Sensitivity-based Parameter Uncertainty Reduction for Structural Reservoir Models

Addy Satija and Jef Caers
Energy Resources Engineering Department, Stanford University, USA

Abstract
Uncertainty in the geological structure significantly influences the overall uncertainty in a reservoir. However, this structural uncertainty is currently still not widely incorporated in actual reservoir forecasting. Structural uncertainty has many sources of uncertainty. These sources of uncertainty can be parameterized with uncertain parameters. These uncertain parameters can be categorical, discrete or continuous. Integrating all these uncertain parameters effectively requires generating a large set of structural reservoir models. This is computational very complex. In our work, instead of considering all the parameters, we only consider those that have a significant impact on the targeted response variable. These can be identified using sensitivity analysis. In considering only significant parameters, we reduce the uncertainty in the less significant parameters. We do this by considering a narrow range of the values of the less significant parameters. This narrowed range is selected so as to not interfere with the effects of interactions with more significant parameters. In a realistic reservoir with uncertainty in the number of faults, fault throws and fault transmissibility; we illustrate how our approach successfully reduces parameter uncertainty for a given water-flooding case. The reduced reservoir model (with reduced parameter uncertainty) is demonstrated to lead to the same water production response uncertainty as Monte Carlo method that spans the entirety of parameter uncertainty.

Introduction
Profitable petroleum production is subject to reliable predictions about reservoir performance. In presence of limited knowledge about reservoir properties and reservoir structure, reliable reservoir predictions depend upon uncertainty quantification. Conventional uncertainty quantification approaches focus on reservoir properties such as facies or porosity or permeability (Caers 2012). However, uncertainty in the geological structure of a reservoir can influence reservoir predictions more significantly through effects on reservoir bulk volume and fluid flow. As a result, modeling the structural uncertainty is necessary for risk modeling, reservoir decision-making and consequently, profitable reservoir production.
When modeling structural uncertainty in a faulted reservoir, it is important to consider both, (1) fault geography and geometry, and (2) what is the effect of these faults on reservoir predictions. Sources of uncertainty in fault geography and geometry in faulted reservoirs have been classified in a hierarchy based on their effect on the uncertainty in fluid flow (Suzuki, Caumon et al. 2008). However, structural uncertainty modeling traditionally has been impracticable due to our inability to generate multiple structural reservoir models automatically. Recently, automated methods that can generate multiple structural models of faulted reservoirs taking into account the structural uncertainty in fault geography and geometry have been developed (Cherpeau, Caumon et al. 2010). Once the geographic and geometric aspects of faults in a reservoir have been modeled, their effect on fluid flow can be modeled using fault transmissibility. Fault transmissibility is parameterized using the Shale Gauge Ratio – a measure of the shale content in the fault smear (Manzocchi, Walsh et al. 1999).

When uncertainty in fault transmissibility parameters is integrated with the uncertainty in fault geometry parameters, we may obtain a large parameter uncertainty. Quantifying its effect on reservoir prediction using traditional Monte Carlo methods involves generating a lot of structural reservoir models. In spite of automation, this is still computationally complex. In this paper, we address this problem by reducing the numerical model of the reservoir by reducing the parameter uncertainty. This requires (1) isolating the parameter whose uncertainty can be reduced because it has little or no impact on a prediction variable, and (2) deciding how to reduce such a parameter uncertainty in a way that doesn’t affect the reservoir prediction. Reducing the parameter uncertainty in this way results in a computationally simpler reduced reservoir model that is, by design, effective for reservoir prediction.

**Methodology**

In order to maintain the integrity of the reservoir prediction when reducing the parameter uncertainty, care must be taken to maintain uncertainty in the reservoir response of interest. The parameters relevant for numerical modeling of the structural uncertainty of a faulted reservoir tend to be very diverse. They can be categorical (such as faulting scenarios) or discrete (such as number of faults) or continuous (such as fault displacement). They also interact in complex ways to influence fluid flow in a reservoir across a fault system. This makes it difficult to isolate “insignificant” parameters whose uncertainty can be reduced without changing the uncertainty in the reservoir response.

For example, fluid flow across a fault depends upon fault transmissibility. Fault transmissibility, in turn, depends upon the fault displacement and shale gauge ratio (Manzocchi, Walsh et al. 1999). Shale gouge ratio itself depends upon the distribution of the reservoir facies. This reservoir facies distribution is a property of the reservoir grid that, in turn, depends upon fault positioning and displacement. Evidently, the parameters interact in very complex ways to determine the fluid flow. As a result, it is very difficult to determine direct causal relationships
between them. Thus, to isolate the parameters that are the most significant (or least significant) to a given reservoir response variable, a generalized sensitivity analysis method is needed.

**Generalized sensitivity analysis**

Fenwick et al 2012 present a distance based generalized sensitivity analysis (DGSA) based on the general idea that narrowing the distribution of a significant parameter changes the response distribution more than narrowing the distribution of an insignificant parameter does. DGSA measures the sensitivity as a dimensionless metric. Unlike traditional sensitivity analysis methods based on response surfaces, this approach applies to non-linear models with stochastic responses and diverse types of input parameters (categorical, discrete and continuous). In addition, this method can also be applied to multi-way asymmetric parameter interaction by using conditional parameter distributions (Fenwick, Scheidt et al. 2012).

**Parameter Uncertainty Reduction**

Intuitively, if narrowing the distribution of an insignificant parameter doesn’t change the response distribution, we should reduce the overall parameter uncertainty by doing so. However, as outlined before, flow responses of faulted reservoir models depend upon complex interactions of the underlying parameters. A one-way sensitivity analysis may result in isolating a parameter that is less significant for the response by itself but interacts significantly with other parameters in the response variable.

To illustrate this, consider a simpler problem of quantifying the uncertainty in

\[ f(x, y, z) = \left\{ \frac{z}{|x(y - 1)|} \right\} \]

Here, \( x, y, z \) are all uniformly distributed random variables between 0 and 1. From a one way generalized sensitivity analysis in Figure 1, the response \( f(x, y, z) \) is very sensitive to \( z \) and less to \( y \). Hence, uncertainty in \( y \) can be reduced without causing a major change in the uncertainty in \( f(x, y, z) \). One way to do this would be to take a median value. When the median value of \( y \) is used, the vertical axis value \( |x(y - 1)| \) is limited to the maximum value of 0.5. This visibly disrupts the overall uncertainty in \( f(x, y, z) \) in Figure 3. Hence, it is important to consider what range for \( y \) to consider.

This undesirable effect is understood from the sensitivity analysis based on interactions. In faulted reservoirs, uncertain parameters often interact in unintuitive ways. Just as uncertainty in \( f(x, y, z) \) was changed significantly when the parameter uncertainty in \( y \) was reduced to the median value, the uncertainty in reservoir response can be significantly impacted by arbitrarily choosing a value of an uncertain structural parameter when doing parameter uncertainty reduction in a faulted reservoir model.
The interactions of other parameters with \( y \) are the most significant in Figure 2. As a result, we can’t narrow the uncertainty in \( y \) using an arbitrarily chosen value. Instead, we must narrow the uncertainty in \( y \) systematically in a way such that the effects of the significant interactions are preserved.

To study the effects of narrowing uncertainty in \( y \) on \( f(x, y, z) \), we observe the sensitivities of the interactions conditional to \( y \) limited to values in 3 representative ranges: \( \{ y \in [0, \frac{1}{3}] \}, \{ y \in [\frac{1}{3}, \frac{2}{3}] \} \) and \( \{ y \in [\frac{2}{3}, 1] \} \). The conditional sensitivity measure \( s(x \mid y \in [\frac{1}{3}, \frac{2}{3}] \) indicates how much will \( f(x, y, z) \) vary if \( y \) is fixed to the range \( y \in [\frac{1}{3}, \frac{2}{3}] \) and \( x \) is varied. The net sensitivity conditional to \( y \) can be calculated as a sum of \( s(x \mid y) \) and \( s(z \mid y) \) since sensitivity, as calculated using DGSa, is dimensionless. From Figure 4, the median value of \( y \) lies in a range with low conditional sensitivity whereas the most variation in \( f(x, y, z) \) is preserved when the values of \( y \in [0, \frac{1}{3}] \). If the uncertainty in \( y \) is reduced to values in the range \( y \in [0, \frac{1}{3}] \), the overall uncertainty in \( f(x, y, z) \) stays unchanged as seen in Figure 3.

This illustrates that uncertainty in the parameter to which the response is least sensitive can be narrowed to a range where the response has maximum conditional sensitivity and there will only be a minimum change in response uncertainty. In the next section, this principle is applied to a realistic faulted reservoir model to demonstrate its applicability in systems where the parameters are diverse and interact in complex ways to influence the fluid flow.
Figure 2: Sensitivity analysis on two-way interactions for \( f(x,y,z) \)

Figure 3: \( f(x,y,z) \) values. All Parameters indicates full range of \( x,y,z \). Reduced Parameter indicates full range of \( x \) and \( z \) but a reduced range of \( y \).

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Case Study

To demonstrate the model reduction methodology, we use a faulted reservoir that has been modeled using 3 facies on a grid with $50 \times 50 \times 20$ cells with an injector well and a producer well. The model seeks to quantify the uncertainty in cumulative water production response in the producer well over 3 years. The structural model is uncertain with respect to the number of faults in the reservoir and their displacements.

This uncertainty is parameterized using a number-of-faults parameter numFaults that takes equiprobable integer values between 1 and 4, and a fault displacement parameter Throw for each fault that takes values between 0 and 10 $z$-direction cell units. For simplicity, the range of Throw are be divided into three categories and a reservoir can be said to have a Low, Medium or High fault displacement in each fault. Geologically, older faults have higher displacement than newer faults. So, there is a joint distribution on the Throw parameters such that

$$\text{Throw}_{i+1} \leq \text{Throw}_i \, \forall \, i \leq \text{numFaults}$$

$$\text{Throw}_{\text{numFaults}+1} = 0$$

The geography and geometry of the faults and the wells is seen in Figure 5. Since the faults are fixed, the numFaults parameter expresses as a categorical grid variable seen in Figure 6. To model the effect of the faults on fluid flow, the concept of fault transmissibility as presented by Manzocchi et al 1999 is used. The fault transmissibility depends on the reservoir permeability field and the flow displacement. As a result the structural uncertainty is dependent upon spatial uncertainty. Spatial uncertainty in the reservoir is represented using multiple point geostatistics.
(MPS). The uncertainty in the marginal distribution of facies is parameterized using two cases seen in the training images in Figure 7. A high sand proportion case with 70% sand and a low sand proportion case with 50% sand are used.

To get a feel for the computational complexity and size of this uncertainty quantification problem, we can do full factorial experimental design on the structural parameters. With 2 cases of marginal proportion, 4 categories from number-of-faults and 3 ranges of values in each fault displacement parameter, a naïve full factorial design leads to $2 \times 4 \times 3^4 = 648$ possible scenarios of parameter-combinations for such a relatively simple reservoir case. Each of these scenarios contains its own spatial uncertainty that needs to be modeled using MPS. In such a problem, the number of models to be generated and flow-simulated for accurate uncertainty quantification can easily get out of hand. Such a problem is a good candidate for sensitivity-based parameter uncertainty reduction.

From one-way sensitivity analysis using DGSA, the least significant parameter to the cumulative water production for 3 years (reservoir response) is the displacement of the oldest fault as seen in Figure 8. Looking at the interaction sensitivities conditional on this parameter, $\text{Throw}_1$ in Figure 9, high values would allow for maximum likelihood of maintaining the uncertainty in the reservoir response. This is geologically consistent since high values in $\text{Throw}_1$ allow for a broader range of values in $\text{Throw}_i \forall i > 1$ that in turn allows for a more complete response uncertainty.

When the uncertainty in $\text{Throw}_1$ is narrowed to only high values, it results in just 29 scenarios of parameter-combinations as opposed to the 648 scenarios from a naïve experimental design. This is a significant computational simplification of an otherwise complex reservoir model. To test this simplified (or reduced) reservoir model, 300 realizations using the reduced parameter uncertainty were sampled, generated and flow-simulated. Their responses were compared with those of 300 randomly sampled realizations. As seen in Figure 10, the reduced parameter uncertainty model is very reliable when it comes to predicting the overall reservoir uncertainty.
Figure 5 Fault and well placement in reservoir

Figure 6 Reservoir grid as a categorical variable based on number-of-faults
Figure 7 Training images with high sand proportion (top) and low sand proportion (below)

Figure 8 One way sensitivity analysis for reservoir case study
Conclusions

We have presented a sensitivity-based parameter uncertainty reduction scheme for model reduction. The case study shows that a reduced model obtained this way can be used to quantify uncertainty in the reservoir response very reliably. This reduced model makes uncertainty quantification based on structural uncertainty in the reservoir computationally feasible. This leads to a better understanding of the reservoir uncertainty, better risk modeling, better reservoir decision-making and consequently, more profitable reservoir production.

Such a reduced reservoir model can lead to possible computational gains over the full reservoir model in any workflow that involves parameter-combination sampling. Preliminary experiments in reservoir flow response inversion are underway. As more production data and other
information about the reservoir become available, updating a reduced reservoir model would present an interesting challenge as well.

References


