Joint integration of 4D seismic and production data using the probability perturbation method: a proposal

Scarlet A. Castro and Jef Caers

Stanford Center for Reservoir Forecasting

May 2004

Abstract

Geostatistical reservoir characterization using 3D seismic data provides a static description of reservoir models. A better description of such models can be obtained by the integration of “dynamic data”. One approach consists of history match production data and obtain a model of the reservoir that honors both static and dynamic data. Considered as dynamic data, 4D seismic data relating changes in seismic attributes, such as amplitude, to changes in the distribution of fluids in a reservoir under to production. The approach proposed here is to history match both production and 4D seismic data using a technique that perturbs the reservoir model in such a way that the geological continuity is preserved.

Traditional history matching techniques perturb gridblock properties of the reservoir model without preserving the geological continuity. A recently developed approach for avoiding this problem uses a multiple-point geostatistics technique that creates reservoir realizations where spatial patterns are inferred using many spatial locations. Instead of perturbing gridblocks, this approach (probability perturbation method), perturbs the probability distribution used by the multiple-point geostatistics algorithm.

In this paper we propose a history matching workflow where the reservoir model will be perturbed in the fine scale by using the probability perturbation method, and which 4D seismic response will be matched as dynamic data. The results obtained in the case study presented show that the proposed algorithm not only matches the 4D seismic data but also makes reasonable predictions of water saturation in the reservoir.
1 Introduction

Laboratory and field measurements have illustrated the dependence of seismic velocities and impedance on pore fluids (Nur and Simmons, 1969; Wang and Nur, 1988). The fluid contained in the pore space changes the overall elastic moduli of the rock and therefore changes the seismic velocities. Intuitively, the less compressible the fluid in the pore space the more resistant to wave-induced deformations the rock is. A rock with a less compressible fluid (such as brine) is stiffer than a rock with a more compressible fluid (such as gas). This fundamental observation is the basis for using 4D seismic data in predicting fluid saturation changes in the reservoir.

Traditional 3D reflection seismic captures the acoustic response of the reservoir at the time the seismic survey was recorded. As production starts the distribution of fluids changes in the reservoir, and 3D seismic has been used to monitor the movement of such fluids (Greaves and Fulp, 1987).

Termed “four-dimensional seismology” (Nur, 1989), 4D seismic data is referred to the set (usually two) of three-dimensional seismic data acquired at different times over the same area, to assess the changes in a producing hydrocarbon reservoir with time. The changes can be observed in fluid location, saturation, pressure and temperature; therefore the main application of 4D seismic is to monitor fluid flow in the reservoir.

Typically, the difference between the two 3D seismic datasets recorded at different times is an indicator of the areas in the reservoir where the distribution of fluids has changed. With such a close relationship between 4D seismic and production “dynamic” data, incorporating 4D seismic data into the history matching workflow seems inevitable.

Integrating 4D seismic and production data is not an easy task and not many works have been published in the literature. Among others, Huang, Meister and Workman (1997) approached a combined seismic and production history matching where the porosity model of the reservoir was perturbed. Although this work shows interesting and encouraging results, additional research on the scale difference between seismic and production data is still warranted. Moreover, in the approach they have used, porosity is the only perturbed factor of the geological model. Other important geological parameters to be perturbed are regional facies proportion variations, facies positions and within facies permeability.
One of the most important issues of integrating 4D seismic and production data is the difference of scales. Traditionally, while doing history matching the fine scale reservoir model (geostatistical scale) is upscaled for flow simulation and perturbed until production data is matched. When integrating the matching of 4D seismic data the issue of scale difference arises: 4D seismic data informs about a scale larger than the geostatistical modelling scale but, the seismic scale may not be the same as the flow simulation scale.

The way by which the reservoir model is perturbed, is also a significant issue in history matching. In practice, the perturbation is applied on the upscaled reservoir model by changing coarse grid block properties without regard to the impact such perturbation may have on the fine scale model. An impressive history match can be achieved with this procedure, although paying the high cost of degrading the geological realism carried over from the fine scale model. The goal of history matching is not only to match production data but also to find a reservoir model that honors hard data, seismic data, as well as geological continuity information (variogram or training image) and is able to provide accurate predictions of reservoir performance.

A reservoir model can only be predictive if one of the main mechanisms for flow, such as geological heterogeneity, is represented as realistically as possible. However, the impact of some geological details should be evaluated from the start; some geological features have more impact on flow than others and they have to be the target of the perturbation procedure.

An alternative procedure for perturbing the fine scale model and maintaining the initial geological heterogeneity was proposed by Caers (2003). This procedure, termed probability perturbation method, consists of perturbing the probability distributions used to populate the reservoir model with petrophysical, or any other, properties. The perturbation is such that it moves the reservoir model closer to matching production, and also maintains the desired geological continuity.

The probability perturbation method has been used for integrating “dynamic” and “static” data with successful results (Caers, 2003; Hoffman and Caers, 2003; Kim and Caers, 2003). Taking advantage of the adaptability of this method this paper proposes the joint integration of 4D seismic with “static” and “dynamic” data using the probability perturbation method.

The paper starts with the detailed description of the proposed workflow
for the integration of 4D seismic and production data, continues with a re-
view of the probability perturbation method, and ends by presenting a 3D
synthetic case study.
2 Proposed workflow

Traditionally, the workflow used for history matching consists of perturbing the upscaled reservoir model until production history is matched. The fine scale model of the reservoir generated with geostatistical techniques often consists of millions of grid cells that practically cannot be managed by actual flow simulators. To make flow simulation feasible, the number of grid cells needs to be reduced, hence the geostatistical model needs to be upscaled to a manageable grid block scale.

This commonly used approach for history matching involves the perturbation of upscaled petrophysical properties and usually an impressive match can be achieved. However, perturbing upscaled properties imply degrading the fine scale model which honors both hard and seismic data. The final result of this traditional approach is a reservoir model that matches production history, but precludes the prediction of future production.

An alternative approach, proposed by Tureyen and Caers (2003), is shown in Figure 1 and consists of generating a fine scale model by sequential simulation, then upscale it and run flow simulation to check for history matching of production data. The fine scale model is subsequently perturbed and the entire procedure is repeated until history matching is achieved. Using this workflow it is possible to obtain a final model that not only matches the production history but also:

- honors important geological hard and seismic data, and
- provides accurate predictions of reservoir performance.

The workflow proposed here also consists of perturbing the fine scale model, however it incorporates two new variants, see Figure 1:

- the fine scale model is created using both hard and 3D seismic data.
- the history matching is accomplished by matching both production data and 4D seismic.

Each of the new variants will be discussed in detail in the following sections.
2.1 Creating the fine scale model

Traditionally, geostatistical techniques capture geological continuity through a variogram. A variogram is a two-point statistical function that describes the level of correlation, or continuity, between any two sample values as separation between them increases. Since the variogram describes the level of correlation between two locations only, it is not able to model continuous and sinuous patterns such like channels or fractures. For modeling such geological features a multiple-point approach should be used, where spatial patterns are inferred using many spatial locations (Strebelle, 2002).

In multiple-point geostatistics, the spatial patterns are inferred from a training image which represents a conceptual reservoir analog with the expected geological heterogeneity. Since it is a conceptual model, the training image is not constrained to any data.

The geostatistical technique that uses a training image to create realizations constrained to reservoir data is proposed by Strebelle (2002). The “single normal equation simulation” (snesim) algorithm is a conditional sequential simulation where the probability distribution is retrieved from the training image and made conditional to a multiple-point data event.
Suppose that $A$ is the event we want to simulate, say channel facies is present or absent at a certain location $u = (x, y, z)$, and $B$ is the set of sample data (hard data) and all previously simulated locations used to constrain $A$. To simulate the current location, the *snesim* algorithm retrieves the probability model $P(A | B)$ from the training image, then draws a value from $P(A | B)$. This allows reproducing the patterns contained in the training image.

The *snesim* algorithm also allows the integration of secondary information, such as seismic data. Following previous notation, $C$ is a secondary data event observed in the neighborhood of the location to be simulated. $C$ could consist of a set of seismic data within a neighborhood near event ‘$A’$. Using a calibration method, a probability $P(A | C)$ can be determined from secondary data at every location $u$.

The *snesim* algorithm is extended to include the seismic derived probability $P(A | C)$. To include the seismic data $C$, a joint conditional probability $P(A | B, C)$ is calculated from the individual probabilities $P(A | B)$ (from training image) and $P(A | C)$ (from seismic calibration). A simulated value is drawn from $P(A | B, C)$.

To combine $P(A | B)$ and $P(A | C)$ into $P(A | B, C)$, Journel (2002) proposes using the following expression:

\[
\frac{x}{b} = \left( \frac{c}{a} \right)^\tau
\]

where:

\[
x = \frac{1 - P(A | B, C)}{P(A | B, C)}
\]

and

\[
a = \frac{1 - P(A)}{P(A)} \quad b = \frac{1 - P(A | B)}{P(A | B)} \quad c = \frac{1 - P(A | C)}{P(A | C)}
\]

$P(A)$ is the global proportion of $A$ occurring and the ratio $a$ can be seen as a measure of prior uncertainty about $A$ or distance to $A$ occurring. Similarly, $c$ can be seen as the distance to $A$ occurring after observing the data event $C$. Subsequently $x$ is a measure of uncertainty about $A$ after observing.
both data events $B$ and $C$. Following Journel’s formulation, the parameter $\tau$ in (1) should be a function of both $B$ and $C$ and controls the influence of the $C$-data event. Choosing $\tau = 1$ is the common used approach, however this assumption calls for conditional independence. The selection of the parameter $\tau$ will be subject of future research.

With $\tau = 1$, the conditional probability $P(A \mid B, C)$ is calculated as follows:

$$P(A \mid B, C) = \frac{1}{1 + x} = \frac{a}{a + bc}$$

Using the *snesim* algorithm, the fine scale model can now be created. Next we elaborate on a simple calibration method to obtain $P(A \mid C)$.

### 2.1.1 Creating $P(A \mid C)$

Seismic data carries valuable information about lithology among other properties of the reservoir. The seismic signal is a record of all reflections registered at a given location due to the propagation of the source wave. Each reflection is registered with a certain strength (amplitude of the signal) and at certain time. The time of the reflection arrival is directly related to the depth of the interface where the wave was reflected back and also with the velocity with which the wave traveled through the medium. The strength of the reflection (amplitude of the signal) is directly related to the contrast in the acoustic properties that characterized that interface.

The contrast in acoustic impedance between two layers is given by:

$$\frac{Z_2 - Z_1}{Z_1 + Z_2}$$

where $Z_1$ is the acoustic impedance of the top layer and $Z_2$ of the bottom layer.

The acoustic impedance has been widely used as the most “physical” seismic attribute with a direct relationship to lithology. By definition the acoustic impedance is expressed as:

$$Z = \rho V_p$$
where $\rho$ is the density of the medium and $V_p$ is the velocity of a compressional wave in that medium. Since lab measurements on different rock types or lithologies have shown different values for density and $V_p$ (Mavko, Mukerji, and Dvorkin, 1998), the acoustic impedance has been traditionally used to identify lithology in the reservoir.

A common practice is to extract the acoustic impedance from the seismic signal through a process called seismic inversion. This process calibrates the seismic with well data and generates an acoustic impedance model of the reservoir by minimizing the difference between its seismic response and the observed seismic data.

In order to use the obtained acoustic impedance model of the reservoir for identifying lithology, well data are used for calibration. At the well locations, density, $V_p$ and facies are often known through well-loging, and a histogram of acoustic impedance for each facies can be obtained. In practice, a deterministic differentiation between facies, based on acoustic impedance, is not feasible due to the ambiguity in acoustic impedance/facies type relationship, hence a probabilistic approach is warranted. We use a method based on Bayes relationship also employed in Bitanov and Journel (2003).

Consider a set of indicator random variable $I(u; s_k)$, modeling the spatial variability of each facies $s_k k=1,\ldots,K$. The quantity $p(u; s_k)$ represents the prior probability for the facies $s_k$ to prevail at location $u=(x,y,z)$:

$$p(u; s_k) = \text{Prob}(I(u, s_k) = 1)$$

This probability can also be expressed as the expected value of the indicator random variable $I(u; s_k)$ at that location:

$$E\{I(u; s_k)\} = \text{Prob}(I(u, s_k) = 1) = p(u; s_k)$$

where the indicator random variable $I(u; s_k)$ is defined as:

$$I(u; s_k) = \begin{cases} 
1 & \text{if facies } s_k \text{ occur at } u \\
0 & \text{otherwise} 
\end{cases}$$

Consider now $\{c(u_\alpha), i(u_\alpha, s_k), s_k = 1,\ldots,K, \alpha = 1,\ldots,n\}$, the set of $n$ joint measurements of acoustic impedance $c$ and facies $s_k$ at well location $u, \alpha = 1,\ldots,n$. In our case the distribution of acoustic impedance for each facies can be observed at the well locations. Acoustic impedance (a continuous
variable) is further categorized into \( l \) classes \( c_l, l = 1, ..., L \).

For each facies \( s_k \) the probability \( P(I(u; s_k) = 1 \mid C(u) \in c_l) \) can be calculated by using Bayes’ relationship:

\[
\phi_k(u) = P(I(u; s_k) = 1 \mid C(u) \in c_l) = \frac{P(C(u) \in c_l \mid I(u; s_k) = 1)}{P(C(u) \in c_l)} \tag{7}
\]

where \( P(I(u; s_k) = 1) \) is the marginal probability (proportion) of facies \( k \), \( P(C(u) \in c_l \mid I(u; s_k) = 1) \) is the probability of acoustic impedance belonging to a certain impedance category \( c_l \) given facies \( s_k \) is observed, and \( P(C(u) \in c_l) \) is calculated as:

\[
P(C \in c_l) = \sum_k P(C \in c_l \mid I(u; s_k) = 1) P(I(u; s_k) = 1) \tag{8}
\]

Using the joint data set \( \{ c(u_\alpha), i(u_\alpha; s_k \ \forall k \ \forall \alpha \} \), one can calculate the probability \( P(C \in c_l \mid I(u; s_k) = 1) \). \( P(I(u; s_k) = 1 \mid C \in c_l) \) is then computed using Bayes’ relationship in Eq. (7).

The function \( \phi_k(u) \) is then applied to every location \( u \) to obtain \( k \) seismic derived cubes of facies presence \( P(A_k \mid C) \), with \( A_k = \{ I(u, s_k) = 1 \} \) and \( C = C(u), k = 1, ..., K \).

### 2.2 Matching 4D seismic

What is referred to as 4D seismic data is nothing more than three-dimensional (3D) seismic data acquired at different times over the same area. Typically, the difference between the two 3D seismic datasets will be an indicator of the areas in the reservoir where the distribution of fluids has changed; therefore, this difference could be considered as “dynamic data”.

It is true that one of the main drawbacks of 4D seismic data interpretation is the issue of repeatability of the 3D seismic data. The fact that the time difference between the acquisition of the two seismic surveys is often 5 to 10 years has an important impact on the technology and also on the level of noise in the area (among others); considering that both surveys were designed before starting production, which is rarely the case. Since the concept of 4D seismic is relatively new, many fields were already in production and comparing new 3D seismic data with old surveys brought many problems of differences in acquisition and processing. The repeatability issue of
time-lapse seismic is a current area of research in geophysics; however it is not a matter of research in this proposal.

As we have discussed previously, the process of history matching consists of finding a reservoir model that not only matches the observed production data but also predicts the future production response. By incorporating the 4D seismic data, the reservoir model should also match the 4D seismic response observed in the field.

The 4D seismic response observed in the field if often considered as the difference between two 3D seismic surveys:

\[ s(u, t_1) \text{ seismic survey recorded at } t = t_1 \]
\[ s(u, t_2) \text{ seismic survey recorded at } t = t_2 \]

where \( t_2 > t_1 \) and \( u \in \text{Reservoir} \).

The difference between the two surveys, defined as:

\[ \Delta s(u, \Delta t) = s(u, t_2) - s(u, t_1) \quad (9) \]

can be directly obtained by subtracting the originally recorded amplitudes or any seismic attribute such as acoustic impedance. Generally speaking, \( s \) can be considered as any attribute obtained from the seismic data.

When trying to match 4D seismic data we will need to simulate \( \Delta s(u, \Delta t) \) for each reservoir realization and minimize the difference with the observed \( \Delta s_{\text{ref}}(u, \Delta t) \) in the field.

In this proposal the used measure of discrepancy between \( \Delta s(u, \Delta t) \) and \( \Delta s_{\text{ref}}(u, \Delta t) \) is the correlation coefficient \( \rho \) obtained from pointwise comparison of \( \Delta s(u, \Delta t) \) and \( \Delta s_{\text{ref}} \) at every location \( u \). The added extra term to the objective function will be \( 1 - \rho \). A match is therefore achieved when \( \rho = 1 \), i.e. when \( \Delta s(u, \Delta t) = \Delta s_{\text{ref}}(u, \Delta t) \).

2.2.1 Creating 4D seismic

In order to create the 4D seismic response, two 3D seismic datasets \( s(u, t_1) \) and \( s(u, t_2) \) are forward modeled using the simple convolutional model.
The convolutional model is the most basic definition of a seismic trace:

\[ s(x, y, t) = r(x, y, t) \ast w(t) \]  \hspace{1cm} (10)

where \( s(x, y, t) \) is the seismic trace at location \((x, y)\), \( r(x, y, t) \) is the reflectivity series at location \((x, y)\) and \( w(t) \) is the source wavelet. The reflectivity series contains the reflection coefficients or acoustic contrasts (Eq. 3) from each layer interface in the subsurface. The source wavelet \( w(t) \) is the impulse we introduce in the subsurface, hence the subsurface response is \( s(t) \). The convolutional model states that the seismic trace is the convolution between the reflectivity series of the earth and the source wavelet.

Using the convolutional model to create the 3D seismic datasets, it is clear that \( s(u, t_1) \) will be created using the acoustic impedance of the reservoir prior to production, while \( s(u, t_2) \) uses the changed acoustic impedance due to the movement of fluids in the reservoir.

Acoustic properties of rocks change depending on the fluid they contain (Nur and Simmons, 1969; Wang and Nur, 1988). It is clear that the density of the rock changes as the fluid contained in its pore space changes. However, compressional \( V_p \) and shear \( V_s \) velocities also change due to the presence of different fluids in the rock, and there is a basic reason for that. Compressional and shear velocities are function of the density and the elastic moduli of the rock, more specifically:

\[ V_p^2 = \frac{K + \frac{4}{3}\mu}{\rho} = \frac{M}{\rho} \] \hspace{1cm} (11)
\[ V_s^2 = \frac{\mu}{\rho} \] \hspace{1cm} (12)

where \( \rho \) is density, \( K \), \( \mu \) and \( M \) are the rock’s elastic moduli. Strictly speaking they are called bulk modulus \((K)\), shear modulus \((\mu)\) and P-wave modulus \((M)\). The elastic moduli define the properties of a rock that undergoes stress, deforms, and then recovers and returns to its original shape after the stress ceases. When the fluid contained in the rock changes the overall elastic moduli of the rock also changes and the seismic velocities are affected.

Intuitively, the less compressible the fluid in the pore space the more resistant to wave-induced deformations the rock is. A rock with a less compressible fluid (such as brine) is stiffer than a rock with a more compressible...
Gassmann (1951) introduced a mathematical transformation that allowed to calculate the elastic moduli of the rock as one fluid displaces another in the pore space:

\[
\frac{K_2}{K_{\text{min}} - K_2} - \frac{K_{fl2}}{\phi(K_{\text{min}} - K_{fl2})} = \frac{K_1}{K_{\text{min}} - K_1} - \frac{K_{fl1}}{\phi(K_{\text{min}} - K_{fl1})}
\] (13)

where \(K_1\) and \(K_2\) are the rock’s bulk moduli with fluids 1 and 2 respectively, \(K_{fl1}\) and \(K_{fl2}\) are the bulk moduli of fluids 1 and 2, \(\phi\) is the rock’s porosity, and \(K_{\text{min}}\) is the bulk modulus of the mineral.

This powerful transformation has been widely used and the procedure is termed “fluid substitution”.

Figure 2 shows how the “fluid substitution” procedure is followed in practice. The process starts with the compressional \(V_p\) and shear \(V_s\) wave velocities measured on rocks saturated with the initial pore fluid and then extracting the bulk and shear moduli:

\[
K_1 = \rho_1(V_p^2 - \frac{4}{3}V_s^2)
\] (14)

\[
\mu_1 = \rho_1V_s^2
\] (15)

The bulk modulus of the rock containing the second fluid \(K_2\) is computed using Gassmann’s relations (Eq. 13); however, the shear modulus \(\mu_2\) remains unchanged \(\mu_2 = \mu_1\) since shear stress cannot be applied to fluids. The density of the rock is also transformed and the density of the rock with the second fluid is computed as:

\[
\rho_2 = \rho_1 + \phi(\rho_{fl2} - \rho_{fl1})
\] (16)

Having transformed the elastic moduli and the density, the compressional and shear wave velocities of the rock with the second fluid are computed as:

\[
V_p = \sqrt{\frac{K_2 + \frac{4}{3}\mu_2}{\rho_2}}
\] (17)

\[
V_s = \sqrt{\frac{\mu_2}{\rho_2}}
\] (18)
The fluid contained in the rock could be a single fluid like brine or oil, however it can also be a mixture of fluids. The bulk modulus of each fluid is needed in order to perform Gassmann’s transformation; when dealing with single fluids, elastic moduli can be gathered from tables (Mavko, Mukerji, Dvorkin, 1998), however this is not the case for mixtures and partial saturations need to be considered.

The most common approach to modeling partial saturation (gas/water or oil/water) or mixed fluid saturations (gas/water/oil) is to replace the set of phases with a single “effective fluid”. The bulk modulus of this “effective fluid” is computed with a weighted harmonic average, termed Reuss average in the rock physics literature:

$$\frac{1}{K_{ff}} = \sum_i S_i \frac{1}{K_i}$$  \hspace{1cm} \text{(19)}$$

where $K_{ff}$ is the effective bulk modulus of the fluid mixture, $K_i$ denotes the bulk moduli of the individual fluid phases, and $S_i$ represents their saturations. This model assumes that the fluid phases are mixed at the finest scale.
The fluid substitution procedure is frequently used in the oil industry, however a common practical problem arises when we wish to estimate the change of $V_p$ during fluid substitution without knowing the shear velocity $V_s$. Strictly speaking, the bulk modulus cannot be extracted from $V_p$ without knowing $V_s$ (see Eqn. 11) and Gassmann’s relations cannot be applied.

Since it is very common in practice that $V_s$ is not available we have used a method suggested by Mavko, Chan and Mukerji (1995), that operates directly on the P-wave modulus $M = \rho V_p^2$. This method performs the same transformation with the P-wave modulus $M$ as it is done with the bulk modulus $K$ in the Gassmann’s relation.

To summarize, Figure 3 shows the general procedure followed to create the 4D seismic response of each realization. The simulated values of density and compressional velocity are used to compute the acoustic impedance of the reservoir prior to production, and subsequently the seismic data $s(u, t_1)$ (amplitude) is created using the convolutional model.

On the other hand, flow simulation is performed in order to obtain the partial saturations and compute the properties of new occupying fluid in the reservoir (after $\Delta t$ years). The fluid substitution procedure is then used to create the changed acoustic impedance $Z_2$ of the reservoir (after $\Delta t$ years), and seismic amplitude data is forward modeled to obtain $s(u, t_2)$.

Having forward modeled the two 3D seismic surveys $s(u, t_1)$ and $s(u, t_1)$ for the considered reservoir realization, their difference $\Delta s(u, \Delta t)$ is computed and compared with the difference observed in the field (reference) $\Delta s_{ref}(u, \Delta t)$. The goal is therefore to compute the correlation coefficient $\rho$ between the difference volumes and maximize it, or minimize $1 - \rho$. 

15
Figure 3: General procedure for creating the 4D seismic response of each realization and generation of the difference volumes. The correlation coefficient between the two difference volumes should be maximized.
3 History matching under geological control

The process of history matching consists of perturbing an initial model realization of the reservoir until production history is matched. For each perturbed reservoir model a flow simulation is performed and production data from the field is compared with the observed production after the simulation.

Many approaches can be taken for perturbing the reservoir model, most of them directly perturb grid block properties such as permeability, based on gradient calculations or sensitivity coefficients that are computed from production data and independently of the geological model. This commonly used approach leads to degrading any prior geological continuity in the reservoir model, which also leads to unrealistic models that match production history but may not predict future performance of the reservoir.

As an alternative to this problem, Caers (2003) suggested a procedure that uses a multiple-point geostatistics algorithm (snesim) and perturbs the probability distribution that this algorithm uses to populate the reservoir model with petrophysical properties. This perturbation moves the model closer to matching production and preserves the desired geological continuity.

3.1 The Probability Perturbation method

Consider the indicator variable \( I(u) \) that describes an event occurring, say sand channel present or not:

\[
I(u) = \begin{cases} 
1 & \text{if the event occurs at } u \\
0 & \text{else}
\end{cases}
\]

Two cases can be considered while matching production data ‘D’: the case where a match is achieved, hence the specific realization \( i^{(0)}(u) \) is kept, or, the case where the realization \( i^{(0)}(u) \) is far from matching the production data and should be perturbed considerably.

In section 2 we discussed the multiple-point geostatistics simulation algorithm (snesim) that simulates values from the joint probability distribution \( P(A \mid B, C) \). The goal of the probability perturbation method is to use the
same algorithm but perturb that joint probability in a geologically consistent fashion, and adding the influence of production data.

The perturbation of \( P(A \mid B, C) \) is achieved by the introduction of the probability \( P(A \mid D) \) defined as:

\[
P(A \mid D) = (1 - r_D)i^{(0)}(u) + r_D P(A)
\]

where \( r_D \) is a parameter that can be chosen between \([0, 1]\), and \( P(A) \) is the overall proportion of the event \( A \) occurring independent of location, hence is the marginal distribution.

The parameter \( r_D \) determines the magnitude of the perturbation:

- if \( r_D = 0 \), then \( P(A \mid D) = i^{(0)}(u) \) and the information ‘\( D \)’ is determined to be fully informative of event ‘\( A \)’. The flow simulation response of the realization \( i^{(0)}(u) \) matches the production data ‘\( D \)’, therefore no perturbation is needed (\( r_D = 0 \)).

- if \( r_D = 1 \), then \( P(A \mid D) = P(A) \), meaning that ‘\( A \)’ is not yet representative of data ‘\( D \)’. The flow simulation response of the realization \( i^{(0)}(u) \) does not match the production data ‘\( D \)’, and one retains a different equiprobable realization \( i^{(1)}(u) \).

In other words, the parameter \( r_D \) defines a perturbation of the initial realization \( i^{(0)}(u) \) towards another independent realization \( i^{(1)}(u) \). Each value of \( r_D \) fully determines the probability \( P(A \mid B) \) at every location \( u \). During sequential simulation with \textit{snesim} \( P(A \mid D) \) is combined with \( P(A \mid B, C) \) to form \( P(A \mid B, C, D) \), from which simulated values are drawn. The resulting realization simulated in this fashion is denoted as \( i^{(1)}_{r_D}(u) \).

The combination of conditional probabilities \( P(A \mid B, C) \) and \( P(A \mid D) \) is achieved using the approach presented in Journel (2002), as follows:

\[
\frac{x}{b} = \left(\frac{c}{a}\right)^{\tau_1} \left(\frac{d}{a}\right)^{\tau_2}
\]

where:

\[
x = \frac{1 - P(A \mid B, C, D)}{P(A \mid B, C, D)}
\]

(20)
and
\[ a = \frac{1 - P(A)}{P(A)} \quad b = \frac{1 - P(A | B)}{P(A | B)} \quad c = \frac{1 - P(A | C)}{P(A | C)} \quad d = \frac{1 - P(A | D)}{P(A | D)} \]

where \( a \) is the distance to \( A \) occurring, \( c \) is the distance to \( A \) occurring after observing the data event \( C \), and similarly \( d \) is the distance to \( A \) occurring after observing the data event \( D \). Subsequently \( x \) is a measure of uncertainty about \( A \) after observing data events \( B, C \) and \( D \).

The original \textit{snesim} algorithm of Strebelle (2002) was modified to accommodate the additional soft probability \( P(A | D) \) (see Eqn. 1). In order to incorporate the additional soft probability \( P(A | D) \) the algorithm was modified using Eqn. 20 instead. A test on the new \textit{snesim} algorithm is presented in the appendix section.

We select \( \tau_1 = 1 \) and \( \tau_2 = 1 \), which is equivalent to conditional independence. The selection of the parameters \( \tau_1 \) and \( \tau_2 \) will be subject of future research.

### 3.1.1 The algorithm

A random seed uniquely defines a realization. In order to perturb this realization \( i^{(0)}(u) \), generated with a random seed \( s \) and the conditional probability distribution \( P(A | B, C) \), a different random seed \( s' \) and the perturbed probability distribution \( P(A | B, C, D) \) are used to simulate the perturbed realization \( i^{(1)}_{pD}(u) \).

This perturbation procedure requires to define the magnitude of the parameter \( r_D \) in order to create \( P(A | D) \). Consequently, the perturbation process consists of finding the \( r_D \) value for which the new realization matches best the production data; this is achieved by a simple 1-D optimization.

Figure 4 shows the basic algorithm for the probability perturbation method which proceeds as follows:

- simulate an initial guess realization using a random seed \( s \) and the conditional probability \( P(A | B, C) \).
• Outer loop: this loop controls the change in the random seed used by the simulation algorithm. The iteration number is \( l = 1, \ldots, \text{lmax} \); the program stops either if \( \text{lmax} \) is reached or if history matching is achieved.

• Inner loop: this loop controls the iterative 1-D optimization problem used to find the best value of \( r_D \).
  - for any value of \( r_D \) \( P(A \mid D) \) is created using:
    \[
P(A \mid D) = (1 - r_D)i^{(l-1)}(u) + r_DP(A)
    \]
  - a new realization \( i^{(l)}_{r_D}(u) \) is simulated using the perturbed conditional probability \( P(A \mid B, C, D) \) and the random seed \( \tilde{s} \) generated in the outer loop.
  - a flow simulation is performed using the perturbed realization \( i^{(1)}_{r_D}(u) \)
  - the objective function is computed as the difference between the simulated dynamic data \( D_s(i^{(1)}_{r_D}(u)) \) and the observed dynamic data \( D \):
    \[
    \mathcal{O}(r_D) = \| D_s(i^{(1)}_{r_D}(u)) - D \|
    \]

In order to find the reservoir model that best matches the observed production data, it is necessary to know not only the best \( r_D \) but also the random seed that uniquely defines the realization. Consequently, the objective function will depend on the random seed \( s \) and the algorithm needs to explore the space of all random seeds by considering two random seeds at a time and performing a 1-D optimization between the two.

To find the best \( r_D \) in the interval \([0,1]\) means that we do a 1-D search in the space of all random seeds. Suppose the initial guess does not match the production data, the random seed \( s \) is changed into \( \tilde{s} \) and different values of \( r_D \) can be used to create a perturbation of the initial guess \( i^{(1)}_{r_D}(u) \). As we discussed previously, the first limit case \( r_D = 0 \) means that no perturbation is needed and the initial realization is retained (simulated with random seed \( s \)), while the limit case \( r_D = 1 \) entails a maximum perturbation and a new realization is retained (simulated with random seed \( \tilde{s} \)). Effectively, the search for the optimum \( r_D \) is performed on a 1-D trajectory between the two random seeds \( s \) and \( \tilde{s} \). For each outer loop in the algorithm presented before this 1-D search is performed.

As discussed in section 2, the workflow for history matching proposed here incorporates two new variants:
• the fine scale model is created using both hard data and 3D seismic.

• the history matching is accomplished by matching both production data and 4D seismic.

These variants were incorporated in the probability perturbation algorithm as shown in Figure 5. The conditional probability $P(A \mid C)$ needs to be computed from seismic calibration as explained in section 2.1.1, and it is used by the \textit{snesim} algorithm to simulate realizations of the reservoir model; the initial guess is also simulated using $P(A \mid C)$.

For each realization of the reservoir model, a time lapse response $\Delta s(u, \Delta t)$ is generated as discussed in section 2.2.1 and the correlation coefficient $\rho$ is computed between $\Delta s(u, \Delta t)$ and $\Delta s_{\text{ref}}(u, \Delta t)$. An extra term $1 - \rho$ is then added to the objective function. The objective now is not only to minimize the difference between the simulated and the observed production data, but also the difference between the simulated and the observed 4D seismic.
Figure 4: Basic algorithm for the probability perturbation method.
Figure 5: Proposed algorithm for the probability perturbation method that incorporates the matching of 4D seismic data. Note the green modules are the newly developed or modified from the original algorithm.
4 Case Study

In order to put in practice the proposed workflow, explained in the previous sections, we developed a controlled synthetic case study were the 3D reference model presented in Figure 6 corresponds to the true model of the reservoir, our unknown in the real practice of history matching. The geological model corresponds to a system of fluvial channels high permeability (10000 mD) oriented E-W; as indicated in the figure, crevasse facies is represented by the red bodies (1000 mD) and mud facies (100 mD) is depicted in blue. We assume that permeability values are known, while the location of the high permeability channels is not known.

![Figure 6: 3D reference model of the reservoir. From top left corner to bottom right corner, each slice represents a different depth level; in this case from the deepest level (top left corner) to the shallowest (bottom right corner). In green are depicted sand channels, red corresponds to crevasse, and blue corresponds to mud.](image)

The only dynamic data matched in this case study was the 4D seismic, since that is precisely what we have added to the traditional approach. The ultimate goal, however for future work, will be to history match both production data and 4D seismic jointly.

An injector well was placed in the lower left corner of the reservoir \((x,y) = (1,1)\) which injects water in at a rate of 1500 STB/day an initially oil saturated reservoir, and a producer well was placed in the upper
right corner \((x, y) = 70, 70\) which produces 2000 STB/day. Using a simple black oil model finite difference simulator (ECLIPSE), the water saturation in the reservoir after 6 months of production was obtained and shown in Figure 7. Comparing Figures 6 and 7 is clear to see how the location of the high permeability channels in the reservoir affects the sweep efficiency.

Figure 7: Reservoir water saturation after 6 months of production, as obtained from flow simulation using the reference model. From top left corner to bottom right corner, each slice represents a different depth level; in this case from the deepest level (top left corner) to the shallowest (bottom right corner).

Having the reference reservoir model and the water saturation in the reservoir after 6 months of production, the reference 4D seismic response was created. Using the facies realization in Figure 6, the reference reservoir model was populated with porosity, compressional velocity and density by using a sequential simulation algorithm:

- porosity was simulated using a target histogram and variogram.
- compressional velocity was computed using Han’s relations (Han, 1986) for sandstones and shaly sandstones, and Gardner’s relation for mudstones (Gardner, et al., 1974).
- density was computed from porosity as:
  \[
  \rho = \phi \rho_{\text{fluid}} + (1 - \phi) \sum_i \rho_m f_i
  \]
where $\rho_{\text{fluid}}$ is the density of oil, since the reservoir is initially fully saturated with oil, $\rho_{m_i}$ is the density of the mineral $m_i$ and $f_i$ is the proportion of the mineral $m_i$ in the rock.

Having the reservoir populated with the petrophysical properties as described before, the 4D seismic response of the reference model was computed as discussed in section 2.2.1:

- a 3D seismic dataset is forward modeled using the petrophysical properties of the reservoir fully saturated with oil.
- a 3D seismic dataset is forward modeled using the petrophysical properties of the reservoir after 6 months of production.

If we were to take the difference of both datasets we will obtain a volume of the 4D seismic response; however, we choose to take the difference between a 2D seismic attribute map obtained from each 3D dataset.

For each seismic trace the instantaneous amplitude was calculated and the average value was taken. This process was applied to all traces in each seismic dataset (before and after production), resulting in a map of average instantaneous amplitude for each of them. The difference between the two maps is referred from now on in this example as the 4D seismic response. Therefore, the reference 4D seismic response is depicted in Figure 8; the goal will be now to find a reservoir model realization which 4D seismic response best matches the reference in Figure 8.

The history matching procedure started with the initial guess reservoir model in Figure 9, whose corresponding water saturation after production is shown in Figure 10 and 4D seismic response is shown in Figure 11.

The correlation coefficient between the 4D seismic response of the initial guess and the reference was computed as $\rho = 0.18$, which means that the value of the objective function was $f(\rho) = 0.82$. The goal is therefore to decrease the value of the objective function to its minimum, hence increasing the correlation coefficient between the reference and the actual 4D seismic response.

The history matching procedure was performed using a maximum of four outer iterations and 6 inner iterations. The best reservoir model found by this procedure is shown in Figure 12, 13 and 14. The correlation coefficient between the 4D seismic response of the best model and the reference was
Figure 8: 4D seismic response (difference *average instantaneous amplitude* map) from reference model.

computed as $\rho = 0.79$, which means that the value of the objective function was $f(\rho) = 0.21$. This is a considerable improvement in terms of the objective function that is reflected by the similarity between the corresponding 4D seismic responses.

Comparing the water saturation predicted by the best model in Figure 13 with the reference water saturation in Figure 7, an adequate agreement is achieved.

Using this resulting model, we can predict the distribution of fluids in the reservoir, and important decision variable in future well planning. A good estimate of the water saturation in the reservoir can help us to identify areas where oil has been bypassed by the injected water, hence reducing the risk involved in placing future wells to produce oil from such areas.
Figure 9: 3D initial guess model. From top left corner to bottom right corner, each slice represents a different depth level; in this case from the deepest level (top left corner) to the shallowest (bottom right corner). In green are depicted sand channels, red corresponds to crevasse, and blue corresponds to mud.

Figure 10: Reservoir water saturation after 6 months of production, as obtained from flow simulation using the initial guess model. From top left corner to bottom right corner, each slice represents a different depth level; in this case from the deepest level (top left corner) to the shallowest (bottom right corner).
Figure 11: 4D seismic response (difference *average instantaneous amplitude* map) from initial guess reference model.

Figure 12: 3D best reservoir model found by the history matching algorithm. From top left corner to bottom right corner, each slice represents a different depth level; in this case from the deepest level (top left corner) to the shallowest (bottom right corner). In green are depicted sand channels, red corresponds to crevasse, and blue corresponds to mud.
Figure 13: Reservoir water saturation after 6 months of production, as obtained from flow simulation using the best model found by the history matching algorithm. From top left corner to bottom right corner, each slice represents a different depth level; in this case from the deepest level (top left corner) to the shallowest (bottom right corner), compare to Figure 7.

Figure 14: 4D seismic response (difference average instantaneous amplitude map) from best model found by the history matching algorithm, compare to Figure 8.
5 Conclusions and future research

- A new workflow for the integration of 4D seismic and production data using the probability perturbation method is proposed. The workflow allows to use 3D seismic data to constrain the space of realizations of the reservoir model and also includes the matching of 4D seismic data into the objective function in a flexible fashion.

- The synthetic case study presented shows encouraging results where the history matching process is not only able to match the 4D seismic response of the reservoir but also is able to make reasonable predictions of the water saturation in the reservoir.

- Future research will focus on studying the scale difference between seismic and production data. In the present proposal, the geological, geophysical and flow simulation scale are taken to be the same. This will not be practical in any realistic reservoir setting. Issues such as downscaling of saturations or upscaling of petrophysical properties will be addressed. Additionally, new methods for approaching fluid substitution that do not require fine scale mixing of fluids, are an important subject to be considered for study.

- A case study with real data is also being considered as an important part of future research.
6 References

• Bitanov, A., Journel, A., 2003. Quantification of net-to-gross uncertainty at the appraisal stage. 16th Annual Report, Stanford Center for Reservoir Forecasting, Stanford, USA.


• Nur, A., 1989. Four-dimensional seismology and (true) direct detection of hydrocarbon: the petrophysical basis. The Leading Edge, V. 8, No. 9, 30-36.


7 Appendix: Handling two soft probabilities with snesim

As discussed previously in section 3, the snesim algorithm was modified in order to incorporate two soft probabilities. Specifically, the soft probability \( P(A \mid C) \) computed from some seismic calibration procedure and the soft probability \( P(A \mid D) \) computed as a perturbation of the initial realization:

\[
P(A \mid D) = (1 - r_D)i^{(0)}(u) + r_D P(A)
\]  

An 2D example is showed in this section to illustrate the results obtained with the new version of the algorithm.

Consider the 2D training image and the 2D conditional soft probability in Figure 15. Using the snesim algorithm with a random seed \( s \) the realization shown in Figure 15 (right) is simulated using only the training image and \( P(A \mid C) \). Used as the initial realization (corresponding to \( r_D = 0 \)), different perturbations can be achieved by computing \( P(A \mid D) \) using Eqn. 21 for different \( r_D \) values.

![Figure 15: 2D training image (left), 2D conditional soft probability \( P(A \mid C) \) (middle), and the initial realization (right) corresponding to \( r_D = 0 \).](image)

Using a new random seed \( \hat{s} \), a perturbation of the initial realization is simulated using the snesim algorithm with the two soft probabilities \( P(A \mid C) \) and \( P(A \mid D) \); the latter computed for the particular \( r_D \) value that defines the magnitude of the perturbation.
Figure 16 shows five perturbations on the initial realization for different values of $r_D$ simulated with the new algorithm and using both soft probabilities as explained before. In this figure we clearly see how the perturbation increases as $r_D$ increases, until for $r_D = 1$ a completely different realization is obtained. This behavior is exactly as expected, indicating that the modified algorithm is handling well the two soft probabilities and correctly computing the joint conditional probability $P(A \mid B, C, D)$.

Additionally, for $r_D = 1$, 50 realizations were generated with the new algorithm and an E-type estimate was obtained (see Figure 17). As expected, the E-type is very similar to $P(A \mid C)$ since for $r_D = 1$ the soft probability $P(A \mid D) = P(A)$ (see Eqn. 21).

Figure 16: Realizations simulated using the joint conditional probability $P(A \mid B, C, D)$ for $r_D$ values of 0.1, 0.3, 0.5, 0.7 and 1.0.

Figure 17: 2D soft probability $P(A \mid C)$ (left) and E-type generated using 50 realizations.