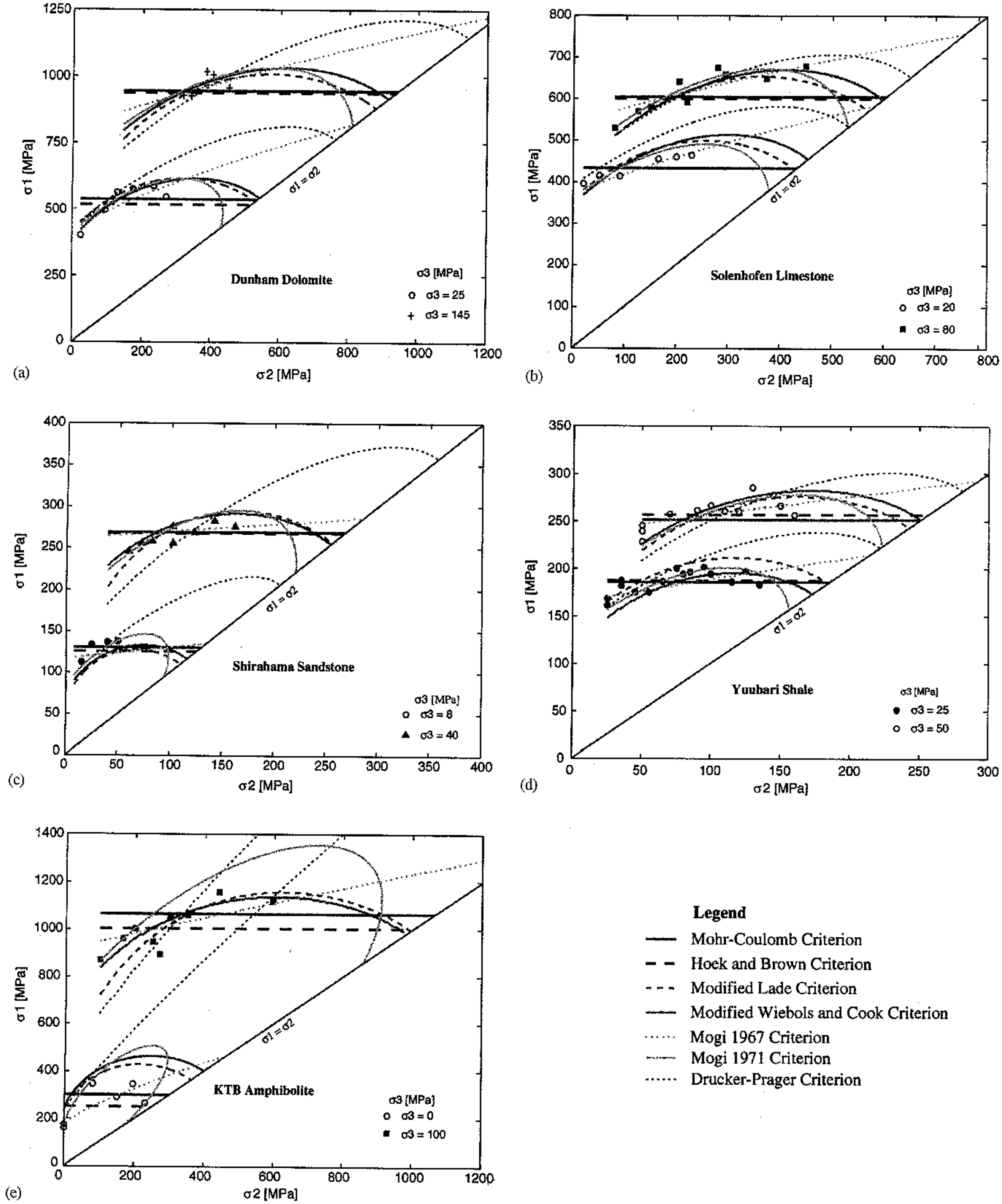
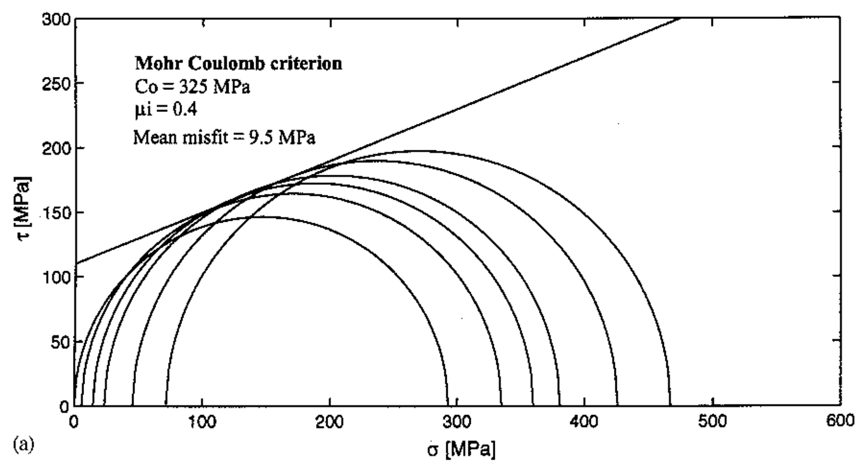
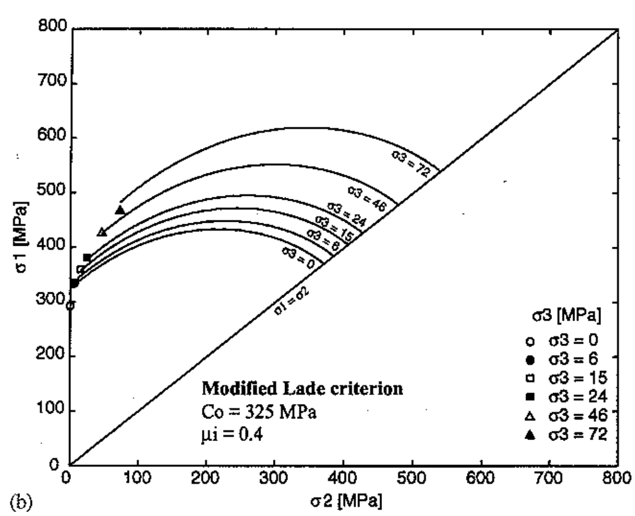



Fig. 16. Summary of the best-fitting solution compared to the actual data for all the failure criteria. The best-fitting parameters ($C_0$ and $\mu_i$) are summarized in Tables 7–11. (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

the Modified Lade criterion and Modified Wiebols and Cook criterion (Figs. 8c and 9c) do not accurately predict the failure stress for this data set. Both give essentially the same $C_0$, and $\mu_i$. It can be seen that as the Modified Lade criterion accounts for a high $\sigma_2$-dependence at low $\sigma_2$ (Fig. 8), the slope of the curve at the beginning is steeper than the slope of the Modified Wiebols and Cook curve (Fig. 9), which is why the latter fits the data better. The Mogi 1967 failure criterion does a good job reproducing the trend of the data. According to the value of $\beta$ found using the Mogi 1967 criterion, the fracture plane is almost parallel ($\sim 3°$) to the $\sigma_2$-direction. This criterion approximates the data better than does the Mogi 1971 failure criterion, which yielded the duality of values of $\sigma_1$ for the higher values of $\sigma_2$ for a specific value of $\sigma_3$, as it can be seen in Fig. 13c. The misfit achieved by the Mogi 1971 criterion was 1.6 times larger than the one achieved by the Hoek and Brown criterion. The Drucker–Prager criterion does not represent the trend of the data set (Fig. 15c) and therefore, it does not predict $\sigma_1$ at failure correctly. The misfit of the Drucker–Prager criterion is approximately 3 times larger than the misfits obtained using the triaxial failure criteria. Fig. 16c shows that the failure criteria fit the data in the same average manner, except for the Drucker–Prager criterion and the Mogi 1971 criterion as indicated above.

The results for the Yuubari shale are summarized in Table 10. The misfits associated to the Mohr–Coulomb, the Hoek and Brown, the Modified Lade and the Modified Wiebols and Cook criteria are all approximately the same. Using the Hoek and Brown criterion we obtained a value of $m = 6.5$, which is within the range of values reported by Hoek and Brown [1,2] for argillaceous rocks. As shown in Fig. 21d the $m$ values that range between 5.5 and 7.5 fit the data almost equally well, as for the lower and upper bounds of $m$, the misfits would be approximately 3 times larger than the misfit for $m = 6.5$. The misfits yielded by the Mogi criteria are also very similar, differing only by 1 MPa. The Mogi 1967 failure criterion reproduces the trend of the data very well as it can be seen in Fig. 11d. The least mean misfit for this rock is achieved using the Mogi 1967 criterion, however, as it does not provide direct information about $C_0$ or $\mu_i$, it would be better to use the Modified Wiebols and Cook criterion which only yielded a mean misfit $\sim 1.2$ times higher than the one yielded by the former criterion. The Mogi 1971 failure



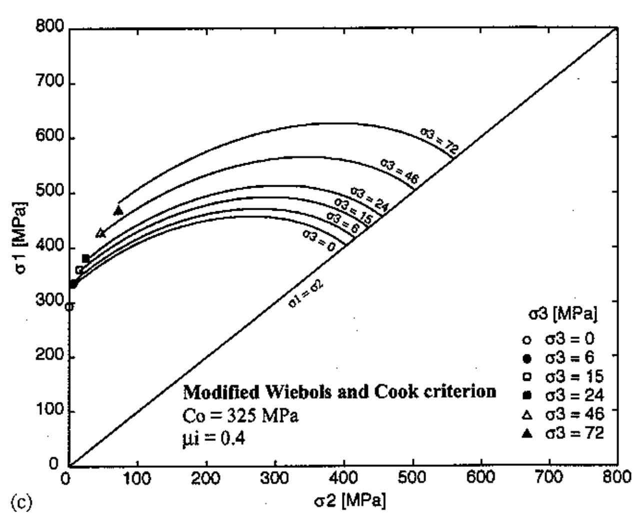




Fig. 17. Best-fitting solution for the Solenhofen limestone using the triaxial test data. (a) Mohr–Coulomb criterion. (b) Modified Lade criterion. (c) Modified Wiebols and Cook criterion. The best-fitting parameters obtained using the Mohr–Coulomb criterion are reported in Table 12.

criterion also does a good job fitting the data (Fig. 13d). Fig. 15d shows that the fitting curve for the Drucker–Prager criterion for $\sigma_3 = 25\,\text{MPa}$ reproduces the trend of the data until $\sigma_2 \approx 100\,\text{MPa}$, but for $\sigma_3 = 50\,\text{MPa}$, the fitting curve does not even represent the trend of the data. Therefore, the Drucker–Prager criterion does not give reliable values of $\sigma_1$ at failure. As it can be seen in Fig. 16d, the triaxial criteria and two of the polyaxial criteria (Modified Lade and Modified Wiebols and Cook), fit the data in approximately the same manner and predict almost equal values of $\sigma_1$ at failure when $\sigma_1 = \sigma_2$.

For the KTB amphibolite, the Mohr–Coulomb criterion represents a good general fit to the data except for $\sigma_3 = 150\,\text{MPa}$ as it can be seen in Fig. 6e. In contrast, the Hoek and Brown criterion represents a good fit to all the experimental data. We found that $m = 30$, which is in the range of values reported by Hoek and Brown [1,2] for coarse-grained polymineralic igneous rocks. As shown in Fig. 21e, the $m$ values that range between 26 and 33 fit the data almost equally well, as for $m = 22$ and the misfit would be approximately 1.5 times larger than the misfit for $m = 30$. Both, Modified Lade and Modified Wiebols and Cook criterion (Figs. 8e and 9e) achieve a similar fit to the data and yield the same value of $C_0$. However, the misfits differ by 20%, which is most likely due to the shape of the failure envelope of each criterion as the slope of the curve for low values of $\sigma_2$ is greater for the Modified Lade criterion than for the Modified Wiebols and Cook criterion. The latter yielded the least mean misfit for this rock as it is reported in Table 11. As it can be seen in Figs. 10e and 12e, both Mogi empirical failure criteria fit the data very well in the Mogi space. However, in the $\sigma_1 - \sigma_2$ space the fitting is different. As can be seen in Fig. 11e, the Mogi 1967 criterion somewhat reproduces the trend of the data. For high $\sigma_2$ on a given $\sigma_3$, the Mogi 1971 failure criterion yield two values of $\sigma_1$ at failure which is physically impossible (see Fig. 13e). The failure criterion that best describes failure on the KTB amphibolite is the Modified Wiebols and Cook criterion and not the Mogi 1971 criterion as proposed by Chang

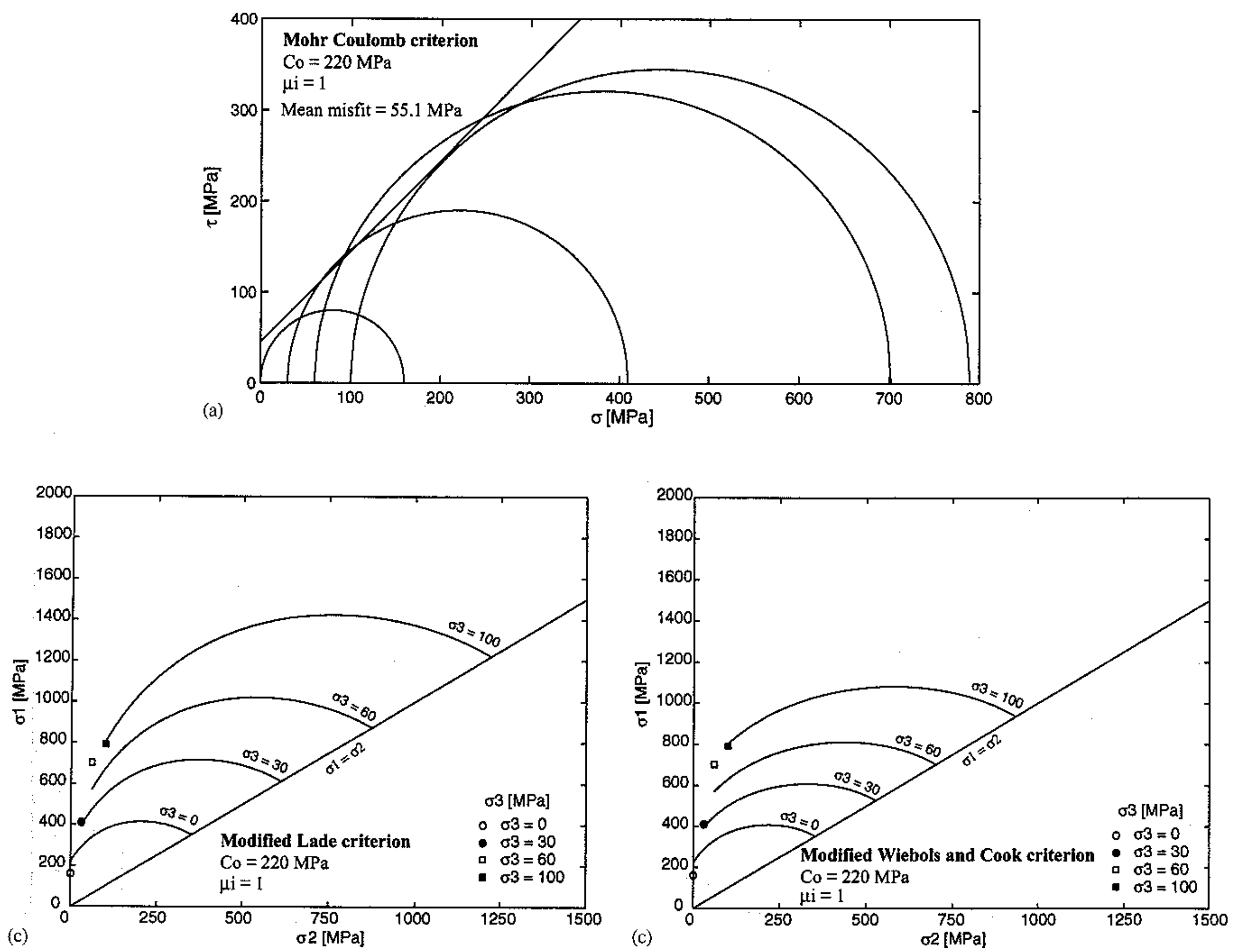



Fig. 18. Best-fitting solution for the KTB amphibolite using the triaxial test data. (a) Mohr–Coulomb criterion. (b) Modified Lade criterion. (c) Modified Wiebols and Cook criterion. The best-fitting parameters obtained using the Mohr–Coulomb criterion are reported in Table 13.

and Haimson [13]. The Drucker–Prager criterion does not reproduce the trend of the data whatsoever as can be seen in Figs. 15e and 16e. It was impossible to find the values of $C_0$ and $\mu_i$ according to the Inscribed Drucker–Prager criterion because the value of $\alpha$ was greater than the asymptotic value of $\alpha$ for this criterion. This might be due to the fact that the Drucker–Prager criterion was originally derived for soils and perhaps should not be applied to strong rocks such as the KTB amphibolite. In Fig. 16e it is possible to see that the Modified Lade and the Modified Wiebols and Cook criteria give a better fit of the data for $\sigma_3 = 100$ MPa than for $\sigma_3 = 0$ MPa. However, the Mohr–Coulomb and the Hoek and Brown criteria give a good average fit of the data for both values of $\sigma_3$.

## 5. Application: How necessary are polyaxial tests?

Polyaxial tests are very difficult to perform and it would be preferable to do triaxial tests. In this section we briefly explore the possibility of working with triaxial test data to see if it is possible to predict the $\sigma_2$-dependence on failure using polyaxial failure criteria.

We used triaxial test data for Solenhofen limestone [8], KTB amphibolite [13] and Dunham dolomite [8]. It is important to remember that these rocks have a large $\sigma_2$-dependence on failure. We performed a grid search to find the best-fitting parameters $C_0$ and $\mu_i$ using the conventional Mohr–Coulomb criterion. We used these parameters to fit the data with the Modified Lade criterion and the Modified Wiebols and Cook criterion and they yielded very good fits of the triaxial data (Figs. 17–19). The best-fitting parameters for each rock are summarized in Tables 12–14. For the Solenhofen limestone and the KTB amphibolite the parameters obtained using the triaxial test data are very similar to those obtained using the polyaxial failure criteria on the polyaxial test data. As shown in Table 12 for the Solenhofen limestone, if we had only triaxial test data, we would have obtained a value of $C_0 \pm 3\%$ those obtained using polyaxial failure criteria on polyaxial test

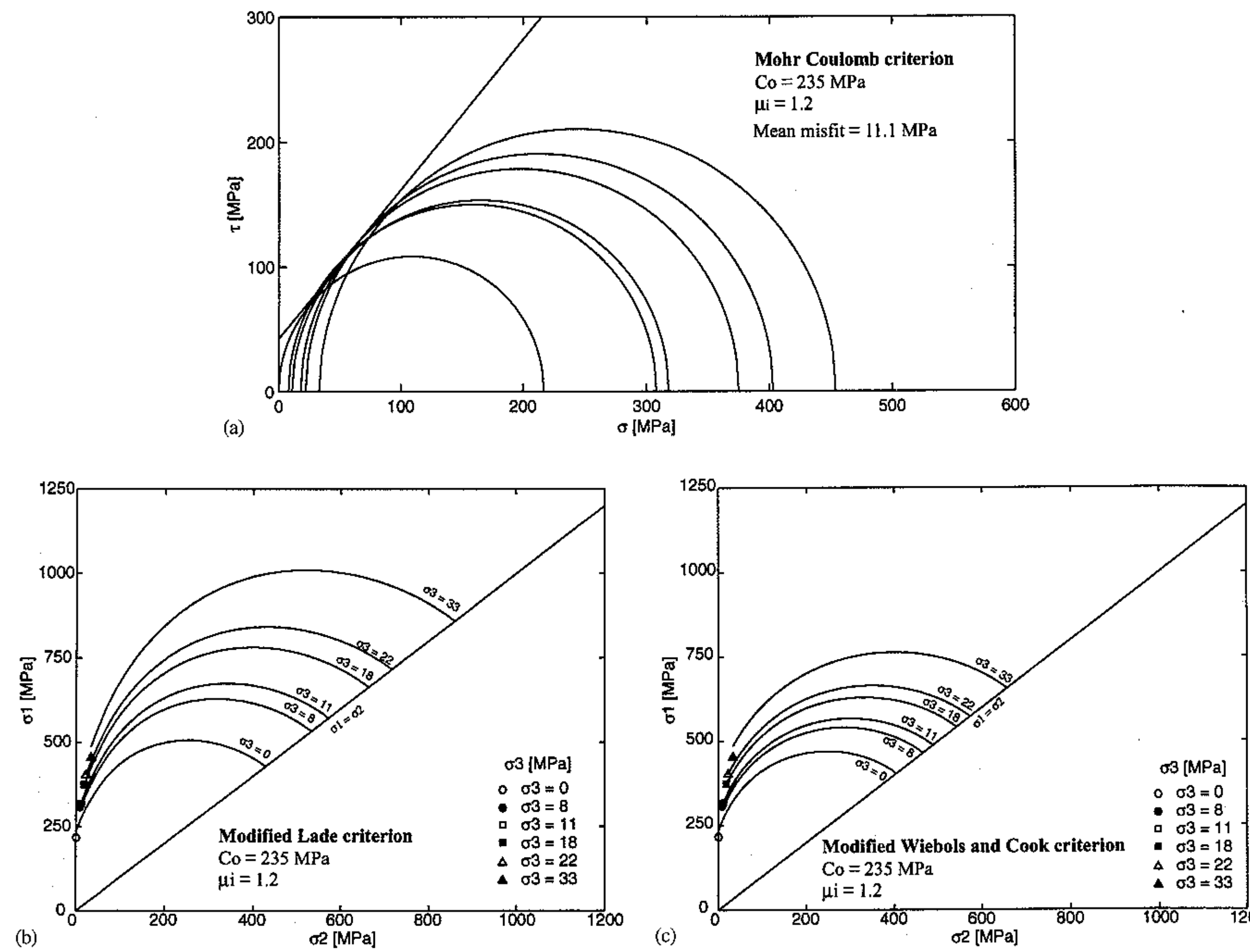



Fig. 19. Best-fitting solution for the Dunham dolomite using the triaxial test data. (a) Mohr–Coulomb criterion. (b) Modified Lade criterion. (c) Modified Wiebols and Cook criterion. The best-fitting parameters obtained using the Mohr–Coulomb criterion are reported in Table 14.

Table 12
Best-fitting parameters for the triaxial test data of the Solenhofen limestone

| Failure criterion | $C_0$ (MPa) | $\mu_i$ | Mean misfit (MPa) | Mean misfit using triaxial parameters in polyaxial criterion (MPa) |
|---|---|---|---|---|
| Triaxial test Mohr–Coulomb | 325 | 0.4 | 9.5 | — |
| Polyaxial test Modified Lade | 320 | 0.45 | 25.5 | 27 |
| Polyaxial test Modified Wiebols and Cook | 335 | 0.4 | 23.2 | 25 |

Table 13
Best-fitting parameters for the triaxial test data of the KTB amphibolite

| Failure criterion | $C_0$ (MPa) | $\mu_i$ | Mean misfit (MPa) | Mean misfit using triaxial parameters in polyaxial criterion (MPa) |
|---|---|---|---|---|
| Triaxial test Mohr–Coulomb | 220 | 1 | 55.1 | — |
| Polyaxial test Modified Lade | 250 | 0.85 | 91.3 | 163 |
| Polyaxial test Modified Wiebols and Cook | 250 | 1 | 77.8 | 82 |

Table 14
Best-fitting parameters for the triaxial test data of the Dunham dolomite

| Failure criterion | $C_0$ (MPa) | $\mu_i$ | Mean misfit (MPa) | Mean misfit using triaxial parameters in polyaxial criterion (MPa) |
|---|---|---|---|---|
| Triaxial test Mohr–Coulomb | 235 | 1.2 | 11.1 | — |
| Polyaxial test Modified Lade | 380 | 0.5 | 27.8 | 550 |
| Polyaxial test Modified Wiebols and Cook | 340 | 0.6 | 27.4 | 300 |

data. The misfit associated with using the triaxial parameters in the polyaxial failure criteria for the Solenhofen limestone was only 10% larger, which is a very reasonable result if we only have to work with triaxial data. For the KTB amphibolite we obtained a $C_0$ 13% smaller than that obtained for the polyaxial test data using the polyaxial failure criteria. The misfits for using the triaxial parameters in the polyaxial criteria were larger, especially for the Modified Lade criterion. For the Modified Wiebols and Cook the misfit was only 5% larger which is still considered to be acceptable. However, for the Dunham dolomite, the $C_0$ obtained with the triaxial test data was $\sim$1.6 times smaller than those obtained using the polyaxial test data and $\mu_i$ is approximately half the value found for the polyaxial test data. As for the misfits, they are approximately 15–20 times larger than the original misfit for the polyaxial

Table 15
Polyaxial test data for the KTB amphibolite (kindly provided by Chang and Haimson)

| $\sigma_1$ (MPa) | $\sigma_2$ (MPa) | $\sigma_3$ (MPa) |
|---|---|---|
| 158 | 0 | 0 |
| 160 | 0 | 0 |
| 176 | 0 | 0 |
| 346 | 79 | 0 |
| 291 | 149 | 0 |
| 347 | 197 | 0 |
| 267 | 229 | 0 |
| 410 | 30 | 30 |
| 479 | 60 | 30 |
| 599 | 100 | 30 |
| 652 | 200 | 30 |
| 571 | 249 | 30 |
| 637 | 298 | 30 |
| 702 | 60 | 60 |
| 750 | 88 | 60 |
| 766 | 103 | 60 |
| 745 | 155 | 60 |
| 816 | 199 | 60 |
| 888 | 249 | 60 |
| 828 | 299 | 60 |
| 887 | 347 | 60 |
| 954 | 399 | 60 |
| 815 | 449 | 60 |
| 868 | 100 | 100 |
| 959 | 164 | 100 |
| 1001 | 199 | 100 |
| 945 | 248 | 100 |
| 892 | 269 | 100 |
| 1048 | 300 | 100 |
| 1058 | 349 | 100 |
| 1155 | 442 | 100 |
| 1118 | 597 | 100 |
| 1147 | 150 | 150 |
| 1065 | 198 | 150 |
| 1112 | 199 | 150 |
| 1176 | 249 | 150 |
| 1431 | 298 | 150 |
| 1326 | 348 | 150 |
| 1169 | 399 | 150 |
| 1284 | 448 | 150 |
| 1265 | 498 | 150 |
| 1262 | 642 | 150 |

failure criteria. We cannot attribute this result merely to the fact that Dunham dolomite has a large $\sigma_2$-dependence because we did not obtain such results for the Solenhofen limestone or the KTB amphibolite, which also have a large $\sigma_2$-dependence. The reason why the results for Dunham dolomite are so unsatisfactory might be due to the fact that the triaxial test data reported by Mogi [8] considered values of $\sigma_3$ up to 33 MPa, while the polyaxial test data reported by Mogi [9] considered values of $\sigma_3$ up to 145 MPa. Therefore, the difference between the $C_0$ and $\mu_i$ obtained using the triaxial test data and the values from the analysis of polyaxial test data might simply be because we are considering such different pressures.

Thus, in two of the three rocks studied, the rock strength parameters yielded by the triaxial test data are very similar to those found using polyaxial test data. This is very helpful because it allows one to perform triaxial tests instead of polyaxial tests to obtain the rock strength parameters and then apply those parameters using a polyaxial failure criterion. However, it is necessary to have a good triaxial test data set covering a wide range of pressures, otherwise the results could be inaccurate as we think was the case for the Dunham dolomite.

Table 16
Polyaxial test data for the Dunham dolomite (digitize from [9])

| $\sigma_1$ (MPa) | $\sigma_2$ (MPa) | $\sigma_3$ (MPa) |
|---|---|---|
| 399.9 | 23.5 | 25 |
| 475.2 | 61.8 | 25 |
| 495.6 | 93.8 | 25 |
| 560.4 | 130.3 | 25 |
| 572.5 | 173.1 | 25 |
| 585.1 | 232.9 | 25 |
| 544 | 268.8 | 25 |
| 485.6 | 42.8 | 45 |
| 566 | 93.7 | 45 |
| 586.4 | 124.3 | 45 |
| 606.9 | 159.3 | 45 |
| 638.7 | 182.5 | 45 |
| 670.5 | 241.3 | 45 |
| 670 | 263.3 | 45 |
| 622.1 | 292.5 | 45 |
| 567 | 62.5 | 65 |
| 636.3 | 113.3 | 65 |
| 641.9 | 152.4 | 65 |
| 687.1 | 207.6 | 65 |
| 683.9 | 258.9 | 65 |
| 725.2 | 306.4 | 65 |
| 701.4 | 390.1 | 65 |
| 620.4 | 83.9 | 85 |
| 682.1 | 125.9 | 85 |
| 718 | 149.7 | 85 |
| 743.3 | 230 | 85 |
| 770.6 | 303.5 | 85 |
| 817.5 | 371 | 85 |
| 798.2 | 440.3 | 85 |
| 680.3 | 103.3 | 105 |
| 776.1 | 165.2 | 105 |
| 784.1 | 202.1 | 105 |
| 804.2 | 264.9 | 105 |
| 822.1 | 330.7 | 105 |
| 838.7 | 350.8 | 105 |
| 820.4 | 411 | 105 |
| 862.5 | 266.2 | 105 |
| 726.3 | 122.7 | 125 |
| 822.6 | 185.8 | 125 |
| 858.8 | 241.1 | 125 |
| 861.6 | 288.1 | 125 |
| 893.3 | 358.8 | 125 |
| 941.7 | 410.5 | 125 |
| 918.4 | 457.8 | 125 |
| 887.1 | 510.1 | 125 |
| 892.1 | 254.2 | 145 |
| 928.5 | 292.3 | 145 |
| 924 | 318.7 | 145 |
| 922 | 341.6 | 145 |
| 1015.7 | 386.6 | 145 |
| 1003.2 | 404.4 | 145 |
| 952.9 | 450.9 | 145 |

## 6. Conclusions

By comparing the different failure criteria to the polyaxial test data we demonstrated that indeed the way

Table 17
Polyaxial test data for the Solenhofen limestone (digitize from [9])

| $\sigma_1$(MPa) | $\sigma_2$ (MPa) | $\sigma_3$ (MPa) |
|---|---|---|
| 395 | 19.1 | 20 |
| 414.4 | 52.2 | 20 |
| 413.3 | 91 | 20 |
| 454.6 | 165 | 20 |
| 459.4 | 203.4 | 20 |
| 463.6 | 230.9 | 20 |
| 442.1 | 40.1 | 40 |
| 455 | 39.9 | 40 |
| 485.6 | 80.4 | 40 |
| 496.1 | 112.8 | 40 |
| 525.8 | 189.6 | 40 |
| 542.2 | 267.2 | 40 |
| 534.3 | 312.4 | 40 |
| 471.9 | 57 | 60 |
| 516 | 87.1 | 60 |
| 535.2 | 99.5 | 60 |
| 529.4 | 111.1 | 60 |
| 572.9 | 162.1 | 60 |
| 550.5 | 196.1 | 60 |
| 556.1 | 271.4 | 60 |
| 529.3 | 80.5 | 80 |
| 568.9 | 124.9 | 80 |
| 580.3 | 149.6 | 80 |
| 641.3 | 205.4 | 80 |
| 591.6 | 220.9 | 80 |
| 674.4 | 280.3 | 80 |
| 658.7 | 293.8 | 80 |
| 647.7 | 373 | 80 |
| 678.2 | 448.1 | 80 |

a failure criterion fits the data will depend on the type of failure criterion (i.e. triaxial, polyaxial) and on the $\sigma_2$-dependence of the rock in question. In general, we found that the Modified Wiebols and Cook and the Modified Lade criteria achieved good fits to most of the test data. This is especially true for rocks with a highly $\sigma_2$-dependent failure behavior (e.g. Dunham dolomite, Solenhofen limestone). The Modified Wiebols and Cook criterion fit the polyaxial data much better than did the Mohr–Coulomb criterion. However, for some rock types (e.g. Shirahama Sandstone, Yuubari shale), the intermediate stress hardly affects failure at some values of $\sigma_3$ and the Mohr–Coulomb and Hoek and Brown criteria fit these test data equally well, or even better, than the more complicated polyaxial criteria.

The values of $C_0$ corresponding to the Inscribed and the Circumscribed Drucker–Prager criterion bounded the $C_0$ value obtained using the Mohr–Coulomb criterion as expected. The values of $C_0$ obtained using the Modified Wiebols and Cook and the Modified Lade criteria were always smaller than the lower bound of the Drucker–Prager criterion, except for the KTB amphibolite for which it was not possible to find both bounds with the Drucker–Prager criterion.

The Mogi 1967 empirical criterion was always able to reproduce the trend of the experimental data for all the rocks. Even though it yielded the least mean misfit for the Yuubari shale, it would be better to use the Modified Wiebols and Cook criterion to fit the data, as the Mogi failure criteria cannot be related to $C_0$ or to other parameters used for characterizing rock strength. The Mogi 1971 failure criterion is mathematically problematic because it yields two values of $\sigma_1$ at failure for the same value of $\sigma_2$ for the Shirahama sandstone, the KTB amphibolite and for low $\sigma_3$ values of Dunham dolomite.

The two triaxial failure criteria analyzed in this study (Mohr–Coulomb and Hoek and Brown) always yielded comparable misfits. Furthermore, the Modified Lade and the Modified Wiebols and Cook criteria, both polyaxial criteria, also gave very similar fits of the data. The Drucker–Prager failure criterion did not accurately indicate the value of $\sigma_1$ at failure and had the highest misfits.

The $\sigma_2$-dependence on failure varies for different rock types but can be very important. We have shown that the use of polyaxial failure criteria can provide

Table 18
Polyaxial test data for the Shirahama sandstone (digitize from [14])

| $\sigma_1$ (MPa) | $\sigma_2$ (MPa) | $\sigma_3$ (MPa) |
|---|---|---|
| 97 | 9 | 5 |
| 99 | 15 | 5 |
| 89 | 29 | 5 |
| 110 | 45 | 5 |
| 95 | 64 | 5 |
| 112 | 15 | 8 |
| 133 | 26 | 8 |
| 136 | 41 | 8 |
| 137 | 51 | 8 |
| 128 | 74 | 8 |
| 160 | 29 | 15 |
| 167 | 61 | 15 |
| 166 | 81 | 15 |
| 164 | 87 | 15 |
| 171 | 97 | 15 |
| 183 | 30 | 20 |
| 173 | 41 | 20 |
| 188 | 51 | 20 |
| 185 | 60 | 20 |
| 198 | 72 | 20 |
| 196 | 85 | 20 |
| 194 | 100 | 20 |
| 187 | 103 | 20 |
| 222 | 49 | 30 |
| 227 | 72 | 30 |
| 233 | 91 | 30 |
| 230 | 112 | 30 |
| 242 | 132 | 30 |
| 226 | 152 | 30 |
| 216 | 172 | 30 |
| 245 | 60 | 40 |
| 255 | 72 | 40 |
| 258 | 82 | 40 |
| 255 | 102 | 40 |
| 275 | 102 | 40 |
| 268 | 123 | 40 |
| 282 | 142 | 40 |
| 275 | 162 | 40 |

Table 19
Polyaxial test data for the Yuubari shale (digitize from [14])

| $\sigma_1$ (MPa) | $\sigma_2$ (MPa) | $\sigma_3$ (MPa) |
|---|---|---|
| 160.975 | 25.673 | 25 |
| 167.713 | 25.558 | 25 |
| 181.677 | 35.567 | 25 |
| 187.369 | 35.947 | 25 |
| 175.436 | 45.417 | 25 |
| 175.05 | 56.153 | 25 |
| 186.264 | 65.469 | 25 |
| 199.69 | 76.48 | 25 |
| 193.765 | 79.118 | 25 |
| 196.405 | 85.347 | 25 |
| 200.678 | 96.286 | 25 |
| 194.04 | 100.093 | 25 |
| 185.64 | 114.289 | 25 |
| 197.359 | 124.28 | 25 |
| 183.191 | 133.23 | 25 |
| 228.364 | 50.194 | 50 |
| 238.904 | 49.941 | 50 |
| 244.782 | 49.652 | 50 |
| 257.171 | 69.38 | 50 |
| 260.564 | 89.876 | 50 |
| 265.544 | 99.982 | 50 |
| 259.646 | 110.003 | 50 |
| 259.761 | 121.581 | 50 |
| 285.345 | 129.164 | 50 |
| 265.797 | 148.138 | 50 |
| 255.91 | 158.967 | 50 |

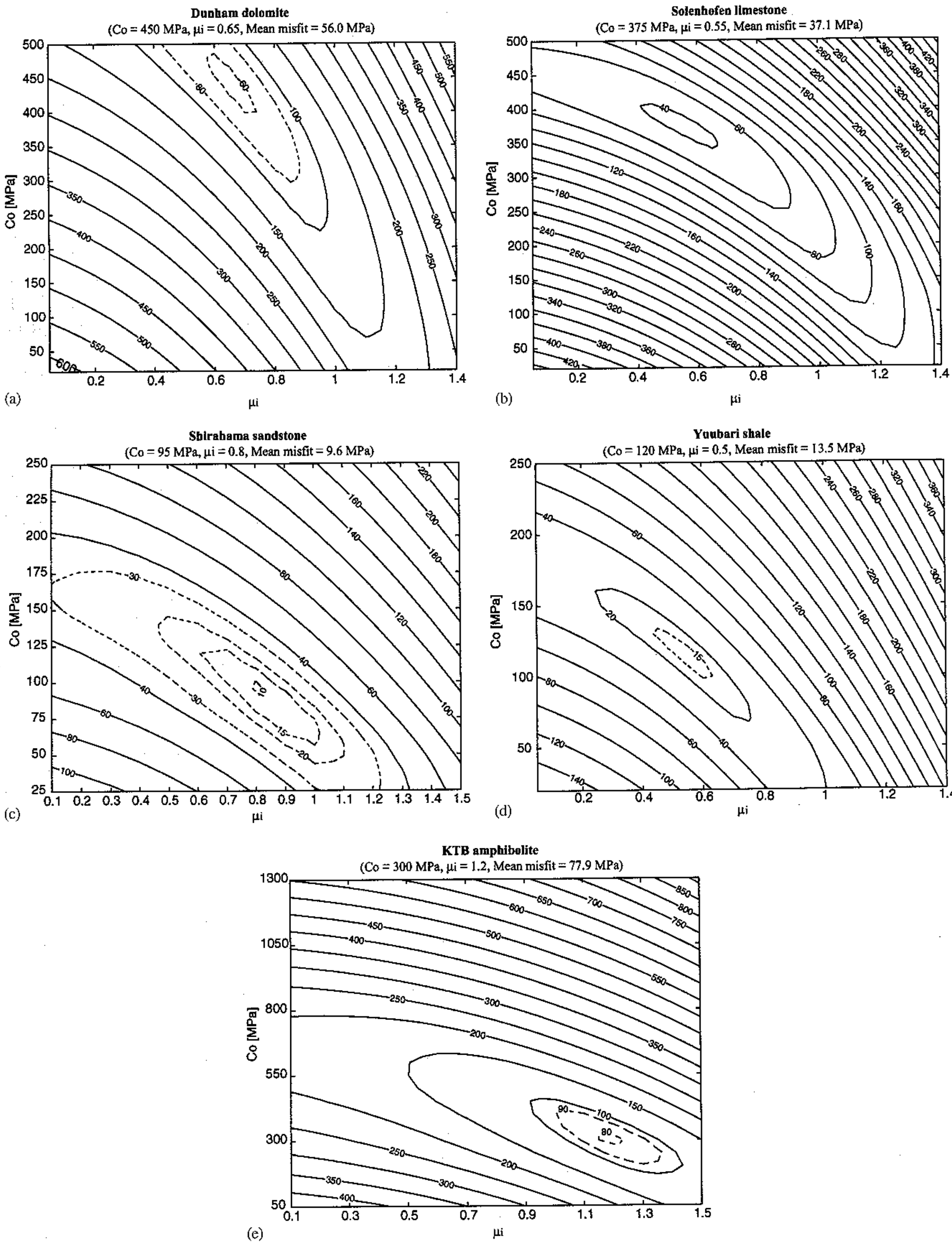



Fig. 20. Misfit contours for the Mohr–Coulomb criterion. (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

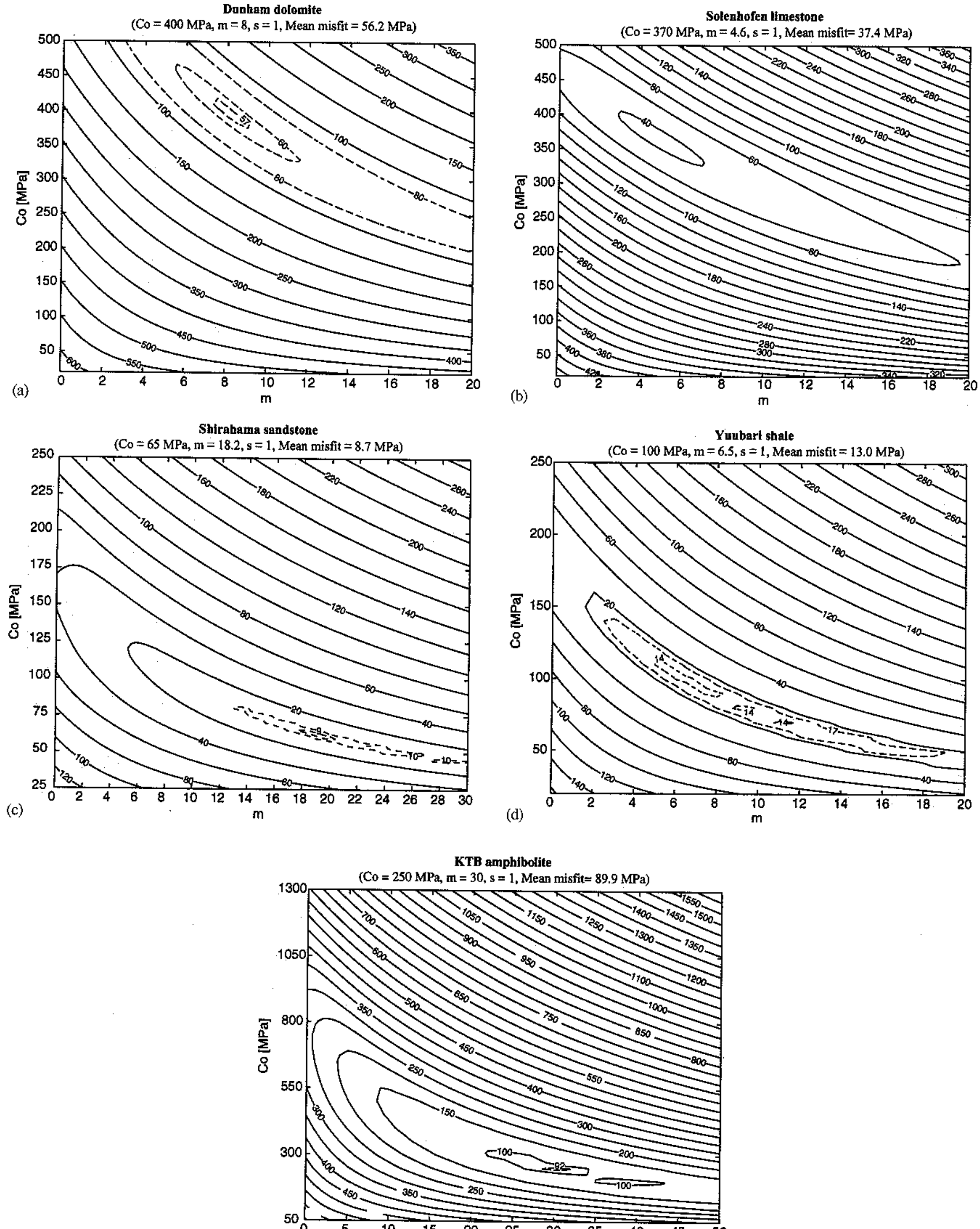



Fig. 21. Misfit contours for the Hoek and Brown criterion. (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

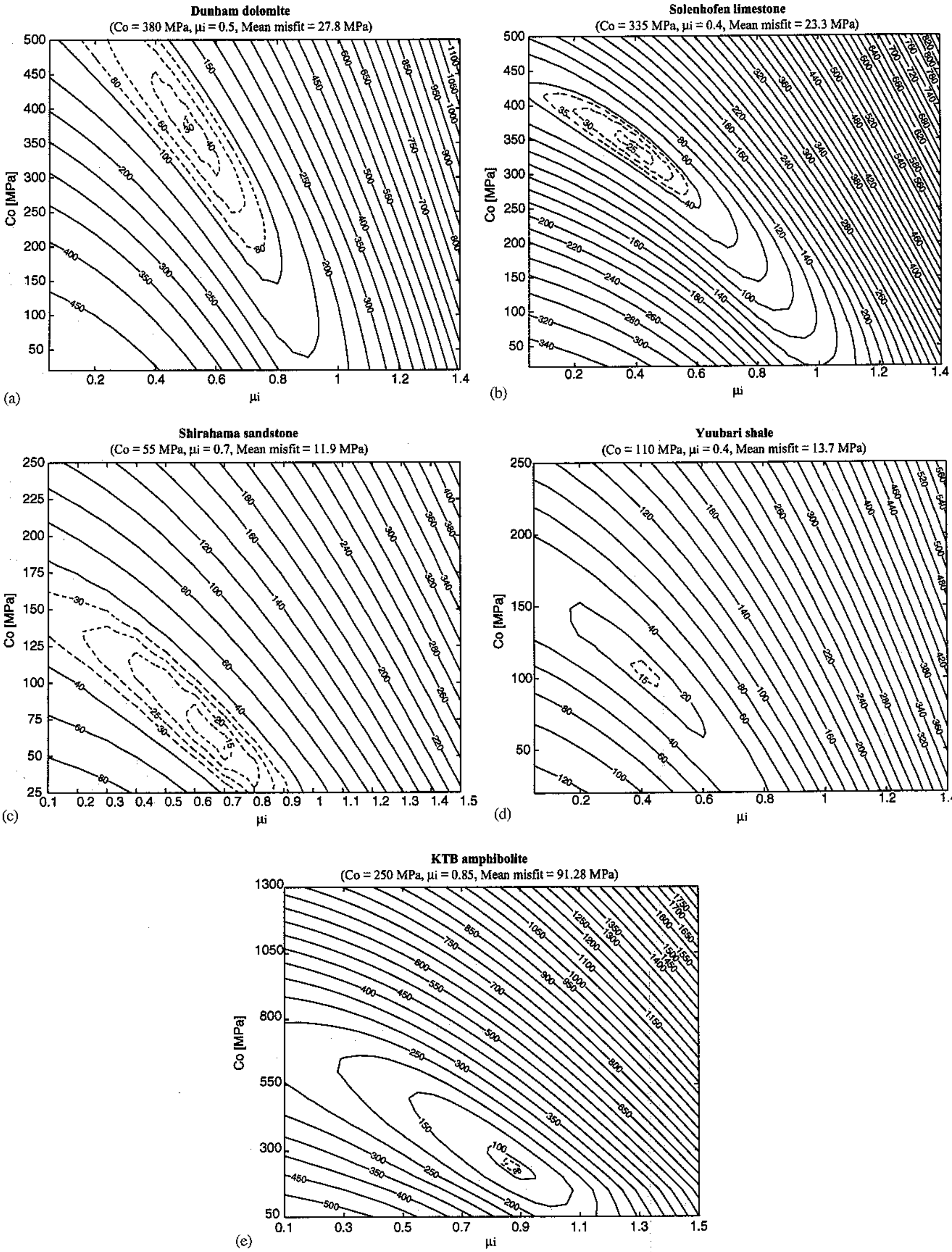



Fig. 22. Misfit contours for the Modified Lade criterion. (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

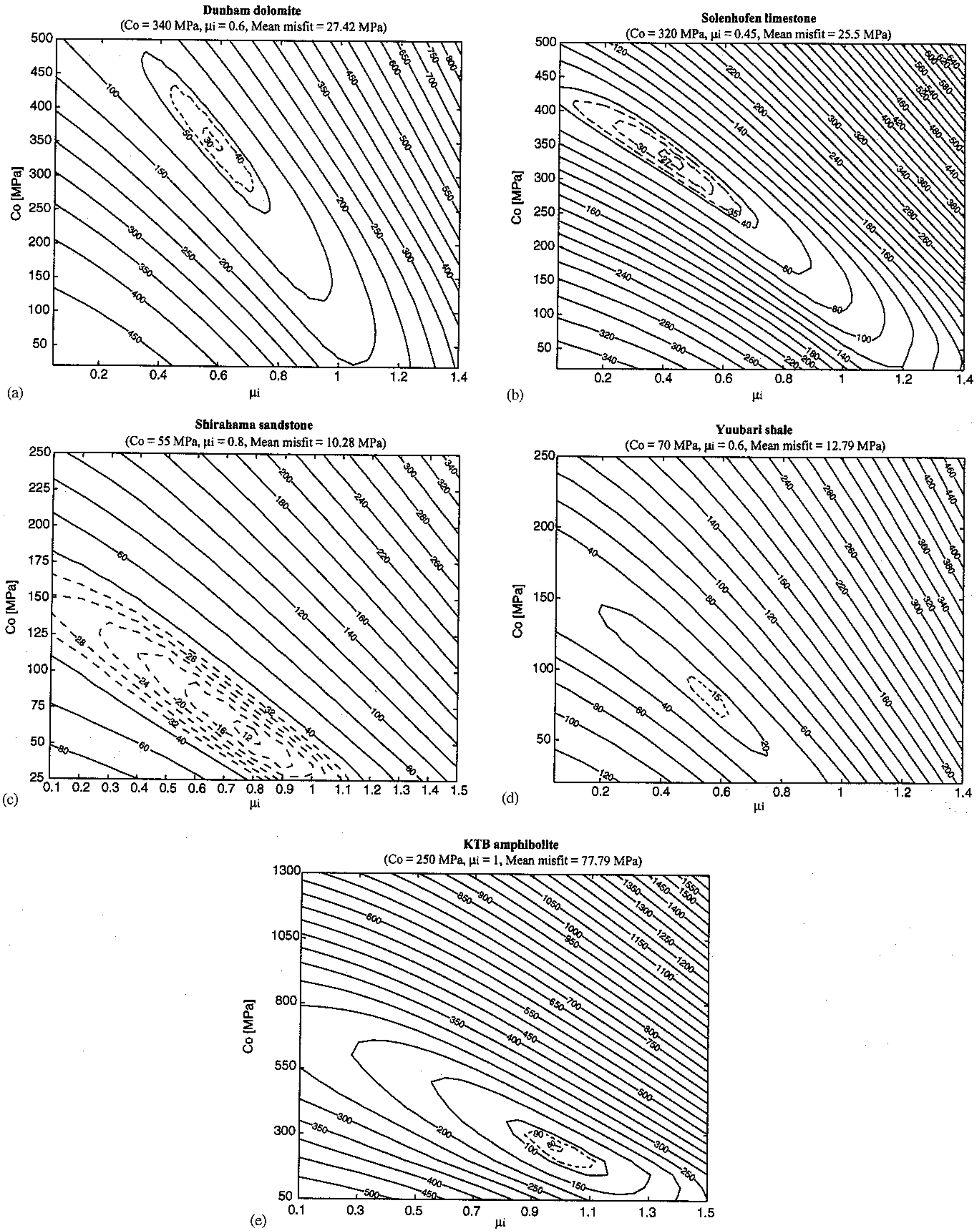



Fig. 23. Misfit contours for the Modified Wiebols and Cook criterion. (a) Dunham dolomite. (b) Solenhofen limestone. (c) Shirahama sandstone. (d) Yuubari shale. (e) KTB amphibolite.

meaningful results even in the absence of polyaxial test data when only triaxial test data are available. The results for two out of three rocks that could be analyzed in this way were encouraging. This finding can be very useful as polyaxial test data is hard to perform and therefore uncommon.

## 7. Recommendations

The use of the Modified Wiebols and Cook criterion is recommended even when polyaxial test data is unavailable, as this criterion did not tend to overestimate the strength of the rock as much as the Mohr–Coulomb criterion ($C_0$ was always ~55–80% lower than those obtained using the Mohr–Coulomb criterion) and it consistently gave low misfits. The Modified Lade criterion also gave very good results.

If only a bound of the rock strength is needed then the Drucker–Prager criterion might be appropriate, as it is able to give the lower and upper bound of $C_0$ with respect to the Mohr–Coulomb criterion, however, the lower bound was always greater than the $C_0$ given by the Modified Wiebols and Cook criterion, that is, the Drucker–Prager criterion tends to overestimate the rock strength.

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## Appendix A. Polyaxial test data

The polyaxial test data of the rocks studied here were obtained from published works as follows: Dunham dolomite and Solenhofen limestone from Mogi [9], Shirahama sandstone and Yuubari Shale from Takahashi & Koide [14] and the data of the amphibolite from the KTB site was kindly provided by Chang and Haimson. Tables 15–19 show the polyaxial test data for each rock.

## Appendix B. Misfit contours plots

Figs. 20–23 show the misfit contours plots for all the rocks for the Mohr–Coulomb criterion, the Hoek and Brown criterion, the Modified Lade criterion and the Modified Wiebols and Cook criterion. These figures show a well-defined minimum, which allowed accurate selection of the $C_0$ and $\mu_i$ that describe the failure of each rock in terms of the respective criterion.

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