Updating joint uncertainty in trend and depositional scenario for exploration and early appraisal

Céline Scheidt¹, Pejman Tahmasebi¹ and Jef Caers¹

¹Energy Resources Engineering department, Stanford University, USA

Corresponding author: Céline Scheidt: scheidtc@stanford.edu

Abstract

The early stage development of a reservoir, facies modeling often focuses on the specification and uncertainty regarding the depositional scenario. However, in addition to well data, facies models are also constrained to a spatially-varying trend, often obtained from geophysical data. While uncertainty in the training image has received considerable attention, uncertainty in the trend/facies proportion receives little to no consideration. In many practical applications, with either poor geophysical data or little hard data, the trend is often as uncertain as the training image, yet is often fixed, leading to unrealistic uncertainty models. In this paper we address uncertainty in the trend jointly with uncertainty in the depositional scenario, represented as a training image in multi-point geostatistics. The problem is decomposed into a hierarchical model. Total model uncertainty is divided into first uncertainty in the training image, then of variability modeled in the trend given that training image. The result is that the joint uncertainty in trend and training image can be easily updated when new information becomes available, such as newly available hard data. In this paper we present the concepts of this approach and apply them to a real-field case study involving wells drilled sequentially in the subsurface, where, as more data becomes available, uncertainty in both training image and trend are updated to improve characterization of the facies.
Introduction

In early exploration and appraisal phase of development of a reservoir, little information on the reservoir is known; in this phase only a few wells are drilled, seismic surveys may not be available or of very limited quality, and no production has started. As a consequence, a significant degree of uncertainty is generally observed, due to the lack of data. The main sources of geological uncertainty in such green fields are typically found in the facies proportions, trends and depositional scenarios. These many sources of uncertainty must be accounted for in the modeling exercise and, once new data becomes available, the models, and the prior beliefs which were used to construct them must be updated to account for this new information. This is an important challenge in the industry today, as rebuilding entirely new models with new data is often impractical, and procedures for updating prior beliefs are rarely if ever employed.

This paper addresses the question of updating the uncertain parameters associated with trend and training image when a new well is drilled. New well information does not only provide local information by adding new conditional data in the model construction. It also may provide useful information of the uncertain parameters. For example, a new well may reveal that some of the prior beliefs made in constructing the models are inconsistent with the new well data. It may also suggest a different understanding of the depositional system, or perhaps that certain parameter choices in generating the models need revising. Under these circumstances, the geoscientists must review their understanding of the reservoir and construct an entire new set of models. On the other hand, if the prior set of models is consistent with the new data, the new well information may inform us that some parameters values are more probable than others. The paper assumes the later alternative and provides a workflow to update the probability of the uncertain parameters.

The proposed methodology is an extension of the workflow developed in Park et al. (2013). These authors developed an approach to update probabilities of depositional scenarios (training images in their application) when new production data becomes available. A Bayesian framework was employed to update the uncertain probabilities and create a set of history matched models. The methodology requires a set of prior models and their forward flow simulation to update the probabilities of the training images.

The method presented in Park et al. (2013) has been extended in several aspects. First, more than one uncertain parameter is considered in this paper, and second, the methodology is extended to continuous parameters. The approach was additionally adapted to well data instead of dynamic data. Finally, an automated procedure was developed to estimate the parameters required by the methodology.
The approach was applied to a real turbidite dataset where only a single well was drilled. The main uncertainties for this field are in the trend (width of the main channel belt containing the geobodies) and in the depositional scenario. Below, a description of the field and its available data is presented, followed by a detailed explanation of the methodology using synthetic well data. The methodology is then applied on the newly drilled well and validated by comparison with rejection sampling. Finally, a randomization procedure is applied to further validate the obtained updated probabilities.

**Reservoir case study description**

The field under investigation is a turbidite reservoir in early stage of development. The dimensions of the field are 8.5x13.5x0.08 km at its largest point, discretized into 170x275x55 grid cells. Only one well (w₁) has been drilled and a second well is scheduled to be drilled in the near future. Both wells are shown in Figure 1. At the time of the initial modeling phase, well w₂ was not drilled yet, hence it was not used in the initial modeling phase of the reservoir. A 3D seismic survey was performed, but is of relatively poor quality. Therefore, very little information is known about the reservoir and the initial modeling phase should account for uncertainty in the facies architecture, locations and proportions, represented by depositional scenarios and trends in the reservoir model.

![Figure 1](image.png)

**Figure 1:** A single layer of the field, showing one realization of the facies, and the locations of the only well drilled (w₁) and the upcoming well (w₂)

**Modeling of uncertainty in depositional scenarios**

Four facies have been identified from well w₁ and the seismic data. They consist of background shales, thin bed sands, bedded sands and massive sands. Due to the low quality of the seismic, the location and the proportion of each facies is highly uncertain. In addition, high uncertainty is present in geological continuity, architecture, dimension of geobodies and facies proportions. Given the little amount of data
available, different depositional hypotheses can be made for this study. The facies are modeled using a multi-point statistics (MPS) algorithm, where the depositional scenarios are represented as training images (TI). Uncertainty in the depositional scenario is represented by three different training images (TI), illustrated in Figure 2. All training images have the four facies mentioned above and all represent a levee channel complex, which was defined as the most probable depositional scenario. The training images differ in the spatial arrangement and proportions of the channels and levee. As an example, only one channel and lower percentage of sand can be observed for TI2, compared to TI1. In addition, some levees that are present in TI1 and TI2 are not present in TI3.

Figure 2: Three different depositional scenarios represented as TIs

It can be seen on Figure 3 that both wells (w1 and w2) are located close to each other. Only a sub-region of the reservoir around the two wells is thus considered in this study, for simplicity. The number of grid cells in the sub-region illustrated in Figure 3 is 120x140x55, which corresponds to approximately 6x7x0.08 kilometers.

Figure 3: Use of a sub-grid around the wells
Modeling of uncertainty in horizontal trend and proportions

Most commonly, trends and proportions in the reservoir are derived from seismic data and are represented as a set of probability maps per facies. Examples of probability maps inverted from seismic are shown in Figure 4. A main channel belt containing the levee-channel complex can clearly be seen, however its width is highly uncertain, due to the poor resolution of the seismic.

However, probability maps are not easily generated from seismic and can be highly uncertain. In particular, one difficulty is that probability maps require the target proportions to be defined for each of the facies, which in itself is highly uncertain. In this work, auxiliary variables (Chugunova and Hu, 2008) are used instead of probability maps. An auxiliary variable is a continuous property that reflects some property of the training image in a certain neighborhood (support). Auxiliary variable can for example represent the facies proportion, object size, orientation, etc. of the training image, at a given location (Chugunova and Hu, 2008). Compared to proportion maps, the advantage of auxiliary variables, in addition to its simplicity, is that only one map needs to be defined, regardless of the number of facies in the training image. In the case study investigated, an auxiliary variable indicating where the levee-channel complex should preferably be located can be used instead of the probability maps shown in Figure 4 without significant loss of information. Examples of auxiliary variables are presented in Figure 5. A high value (red) of the auxiliary variable implies that levees and channels are highly probable, whereas a small value for the auxiliary variable implies most likely the presence of shale at the given location.

As mentioned earlier, significant uncertainty lies in the width of the belt that contains the levee-channel complex. A simple low-dimensional parameterization is used in this paper to account for uncertainty in trend. By using a single parameter \( w \), a set of auxiliary variables with varying widths can be generated (Figure 5). The uncertainty on \( w \) is taken uniform between 70 and 140 (the prior distribution of \( w \)) and defines the width of the auxiliary variable along the x-axis measured in number of grid blocks, equivalent
to between approximately 3.5 km and 7 km in width. Note that the auxiliary variables shown in Figure 5 vary in depth to respect the vertical trend observed in the seismic. The auxiliary variable guides the MPS simulation to place channels in regions inside the belt. The auxiliary variable additionally accounts for uncertainty in the facies proportions, which is a crucial advantage when the proportions are uncertain. A narrower belt will contain less channels and levee (and more shale) than a wider belt.

Figure 5: Example of auxiliary variables with different width \( w \).

**Creation of a set of initial/prior models**

Having defined the training images and a set of auxiliary variables, a series of reservoir models are generated. The multi-point algorithm CCSIM (Tahmasebi et al, 2012) is used. CCSIM is a pattern-based technique that uses cross-correlations on an overlapping region between a previously simulated pattern and the pattern to be simulated to ensure spatial continuity of geological features. Since searching patterns in a large TI is very CPU demanding, a multi-scale representation of the training image in the Fourier domain is used (MS-CCSIM, Tahmasebi et al, 2014), where the large TI is transformed into sequential coarse grid training images.

Note that the models are only conditioned to the well \( w_1 \), since at the time of the modeling study, the second well had not yet been drilled. A set of 100 models was generated for each TI and with varying width \( w \) sampled from its prior distribution (uniform). Examples of a few models are shown in Figure 6. Note that as expected, realizations generated with a small \( w \) concentrate geobodies in the center, whereas realizations generated with a large \( w \) place geobodies more widely.
The next section describes in detail the proposed methodology to update the prior probabilities of the TI and trend when new well data becomes available. Updating the prior beliefs recognizes that some of the trend values or TIs that were thought possible after drilling the first well may now be highly unlikely given the information from the new well. Illustration of the method is provided using a synthetic well data (shown in Figure 7).

**Methodology**

**Bayesian approach**

A Bayesian formulation is used to update the probabilities of the uncertain parameters. In this context, the term prior uncertainty refers to the probability distributions of the uncertain parameters that express the beliefs and uncertainties in the model parameters (in this case TI and trend) before drilling the new well $w_2$. The set of models $M$ that were generated in the study prior to drilling $w_2$ are denoted as prior models and are used to update the prior uncertainty given new observed data. The traditional Bayesian formulation can be expressed as:

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

In the presence of uncertainty in parameters $\theta$, the posterior distribution $P(M|D)$ can be written as:
\[ P(M|D) = \int_\theta P(M|\theta,D)P(\theta|D)d\theta \]  

(1)

In this paper, we focus the study on the latter term, \( P(\theta|D) \) which corresponds to updating the prior probabilities of the uncertain parameters \( \theta = (TI, TR) \) in this example) given new observed well data. Based on the values of \( P(\theta|D), P(M|\theta,D) \) can then be determined through a sampling procedure.

The notations used throughout this paper are the following:

- **TR**: random variable representing the trend (described by the belt width), with outcomes \( w \in [70,140] \).

- **TI**: random variable representing the training image choice, with outcomes \( t_{i_k}, k = 1,...,K, \) \( K \) representing the number of training images.

- **d**: response from the set of prior models: in this application, well data extracted at the well location from the prior models.

- **\( d_{obs} \)**: observed data at the newly drilled well \( w_2 \)

- \( f_{TI,TR|d}(t_{i_k},w|d_{obs}) \): joint probability density of TI and TR given data

- \( f_{ti_k}(w|d_{obs}) = f_{TR|TI,d}(w|t_{i_k},d_{obs}) \): probability density of the trend, given \( t_{i_k} \) and \( d_{obs} \).

The probability \( P(\theta|D) \) in this real field application is a probability density function (pdf) since the trend is a continuous variable. It represents the density of the trend and TI given the observed data and is expressed as: \( f_{TI,TR|d}(t_{i_k},w|d_{obs}) \). A sequential approach is used to estimate \( f_{TI,TR|d}(t_{i_k},w|d_{obs}) \). By using conditional probability rules, the joint probability can be decomposed into two parts:

\[ f_{TI,TR|d}(t_{i_k},w|d_{obs}) = f_{TR|TI,d}(w|t_{i_k},d_{obs})P(TI=t_{i_k}|D=d_{obs}) \]  

(2)

First, the probability of the discrete variable \( P(TI=t_{i_k}|D=d_{obs}) \) given the observed data is evaluated and then, for each outcome \( t_{i_k} \) of the discrete variable, the densities of the continuous variable (trend in this case) given the observed data are evaluated: \( f_{TR|TI,d}(w|t_{i_k},d_{obs}) \). Both terms in Eq. 2 are evaluated using a combination of distance-based modeling (Scheidt and Caers, 2009a, 2009b) and kernel smoothing (Silverman, 1986). Details of how to evaluate both terms from Eq. 2 are provided in the two next sections.
Modeling of the probability of the training image given the data

First, the probability of the TI given the data \( P(TI = t_{i_k} | D = d_{obs}) \) is evaluated using the methodology presented in Park et al. (2013). For completeness, a brief review of this work is presented, which will then allow contrasting with what is presented here.

To estimate \( P(TI = t_{i_k} | D = d_{obs}) \), Park et al. (2013) use Bayes’ rule as follows:

\[
P(Ti = t_{i_k} | D = d_{obs}) = \frac{f_{t_{i_k}}(d_{obs})P(Ti = t_{i_k})}{\sum_{k=1}^{K} f_{t_{i_k}}(d_{obs})P(Ti = t_{i_k})}
\]  

(3)

Where \( f_{t_{i_k}}(d_{obs}) = f(d_{obs} | t_{i_k}) \) represents the density of the observed data given the training image \( t_{i_k} \). Only the probability densities \( f_{t_{i_k}}(d_{obs}) \) need to be estimated since the priors \( P(Ti = t_{i_k}) \) are specified by the user. The density \( f_{t_{i_k}}(d_{obs}) \) cannot be calculated directly and can only be estimated using a set of prior models with response \( d \) (flow response in their application). However, since \( d \) can be a high-dimensional variable, a low-dimensional representation of the model responses \( d \) is constructed. A metric space is created where Park et al. define the distance as the difference in production response between any two models from the prior. The observed data \( d_{obs} \) can be represented in this metric space as well, as its distance to any other models can be evaluated. Multi-dimensional scaling (MDS, Borg and Groenen, 1997) is then applied to create an equivalent Euclidean space on which \( f_{t_{i_k}}(d) \) can be approximated. The underlying assumption of the method is that \( f_{t_{i_k}}(d) \) can be approximated by the density of points/models for a given TI at the location of \( d_{obs} \) in the low-dimensional MDS space. Park et al. employed an adaptive kernel density estimation method where the bandwidth is determined by clustering to obtain an approximation of \( f_{t_{i_k}}(d) \).

In this paper, a similar procedure is followed to obtain \( P(TI = t_{i_k} | D = d_{obs}) \). One major difference is the purpose of the modeling study, which is to update the trends and TIs given the facies profile measured at the newly drilled well, as opposed to the use of production data in Park et al. As in all distance-based modeling approaches, the distance needs to be tailored to the response of interest. When evaluating \( P(TI = t_{i_k} | D = d_{obs}) \), uncertainty in training images is of interest, therefore the distance should be designed to distinguish between different patterns at the newly-drilled well. To evaluate a distance between prior models and observed well data, well facies must be extracted from the prior models at the well location \( (w_2) \). A multi-point histogram approach is used (MPH, see for example, Deutsch and Gringarten 2000; Lange et al. 2012), where histograms of patterns found at the new well location for each reservoir model and the observed well are computed. The histogram represents the frequency
distribution of the patterns that appears in the well. A J-S divergence distance is used to evaluate the differences in pattern distribution and to project the models in metric space. Based on the pair-wise MPH distance, MDS can be applied to represent in metric space differences in patterns found at the wells. Figure 7 (left) shows, for each training image, a 2D projection of the prior models (colored dots) and the data (black cross) in MDS space.

Having defined a low-dimensional representation of \( \mathbf{d} \), which is denoted \( \mathbf{d}' \), the next step is to apply kernel smoothing to estimate \( f_{\text{d}_{ij}}(\mathbf{d}'_{\text{obs}}) \). In this work, an automated procedure to define the bandwidth has been implemented described in Appendix. Figure 7 (right) shows an illustration of the densities \( f_{\text{d}_{ij}}(\mathbf{d}') \) estimated for each training image. Note that for illustration purposes, the densities are evaluated in the space of \( \mathbf{d}' \), but to estimate \( f_{\text{d}_{ij}}(\mathbf{d}'_{\text{obs}}) \) the densities only need to be evaluated at the location of the new data, i.e. at \( \mathbf{d}'_{\text{obs}} \).

![Figure 7](image)

Figure 7: (left): Low dimensional representation of the well facies \( \mathbf{d} \) extracted at the new well location for the prior models, for each TI. The observed well is illustrated and its location in space is shown by the black cross. (right) Corresponding probability density \( f_{\text{d}_{ij}}(\mathbf{d}') \).

The probabilities of each TI given the observed well data are then obtained by assuming \( f_{\text{d}_{ij}}(\mathbf{d}_{\text{obs}}) \equiv f_{\text{d}_{ij}}(\mathbf{d}'_{\text{obs}}) \) and using Eq. (3). Table 1 shows the resulting probabilities. It can be observed that TI2 has a very low probability and can be rejected from subsequent modeling. In addition, TI3 has a much higher probability than TI1 when such a well is observed.

<table>
<thead>
<tr>
<th>( \text{T}_1 )</th>
<th>( \text{T}_2 )</th>
<th>( \text{T}_3 )</th>
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<tbody>
<tr>
<td>( P(\text{T}_1</td>
<td>\mathbf{d}_{\text{obs}}) )</td>
<td>0.17</td>
</tr>
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Table 1: Updated probabilities for each training images.
Now that the second term in Eq. 2 is determined, the first term, namely \( f_{TR|TI,d}(w|t_k, d_{obs}) \), needs to be estimated. This term consists of estimating for each training image the values of the trend that are most likely plausible with the observed well.

**Modeling of the probability of the trend given the training image and the observed data**

The method presented in the previous section is limited to only one type of uncertainty (the TI) which has discrete outcomes. In this study, the approach needs be generalized in two aspects. First, a joint probability distribution must be evaluated since uncertainty is present in the trend and the TI. Second, the parameter \( w \) which defines the trend is continuous, hence a joint probability of “mixed” parameters (continuous and discrete) must be calculated.

As before, the probability density of the trend given the data and the TI \( f_{TR|TI,d}(w|t_k, d_{obs}) \) can only be estimated using a low-dimensional representation of the well data \( d \) extracted from the set of prior models. The density is obtained in the similar manner as described above, namely, creation of a metric space, followed by construction of the reduced space \( d^r \) using multi-dimensional scaling. For \( f_{TR|TI,d}(w|t_k, d_{obs}) \), the distance must be designed to distinguish between different values of the trend. Models generated with a small \( w \) will most likely show no geobodies at the new well location. As the width increases, more geobodies will be observed. As a consequence, a good measure to distinguish the value of \( w \) is the proportion of each facies in the well extracted from the models. The distance is therefore defined as the root mean-squared sum of the difference in proportion of each facies between any two wells. An example of the resulting MDS plot for all three TIs is provided in Figure 8, where points are colored according to the value of the trend. The percentages shown on the axes represent the variance explained by each dimension of the MDS map.

![Figure 8: MDS representation of the prior models. The distance is the difference in the proportion for each facies.](image)
Figure 8 shows that, large values of w (red points, wide belt) tend to be grouped on the right side of the graph, close to the observed data (black cross), whereas small values of w tend to be located further on the right side. As a consequence, it expected that the probability density of w is high for large values of w and then gradually decreases as w decreases. A mathematical evaluation is presented next, where the probability density is estimated for each TI.

The main difficulty compared to the previous section lies in the continuity of the trend. Contrary to the case of discrete uncertainty, a probability density must be calculated instead of a probability. One way to address this challenge is to use an additional dimension to the MDS space representing the values of w. An example of such a space is illustrated in Figure 9. Note that only one dimension is used to represent the MDS (low dimensional representation of the data, d') in Figure 9 for illustrative purposes. In reality, its dimension can be higher (2-5D in most problems, 3D in the example using the synthetic well data).

Figure 9: Prior set of models in joint space (d', TI, w). Points are colored according to the value of the w.

Evaluating $f_{\text{MDS},d}(w|t_{ik}, d_{ob})$ can be done in the space represented in Figure 9. Since the TI is a discrete parameter and its value is assumed fixed to $t_{ik}$ the density $f_{\text{MDS},d}(w|t_{ik}, d_{ob})$ can be evaluated independently for each TI. For simplification, the probability density is denoted as $f_{T_{ik}}(w|d_{ob})$. Again, kernel density estimation is used to evaluate $f_{T_{ik}}(w|d_{ob})$. Since the kernel smoothing is applied to both d and w, a bandwidth for the trend must be evaluated as well. Details on how to compute the bandwidth automatically are provided in Appendix. Figure 10 shows the probability density $f_{T_{ik}}(w|d_{ob})$ (left) and $f_{T_{ik}}(w|d_{ob})$ (right).
At this point, both terms of Eq. 2 have been estimated. Remaining is the multiplication of those terms, to obtain the final probability density \( f_{T_I, T_{R, j}}(t_k, w | d_{\text{obs}}) \), which is presented in Figure 11 (left).

Since the total proportion \( p \) of all geobodies in the models and the auxiliary variable width \( w \) are highly dependent on each other, one can determine the updated joint probability density \( f_{T_I, p, d}(t_k, p | d_{\text{obs}}) \) in the exact same way as for the width \( w \). The only difference is that the density is estimated in the joint space \((d', T_I, p)\) instead of \((d', T_I, w)\). The updated joint probability density \( f_{T_I, p, d}(t_k, p | d_{\text{obs}}) \) is shown in Figure 11 (right).

Given the observed well data shown in Figure 7, the proposed approach shows that TI2 is not likely to occur. The depositional setting represented in TI2 could thus be removed from the study. In addition, only large values for \( w \) are possible, which indicates a wide belt containing the geobodies. Not
surprisingly, the proportion of geobodies $p$ in the model is quite high, with values varying between 40% and 75%. This confirms what was expected, as the well contains mostly sand facies.

The goal of this study is to update uncertainty based on new well information. Details on the method were presented taking a synthetic well as the observed data. In the next section, the methodology is applied using the real observed data from well $w_2$, which was just recently drilled.

**Results**

Unfortunately, the actual well $w_2$ drilled encountered only shale; no producing sand was found. In this section, first the probabilities of the trend and TI are updated given that the new well is 100% in shale. Then, the updated probabilities are validated by comparison with rejection sampling. Finally, a resampling procedure is applied to validate the proposed method.

**Application to the real observed data**

The proposed approach is applied to the new observed well (100% shale). First the probability of the TI given the observed well data was computed and then the probability densities of the trend for a given TI and $d_{obs}$ were estimated. The definitions of the distances remain the same for each expression of the probability; the only change is the location of the observed data in MDS space. Illustrations of the MDS spaces including the observed data for both the training image (left) and the trend (right) are displayed in Figure 12. In both maps, the location of the new well is right at the edge of the cloud of points, which is not surprising given that it traversed 100% shale.

![Figure 12](image)

Figure 12: Low dimensional representation of the well values extracted from the prior set of models and the observed well data (black cross) for different distances: (left): MPH and (right): difference in proportions.

The updated probabilities of each TI given the new well data are shown in Table 2. It can be concluded that the new well is not very informative on the training image, although TI3 shows a slightly higher probability than TI1 and TI2.
The joint probabilities of the TI and the trend (w and p) given the observed well data are displayed in Figure 13

![Figure 13](image)

Figure 13 Joint probability density function: (left) $f_{TI_{,TI|d}}(t_{i_k}, w | d_{obs})$ and (right) $f_{TI_{,P|d}}(t_{i_k}, p | d_{obs})$

Given that the newly drilled well did not find producing sands, the updated probabilities suggest that narrow belts (small values of w) are more likely to occur than wide belts. Note that it is possible to obtain a 100% shale well for a wide belt for TI1 and TI2. This observation highlights one main advantage of the procedure; it accounts for the unlucky possibility that a dry well can be obtained even with wide belt, due to channel sinuosity, architecture, or simple bad luck. Interestingly too, the updated probabilities of the proportion of geobodies show that a larger proportion for TI3 can be obtained, compared to TI1 and TI2. The larger proportion of geobodies for TI3 can be explained by the fact that in general TI3 contains more channel-levees than TI1 and TI2, but they are of smaller size, thus increasing the possibility of missing the sand body. Finally, the new well does not provide significant information on the type of depositional scenario. TI3 is shown to be slightly more probable that TI1 and TI2. In the next section, rejection sampling is performed to determine the “true” joint probability density of the trend (and proportion) and the TI, which is then used to validate the proposed approach.

**Real observed data: comparison with rejection sampler**

In most situations, rejection sampling is not possible because it requires considerable CPU time. Here, since the well has only shale, it is relatively easy to obtain models that contain only shale at the well location. This would evidently not be the case for variable facies profiles at the well, such as the synthetic well data used above. Rejection sampling is applied as follow:
1. Draw randomly a $T_i$ from the prior
2. Draw randomly a $w$ from the prior
3. Generate a single model $m$ with that $T_i$ and $w$
4. Extract the well data from the models at the well location
5. If the well is in all shale, keep the model, otherwise reject it.

Rejection sampling was applied until 850 wells with 100% shale were obtained. The frequency distribution of the uncertain parameters $T_i$ and $w$ are presented in Figure 14 (left). In Figure 14 (right), the kernel smoothing densities obtained by the proposed approached are displayed again for comparison. Both methods provide similar distributions. We can however see a slight edge effect for small values of $w$ in the kernel smoothing. This is due to the lack of models on the other side of the boundaries. Figure 15 confirms the validity of the updated joint probability of the $T_i$ and proportion of geobodies $p$. In particular, the density of $p$ for $T_i3$ is much wider than for $T_i1$ and $T_i2$.

![Figure 14: Joint frequency distribution for $(T_i, w)$ obtained by rejection sampling. Probability density distribution of $(T_i, w)$ obtained by the proposed methodology.](image1)

![Figure 15: Joint frequency distribution for $(T_i, p)$ obtained by rejection sampling. Probability density distribution of $(T_i, p)$ obtained by the proposed methodology.](image2)
Now that the joint probability of the trend and width has been obtained, the next step is to update the set of existing models to reflect the joint probability and, if necessary, create new models.

**Selection of existing models consistent with the observed well data**

Updating prior probability on the depositional scenario and the trend/proportion is not a goal on its own. These probabilities should be accounted for when generating new models that are conditioned to all the well data (new and old). For example for this field, only a few models created with a wide channel belt and TI1 and TI2 should be generated, with a much larger proportion of models with a narrow belt. In this particular case of a well drilled in all shale, some of the existing models may be valid and already honor the new well data. These models should therefore be recycled in accordance to the updated probabilities and used for the next modeling phase. Here, 76 models were valid out of the 300 initial models and will be used in for the next modeling phase. If more models are needed, then one can sample additional values of TIs and w from the updated joint distribution and generate the models by conditioning at both wells.

**Validation using a resampling procedure**

As mentioned earlier, rejection sampling is much more difficult to apply for non-shale wells. In order to confirm the validity of the obtained joint probability density, a validation procedure is applied. The idea underlying this section is based on the total probability formula:

\[
    f_{TI,TR}(ti_k, w) = \int f_{TI,TR|d}(ti_k, w | d) f(d) dd \tag{4}
\]

One can see in Eq. 4 that if a randomization is performed on d, the integration of the conditional probabilities should average out to the prior probabilities. As a consequence, one way to validate the proposed approach is to do a randomization of d, evaluate the conditional density \( f_{TI,TR|d}(ti_k, w | d) \) given that d using the proposed approach and integrate it over many d (right-hand side in Eq. 4). The validity of the procedure to estimate the joint probability \( f_{TI,TR|d}(ti_k, w | d) \) is then verified if the procedure can retrieve the prior joint density (left-hand side in Eq. 4) densities.

The procedure is the following:

1. Draw a TI and a w from their prior distribution (uniform for both variables)
2. Generate a single model m with that TI and w
3. Extract well data and take it as observed well: \( d_{obs} \)
4. Evaluate \( f_{TI, TR|d}(t_i, w|d_{obs}) \) \( f_{TI, Pl|d}(t_i, p|d_{obs}) \) using the methodology presented above

5. Sample repeatedly from the resulting distribution \( f_{TI, TR|d}(t_i, w|d_{obs}) \) \( f_{TI, Pl|d}(t_i, p|d_{obs}) \)

6. Repeat the procedure many times

The densities \( f_{TI, TR|d}(t_i, w|d_{obs}) \) and \( f_{TI, Pl|d}(t_i, p|d_{obs}) \) are evaluated using the same set of 300 prior models (100 per TI, with a channel belt width varying uniformly) as for the above studies. The procedure was repeated 300 times, with different \( d \). For each iteration of the procedure, 1000 samples were drawn from the conditional density distributions (step 5). Figure 16 displays the RHS (left) and LHS (right) of Eq. 4 for the auxiliary variable \( w \) (top) and the proportions of geobodies (bottom).

One can observe that distributions close to the prior distributions are retrieved, which confirms the validity of the evaluation of \( f_{TI, TR|d}(t_i, w|d_{obs}) \) and \( f_{TI, Pl|d}(t_i, p|d_{obs}) \). Because of the approximate nature of this procedure, one cannot expect to obtain a perfect similarity between the RHS and LHS of Eq. 4. In particular, for this example, only 300 prior models were to estimate the conditional densities. Such a limited set may already contain some sampling error. In addition, the border effects from the kernel smoothing can be observed. Even though not applied for this case, a correction procedure can be used to overcome this.
Figure 16: Resampling procedure: prior frequency distribution of the uncertainty parameters (left) and frequency distribution resulting from the resampling procedure (right) for $w$ (top) and $p$ (bottom).

This randomization procedure confirms that the probability density of uncertain parameters can be approximated by the density of points in the reduced joint space $(d', T1, w)$. In addition, it confirms as well that the bandwidth estimation is robust and leads to reasonable density estimations.

**Conclusions**

A methodology to update prior uncertainty when new data becomes available has been proposed in this paper. This methodology is designed for fields in early development, with little available data (only a few wells, no production) and hence considerable uncertainty. One major advantage of this approach is that the workflow has been fully automated, rendering it practical for geoscientists. Updating the uncertain parameters and accounting for their probabilities in the modeling exercise is a crucial part of a successful modeling effort, and leads to better decision making.

The proposed method extends the idea of Park et al. (2013) in several aspects. It has been adapted to handle multiple uncertain parameters, as well as a “mixture” of continuous and discrete parameters. Even though only two uncertain parameters were used in the application, the methodology can be easily applied to more uncertain parameters. In addition, a procedure to estimate automatically the bandwidth in the kernel smoothing was developed as the bandwidth choice may influence significantly
the density estimates. A validation of the approach was provided through a resampling technique, which confirms the robustness of the proposed automated bandwidth calculation.

The method was applied successfully to a real field where new data was obtained through well log measurements at a new well. The prior probabilities of uncertain parameters (in this case, depositional scenario and trend) were updated, given that the new well was drilled entirely into shale. A combination of distance-based modeling and kernel smoothing was used to successfully evaluate the joint probability density. The joint probability was validated by rejection sampling.

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Appendix

Kernel Smoothing: estimation of the kernel bandwidth
The proposed workflow relies on estimating probability density functions using a kernel smoothing approach (Silverman, 1986). The kernel density estimate is formulated as follow:

\[
\hat{f}_{H}(x) = \frac{1}{n} \sum_{i=1}^{n} K_{H}(x - x_{i})
\]

where

- \( x = (x_1, x_2, ..., x_d)^T \), \( x_i = (x_{i1}, x_{i2}, ..., x_{id})^T \), \( i = 1, 2, ..., n \) are \( d \)-vectors;
- \( H \) is the bandwidth \( d \times d \) matrix which is symmetric and positive definite;
- \( K \) is the kernel function which is a symmetric multivariate density: \( K_{H}(x) = |H|^{-1/2} K(H^{-1/2}x) \).

The choice of the kernel function \( K \) is not as crucial to the accuracy of kernel density estimators as the bandwidth \( H \) (Wand and Jones, 1995). As a consequence, the standard multivariate normal kernel function is used: \( K(x) = (2\pi)^{-d/2} \exp(-1/2x^T x) \).

Estimation of the kernel bandwidth when estimating \( f(d_{obs}|t_i) \)
Since only a neighborhood around the observed data is of interest when evaluating \( f_{t_i}(d) \) or \( f_{t_i}(w|d) \) it is sufficient to evaluate the bandwidth only for points in the vicinity of the data. A clustering technique is used to this end, where the new observed data is fixed as a medoid. The bandwidth is then evaluated using only the points belonging to the cluster containing the observed data. Silverman rule’s of thumb (Silverman, 1986) is then used to estimate the bandwidth:
\[
\sqrt{H_{ij}} = \left( \frac{4}{d+2} \right)^{\frac{1}{d+4}} n^{\frac{-1}{d+4}} \sigma_i, \quad \text{where } \sigma_i \text{ is the standard deviation of the } i^{\text{th}} \text{ variable, } d \text{ is the dimension of the space and } H_{ij} = 0, i \neq j. \]  

This bandwidth was proven to be good estimation for Gaussian kernel distributions. Even though the distribution of points in MDS is not necessarily Gaussian, tests have shown that it is a valid estimation.

**Estimation of the kernel bandwidth when estimating** \( f_{t_i} (w | d) \)

Since the trend is a continuous variable, a probability density is obtained instead of a probability, hence smoothing along \( w \) is required as well. As a consequence, two separate bandwidths for \( d \) and \( w \) must to be estimated. For \( d \), the approach described in the previous section is applied, the resulting bandwidth is denoted by \( H_d \). As before, Silverman’s rule of thumb is used to define the bandwidth during the kernel smoothing. For \( w \), to estimate the bandwidth \( H_w \), it makes sense to use values of \( w \) in a neighborhood of the data, as only those values will be in fact considered when evaluating the joint density. Hence, the same clustering that was defined for \( d \) is used and Silverman’s rule of thumb is applied on the values of \( w \) within the cluster containing the history. The kernel smoothing is subsequently applied in the joint space \((d', w)\), using the bandwidth:

\[
H_{d,w} = \begin{pmatrix} H_d & 0 \\ 0 & H_w \end{pmatrix}
\]

**References**


