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A Streamline Approach for History-Matching Production Data

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Abstract

This study proposes and develops a streamline approach for inferring field-scale effective permeability distributions based on dynamic production data including producer water-cut curve, well pressures, and rates. The approach simplifies the history-matching process significantly.

The basic idea is to relate the fractional-flow curve at a producer to the water breakthrough of individual streamlines. By adjusting the effective permeability along streamlines, the breakthrough time of each streamline is found that reproduces the reference producer fractional-flow curve. Then the permeability modification along each streamline is mapped onto cells of the simulation grid. Modifying effective permeability at the streamline level greatly reduces the size of the inverse problem compared to modifications at the grid-block level. The approach outlined here is relatively direct and greatly reduces the computational work by eliminating the repeated inversion of a system of equations. It works well for reservoirs where heterogeneity determines flow patterns. Example cases illustrate computational efficiency, generality, and robustness of the proposed procedure. Advantages and limitations of this work, and the scope of future study, are also discussed.

Introduction

History-matching plays an important role in monitoring the progress of displacement processes and predicting future reservoir performance. Historical production data is routinely collected and it carries much information, although convoluted, that is useful for reservoir characterization and description of reservoir heterogeneity[1,2]. In this paper, the concept of streamlines is applied to develop an automatic method for inferring the permeability distribution of a reservoir based on the history of pressure, flow rate, and water cut at producers.

The properties of streamlines are used in deriving the inverse method, and so a brief review of streamline methodology follows. Streamline and streamtube techniques are approximate reservoir simulation methods proposed some years ago[3-6] that have undergone recent intense study[7-9]. They are most accurate when heterogeneity determines flow paths and the recovery process is dominated by displacement (viscous forces) as opposed to gravity or capillarity[10]. A streamline is tangent everywhere to the instantaneous fluid velocity field and, for a symmetric permeability tensor, streamlines are perpendicular to isobars or iso-potential lines. Streamlines bound streamtubes that carry fixed volumetric flux, and in some cases, flow rate is assigned to streamlines[8,11]. For this reason, we use the terms streamline and streamtube interchangeably.

Displacement along any streamline follows a one-dimensional solution, and there is no communication among streamlines. Thus, the flow problem is decomposed into a set of one-dimensional flow simulations linked by common boundary conditions. A streamline must start and end at a source to maintain continuity. In a streamline-based approach, pressure equations are solved independent of saturation equations. The decoupling of pressure equations from saturation equations speeds up significantly the simulation by reducing the number of times that the pressure field must be updated and greatly reduces the number of equations to solve.

For unit mobility ratio and constant boundary conditions, the streamline distribution remains unchanged throughout the displacement process. Therefore, the pressure field or streamline distribution only needs to be solved once and saturation solutions can be mapped along streamlines. For non-unit mobility ratio, there are two common approaches to treat streamlines. One is to fix the streamline geometry and allow the flow rate to change during the displacement process[12,13]. The other is to update periodically the streamline distribution and distribute the flow rate equally among the streamtubes[14,15]. In the second case, the pressure field and streamline geometry must be updated periodically.

For complete descriptions of the various streamline formulations and inclusive reviews of the history of streamlines for predicting reservoir flow, please refer to references[7,10,12,16].

In this study, the streamline simulator 3DSL by Batycky *et al*[11] is employed for forward simulation. In short, after solving the pressure field and the streamline distribution, 3DSL assigns equal flow rate to each streamline. Then a one-dimensional saturation solution, either analytical or numerical, is solved along the streamlines. Periodically, the streamline saturation distribution is mapped onto the multidimensional grid, the pressure equation is resolved, and streamline geometry redetermined. There is no detectable deduction in accuracy with this technique[17].

Previous work

Most approaches to history-matching field data manipulate permeability at the grid-block level, and hence, demand a great amount of computational work because there are many grid-blocks in a typical simulation. Integration of production data with reservoir description remains an important issue because of the prevalence of production data and the information that it carries about the reservoir. In this brief review, we focus on work that is most similar to our method to follow. Other approaches to history matching are based upon simulated annealing[18], sensitivity coefficients[19,20,21], and parameter estimation approaches[22].

Sensitivity coefficient techniques compute the sensitivity of the objective function to the change of permeability of a cell or a set of cells and solve an inverse system that can be very large and somewhat difficult to construct[21,22]. Sensitivity coefficient methods might also be computationally expensive if the sensitivity coefficients are evaluated numerically by running multiple simulations. Chu et al[21] developed a generalized pulse spectrum technique to estimate efficiently the sensitivity of wellbore pressure to gridblock permeability and porosity. Other work employed sensitivity coefficients in the integration of well test information, production history, and time-lapse seismic data[22].

Vasco et al[20] combined streamlines and a sensitivity coefficient approach while integrating dynamic production data. They employ streamlines to estimate sensitivity coefficients analytically thereby greatly speeding up the procedure. The streamline analysis allows them to "line up" the first arrival of injected fluid at production wells and then match the production history. This technique remains a grid-block-level optimization approach as all of the cells from the flow simulation are used to describe reservoir heterogeneity.

Sensitivity coefficients have also been employed in a scheme to identify the geometry of geological features such as faults and the dimensions of flow channels[23]. In essence, the technique is to minimize an objective function incorporating single and/or two-phase production data. The parameters for minimization are the size and interfacial area of geological bodies rather than grid-block parameters.

Simulated annealing, as applied by Gupta et al[18], perturbs permeability in a set of grid-blocks and evaluates energy objective functions or the degree of misfit between simulated and desired results. This process is stochastic and it is not guaranteed that a perturbation will decrease the energy level. The decision of whether or not to accept the perturbation is based on the change of energy caused by this perturbation. Perturbations that increase the degree of mismatch are accepted with a frequency that decreases with increase in the error. Many iterations are usually required to obtain an acceptable solution. In general, the computational costs of incorporating production data using simulated annealing become very large if a reservoir simulation must be conducted for each iteration.

Reservoir characterization approaches such as geostatistics do not explicitly account for field production data. It is a major task to determine the geostatistical realizations that are consistent with injection and production data. Generally, this involves conducting flow simulations for many different geostatistical simulation realizations. Efficient conditioning of permeability fields to both a geostatistical model and production data is discussed by Wu et al[24]. In other work, Wen et al[19, 25] also present a geostatistical approach to the inverse problem of integrating well production data. They adapt the sequential self-calibration (SSC) inverse technique to single-phase, multi-well, transient pressure and production rate data. The SSC method is an iterative, geostatistically-based inverse method coupled with an optimization procedure that generates a series of coarse grid two-dimensional permeability realizations. In later work, they combine SSC with analytical computation of sensitivity coefficients using streamline distributions[19]. The output realizations correctly reproduce the production data. In both instances, this approach is applied for single-phase flow.

An interesting question that arises with any match to production data is the accuracy of the match. Lepine et al[26] combine error analysis with a gradient-based technique to compute the uncertainty in estimates of future performance based upon history-matched models. Their method helps to identify and select the parameters of a given reservoir model that most sensitively affect the match.

The remainder of this paper presents the development of a streamline-based history-matching approach where streamline effective permeability is manipulated rather than grid-block level data. An algorithm for mapping inverse streamline information onto the grid is also discussed. Next, several synthetic cases are used to explore the proposed approach. Discussion and conclusions complete the paper.

Method

We propose a two-step method to match dynamic production data and infer reservoir heterogeneity. The first step is to modify the permeability distribution at the streamline level based on the difference between simulation results and field data for water cut, pressure drop, and flow rate. By matching the fractional-flow curve through manipulation of the permeability field, we try to capture reservoir heterogeneity. The second step is to map the streamline permeability modification onto the grid-blocks. Then flow simulation is performed to check the match. The above process is iterated until convergence.

In a multi-phase displacement process, each streamline breakthrough contributes a small amount to the fractional-flow curve at a producer. Throughout, we use the terms water-cut and fractional flow interchangeably. Because of the equal flow rate property of streamlines in forward flow simulation with 3DSL, every streamline breakthrough contributes the same amount to the fractional flow. Therefore, by ordering the streamlines with respect to their breakthrough time, we discretize the fractional-flow curve and relate various segments of the fractional-flow curve to the breakthrough of individual streamlines. In this sense, our proposed method is similar to the work of Vasco et al[20]; however, we do not compute sensitivity coefficients nor is our formulation of the inverse problem similar. When the fractional-flow curve of the simulation result for a given permeability does not match the field water-cut curve, we infer the streamlines responsible for the difference between the fractional flow curves. Then based on the relation between streamline breakthrough time and effective permeability, a modification of effective permeability along streamlines can be computed to match the production data. The objective function, as defined below, indicates the error of the simulation result compared to the field data, and therefore is minimized.

$$E = \sum_{n=1}^{N_p} (E_{t,n} + E_{p,n} + E_{q,n}) \qquad (1)$$

In Equation (1) the subscript *n* is the index of the producer, N_p is the total number of producers in the study, $E_{t,n}$, $E_{p,n}$, and $E_{q,n}$ are errors in the dimensionless breakthrough time of individual streamlines, pressure, and flow rate at producer *i*, respectively.

Because streamtubes do not communicate except at producers, it is logical to organize subsystems for each producer. It is also computationally efficient if each producer is treated independently because solving several small sub systems is much cheaper numerically than solving a large system.

To match pressure and flow rate, the effective permeability of the entire region is modified. However, to capture heterogeneity, we need to solve an inverse system to match the fractional flow curve. This is the most important part of this study and is therefore discussed in detail.

In the following development, the index of producer *n* is omitted. The degree of mismatch between reference and history-matched results is computed as

$$E_t = \frac{1}{N_{sl}} \sum_{i=1}^{N_{sl}} E^2_{t_{BT},i} \qquad (2)$$

where N_{sl} is the number of streamlines connected to the producer. The error $E_{t_{BT},i}$ refers to breakthrough time of streamline *i* as defined below

$$E_{t_{BT},i} = t^C_{D,BT,i} - t^R_{D,BT,i} \qquad (3)$$

where $t^C_{D,BT,i}$ and $t^R_{D,BT,i}$ are the computed, C, and reference, R, dimensionless breakthrough times (pore volume injected) for the i^{th} streamline, respectively.

Inverse system

Permeability modification of the i^{th} streamline alters the breakthrough time of not only the i^{th} streamline, but also the other streamlines. Therefore, all the streamlines must be considered and a system of equations has to be solved.

The system to solve for each producer is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2N} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3N} \\ \cdots & & & & \\ a_{N1} & a_{N2} & a_{N3} & \cdots & a_{NN} \end{bmatrix} \begin{bmatrix} \Delta k_1 \\ \Delta k_2 \\ \Delta k_3 \\ \cdots \\ \Delta k_N \end{bmatrix} = - \begin{bmatrix} E_{t,BT,1} \\ E_{t,BT,2} \\ E_{t,BT,3} \\ \cdots \\ E_{t,BT,N} \end{bmatrix} \qquad (4)$$

where $E_{t,BT,i}$ is defined in Eq. (3), Δk_j is the modification of effective permeability along streamline *j* required to get a match, and $\boldsymbol{a_{ij}}$ is the sensitivity of breakthrough time (dimensionless) of the $\boldsymbol{i^{th}}$ streamline to the effective permeability of the $\boldsymbol{j^{th}}$ streamline. These derivatives are defined as

$$a_{ij} = \frac{\partial t_{D,BT,i}}{\partial k_j} \qquad (5)$$

where $t_{D,BT,i}$ is the dimensionless breakthrough time of streamline ***i***, and $\boldsymbol{k_j}$ is the effective permeability along streamline ***j***. Because streamlines are non-communicating, the derivatives can be approximated by applying Dykstra and Parsons[27] method for non-communicating layers of different length. The method relates the breakthrough time of different layers to the effective permeability of each layer. For unit mobility ratio and piston-like displacement, the approximation is exact.

The breakthrough time for streamline *i* is calculated as

$$t_{D,BT,i} = \frac{\sum_{k=1}^{N_{sl}} (\overline{A}\,\overline{\phi}\, l)_k \, x_{D,k}}{\sum_{k=1}^{N_{sl}} (\overline{A}\,\overline{\phi}\, l)_k} \qquad (6)$$

where *l* is the length of a streamline, $x_{D,k}$ is the dimensionless position of the displacing phase front of the k^{th} streamline when the i^{th} streamline breaks through. The term $x_{D,k}$ can also be viewed as the fraction of pore volume of streamline *k* swept when streamline *i* breaks through (i.e., $x_{D,i} = 1$). For those streamlines that break through earlier than streamline *i*, $x_{D,k}$ can be greater than 1. The symbols $\overline{\phi}_k$ and $\overline{A}_k$ represent the average porosity and average cross-sectional area of streamline k respectively. They are defined as

$$\overline{A} = \int_0^1 A(x_D)dx_D \tag{7a}$$

$$\overline{\phi} = \int_0^1 \phi(x_D)dx_D \tag{7b}$$

Now, define the ratio of pore volume of streamline k over the total pore volume as

$$V_{D,k} = \frac{(\overline{A}\,\overline{\phi}\,l)_k}{\sum_{k=1}^{N_{sl}}(\overline{A}\,\overline{\phi}\,l)_k} = \frac{V_k}{V_T} \tag{8}$$

where V_k is the pore volume of streamline k, the subscript D denotes dimensionless, and V_T is the total pore volume. Equation (6) can be rewritten as

$$t_{D,BT,i} = \sum_{k=1}^{N_{sl}} V_{D,k}x_{D,k} \tag{9}$$

Then by applying the chain rule, Eq. (5) is evaluated:

$$\frac{\partial t_{D,BT,i}}{\partial k_j} = \sum_{k=1}^{N_{sl}}\frac{\partial t_{D,BT,i}}{\partial x_{D,k}}\frac{\partial x_{D,k}}{\partial k_j} = \sum_{k=1}^{N_{sl}}V_{D,k}\frac{\partial x_{D,k}}{\partial k_j} \tag{10}$$

Dykstra and Parsons(1950)[27] method provides $x_{D,k}$ in terms of the effective permeability of all the streamlines; hence, the summation above must be computed for all k streamlines. The formula for calculating $x_{D,k}$ for unit mobility ratio is slightly different from that for non-unit mobility ratio.

For unit mobility ratio, the pressure field as well as the streamline distribution remains unchanged throughout the displacement process for constant boundary conditions. When breakthrough happens at streamline i, the front position at streamline k is calculated by

$$x_{D,k} = c_{ik}\frac{k_k}{k_i} \tag{11}$$

where c_{ik} is a constant related to the length of the streamline i and k. Applying this definition and completing the partial derivative indicated in Eq. (7) yields

$$\frac{\partial t_{D,i}}{\partial k_j} = \sum_{k=1}^{N_{sl}}V_{D,k}\frac{\partial x_{D,k}}{\partial k_j} = \begin{cases} -\dfrac{1}{k_i^2}\displaystyle\sum_{k=1,k\neq i}^{N_{sl}}c_{ik}V_{D,k}k_k, & if\ i=j \\ c_{ij}V_{D,j}/k_i, & if\ i\neq j \end{cases} \tag{12}$$

This procedure can be repeated for non-unit mobility ratios given the standard Dykstra-Parsons result[28], where $x_{D,k}$ is a function of the permeability of streamlines i and k, and the end-point mobility ratio.

It is an approximation to treat streamlines as non-evolving for the non-unit mobility ratio cases. This assumption works well if heterogeneity is a dominant factor during displacement. Therefore, it is most applicable to unfavorable mobility ratios and heterogeneous permeability fields. Although streamlines evolve during a non-unit mobility ratio displacement, for a heterogeneity dominant reservoir, the streamlines evolve little. An important fact is that the order of breakthrough of streamlines is preserved even though length and volume may change. That is, if a streamline passes through a high permeability channel at the start of the displacement process, that channel will remain a channel throughout the entire process. The streamlines with the smallest pore volume or time of flight always break through earliest, no matter how they evolve.

Simplifying the inverse system

For unit mobility ratio, the inverse system can be simplified by defining relative or normalized parameters such as

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1N} \\ b_{21} & b_{22} & b_{23} & \cdots & b_{2N} \\ b_{31} & b_{32} & b_{33} & \cdots & b_{3N} \\ \cdots & & & & \\ b_{N1} & b_{N2} & b_{N3} & \cdots & b_{NN} \end{bmatrix}\begin{bmatrix}\delta k_1 \\ \delta k_2 \\ \delta k_3 \\ \cdots \\ \delta k_N\end{bmatrix} = -\begin{bmatrix} e_{t,BT,1} \\ e_{t,BT,2} \\ e_{t,BT,3} \\ \cdots \\ e_{t,BT,N}\end{bmatrix} \tag{13}$$

where $e_{t,BT,i}$ is normalized error in breakthrough time of streamline i to be defined in Eq. (14), δk_j is the normalized modification of effective permeability along streamline j, and b_{ij} is the derivative of $e_{t_{BT},i}$ with respect to δk_j. For the purpose of discussing updates from one iteration to the next, let $k_j^{\lambda+1}$ be the new value of $\boldsymbol{k_j}$, and likewise, k_j^{λ} be the previous value. In equation form, the normalized variables are

$$e_{t_{BT},i} = E_{t_{BT},i}/t^R_{D,BT,i} = (t^C_{D,BT,i} - t^R_{D,BT,i})/t^R_{D,BT,i} \tag{14}$$

$$\delta k_j = \frac{k_j^{\lambda+1} - k_j^{\lambda}}{k_j^{\lambda}} = \frac{\Delta k_j}{k_j^{\lambda}} \tag{15}$$

$$b_{ij} = \frac{\partial e_{t_{D,BT},i}}{\partial(\delta k_j)} = \frac{k_j^{\lambda}}{t^R_{D,BT,i}}\frac{\partial t_{D,BT,i}}{\partial k_j^{\lambda+1}} \tag{16}$$

Substitute Eq. (12) into (16),

$$b_{ij} = \begin{cases} \dfrac{k_i}{\sum_{k=1}^{N_{sl}}c_{ik}V_{D,k}k_k/k_i}\left(-\dfrac{1}{k_i^2}\displaystyle\sum_{k=1,k\neq i}^{N_{sl}}c_{ik}V_{D,k}k_k\right) \approx -1, & if\ i=j \\ \dfrac{k_i}{\sum_{k=1}^{N_{sl}}c_{ik}V_{D,k}k_k/k_i}\left(\dfrac{c_{ij}V_{D,j}}{k_i}\right) = \dfrac{c_{ij}V_{D,j}k_i}{\sum_{k=1}^{N_{sl}}c_{ik}V_{D,k}k_k} \approx \dfrac{1}{N_{sl}}, & if\ i\neq j \end{cases} \tag{17}$$

With the normalization above, the elements of the matrix are now functions of only matrix size N_{sl} for a given set of streamlines. Therefore, the inverse of the matrix is also solely a function of N_{sl}. Because the matrix is symmetric, diagonal-element dominant, and positive definite, it is easy to compute. Thus all elements in the inverse of the matrix are known functions of N_{sl}. These known functions can be derived in general and do not vary from iteration to iteration. All of the elements in the inverse of the matrix are calculated directly. A good approximation of this inversion is a unit matrix. This is verified by practice as discussed later. Here, the inverse process is simply a computation of the product of the inverted matrix and the error vector.

The above simplification works well for unit mobility ratio, as will be demonstrated. For non-unit mobility ratios, it also works to some extent, especially for cases where the mobility ratio is close to unity or heterogeneity is the dominant factor.

In non-unit mobility ratio cases, the elements $\boldsymbol{a_{ij}}$ of matrix **A** are functions of mobility ratio M. If we repeat the above process, we do not get a matrix as simple as **B** in Eq. (17). An alternative for non-unit mobility ratio cases is to solve the system of equations in Eq. (4) by inverting the matrix **A**. However, the streamlines evolve during the displacement process. To obtain a good history match with such a procedure, we may need to select several different streamline distributions over the time period of interest. Each of the distributions could be used to match a segment of the fractional flow curve. This is computationally intensive and so we have not implemented this alternative yet. The case studies indicate that it is appropriate to apply the simplified system developed in Eq. (13) for unit mobility ratio to non-unit mobility ratio cases.

Steps of the inverse streamline approach

This approach can be implemented in a relatively simple procedure that involves no modification to the forward streamline simulator. This is especially helpful when source code is not available. The steps include:

1. Obtain an initial permeability field by guess or geostatistical realization;
2. Run a simulation on the initial permeability field. Check whether the simulation results match the field data (reference data) including fractional-flow curve at the producers, flow rate, and pressure. If not, modify the permeability as in the following steps;
3. Work on the streamlines. Calculate the time of flight (or the associated pore volume) for all streamlines. Sort the streamlines in ascending order of the pore volume;
4. Compute the difference in fractional flow, flow rate, and pressure between the simulation result and the reference. Relate differences in the fractional-flow curve to the corresponding streamline;
5. Solve the system described above (either Eq. (13) or Eq. (4)) to compute the modification of the effective permeability of each streamline required for a good match to available data;
6. Map the modification at the streamline level onto grid-blocks honoring the effective permeability of each streamline;
7. Iterate steps 2 to 6 until a satisfactory match is achieved.

Example Applications

Several cases have been tested with this approach and the results show that this method is robust and converges quickly. Five cases are presented below. First, a reference permeability field is generated, and then 3DSL run with constant injection rate, except for the last case, to obtain the reference production data. Water cut and pressure drop are the data from the reference case that we try to match. An initial permeability field is guessed to start the procedure. The permeability field is either uniform or generated by a geostatistical sequential Gaussian simulation method[29] where the field is conditioned to sparsely distributed data. All the cases discussed in this paper deal with a quarter of a five-spot pattern, although our method is not restricted to this pattern, nor is the method restricted to two-dimensional areal cases. For all of the cases, the injector is located at the lower left corner, and the producer at the upper right corner of the pattern. The pressure at the producer is constant and the fluid is incompressible.

1. Large scale trends, Cases 1, 2 and 3

The three cases discussed in this subsection show how a large-scale trend is retrieved. By large scale, we mean a permeability feature that nearly spans from injector to producer. For all the three cases, the mobility ratio is unity, the water injection rate is constant, and the initial guess for the permeability field is uniform. The numerical solution of the Buckley-Leverett equation is mapped along the streamlines.

In Case1, the reference permeability field contains a high permeability channel connecting the injector and producer as shown in Fig. 1 (a). In Fig. 1, light gray indicates low permeability and dark gray high permeability. Starting from a uniform permeability field of 500 md, four iterations are required to match water-cut and pressure data. Each iteration includes running the flow simulation, computing errors, modifying the permeability of the streamlines, and mapping the modification onto the grid-blocks. The inferred permeability field is shown in Fig. 1 (b). Although somewhat more diffuse, the feature linking injector and producer is reproduced. Figure 2 shows the evaluation of the match of water-cut, and Fig. 3 shows the normalized error for fractional flow and pressure. After four iterations, the match is very good. The inversion converges quickly and the relative error is less than 1%. Most of this error is associated with the match of the water-cut curve. The computational time is spent mainly in running the forward flow simulation. The inversion requires only a few seconds because the solution of the inverse system of equations is simplified.

The configuration of Case 2 is the same as that in Case 1, except a low permeability barrier replaces the high permeability channel. The procedure is the same as above. A very good match to production data is obtained, and the relative error of each parameter is less than 1% after one iteration. Figure 4 shows the reference and inferred permeability fields. Again the light gray indicates low permeability and dark high. The feature is reproduced, but some detail is lost as in the previous case. Figure 5 plots the fractional flow for reference and simulation results, and Fig. 6 plots errors in fractional flow and pressure. Note that the reproduction of water-cut behavior is excellent and achieved in two iterations.

In the first two cases, permeability features were aligned along flow paths between injector and producer. In Case 3, an off-flow-trend barrier exists as shown in the reference permeability field of Fig. 7a. With the same procedure as in Cases 1 and 2, a very good match to fractional flow and pressure behavior is achieved with two iterations as illustrated in Figs. 8 and 9. However, the geometry and size of the barrier are not reproduced very well. Comparing the reference permeability and the inferred permeability distributions, the inferred field has a diffuse low-permeability region that nearly spans from the injector to the producer. This is the worst case that we can formulate to test reproduction of true permeability field. The reason for this result is explored in the discussion section.

Effect of mobility ratio, Case 4

In this case, the end point mobility ratio is an unfavorable 2.5 such that the displacing phase is more mobile than the displaced phase. In forward flow simulations, the Buckley-Leverett solution is again mapped along the streamlines.

To generate more realistic reference and initial permeability fields, scattered permeability data were chosen and the two permeability field realizations shown in Fig.10 were generated by the routine SGSIM[29]. One realization, Fig. 10a, is used as the reference, the other, Fig. 10b, is used as the initial permeability field. The flow simulation results from these two permeability fields show obvious differences in the producer fractional-flow curve as illustrated in Fig. 11 because the realizations are not constrained by production data.

Three iterations were performed to match the reference water-cut data, and the results are illustrated in Figures. 10 to 13. The match is excellent in terms of reproducing fractional flow, Fig.11, and pressure characteristics. Figure 12 shows that most of the error remains in matching the water-cut curve, but this error is less than 1%. The match of permeability distribution is acceptable, as shown by comparing Fig. 10 (a) and (c). Note that the inferred permeability field is the best that we can achieve given the producer information and initial guess.

The match obtained above, Fig. 10 (c), was tested with other mobility ratios to demonstrate that the match does not depend on mobility ratio M. Flow simulations were run on the reference and inferred permeability fields at M=1, M=0.25, and M=5. Figure 13 shows the comparison. It indicates that the permeability field inferred by matching production data at one mobility ratio can be used to predict displacement processes at other mobility ratios and that our inverse approach is independent of the mobility of the injected fluid. This is very useful when a tracer study is used to characterize reservoir heterogeneity and the inferred permeability distribution is then used to predict the reservoir's future performance.

3. Variable injection rate, Case 5

In this case, the injection rate undergoes several step changes during the course of the displacement process. The endpoint mobility ratio is 2.5 and the fluids are again incompressible. The reference permeability field is generated, in a deterministic manner, with an off-trend low permeability region as shown in Fig. 14a. Then six permeability values were collected, two at the wells, two at the other two corners, one inside the low permeability region and one outside the low permeability region. Sequential Gaussian simulation was performed on these data[28]. One realization is chosen as our initial guess for the permeability field (Fig. 14b).

With the procedure given above and fixing the injection rate schedule to match the history of the reference case, a very good match to the fractional flow curve and the pressure at the injector is obtained as shown in Figs. 15 and 16, respectively. The error during iteration is given in Fig. 17. Only two iterations are needed to converge despite the relatively complicated injection conditions. The inferred permeability field is shown in Fig. 14c. Visually the final match appears to be closer to the reference permeability field of Fig. 14a than is the initial permeability in Fig. 14b.

Discussion

The example applications above show that this approach is robust and converges quickly. Each iteration significantly reduces the errors in fractional flow, pressure drop, and flow rate. In all of the examples, an acceptable match to the production data is obtained within a small number of iterations. One strength of this approach is that the input of the inverse problem is simply the usual output of the streamline simulation. Namely, streamline coordinates, production data including fractional flow, flow rate, and pressure. No internal modifications to the streamline simulator are required for implementation. Therefore, this approach stands alone. Importantly, it appears to be expandable to integrate geostatistical data or to honor geological and seismic data. We are actively pursuing this task.

Another strength is that repeated inversion of a matrix is not needed. The permeability modification for every iteration is computed directly based on differences between the streamline breakthrough time from field observation and the simulation result. Therefore, this process takes very little computational time. All of the examples illustrated here used the simplified inverse approach derived for unit mobility ratio cases. It appears that this idea works well for flow with unfavorable mobility ratios in heterogeneous reservoirs.

The computation of the error in water-cut for both unit and non-unit mobility ratio cases is performed using fractional-flow curves from the simulation and the reference results. The pore volume associated with each streamline is calculated for the purpose of sorting streamlines. The accuracy of computing streamline pore volume does not affect inversion results as long as the correct ordering of the breakthrough time is obtained. Therefore, for non-unit mobility ratio cases, the method works well as long as the order of streamline breakthrough time is preserved, even though the streamlines evolve.

Streamline permeability modifications are mapped directly to grid-blocks by propagating the indicated change through all of the grid-blocks that a particular streamline passes. For example, when the effective permeability of a streamline needs to be increased by 5 percent, then the permeability of all the grid-blocks along that streamline are increased by 5%. Mapping is not unique and different mapping schemes can produce the same effective streamline permeability. Necessarily, however, the water-cut curve is matched and the inferred field is independent of mobility ratio. In the mapping process, it is possible to enforce constraints to honor observed geological information, if any.

Examples shown here are for two-dimensional flow problems. We continue to work on the generalization to three-dimensional problems with gravity. Additionally, this approach was originally designed for incompressible two-phase flow or tracer flow studies. That is, the fractional flow curve or tracer breakthrough curve at the producers is required to infer the heterogeneity of the reservoir. More study is needed to extend this procedure to compressible single-phase flow scenarios such as well test problems.

In Cases 1 and 2, the permeability feature is on trend, and we find that it is easy to retrieve the features of the distribution. However, if the heterogeneity is off trend, then the permeability feature may not be retrieved as easily by this approach. Case 3, as shown in Fig. 7, indicates that off-trend barrier is reproduced as diffuse, scattered regions of relatively low permeability if no constraints of mapping are enforced. This result occurs because during the update of permeability along a streamline, all of the grid-blocks along that streamline are multiplied by the same factor. This problem can be overcome by applying geological information while mapping or generating a better initial permeability field. We are currently working on this extension. Nevertheless, the effective permeability of each streamline in the inferred permeability field is very close to that of the corresponding streamline in the reference permeability field. As a result, the fractional flow curve and pressure drop obtained for the inferred permeability field are quite close to the reference values.

Conclusions

This approach relates producer water-cut curves to the breakthrough time of individual streamlines. The effective permeability along streamlines is modified directly to history-match the fractional flow curve, pressure drop, and flow rate information. No matrix inversion is involved in the inverse process and therefore it is quite fast. The forward flow simulation with 3DSL is also fast and so, for the examples examined, the entire process appears to be computationally efficient.

The current work examined 2-D areal porous media, where the effect of gravity is not important, as well as incompressible two-phase flow. This approach works well for reservoirs where heterogeneity is a dominant factor. It also works well for unfavorable mobility ratios because the effects of heterogeneity are exacerbated by the unstable displacement. Although streamlines evolve for non-unit mobility ratio cases during the displacement process, we find it feasible to choose one streamline distribution and apply the simplified inverse system derived for unit-mobility ratio cases.

Importantly, the history matching results are independent of mobility ratio. A permeability field inferred by matching production data for one mobility ratio can be used to predict reservoir performance for displacement processes at other mobility ratios.

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Nomenclature

A	cross-sectional area of a streamtube, L^2
c	constant
E	absolute error
e	relative error
f_w	fractional flow of water
k	permeability, L^2
l	streamline length
M	mobility ratio
p	pressure, $M/(LT^2)$
q	flow rate, L^3/T
t_D	dimensionless time
V	pore volume, L^3
V_D	dimensionless volume (fraction of pore volume)
x_D	dimensionless length
ϕ	porosity

Subscripts:

n	producer index
i, j, k	streamline index
sl	streamline
BT	breakthrough
D	dimensionless

Superscripts:

C	Computed
R	Reference
λ	iteration index

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