

MULTIRESOLUTION REPARAMETERIZATION AND PARTITIONING OF
MODEL SPACE FOR RESERVOIR CHARACTERIZATION

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF PETROLEUM ENGINEERING
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Isha Sahni
August 2006

© Copyright by Isha Sahni 2006
All Rights Reserved

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

Roland N. Horne Principal Adviser

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

André Journal

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

Hamdi Tchelepi

Approved for the University Committee on Graduate Studies.

Abstract

This work develops a generalized wavelet-based methodology for stochastic data integration in complex reservoir models. This is an extension of our earlier work for simpler reservoir descriptions. A single history-matched reservoir permeability model is combined with a stochastic geological description to obtain multiple equiprobable reservoir descriptions using wavelet transforms of the parameter distribution (permeability). The algorithm has been extended and generalized to be usable with commercial reservoir simulation software and to enable handling of three-dimensional models and production scenarios. We also conducted a study of sensitivity coefficient distributions, thresholding and averaging techniques, and a comparison of different Haar wavelet implementations.

Wavelet coefficients of reservoir parameter distributions can, to some extent, be partitioned into sets of history-matching and geologic coefficients and modified independently. Inverse transformation of these coefficients yields multiple reservoir model results, all of them matched to history. A significant reduction in time can be obtained for stochastic modeling of reservoirs by the decoupling of production data and other parameters, since only a single history match is required.

Thus the proposed algorithm addresses the issue of stochastic modeling of complex reservoirs by integrating all available sources of information. From a single history-matched model we obtain a set of distinct equiprobable reservoir models that can then be used to evaluate uncertainty and make future production predictions and reservoir management decisions.

Acknowledgements

I would like to thank my advisor Professor Roland N. Horne for his advice, guidance, and encouragement during the course of this research. It was indeed an honor and a privilege to have had the opportunity to work under his guidance.

I would also like to extend my warm appreciation to Professor André Journal for his encouragement and constructive critique of my work over the years.

Financial support received from the Stanford Graduate Fellowship, H.L. and Janet Bilhartz-ARCO Fellowship is gratefully acknowledged. I am also thankful for financial support from Stanford University Petroleum Research Institute (SUPRI-D) and the Department of Petroleum Engineering.

I wish to express my appreciation for the help extended to me by Jorge Landa, and for his generosity and willingness to share ideas and techniques for this research. I am also thankful for many useful discussions and insights provided by my colleagues Pengbo Lu, Sunderrajan Krishnan, Inanc Tureyen and Burc Arpat.

I would like to thank my family for their love and support. A special thanks to my brother, Akshay for always pushing me to do my best, and to my parents for encouraging me to set my sights high. Warm recognition to my fiancé, Mayank, for his undying help and support during the course of my Ph.D. and for always believing in me.

Contents

Abstract	v
Acknowledgements	vi
1 Introduction	1
1.1 Statement of Problem	3
1.2 Literature Review	5
1.2.1 Reservoir Characterization and History Matching	5
1.2.2 Multiresolution Wavelet Analysis	7
1.2.3 Geostatistics and Data Integration	7
1.3 Overview	9
2 Mathematical Preliminaries	11
2.1 Modeling and Analysis of Physical Systems	11
2.1.1 Inverse Problems	11
2.1.2 Notation and the Objective Function	13
2.2 Optimization Techniques	16
2.2.1 Gradient-Based Optimization	16
2.2.2 Nongradient techniques	21
2.3 Concepts of Wavelet Analysis	22
2.3.1 The Wavelet Transform	23
3 Reservoir Modeling and Characterization	25
3.1 Multiresolution Description	25
3.1.1 Comparison of Pixel-based vs. Wavelet-based Algorithms	26
3.1.2 Data Compression using Fourier, SVD and Wavelet Analysis	27

3.1.3	Exploring Reservoir Models in Wavelet Space	41
3.2	Haar Wavelet Implementation Methodologies	55
3.2.1	Standard and Nonstandard Wavelet Decomposition	56
3.3	Sensitivity Calculations for Reservoir Parameters	61
3.3.1	Wavelet Reparameterization	65
3.4	Chapter Summary	66
4	Production Data Integration	68
4.1	Parameter Estimation	68
4.1.1	History Matching Algorithm	69
4.1.2	Gauss-Newton Method for Parameter Estimation	69
4.2	Sensitivity Thresholding Schemes	70
4.2.1	Sensitivity Coefficient Values as a Function of Time Step Number .	70
4.2.2	Effect of Thresholding Technique	75
4.2.3	Well by Well Thresholding	87
4.2.4	Thresholding Based on Data Type	110
4.2.5	Grayscale-based Thresholding	111
4.3	Three-Dimensional Data Integration	111
4.4	Chapter Summary	121
5	Geostatistical Data Integration and Extensions	126
5.1	Wavelet Decoupling and Geostatistical Data Integration	126
5.1.1	Simulated Annealing	128
5.1.2	Grayscale - Probabilistic History Matching	130
5.1.3	Analytical Development for Gaussian Distribution of Parameters . .	139
5.2	Logarithm Permeability Model	157
5.3	Changing Geological Scenario after History Match	158
5.4	Downscaling	162
5.5	Chapter Summary	168
6	Discussion and Future Directions	170
6.1	Directions for Further Study	171
6.1.1	Multipoint Statistics	171
6.1.2	Integration of Well Test and Seismic Data	173

A Reparameterization Techniques	175
A.1 Wavelets	175
A.2 One-Dimensional Haar Wavelet	176
A.2.1 One-Dimensional Haar Basis Functions	176
A.2.2 Wavelet Transform and Reconstruction	178
A.3 Two-Dimensional Haar Wavelet	179
A.4 Other Techniques for Data Compression	182
A.4.1 SVD-based method	183
A.4.2 Transform Compression	183
B List of Example Cases	187
B.1 Reservoir G1	187
B.2 Reservoir G1b	187
B.3 Reservoir 3A	187
B.4 Case 2B	188
Bibliography	198

List of Figures

1.1	(a) Reference permeability field (b) History-matched model using streamline algorithm shows streamline artifacts from Wang [5].	2
3.1	Original data distribution and singular value decomposition compression result for image compression. Compression ratio 0.2, Norm 2 Error = 22.0864.	28
3.2	Fourier transform compression result for image compression. Compression ratio = 0.2, 2-norm error = 21.7248.	29
3.3	Wavelet analysis compression result for image compression. Compression ratio = 0.20, 2-norm error = 20.1813.	29
3.4	Singular value decomposition compression result for image compression. Compression ratio 0.05, 2-norm error 40.2009.	30
3.5	Fourier transform compression result for image compression. Compression ratio = 0.05, 2-norm error = 34.2512.	30
3.6	Wavelet analysis compression result for image compression. Compression ratio = 0.05, 2-norm error = 30.778.	31
3.7	Singular value decomposition compression result for image compression. Compression ratio 0.01, 2-norm error 62.6898.	31
3.8	Fourier transform compression result for image compression. Compression ratio = 0.01, 2-norm error = 49.0244.	32
3.9	Wavelet analysis compression result for image compression. Compression ratio = 0.01, 2-norm error = 50.2461.	32
3.10	Comparison of 2-norm error magnitudes for SVD, HWT and FT compression of a Gaussian distribution.	33
3.11	Original data distribution and singular value decomposition compression result for image compression. Compression ratio 0.2, 2-norm error = 6.08.	34

3.12	Fourier transform compression result for image compression. Compression ratio = 0.2, 2-norm error = 9.22.	35
3.13	Wavelet analysis compression result for image compression. Compression ratio = 0.20, 2-norm error = 5.0E-14.	36
3.14	Singular value decomposition compression result for image compression. Compression ratio 0.05, 2-norm error 17.18.	36
3.15	Fourier transform compression result for image compression. Compression ratio = 0.05, 2-norm error = 12.34.	37
3.16	Wavelet analysis compression result for image compression. Compression ratio = 0.05, 2-norm error = 8.51.	37
3.17	Singular value decomposition compression result for image compression. Compression ratio 0.01, 2-norm error 31.81.	38
3.18	Fourier transform compression result for image compression. Compression ratio = 0.01, 2-norm error = 17.41.	38
3.19	Wavelet analysis compression result for image compression. Compression ratio = 0.01, 2-norm error = 18.73.	39
3.20	Comparison of 2-norm error magnitudes for SVD, HWT and FT compression of a channel distribution.	40
3.21	Sorted sensitivity magnitudes showing 45% of the highest valued coefficients being retained.	42
3.22	Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.	43
3.23	Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.	44
3.24	Sorted sensitivity magnitudes showing 35% of the highest valued coefficients being retained.	44
3.25	Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.	45
3.26	Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.	46
3.27	Sorted sensitivity magnitudes showing 25% of the highest valued coefficients being retained.	46

3.28	Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.	47
3.29	Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.	48
3.30	Sorted sensitivity magnitudes showing 15% of the highest valued coefficients being retained.	48
3.31	Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.	49
3.32	Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.	50
3.33	Sorted sensitivity magnitudes showing 5% of the highest valued coefficients being retained.	50
3.34	Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.	51
3.35	Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.	52
3.36	Producer 1 BHP and WCT results after thresholding compared with the historical production data.	53
3.37	Producer 2 BHP and WCT results after thresholding compared with the historical production data.	54
3.38	Producer 3 BHP and WCT results after thresholding compared with the historical production data.	55
3.39	Injector BHP results after thresholding compared with the historical production data.	56
3.40	Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.	57
3.41	Thresholded log permeability distribution based on sensitivity to production data using Standard implementation.	58
3.42	Standard and Nonstandard sensitivity coefficients to production data sorted in decreasing order.	59
3.43	Injector BHP comparison of Standard and Nonstandard implementation results with respect to historical production data.	60

3.44	Producer 1 WCT and BHP comparison of Standard and Nonstandard implementation results with respect to historical production data.	61
3.45	Producer2 BHP and WCT comparison of Standard and Nonstandard implementation results with respect to historical production data.	62
3.46	Producer3 BHP and WCT comparison of Standard and Nonstandard implementation results with respect to historical production data.	63
3.47	Venn diagram showing the complete set of wavelet coefficients corresponding to a permeability field, highlighting the fact that there exists a subset that constrains the model to production data.	67
4.1	Sensitivity map in wavelet space. Blue dots represent the complete set of wavelet coefficients. Red stars represent the subset of wavelet coefficients for which the sensitivity to BHP and WCT are plotted with time in Figures 3.40 through 3.42	71
4.2	Producer BHP sensitivity coefficient profile with time also showing the evolution of producer BHP (as closed circles).	72
4.3	Injector BHP sensitivity coefficient profile with time also showing the evolution of injector BHP (as closed circles).	73
4.4	Producer WCT sensitivity coefficient profile with time also showing the evolution of producer WCT (as closed circles).	74
4.5	Sensitivity map in wavelet space. Blue dots represent the complete set of wavelet coefficients. Red stars represent corresponds to the location of wavelet coefficient $w(14,3)$ for which the sensitivity to BHP and WCT are plotted with time in Figures 4.6 and 4.8.	75
4.6	Producer BHP sensitivity coefficient profile with time also showing the evolution of producer BHP.	76
4.7	Producer WCT sensitivity coefficient profile with time also showing the evolution of producer WCT.	77
4.8	Injector BHP sensitivity coefficient profile with time also showing the evolution of injector BHP.	78
4.9	Area under the curve and cutoff limit for producer BHP sensitivity to wavelet coefficient $w(14,3)$ with time.	79

4.10	Area under the curve and cutoff limit for injector BHP sensitivity to wavelet coefficient $w(14,3)$ with time.	80
4.11	Area under the curve and cutoff limit for producer WCT sensitivity to wavelet coefficient $w(14,3)$ with time.	81
4.12	Nonzero sensitivity maps using methodology 1 (area-under-the-curve) for thresholding.	82
4.13	Nonzero sensitivity maps using methodology 2 (minimum cutoff) for thresholding.	83
4.14	Producer BHP with time for reservoir HM1, showing the production history data along with results from the two thresholding techniques.	84
4.15	Injector BHP with time for reservoir HM1, showing the production history data along with results from the two thresholding techniques.	85
4.16	Producer WCT with time for reservoir HM1, showing the production history data along with results from the two thresholding techniques.	86
4.17	Reservoir G1b - sensitivity coefficients of all production data with respect to wavelet parameters, sorted in descending order, highlighting in black the top 25% sensitivity coefficients in magnitude.	87
4.18	Reservoir G1b - thresholded permeability field (md) using the top 25% wavelet coefficients of the permeability field that are highly sensitive to the overall field production history.	88
4.19	Producer 1 - production data match for permeability field shown in Figure 4.18.	89
4.20	Producer 2 - production data match for permeability field shown in Figure 4.18.	90
4.21	Producer 3 - production data match for permeability field shown in Figure 4.18.	90
4.22	Injector - production data match for permeability field shown in Figure 4.18.	91
4.23	Sorted sensitivity coefficients by well, highlighting in black the percentage of coefficients constraining data from each well.	93
4.24	Sensitivity coefficient maps by well, showing the subset of coefficients constraining data from each well.	94
4.25	Permeability distribution (md) corresponding to thresholding separately for each individual well as shown in Figure 4.24.	95

4.26	Reservoir G1b - sensitivity coefficients of all production data with respect to wavelet parameters, sorted in descending order, highlighting in black the top 12.5% sensitivity coefficients in magnitude.	96
4.27	Reservoir G1a Sensitivity Coefficient map showing location of subsets of highest sensitivity wavelet coefficients with respect to production from each well.	97
4.28	Reservoir G1b - thresholded permeability field (md) using the top 12.5% wavelet coefficients of the permeability field that are highly sensitive to the overall field production history.	98
4.29	Producer 1 - production data match for permeability field shown in Figure 4.28.	99
4.30	Producer 2 - production data match for permeability field shown in Figure 4.28.	99
4.31	Producer 3 - production data match for permeability field shown in Figure 4.28.	100
4.32	Injector - production data match for permeability field shown in Figure 4.28.	100
4.33	Sorted sensitivity coefficients by well, highlighting in black the percentage of coefficients constraining data from each well.	102
4.34	Sensitivity coefficient maps by well, showing the subset of coefficients constraining data from each well.	103
4.35	Permeability distribution (md) corresponding to thresholding separately for each individual well as shown in Figure 4.34.	104
4.36	Overall sensitivity coefficient magnitudes sorted in descending order, highlighting how the coefficients chosen by well correspond to the overall sensitivity distribution.	105
4.37	Reservoir G1a - Sensitivity coefficient map showing location of subsets of highest sensitivity wavelet coefficients with respect to production from each well.	106
4.38	Reservoir G1b - Thresholded permeability (md) using individual well thresholds set at [16% 19.9% 1.0% 6.8%] for each well respectively.	107
4.39	Producer 1 BHP and WCT production history match for thresholded permeability distribution as shown in Figure 4.38.	108

4.40	Producer 2 BHP and WCT production history match for thresholded permeability distribution as shown in Figure 4.38.	108
4.41	Producer 3 BHP and WCT production history match for thresholded permeability distribution as shown in Figure 4.38.	109
4.42	Injector BHP production history match for thresholded permeability distribution as shown in Figure 4.38.	109
4.43	Sensitivity coefficient maps showing location of subsets of highest sensitivity wavelet coefficients with respect to BHP data (top) and WCT data (bottom).	112
4.44	BHP (top) and WCT (bottom) sensitivity coefficient magnitudes sorted in descending order, highlighting the coefficients retained during the thresholding process.	113
4.45	Permeability distributions (md) obtained by thresholding based individually on BHP data (top) and WCT data (bottom).	114
4.46	Reservoir G1b location of subsets of highest sensitivity wavelet coefficients with respect to BHP and WCT production profiles.	115
4.47	Permeability distribution (md) corresponding to thresholding separately for each individual well as shown in Figure 4.45.	116
4.48	Producer 1 - production data match for permeability field shown in Figure 4.47.	117
4.49	Producer 2 - production data match for permeability field shown in Figure 4.47.	118
4.50	Producer 3 - production data match for permeability field shown in Figure 4.47.	119
4.51	Injector - production data match for permeability field shown in Figure 4.47.	120
4.52	Sensitivity coefficient magnitudes sorted by absolute value for Reservoir 3A.	121
4.53	Log permeability distribution by layers for layers 1 through 8 for Reservoir 3A computed using 35% of the wavelet coefficients with the highest sensitivity to production data (B.3).	123
4.54	WCT (%) and BHP (psi) with time for production from oil producing well Prod 1.	124
4.55	WCT (%) and BHP (psi) with time for production from oil producing well Prod 2.	125
4.56	BHP (psi) with time for production from water injection well INJ.	125

5.1	Permeability distributions with oriented artifacts caused by modifying sets of wavelet coefficients constraining only the corresponding orientations. . .	127
5.2	Reservoir model results obtained using random traversal to avoid oriented artifacts.	129
5.3	Binary wavelet mask. Probability of perturbation of ‘red’ wavelet coefficients is zero and ‘gray’ wavelet coefficients is one.	130
5.4	Grayscale wavelet mask. Probability of keeping a wavelet coefficient fixed for history-match may lie between zero and one.	131
5.5	Thresholded permeability distribution (log md) based on sensitivity to production data using Nonstandard implementation (refer to Section 3.2). . .	132
5.6	Random traversal showing number of visits to a particular wavelet coefficient node using the grayscaling method along with the nodes constrained to production data in the deterministic method.	133
5.7	Random traversal showing the perturbed and unperturbed wavelet coefficient node using the grayscaling method.	133
5.8	Reservoir model result (log-permeabilities in md) using grayscale sensitivity coefficients.	134
5.9	Variograms for the prior and history-matched model and variogram results for permeability fields obtained after optimization.	135
5.10	Producer 1 - production data match for permeability field shown in Figure 5.8.	136
5.11	Producer 2 - production data match for permeability field shown in Figure 5.8.	137
5.12	Producer 3 - production data match for permeability field shown in Figure 5.8.	137
5.13	Injector - production data match for permeability field shown in Figure 5.8.	138
5.14	Variance between the reference and resulting log permeability distributions.	138
5.15	Data integration: Methodology for Multivariate Gaussian permeability distributions.	142
5.16	Reservoir model result (log-permeabilities in md) using wavelet based <i>sgsim</i> .	144
5.17	Variograms for the prior and history-matched model and variogram results for permeability fields obtained after optimization.	145
5.18	Producer 1 - production data match for permeability field shown in Figure 5.16.	145
5.19	Producer 2 - production data match for permeability field shown in Figure 5.16.	146

5.20	Producer 3 - production data match for permeability field shown in Figure 5.16.	146
5.21	Injector - production data match for permeability field shown in Figure 5.16.	147
5.22	Difference between wavelet coefficients of reference permeability distribution and Result 1, showing also the wavelet mask.	147
5.23	Difference between history-matched permeability distribution and Result 1.	148
5.24	Variance between the reference and resulting log permeability distributions.	148
5.25	Cumulative production data match for permeability field shown in Figure 5.16.	149
5.26	Reservoir 2B: Sorted sensitivity coefficients.	150
5.27	Reservoir 2B: Thresholded permeabilities.	151
5.28	Reservoir 2B: Result 1.	152
5.29	Reservoir 2B: Result 2.	153
5.30	Reservoir 2B: Variograms.	154
5.31	Reservoir 2B: Prod1 BHP history data and projections.	155
5.32	Reservoir 2B: Prod1 WCT history data and projections.	155
5.33	Reservoir 2B: Injector BHP history data and projections.	156
5.34	Reservoir 2B: Difference between truth case (see Appendix B.4) and Result 2 (Figure 5.29).	157
5.35	Initial (isotropic) and prior (anisotropic) log permeability fields along with corresponding variograms in (1,1,0) and (-1,1,0) directions.	159
5.36	Log permeability field results for integration of anisotropic variogram in a history matched model with isotropic prior.	160
5.37	Variogram match results for integration of anisotropic variogram in a history matched model with isotropic prior. Black curves show the initial variogram and red curves show the target variogram and the matches obtained.	161
5.38	Standard deviation map of log-permeability results.	161
5.39	Coarse scale log-permeability distribution.	163
5.40	Permeability distribution substituted as a subset of a larger wavelet coefficient set along with wavelet mask for simulated annealing.	164
5.41	Complete wavelet coefficient set after downscaling using simulated annealing algorithm.	165
5.42	Downscaled log-permeability distribution obtained by inverse wavelet transform of full set of wavelet coefficients as shown in Figure 5.41.	166

5.43	Variograms for initial coarse scale permeability distribution, target fine scale variogram model and final variogram match after downscaling.	167
5.44	Venn diagram showing the total available space of wavelet coefficients for a reservoir model, highlighting the fact that there exists a subset that constrains the model to production data and one that constrains to the geostatistical properties of the property distribution.	169
6.1	Wavelet description of a channel reservoir: (top left) Reference reservoir training image as binary field (top right) wavelet coefficients corresponding to training image (bottom left) Reference reservoir training image as continuous field (bottom right) Showing the non-zero wavelet coefficients out of all wavelet coefficients on top right	174
A.1	Standard two-dimensional Haar wavelet basis (from [72]).	185
A.2	Nonstandard two-dimensional Haar wavelet basis (from [72]).	186
B.1	Permeability distribution (in md) for Reservoir G1 with well locations. . . .	188
B.2	Log permeability distribution (in md) for Reservoir G1 with well locations.	189
B.3	Isotropic variogram for Reservoir G1.	189
B.4	Reservoir G1b: BHP and WCT data for well Prod1.	190
B.5	Reservoir G1b: BHP and WCT data for well Prod2.	190
B.6	Reservoir G1b: BHP and WCT data for well Prod3.	191
B.7	Reservoir G1b: BHP and WCT data for well Inj.	191
B.8	Permeability distribution by layers for layers 1 through 8 for Reservoir 3A .	192
B.9	WCT (%) and BHP (psi) with time for production from oil producing well Prod 1.	193
B.10	WCT (%) and BHP (psi) with time for production from oil producing well Prod 2.	194
B.11	BHP (psi) with time for production from water injection well INJ.	195
B.12	Reservoir 2B: Permeability distribution by layers for layers 1 and 2.	196
B.13	Reservoir 2B: Variogram.	196
B.14	Reservoir 2B: Producer BHP and WCT.	197
B.15	Reservoir 2B: Injector BHP.	197

Chapter 1

Introduction

Reservoir characterisation is the process of developing a reservoir property model, specifically to determine the spatial distributions of properties such as porosity and permeability that are crucial to oil and gas production. History matching can be described as the process of modifying reservoir model properties in order to make sure that the simulated production data *matches* the actual field production data as closely as possible. The aim is to develop a reservoir model that would give the same production profile, historical and future, as the actual subsurface reservoir. Production data is just one type of data used to develop the reservoir model, forming part of the *dynamic* data which may also include pressure transients, long term pressure history, tracer tests etc. There are other types of data (well logs, core samples, three-dimensional seismic and geologic information), that are referred to as *static* data. Most of these are indirect sources of information about the reservoir. Thus we see that reservoir characterization is an inverse problem (see Section 2.1) since we need to *infer* reservoir properties using mostly indirect measurements along with very sparse direct core data at the well locations. Since the problem of reservoir characterization can be posed as an inverse problem, we can make use the extensive work done in the fields of inverse problem and optimization in order to solve our problem.

The ability to include both geological and production data uncertainty into the reservoir model automatically is of great consequence to reservoir modeling. A more complete and realistic reservoir model will lead to better reservoir production and development decisions. Thus, reservoir modeling is an important step in forecasting the performance of a reservoir, forming the basis for reservoir management, risk analysis and for making key economic decisions. A history match, however, is not a sufficient condition for a reservoir to make

better predictions for future production. The model should at least conform to all the available data and the geologists prior conception of the reservoir. Thus, the purpose of reservoir modeling is to use all available sources of information to develop such a reservoir model. This model then can be used to forecast future performance and optimize reservoir-management decisions.

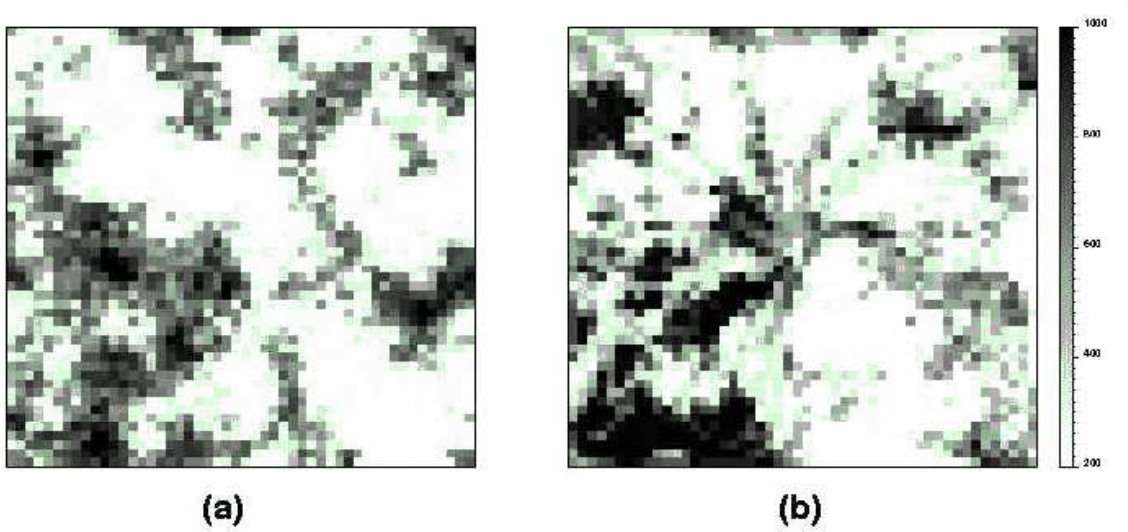


Figure 1.1: (a) Reference permeability field (b) History-matched model using streamline algorithm shows streamline artifacts from Wang [5].

In general practice, a reservoir model is first built using all the other information available, and the production data are then superimposed on the existing model by way of history matching. It has been shown [1, 2, 3, 4] that some methods of history matching might destroy (or remove) previously integrated geologic information and/or produce artifacts that are nongeologic in nature (Figure 1.1). The resulting reservoir models will then, as a result, match production data but may no longer be consistent with the geologic data that were integrated previously. As mentioned before, the purpose of reservoir modeling and history matching is not limited to building a model that is consistent with the production data currently available, but one that gives good predictions of its future behavior. Reservoir models that are inconsistent with the geology are not likely to give good forecasts. Therefore, it is essential to develop reservoir models that conserve geologic information while being consistent with production-history data at the same time. It should be noted here that just like production data, the geological *prior* also has some uncertainties associated with it, and

hence we need to be wary of generating unrealistic or oversimplistic high entropy geological models.

History matching and reservoir data integration have always had the reputation of being extremely slow processes. Even the faster, more efficient assisted/automatic history matching methods sometimes suffer from algorithmic artifacts, geological inconsistency and other limitations on the reservoir and fluid properties. Recent advancements in computational speed and memory have been trying to keep up with the more detailed reservoir property and fluid descriptions that are now being used to build reservoir models. Also, as real-time data acquisition becomes more and more popular, it is important to have a methodology that allows for the introduction of new data as it comes in, without disturbing the match to the data already integrated in the model.

1.1 Statement of Problem

The focus of this research has been to develop an automated way to generate multiple history-matched reservoir models with the inclusion of both geological uncertainty and varying levels of trust in the production data, using wavelet methods. As opposed to many previously developed automated history-matching algorithms, this methodology not only ensures geological consistency in the final models, but also includes uncertainty in the production data. A data distribution, say a permeability field, can be (reversibly) transformed into wavelet space where it is fully described by a set of wavelet coefficients. It was found that different subsets of the collection of wavelet coefficients can be constrained to: (a) the production history (*dynamic data*), and (b) the geological constraints (*static data*). This means the history match need only be performed once, using the first subset of coefficients, after which multiple realizations can be generated by adjusting just the second subset of coefficients. The methodology presented uses wavelets and flow simulation to interpret the production data as influencing a spatial distribution of wavelet coefficients. As compared to the direct integration of production data, this constraint on a subset of wavelet coefficients is easier to integrate with other sources of reservoir data such as seismic or well logs. It was found that as a result of this transformation, production data constrains a particular subset of the wavelet coefficients of the given reservoir model. This is analogous to the way that hard data from cores and soft data from seismic surveys constrain different regions of the reservoir.

Wavelets have been used in the petroleum industry [6] mostly for the analysis of temporal data (Athichanagorn et al. [7, 8]) and more recently for two- or higher dimensional reservoir description [9, 10, 11, 12, 13, 14]. The data-integration algorithm (as described in [1, 2, 3, 4] and in this work) uses multiresolution wavelet analysis for the efficient integration of different data into the reservoir model at appropriate scales. The algorithm also has the advantage of a drastic reduction in the number of parameters required for data integration. Moreover this approach allows for the partitioning of the parameters into subsets based on their sensitivity to the data to be integrated. Each set can be perturbed independently of the other to constrain to the corresponding data. In particular, once a reservoir is constrained to production data, other data, for example geostatistical information, or even subsequent production data can, to some degree, be integrated independently, without destroying the current history match. Stochastic modeling is performed, yielding several equiprobable reservoirs model solutions, all of which are constrained to all available sources of data.

Parameter reduction and sequential integration of production and geostatistical data is made possible in the proposed algorithm through the calculation of sensitivity coefficients. A sensitivity coefficient can be described as a derivative of the production data with respect to a single model parameter [15, 16]. As such it measures the significance or sensitivity of that model parameter to the production data. The efficient integration of all different sources of reservoir information, including geostatistical data and production history improves the overall reservoir description [17, 18, 5, 19, 20, 21]. Stochastic modeling enables the inclusion of some degree of uncertainty in the prediction of reservoir production for infill drilling, or secondary production strategies in mature fields. The key to stochastic parameter estimation comes with the use of the wavelet transform of the parameter distribution in place of the original parameter.

In this study, the types of data considered were: hard well data, production data and statistical data (histogram and variogram). Each of these different types of data inherently provide information about different support or resolutions in the reservoir, as is captured well by multiresolution wavelet analysis. In many cases, there are seismic and well test data available - that are at different resolutions yet. It is an inherent property of wavelets that enables them to be used to manipulate different resolutions of the problem independently and at the same time. Thus, using wavelets, these new types of data can potentially be integrated at the expected resolutions directly, thereby making the wavelet algorithm much more efficient than existing pixel-based methods (see Section 3.1.1). A more detailed

overview of these avenues of research is described in the following sections of the chapter.

For the integration of geological data (the variogram) the iterative optimization technique of simulated annealing was used. The objective function was defined as the absolute difference between the current and true variogram in permeability space. Thus the technique involved modifying wavelet coefficients in order to optimize the variogram in permeability space. A number of example reservoirs were used to test the algorithm. The resulting realizations all matched the production history response as well as the variogram constraint. A different and more efficient method can be applied for the special case in which the reservoir permeability model is a Gaussian random variable (see Section 5.1.3). This methodology is based on a theoretical calculation of relevant statistics in wavelet space. As such, the modification of coefficients is based on sequential rather than undirected iterative techniques and hence much faster and more intuitive.

1.2 Literature Review

1.2.1 Reservoir Characterization and History Matching

During the early days of oil production, little was understood about the subsurface reservoir and its properties. However, as technology progressed with time, it became possible to measure and gather indirect data about the subsurface and make use of physical laws in order to estimate reservoir characteristics. This in turn made it possible to make good guesses or predictions about reservoir performance and hence make better reservoir management decisions. For many years, this process was carried out by hand and subsequently using analog computers.

Variational Analysis In the early 1960s, Jacquard and Jain [22] revolutionized the process of data integration for the petroleum engineering by applying network and variational analysis concepts from electrical engineering for solving the inverse problem that is reservoir parameter estimation and production history matching. The 1970s saw the advent of gradient-based techniques for production history matching as Carter, Pierce, Kemp and William [23] found an efficient way of calculating derivatives of pressure data with respect to reservoir parameters like porosity and permeability for linear problems (the diffusion equation). These derivatives are called *sensitivity coefficients* and form a basis of this work (see Section 3.3).

Optimal Control Theory Gradient-based methods had been used for a long time to solve inverse problem in other fields and hence quickly gained popularity in reservoir characterization problems. Following this development, in 1973, Chen, Gavalas, Seinfeld and Wasserman [24] and Chavent, Dupuy and Lemonnier [25] used optimal control theory for the direct computation the gradient of the history matching objective function E (see Section 2.14) with respect to permeability and porosity. This method was based on using the reservoir flow equations along with their *adjoint* equations and did away with the need for calculating relatively more expensive sensitivity coefficients and was extendable to nonlinear cases. This method was later used by Watson, Seinfeld, Gavalas and Woo [26] and Yang and Watson [27] though it remained limited by the fact that is was not only difficult to implement, but also could only be used with optimization techniques for parameter estimation that were less efficient than those using sensitivity coefficients, like the Gauss-Newton method.

Gradient Simulator In 1989 Anterion, Eymard and Karcher [15] developed a methodology for the calculation of sensitivity coefficients which later came to be known as the gradient simulator. This method was applied successfully to a couple of test cases by Bissel, Sharma and Killough [28]. In 1991 Tan and Kalogerakis [29] used the approach elaborated by Anterion et al. [15] to compute sensitivity coefficients from an implicit numerical simulator. The implementation of this method was further improved by Tan in 1995 [30] and in 1996 its scope extended for application to object modeling [21, 17]. Lu [13, 14] developed a parallelized gradient simulator that he used for history matching using wavelet reparameterization.

Streamline simulation Streamline simulation [31] is a promising technique that offers computational efficiency while minimizing numerical diffusion in comparison to traditional finite-difference techniques. This method has gained popularity for its fast integration of dynamic data into those reservoir models and production scenarios that can be modelled using streamline simulators [20, 32].

Even today, as techniques for history matching such as *assisted* and *automatic history matching* gain more and more popularity, manual data integration is still commonly practiced in industry.

1.2.2 Multiresolution Wavelet Analysis

The development of multiresolution wavelet analysis revolutionized the fields of image analysis, signal processing and data compression within a few decades of their early applications [33]. In reality, most of the development of the wavelet theory was done in the 1930s, though at that time it was not part of a coherent theory. In recent times, the foremost expositions on publications on the theory, implementation and application of different types of wavelets are [34, 35, 36, 37]. Statistical and data analysis applications of wavelets were highlighted in particular in Ogden's work in 1997 [38]. Some important papers in the field of geophysics and multiscale analysis of rock structures have been reviewed in the work of Foufoulou-Georgiou and Kumar [39].

Wavelets set foot in the world of reservoir engineering in a significant way in the late 1990s, in which period some papers were published in the areas of reservoir data analysis and property upscaling [11, 40]. Around the same time, Kikani and He [41] and Athichanagorn et al. [7, 8] applied multiresolution wavelet analysis to long-term pressure data obtained from permanent downhole gauges. In 2000 and 2001, Lu developed the wavelet-based gradient simulator that used a reduced wavelet parameter set for history matching [13, 14]. The current work is partly an extension of Lu's research, incorporating the integration of other sources of data besides production history.

1.2.3 Geostatistics and Data Integration

It is essential to integrate all the different sources of data to provide the most complete reservoir model or models [5, 17, 18]. Our model certainty is always limited by the data available to us. As such, it is never possible to infer or develop a reservoir model with full certainty. However, the optimal use of all consistent data available will yield reservoir models that are less and less uncertain. Herein lies the significance of methodologies that can integrate different sources of reservoir information realistically and efficiently.

Geophysical Inverse Theory In 1986, Fasanino, Molinard, and de Marsily [42] made use of the kriging algorithm along with reservoir pilot points in an optimal control based history-matching procedure in order to integrate geostatistical data in their resulting model. Another significant development in the direction of integrating geostatistical data into history matching took place when in 1982 by the work of Tarantola and Valette [43].

Tarantola and Valette developed a geophysical inverse theory - a set of algorithms for generalized nonlinear inverse problems as are found in geophysical systems, using the least-square criterion. This field was further developed by Tarantola [44], Menke [45] and Parker [46]. Tarantola [44], described methods for data fitting and model parameter estimation in an inverse theoretical framework. Assuming multi-Gaussianity, Tarantola included *a priori* information into the optimization, by building it into the definition of the objective function.

Generalized Pulse Spectrum Technique A significant development for reservoir characterization and history matching was GPST - Generalized Pulse Spectrum Technique [24, 47, 48]. The GPST formulation did not involve the calculation of sensitivity coefficients and was limited in its application to certain inverse problems. This method [50, 49] was later used successfully for reservoir parameter estimation from pressure transient data, while constraining the model to geostatistical data by including it in the definition of the least square formulation of the objective function (see Equation 2.14 and [44, 45]). Between 1994 and 1998, this method was modified and extended to able to calculate sensitivity coefficients. In [51, 16, 52, 53, 54] it was shown how GPST could be used to compute sensitivity coefficients which can in turn be used in the Gauss-Newton algorithm for parameter estimation while using Tarantola's geophysical inverse theory to include *a priori* geostatistical data. This was the first noted integration of static geostatistical data with dynamic production data in a probabilistic framework.

Iteration-based Optimization Ounes, Bréfort, Meunier and Dupéré [55] used the iterative nongradient-based/nondirectional method of simulated annealing (see Section 2.2.2) for automatic history matching especially for problems in which derivatives are hard to compute. This method is known for its low computational efficiency, but in 1994 [56] and 1995 [57] Sultan, Ounes and Weiss showed how parallel computing could be employed to improve the performance of this method. The Genetic algorithm [58], was another technique applied for reservoir parameter estimation while including geostatistical constraints. This method suffered from some of the same shortcomings as the simulated annealing algorithm - slow convergence and computational inefficiency. However these were in many cases offset by the ease of implementation and integration of *static* data, as well as convergence to the global minimum of the objective function whereas getting trapped in local minima is a common stumbling block for the more efficient gradient based methods.

Reservoir data are, generally speaking, divided into two categories: production data, such as pressure and water-cut histories from wells, and all other sources of data, such as core samples, seismic, and well logs. This second category of data depends on reservoir properties like porosity and permeability in a relatively direct way. Core samples can be used to provide porosity and permeability measurements at specific locations (well locations); semivariograms [59, 60] obtained from outcrops, for example, act as spatial statistics information, and seismic surveys may provide three-dimensional impedance distributions that can be inverted and used as soft-conditioning data at the corresponding locations. These different sources of data can be combined together with different approaches (e.g., Bayesian probability techniques [21, 52, 61]) to give a single set of probabilities.

Production data (and well-testing data), however, are of a fundamentally different nature and can be looked at as reservoir response data. If the fluid-flow model is thought of as a function/operator and the reservoir features are parameters, then production data would be the result of applying this function to a given input or stimulus (for example, a well test or oil production). The function linking the production data and reservoir properties is based on flow equations and simulation, which renders the task of conditioning reservoir models to production data much harder than direct conditioning to hard data. Automated history-matching algorithms usually require iterative optimization techniques to match or honor production data [23, 24, 22]. A collection of production data is not easy to transform into a probability distribution in the Bayesian framework. This is the reason why the integration of these data types is difficult to do simultaneously [19, 20, 21].

1.3 Overview

The chapters of this thesis are organized in the order of flow of the procedure. Chapter 2 describes the mathematical theory of wavelet transforms and their different implementations and optimization techniques for solving general inverse problems. Chapter 3 explains wavelet analysis in the context of reservoir modeling and illustration of different Haar wavelet implementations with the help of an example reservoir model. Theoretical derivation for the applicability of *sgsim* in wavelet space is discussed. Production data integration methodology and various different ways of parameter reduction and partitioning (thresholding) are considered in Chapter 4. Chapter 5 shows how geostatistical data

is integrated using the partitioned set of wavelet coefficients, along with probabilistic history matching and practical applications. Key results are summarized and further research avenues are discussed in Chapter 6.

Chapter 2

Mathematical Preliminaries

In this chapter we provide a background into the various mathematical tools used in this work for the study of physical systems. The study of physical systems is a well established field with a vast literature, and is still very much an active area of research. We first describe the methodology that we adopt for analyzing a general physical system and will then specialize that procedure to the study of a reservoir. This procedure will require as input some important reservoir parameters (observed or calculated numerically). Hence, we will outline some of the algorithms required to implement the procedure and characterize the system behavior.

2.1 Modeling and Analysis of Physical Systems

In the investigation of physical systems, the prediction of observations is a forward problem while the use of actual observations to infer the properties of a model is an inverse problem. Inverse problems are difficult because they may not have a unique solution. Further, uncertainties play a central role in inverse problem theory and they are described mathematically under the framework of probability theory. We will describe the various aspects of inverse problem theory in the following sections.

2.1.1 Inverse Problems

The first step in the study of a physical system is to construct a mathematical model using the fundamental physical laws relevant to the problem. The purpose of the mathematical model is to predict with reasonable accuracy the behavior of the system under different

conditions. The problem of computing the response of the mathematical model to an external perturbation is referred to as the *forward* problem. The physical properties that remain invariant for different problems are referred to as *parameters* of the system. The properties that change are referred to as *variables*. The converse problem, the *inverse* problem consists of finding parameter values such that the system behavior predicted by the model mirrors the observed behavior under the same set of external conditions. In general, the standard methodology for the study of a physical system can be enumerated as follows [44]:

1. *Parameterization*: identification of a minimal set of model parameters that characterize relevant properties of the system accurately.
2. *Forward modeling*: prediction of the results of measurements on some observable parameters, for given values of the model parameters.
3. *Inverse modeling*: use of actual results of some measurements of the observable parameters to infer the values of the model parameters.

The specific physical systems being studied in this work are reservoirs. Forward modeling is a relatively mature field and numerical reservoir simulators have been developed and are widely used in the industry. In this work, we used a standard finite-difference reservoir simulator as our forward modeling tool as well as the sensitivity-coefficient generator. We therefore restrict our focus here to the parameterization and inverse modeling steps.

The process of inversion to determine values of reservoir parameters, such as permeability and porosity, from indirect measurements is referred to as a *parameter estimation* problem. The usual approach to solving the parameter estimation problem in general is by three major steps:

1. Construct a mathematical model.
2. Define an objective function.
3. Apply a minimization algorithm.

Once the mathematical model has been constructed, the objective function has been defined, and the minimization algorithm has been chosen, the procedure for inversion works in the following way:

1. Assign an arbitrary, but reasonable, value to the unknown set of parameters.
2. Compute the response of the system with the mathematical model.
3. Compute the objective function, which compares the calculated response of the system to the actual set of measurements. STOP if the objective function is less than a certain predetermined value.
4. Use the minimization algorithm to compute a change in the set of parameters. If the change in the set of parameters is less than a certain predetermined value then STOP.
5. Return to Step (3).

We will provide more details on the mathematical model in Chapter 3.

2.1.2 Notation and the Objective Function

We use the following notation: let N_{par} be the number of parameters that define the system, and N_{obs} be the number of observations. Then,

- $\vec{\alpha} \in \mathbb{R}^{N_{par}}$ is vector of system parameters:

$$\vec{\alpha} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_{N_{par}} \end{bmatrix}$$

- $\vec{d}^{obs} \in \mathbb{R}^{N_{obs}}$ and $\vec{d}^{cal} \in \mathbb{R}^{N_{obs}}$ are respectively the vectors of measurements and their corresponding values calculated by the mathematical model.

The objective function is a measure of the discrepancy between the measurement data and the system response as calculated by the mathematical model using the current set of parameters. There are different ways of quantifying the discrepancy and we choose for the purpose of this work the *Generalized Least Squares* (GLS) formulation. The GLS formulation allows one to introduce into the more standard least-square formulation both the *a priori* and the statistical information about the parameters of the system. The formulation is motivated by probabilistic considerations as we shall see in Section 2.1.2. A detailed derivation and exposition of related concepts can be found in [45] or in [44].

Probabilistic Uncertainty Model

A random vector $\vec{x} \in \mathbb{R}^N$ is said to be distributed as a Gaussian with mean $\vec{\mu}$ and covariance $C_{\vec{x}}$ if its probability density function is given by:

$$\mathcal{N}_{\vec{x}}(\vec{\mu}, C_{\vec{x}}) = \frac{1}{\sqrt{(2\pi)^N |C_{\vec{x}}|}} \exp\left[-\frac{1}{2}(\vec{x} - \vec{\mu})^T C_{\vec{x}}^{-1}(\vec{x} - \vec{\mu})\right]. \quad (2.1)$$

The model parameter space is denoted by M and the space of observable data by D . Thus, $\vec{\alpha} \in M$ and $\vec{d} \in D$. Now the measured data $\vec{d}^{obs}$ will contain some information regarding the true value of the observable data $\vec{d}$, which in turn will have some probabilistic dependence on the model parameter vector α . Let $f_{D,M}(\vec{d}, \vec{\alpha})$ be the joint distribution of the parameters $(\vec{d}, \vec{\alpha})$ with marginal densities given by:

$$f_M(\vec{\alpha}) = \int_D f_{D,M}(\vec{d}, \vec{\alpha}) d\vec{d}, \quad f_D(\vec{d}) = \int_M f_{D,M}(\vec{d}, \vec{\alpha}) d\vec{\alpha}, \quad (2.2)$$

and conditional densities given by:

$$f_{M|D}(\vec{\alpha}|\vec{d}^{obs}) = \frac{f_{D,M}(\vec{d}^{obs}, \vec{\alpha})}{f_D(\vec{d}^{obs})} \quad (2.3)$$

$$f_{D|M}(\vec{d}|\vec{\alpha}) = \frac{f_{D,M}(\vec{d}, \vec{\alpha})}{f_M(\vec{\alpha})}. \quad (2.4)$$

Combining Equations 2.2, 2.3 and 2.4 above we obtain the Bayesian formula:

$$f_{M|D}(\vec{\alpha}|\vec{d}^{obs}) = \frac{f_{D|M}(\vec{d}^{obs}|\vec{\alpha})f_M(\vec{\alpha})}{f_D(\vec{d}^{obs})}. \quad (2.5)$$

Next let $\theta(\vec{d}|\vec{\alpha})$ be the conditional probability density describing $\vec{d}$ as a function of the parameters $\vec{\alpha}$, and let $\nu(\vec{d}^{obs}|\vec{d})$ be the density function of the measurement output $\vec{d}^{obs}$ when the true value is $\vec{d}$. Then one can rewrite Eq. (2.5) as:

$$f_{M|D}(\vec{\alpha}|\vec{d}^{obs}) = \frac{f_M(\vec{\alpha})}{f_D(\vec{d}^{obs})} \int_D \theta(\vec{d}|\vec{\alpha}) \nu(\vec{d}^{obs}|\vec{d}) d\vec{d}. \quad (2.6)$$

A common assumption in the research literature is to consider the error in measurement to be independent of the true data, and the error in the theoretical prediction to be independent of the model parameter values. This allows one to simplify the form of the conditional error

probabilities as follows:

$$\theta(\vec{d}|\vec{\alpha}) = f_T(\vec{d} - \vec{d}^{cal}), \quad (2.7)$$

$$\nu(\vec{d}^{obs}|\vec{d}) = f_d(\vec{d}^{obs} - \vec{d}), \quad (2.8)$$

with density functions $f_d(\cdot)$ and $f_T(\cdot)$, and where $\vec{d}^{cal} = g(\vec{\alpha})$ is the prediction of the theoretical model. In addition, it is standard to treat the modeling and measurement errors as being Gaussian random vectors. That is,

$$\theta(\vec{d}|\vec{\alpha}) = \mathcal{N}_{\vec{d}}(\vec{d}^{cal}, C_T) \quad (2.9)$$

and

$$\nu(\vec{d}^{obs}|\vec{d}) = \mathcal{N}_{\vec{d}^{obs}}(\vec{d}, C_d) \quad (2.10)$$

which gives us

$$f_{D|M}(\vec{d}^{obs}|\vec{\alpha}) = \mathcal{N}_{\vec{d}^{obs}}(\vec{d}^{cal}, C_D) \quad (C_D = C_d + C_T). \quad (2.11)$$

Here, C_D is understood to be the covariance matrix for the data and it provides information about the correlation among the observations. In general, it is assumed that the different measurements are independent of each other in which case the covariance matrix is diagonal with the nonzero elements being the variance of the data (the square of the standard deviation σ_d^2). If one was to further assume that the model parameters also follow the Gaussian distribution, i.e.

$$f_M(\vec{\alpha}) = \mathcal{N}_{\vec{\alpha}}(\vec{\alpha}^{pri}, C_M) \quad (2.12)$$

then Eq. (2.6) becomes:

$$f_{M|D}(\vec{\alpha}|\vec{d}^{obs}) \propto \exp[-E(\vec{\alpha})] \quad (2.13)$$

where using standard formulae for conditional Gaussian random variates we have:

$$E(\vec{\alpha}) = \frac{1}{2}[(\vec{d}^{cal} - \vec{d}^{obs})^T C_D^{-1}(\vec{d}^{cal} - \vec{d}^{obs}) + (\vec{\alpha} - \vec{\alpha}^{pri})^T C_M^{-1}(\vec{\alpha} - \vec{\alpha}^{pri})]. \quad (2.14)$$

C_M is the covariance matrix of the parameters of the mathematical model and α_{pri} is *a priori* information about the parameters. α_{pri} is obtained before the application of the procedure for inversion and may come as the result of a previous inverse problem.

The goal of the inverse problem under the probabilistic uncertainty model is to find

the *maximum likelihood estimate* for α given $d^{\vec{obs}}$. This is equivalent to maximizing the conditional probability of α given $d^{\vec{obs}}$ which we calculated in (Equation 2.13). But this is the same as minimizing the function $E(\alpha)$ (given in Equation (2.14)), which we therefore define as the *objective function*. An early use of this approach in reservoir parameter estimation can be found in [50, 49].

2.2 Optimization Techniques

All optimization methodologies require the construction of an objective function which quantifies the degree of optimality of a solution. We saw in the previous section that the parameter estimation problem can be expressed in form of a minimization of a discrepancy term which is a function of the unknown parameters to be estimated. Thus, parameter estimation problems can be reduced to optimization problems, and hence we have all the various optimization techniques developed in other fields at our disposal for application to reservoir modeling. In particular, reservoir parameter estimation problems share the characteristic that the objective function $E(\cdot)$ such the one defined in Equation (2.14) is a *nonlinear* function of the underlying model parameters. Thus the algorithms are required to be iterative in nature, starting from an initial guess of parameters and progressing in the direction of decreasing objective function by successive modifications. There are different approaches to solving optimization problems which can be broadly classified as *gradient-based* and *nongradient*, depending on whether they use the gradient of the objective function or not.

2.2.1 Gradient-Based Optimization

As the name suggests, gradient-based algorithms [62] make use of the the derivative (or gradient) of the objective function $E(\vec{\alpha})$ with respect to the parameter $\vec{\alpha}$. The gradient of the objective function $E(\vec{\alpha})$ is defined as:

$$\nabla E(\vec{\alpha}) \equiv \left(\frac{\delta E}{\delta \vec{\alpha}} \right)^T. \quad (2.15)$$

Gradient-based algorithms are based on the principle that given an initial *nonzero* value of $\nabla E(\vec{\alpha}_0)$ it is always possible to reduce the value of E from its current value by introducing a step change in the value of the parameter $\vec{\alpha}$ in a *descent* direction. Mathematically, given

that $\nabla E(\vec{\alpha}_0) \neq 0$ there exists a unit vector $\vec{p}$ and a scalar $\rho > 0$ such that:

$$E(\vec{\alpha}_0 + \rho\vec{p}) < E(\vec{\alpha}_0). \quad (2.16)$$

The vector $\vec{p}$ specifies a direction in which the value of $E(\cdot)$ decreases and ρ is a positive scalar that specifies the *step size* in the direction $\vec{p}$. That a suitable $\vec{p}$ and ρ exists is easily seen from the 1st-order Taylor expansion of $E(\cdot)$ about $\vec{\alpha}_0$:

$$E(\vec{\alpha}_0 + \rho\vec{p}) = E(\vec{\alpha}_0) + \rho \nabla E(\vec{\alpha}_0)^T \vec{p} + \text{second order terms}. \quad (2.17)$$

Thus, it can be seen that it is always possible to find a positive value of ρ that will reduce $E(\cdot)$ provided $\vec{p}$ satisfies the following condition:

$$\nabla E^T \vec{p} < 0. \quad (2.18)$$

When $\vec{p}$ satisfies Eq. 2.18, it is said to be a *direction of sufficient descent*. Clearly, when $\nabla E \neq 0$, the existence of a suitable vector $\vec{p}$ and positive stepsize ρ is guaranteed.

In essence, a gradient-based algorithm works as follows:

```

procedure  GRADIENT
Initialize   $\vec{\alpha} := \vec{\alpha}_0$ 
Calculate   $\nabla E(\vec{\alpha})$ 
while     $\|\nabla E(\vec{\alpha})\| > \epsilon$ 
    Compute a direction of sufficient descent   $\vec{p}$ 
    Compute an adequate step size   $\rho$ 
     $\vec{\alpha} := \vec{\alpha} + \rho\vec{p}$ 
end while
end procedure
```

Here $\epsilon > 0$ is an arbitrarily small number which acts as a stopping condition since rarely is the numerically computed value of ∇E identically equal to 0. These methods are computationally very efficient and yield good convergence rates, though the gradient calculation is an expensive overhead. Gradient-based methods also suffer from the shortcoming of often converging to a local minimum instead of seeking out the globally optimum parameter values.

For gradient-based methods to be applicable it is required that the objective function E be sufficiently smooth and gradient calculations be possible. If we have an unconstrained optimization problem with a smooth objective function then the necessary conditions for optimality at a point $\vec{\alpha}^*$ are that $\nabla E(\vec{\alpha}^*) = 0$ and:

$$\vec{x}^T H^* \vec{x} > 0, \quad \forall \vec{x} \in \mathbb{R}^{N_{par}} \neq 0 \quad (2.19)$$

where H^* is the Hessian matrix evaluated at $\vec{\alpha}^*$ and defined as:

$$H^* = \left. \frac{\partial \nabla E}{\partial \vec{\alpha}} \right|_{\vec{\alpha}^*}. \quad (2.20)$$

Condition (2.19) is called the *positive-definiteness* property.

To describe the procedure for calculating a direction of sufficient descent we need some further analysis. Assuming that the objective function is smooth enough, it can be approximated in the neighborhood of $\vec{\alpha}_0$ using the Taylor expansion, i.e. for $\vec{\alpha} = \vec{\alpha}_0 + \delta \vec{\alpha}$:

$$\begin{aligned} E(\vec{\alpha}) &= E(\vec{\alpha}_0 + \Delta \vec{\alpha}) \\ &= E(\vec{\alpha}_0) + \nabla E(\vec{\alpha}_0)^T \Delta \vec{\alpha} + \frac{1}{2} \Delta \vec{\alpha}^T H_0 \Delta \vec{\alpha} + O(\Delta \vec{\alpha}^3) \\ &\approx E(\vec{\alpha}_0) + \nabla E(\vec{\alpha}_0)^T \Delta \vec{\alpha} + \frac{1}{2} \Delta \vec{\alpha}^T H_0 \Delta \vec{\alpha}. \end{aligned} \quad (2.21)$$

where H_0 is the Hessian of the function $E(\cdot)$ calculated at $\vec{\alpha}_0$.

The Hessian matrix H is the second derivative or curvature matrix of the objective function E . Another matrix of importance is the *sensitivity matrix* G defined as:

$$G = \frac{\partial \vec{d}^{cal}}{\partial \vec{\alpha}}, \quad (2.22)$$

which is shorthand for the matrix with elements:

$$g_{i,j} = \frac{\partial d_i^{cal}}{\partial \alpha_j}. \quad (2.23)$$

Thus the magnitude of $g_{i,j}$ is an indication of how much a change in α_j affects d_i^{cal} . Recall that the basic principle behind all gradient-based algorithm is to find a step change $\delta \vec{\alpha}$ that will reduce the objective function $E(\vec{\alpha})$ on the basis of the gradient ∇E . The following are some notable algorithms in the least-square minimization framework along with their

corresponding choices for $\vec{p}$:

- Steepest Descent:

$$\vec{p} = \frac{\nabla E}{\|\nabla E\|}. \quad (2.24)$$

- Gauss-Newton: $\vec{p}$ solves

$$H_{GN} \vec{p} = -\nabla E \quad (2.25)$$

where H_{GN} is the Gauss-Newton Hessian and is given by $H_{GN} = G^T C_D^{-1} G + C_M^{-1}$ for the GLS formulation.

- Singular Value Decomposition: $\vec{p}$ is the SVD-based solution to

$$G \vec{p} = \vec{d}^{obs} - \vec{d}^{cal}. \quad (2.26)$$

There are many other methods which we will only mention, chief among which are the Conjugate Gradient and Quasi-Newton, both of which have the common feature that they do not require the computation of the Hessian in order to obtain a descent direction. The proofs of the correctness, applicability and weaknesses of these algorithms have been studied in detail in the literature and choosing which algorithm to choose is still very much an art (see [62]). Some of these algorithms are summarized in the context of reservoir engineering in [63].

The history-matching algorithm works as follows:

1. Evaluate C_M and C_D . Determine d_{obs} . Determine $\alpha_{initial}$. Iteration n :
2. Run sensitivity calculation using α_{n-1} to evaluate d_{calc} and G_n (sensitivity Coefficients).
3. Evaluate F_n and H_{GN} .
4. Evaluate $\nabla(\alpha)$. update $\alpha_n = \alpha_{n-1} + \nabla(\alpha)$
5. $n = n + 1$. Repeat, until converged. = does that mean $F_n \rightarrow 0$.

Gauss-Newton Method for Parameter Estimation

The Newton method and its variation, the Gauss-Newton Method, are both gradient-based methods of optimization. In the previous section we saw the form of the Gauss-Newton update. We will be using the Gauss-Newton algorithm and an extension of it called the *Levenberg-Marquardt* method to solve our parameter estimation (history-matching) problem.

The objective function in the Generalized Least Squares formulation (from Equation (2.14) is:

$$E(\vec{\alpha}) = \frac{1}{2}[(\vec{d}^{cal} - \vec{d}^{obs})^T C_D^{-1}(\vec{d}^{cal} - \vec{d}^{obs}) + (\vec{\alpha} - \vec{\alpha}^{pri})^T C_M^{-1}(\vec{\alpha} - \vec{\alpha}^{pri})]. \quad (2.27)$$

Then we can calculate respectively, the gradient and the Hessian at some point $\vec{\alpha}$ as follows:

$$\vec{F} = \nabla E = G^T C_D^{-1}(\vec{d}^{cal} - \vec{d}^{obs}) + C_M^{-1}(\vec{\alpha} - \vec{\alpha}^{pri}) \quad (2.28)$$

and:

$$H = \nabla \vec{F} = G^T C_D^{-1} G + C_M^{-1} + \nabla G^T C_D^{-1}(\vec{d}^{obs} - \vec{d}^{cal}), \quad (2.29)$$

where:

$$G = \frac{\partial \vec{d}^{cal}}{\partial \vec{\alpha}} \quad (2.30)$$

is the sensitivity matrix. By ignoring the second-order term in (2.29) we obtain the Gauss-Newton Hessian matrix:

$$H^{GN} = G^T C_D^{-1} G + C_M^{-1}. \quad (2.31)$$

The Gauss-Newton algorithm works by starting with $\vec{\alpha} = \vec{\alpha}_0$ and iterating as follows:

$$\vec{\alpha}_{n+1} = \vec{\alpha}_n + \nabla \vec{\alpha}_n \quad (2.32)$$

where $\nabla \vec{\alpha}_n$ solves:

$$H_n^{GN} \nabla \vec{\alpha}_n = -\vec{F}_n. \quad (2.33)$$

Thus each iteration can be written in form of the following update:

$$\vec{\alpha}_{n+1} = \vec{\alpha}_n - \mu_n H_n^{-1} \nabla E_n \quad (2.34)$$

for an appropriately chosen step size $\mu_n > 0$. The Levenberg-Marquardt variation of the update defines the search direction in the following manner:

$$(H_n^{GN} + \nu_n I) \nabla \vec{\alpha}_n = -\vec{F}_n, \quad (2.35)$$

ν_n being a nonnegative scalar number. Adding a scaled identity matrix to the Gauss-Newton Hessian helps improve its condition number. The Levenberg-Marquardt method is most suitable for nonlinear least squares problems and is faster, requires fewer iterations and function evaluations and gives a result of the same level of accuracy as the other algorithms. Levenberg-Marquardt is a combination of the Steepest Descent method (slow but sure convergence) and Newton's Method (fast convergence close to optimum).

2.2.2 Nongradient techniques

A short but clear description of the use of nongradient methods such as *simulated annealing* and genetic algorithms for reservoir description can be found in [64]. These methods are attractive since they are relatively simple to implement, and do not require the computation of either ∇E or the sensitivity coefficients. Moreover, when the objective function E has numerous local minima, nongradient algorithms are often better able to reach a global minimum. The main disadvantage is that they are very expensive from the numerical point of view since they require a very large number of functions evaluations, and this may become critical when such functions evaluations involve the use of a numerical reservoir simulator.

Simulated Annealing

Simulated Annealing (SA) is a probabilistic metaheuristic for solving global optimization problems, that is, it is an algorithm that helps in finding an approximation to the global optimum of a general function. Simulated Annealing is typically used when the search space (domain of the function) is very large and/or the function being optimized lacks sufficient structure that can be exploited to differentiate global optima from local optima. SA's major advantage over other metaheuristics is its ability to avoid being trapped in a local extrema. This is because SA employs a random search strategy which, in the case of a minimization problem, not only accepts changes that decreases the objective function, but also some changes that cause it to increase.

The invention of SA [65, 66] was inspired by the *annealing* process in metallurgy where

cycles of controlled heating and cooling are used to enhance the crystalline nature of materials and reduce defects. SA is based on the Metropolis algorithm [67]. By analogy with the physical process, the SA algorithm modifies the current iterate of the optimization problem as follows: at each step the current solution is replaced by a random “nearby” solution, chosen with a probability that depends on the difference between the corresponding function values and on a global parameter T (called the temperature), that is gradually decreased during the process. The dependency is such that the current solution changes almost randomly when T is large, but progressively adopts a *downhill* trend as $T \rightarrow 0$. The allowance for *uphill* moves prevents the algorithm from becoming stuck at local minima, something which greedy or descent methods are unable to achieve in general.

A pseudocode for SA is as follows:

```

 $s := s_0; e := E(s)$   (Initial state, energy)
 $k := 0;$   (Energy evaluation count)
while  $k < k_{max}$  and  $e > e_{max}$ ,  (While time remains & not good enough)
 $s_n := neighbour(s);$   (Pick some neighbour)
 $e_n := E(s_n);$   (Compute its energy)
if  $e_n < e_b$  then  (Is this a new best?)
 $s_b := s_n; e_b := e_n$   (Yes, save it)
if  $random() < P(e, e_n, temp(k/k_{max}))$  then  (Should we move to it?)
 $s := s_n; e := e_n;$   (Yes, change state)
 $k := k + 1;$   (One more evaluation done)
returns;  (Return current solution).
```

2.3 Concepts of Wavelet Analysis

The data integration algorithm is based on the reparameterization of a reservoir parameter (for example, permeability) in terms of wavelet coefficients. We use the logarithm of the original parameter in order to perform wavelet analysis. Taking the logarithm of the permeabilities yields a set of unbounded real-valued Gaussian parameters. This has an added advantage since the evaluation of parameters in wavelet space may yield positive or negative values. Also, note that permeability being a Jeffreys parameter [44], upon taking logarithm it yields a Cartesian parameter.

2.3.1 The Wavelet Transform

Wavelets are mathematical functions with some special properties that were developed mathematically in the last 20 years and are being increasingly used in many different application areas [33]. In the most general terms, a wavelet transform presents a different way of storing and analysing data in terms of averages and differences. This special way of representing data turned out to be very useful in a number of applications and wavelets quickly became popular in a number of different fields of study. Some of the useful properties of wavelets are listed here.

- **Stability and Invertibility:** Wavelet functions are stable and invertible given the following condition:

$$C_\psi = \int_0^\infty \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty, \quad \psi(0) = 0 \quad (2.36)$$

where $\hat{\psi}$ is the Fourier Transform of ψ , the wavelet function. This ensures that the wavelet transform exists and is bounded. Also, this condition ensures that we can obtain an exact reproduction of original image by the inverse transform, without any loss of information.

- **Translation Invariance:** This means that translating the function is equivalent to translating the transform. In other words, each set of wavelet coefficients at a given resolution contains spatial information from the original image. Thus, if we know the spatial (or temporal) location of a data point in the function, we can pin-point the location of the corresponding wavelet coefficients that are associated with that particular data point. This property is not only helpful in local conditioning of data, but can also be used for spatial simulation of wavelet coefficients themselves for Gaussian distributions as described in Section 5.1.3. This is an important property for a transform and is absent in transforms such as the Singular Value Decomposition (SVD).
- **Time Frequency Localization:** The wavelet transform is designed such that it extracts information from objects (signals, functions or data) at different scales or frequencies. The scale and resolution are frequency dependent. That is, the wavelet transform picks out high frequency information at high resolution using a narrow template window and low frequency information at low resolution. Thus the wavelet transform window size adapts to suit the scale at which the information is to be stored. This is different

from say the Fourier Transform which picks out all the frequencies of information at all the scales. This inherent zooming in/out property of the wavelet transform enables multiscale, nonuniform parameter reduction (or ‘upscaling’, see Section 3.1.2) whereas Fourier transforms are limited to uniform parameter reduction.

- **Multiresolution Depiction:** Multiresolution analysis is the key property of wavelets that makes them very useful in many applications including the current one of parameter reduction and estimation. The aim of data compression is to reduce an enormous data set, saving only the most important and representative elements of the data set, while minimizing the loss of information or accuracy. Wavelets allow a direct encoding of data based on the resolution of details making it especially suited for the efficient analysis and parameter reduction of discontinuous functions. Information is stored at only the scales and locations at which it is significant and all redundant information can be discarded. This aspect of wavelets has made tremendous contributions in signal and image processing [37]. Wavelets are also useful for regression analysis for a very broad class of functions. For example, in linear regression, it is important to choose the simplest model that represents the data adequately so that there are fewer *parameters* to match. Wavelets offer this reduction of parameters while retaining information at the scales at which it is important. Wavelets also enable automatic multigrid representation and manipulation and the direct application of linear *block constraints*.

A more detailed presentation of the theory of wavelets can be found in Appendix A.1.

Chapter 3

Reservoir Modeling and Characterization

In Chapter 2 we explained in the most general terms the mathematical theory and tools that we used in the development of our algorithms. In this chapter, we will explain how the mathematical theory applies specifically to our current problem of reservoir characterization. In particular, the advantages of wavelet transforms have been widely studied in the fields of signal and image analysis. In this exposition, we will explore how these properties of wavelets interact with reservoir descriptions and parameter estimation.

3.1 Multiresolution Description

For estimation problems throughout this study we made use of the multiresolution Haar wavelet transform of the parameters instead of estimating the parameters directly. That is, instead of estimating values of permeability elements of the reservoir grid, we estimated the wavelet coefficients that correspond to the (log) permeability distribution. Here we describe what it means to take a wavelet transform of a permeability distribution, how the transformation relates to the production data profiles and two different ways of the implementing Haar wavelet transform in two dimensions.

3.1.1 Comparison of Pixel-based vs. Wavelet-based Algorithms

There are number of reasons why wavelet transformed parameters offer a big advantage over actual pixel parameters. The particular advantage of using wavelets is that the approach has the ability to constrain multiple scales of data simultaneously. This is useful because different sources of data provide information about the reservoir at different scales. In terms of data integration using wavelets, this implies that different types of data will potentially constrain different sets of wavelet coefficients that describe the reservoir at different resolutions [36, 37]. This technique is superior to purely pixel-based techniques [1, 2, 3, 4] because, besides providing the power to change the model at the highest resolution (pixel level) it also provides a more realistic higher level support for the different data types. In other words, the technique provides more degrees of freedom that can be modified independently for the purpose of constraining to geostatistical and production data [13, 14].

- Wavelet-based algorithms significantly reduce the number of parameters to be used for estimation, since they focus only on the significant coefficients at the appropriate resolution (see Section 2.3.1).
- Pixel-based methods are based on uniform grids, whereas wavelet based methods, being multiresolution in nature, enables us to obtain a nonuniform resolution in the parameter estimation. In other words, it is observed in most cases that in areas of the reservoir close to the well-bore, greater detail is retained, whereas in areas further away from the well-bores, only certain areal averages of parameters values that significant for a production match are conserved. Pixel-based methods on the other hand work only on a single scale, the pixel scale, and hence it would be impossible to constrain an areal average directly without constraining all the pixels it is composed of as well.
- In reservoir problems, not unlike any other modeling problem, we observe that different types of data may provide information at very different scales. For example in signal processing problems for discontinuous or ‘spikey’ signals, wavelets are able to resolve the information at different resolutions, at scales that are appropriate for the scales of the disturbance. Wavelet-based algorithms allow the flexibility of constraining parameters only at the appropriate resolutions, without disturbing constraints put by other data types at different scales.

3.1.2 Data Compression using Fourier, SVD and Wavelet Analysis

Many different techniques have been used for data compression in the fields of signal and data processing. The idea behind most of these applications is to be able to store or transmit the object (signal or data) using as few parameters as possible with the minimum amount of loss of information. One main factor for consideration is that this useful information may exist at different scales or frequencies within the object.

Using examples, we demonstrate the data compression properties of the following three mathematical tools:

- Singular Value Decomposition (SVD).
- Discrete Fourier Transform (FT).
- Haar Wavelet Transform (HWT).

Two example distributions are used for this demonstration. The first example consists of a Gaussian distribution while the second is based on a channelized reservoir model.

Gaussian Distribution Figure 3.1 shows the original permeability distribution that is used for the demonstration. This distribution is of size 64×64 and hence contains a total of 4096 individual data points. The objective is to obtain a good reproduction of this initial distribution using a small fraction of the total parameters. The key to meeting this objective is to be able to pick out the the main features of the distribution (or image) and reproduce those while ignoring the less significant features. It is important to note that in the current study of data compression, this distinction between *significant* and *insignificant* parameters is directly dependent on the data distribution itself. In reservoir engineering terms, if this distribution of permeabilities is considered to be a *prior* of a reservoir model, the data compression process will try to pick out and retain the key features of the *prior*. In other words, the parameter distribution obtained in the end will be a form of *static* upscaling [9, 68].

We start with a compression ratio of 0.2, which means that 20% of the original number of parameters are retained to generate the compressed image using SVD, FT and HWT respectively. In other words, a total of 820 parameters (out of a total of 4096) are chosen while the rest are ignored (set to zero values) in order to obtain a compressed reproduction of the image. These compressed images are shown in Figures 3.1 through 3.3. Figures 3.2

and 3.3 also show a map of the most significant Fourier and Wavelet coefficients respectively that were used in the inverse transform to obtain the compressed or upscaled representations - the remaining coefficients were set to zero values.

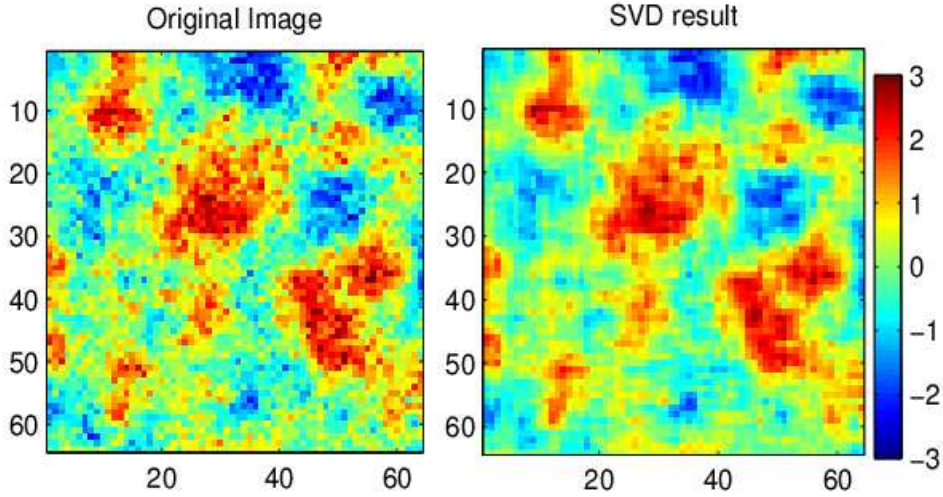


Figure 3.1: Original data distribution and singular value decomposition compression result for image compression. Compression ratio 0.2, Norm 2 Error = 22.0864.

We define the error corresponding to each compressed image as the 2-norm difference between the reproduction and the original distributions. According to this measure of error, we see that using 20% of the total number of parameters in each case, the HWT is able to generate a better approximation of the initial permeability distribution, followed by the FT and the SVD. We repeat this experiment using compression ratios of 0.05 (Figures 3.4 to 3.6) and 0.01 (Figures 3.7 to 3.9) for all three reparameterization methods respectively.

We observe that for this Gaussian distribution compression example, the HWT and FT perform better than SVD. Figure 3.10 compares the error associated with the reproduction of the Gaussian distribution using of each of three methods as a function of compression ratio. We observe that for this example, at a lower degree of compression (that is, at higher compression ratios), all three methods are comparable in performance, though the HWT does marginally better than the other two methods. As the data is compressed further, the SVD fails to give a good reproduction of the reference image, whereas the HWT and FT are comparable in performance, with the HWT again doing a marginally better job.

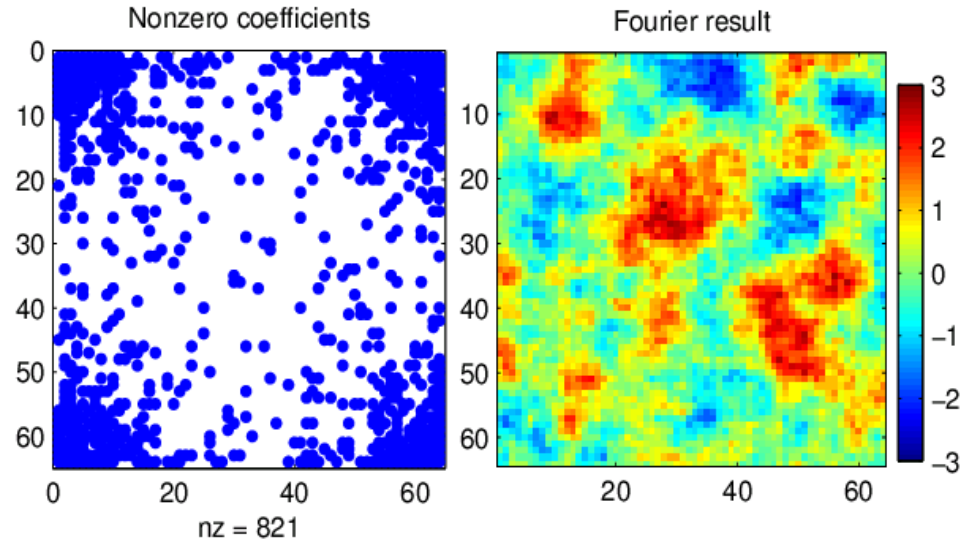


Figure 3.2: Fourier transform compression result for image compression. Compression ratio = 0.2, 2-norm error = 21.7248.

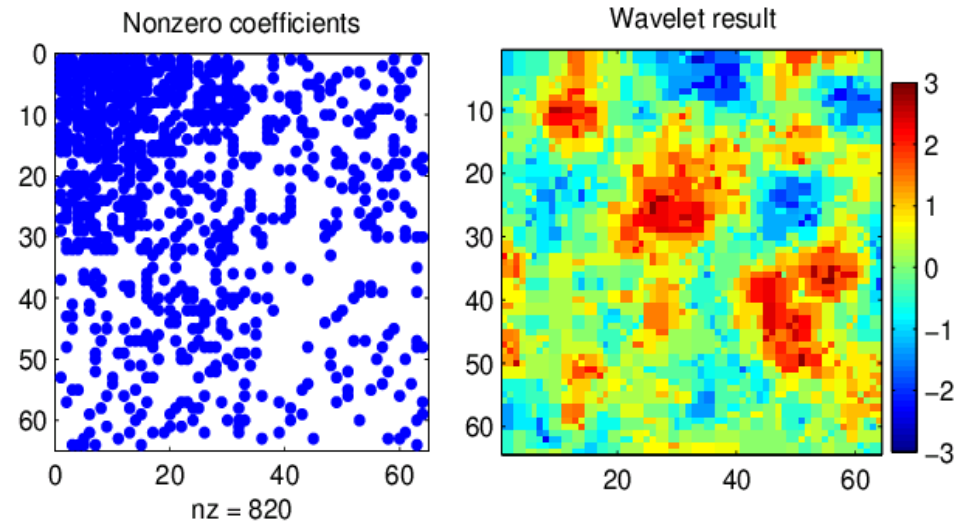


Figure 3.3: Wavelet analysis compression result for image compression. Compression ratio = 0.20, 2-norm error = 20.1813.

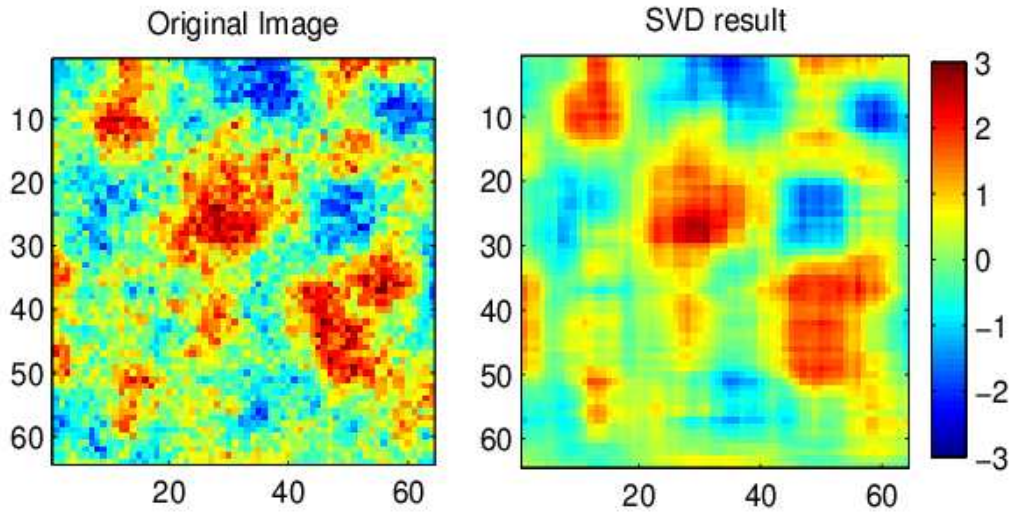


Figure 3.4: Singular value decomposition compression result for image compression. Compression ratio 0.05, 2-norm error 40.2009.

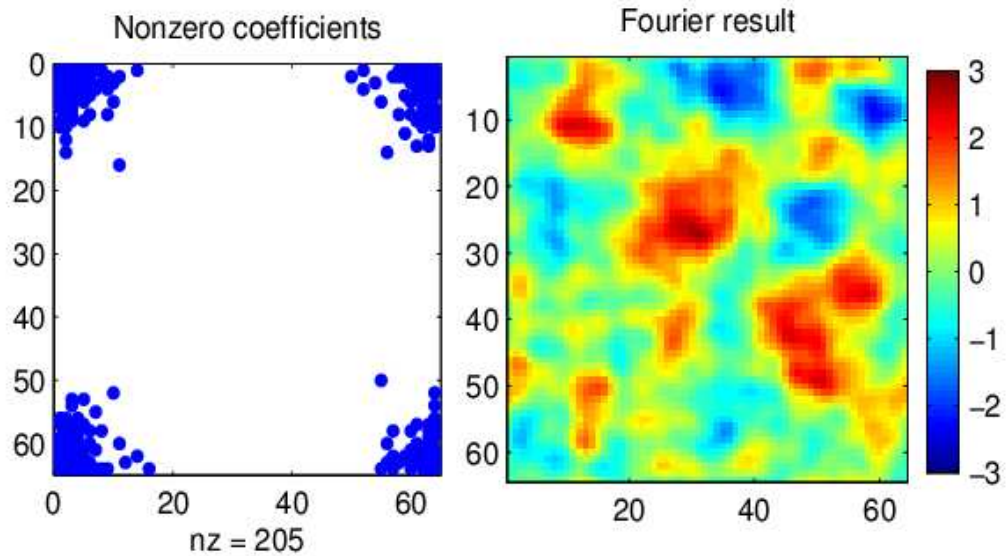


Figure 3.5: Fourier transform compression result for image compression. Compression ratio = 0.05, 2-norm error = 34.2512.

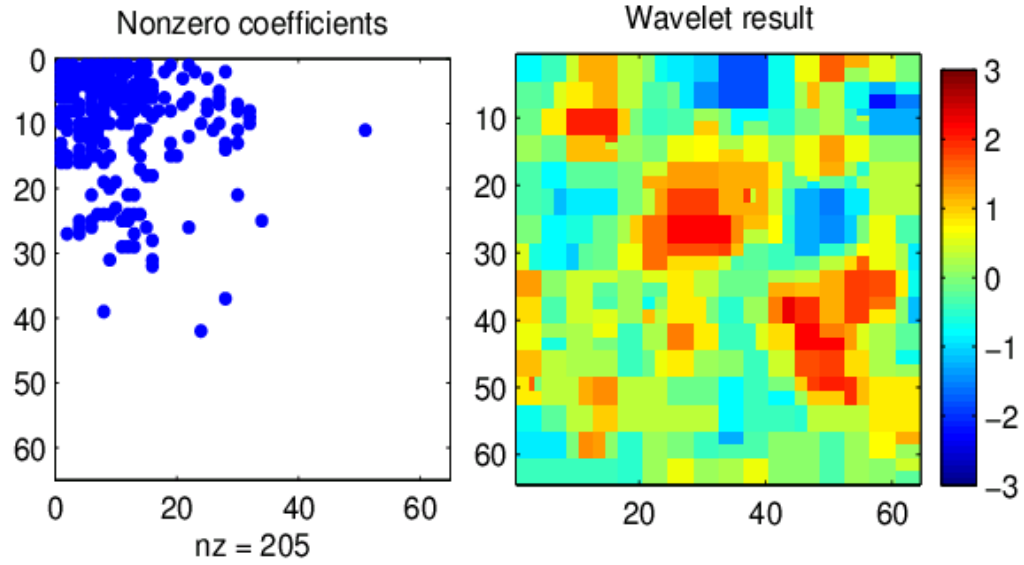


Figure 3.6: Wavelet analysis compression result for image compression. Compression ratio = 0.05, 2-norm error = 30.778.

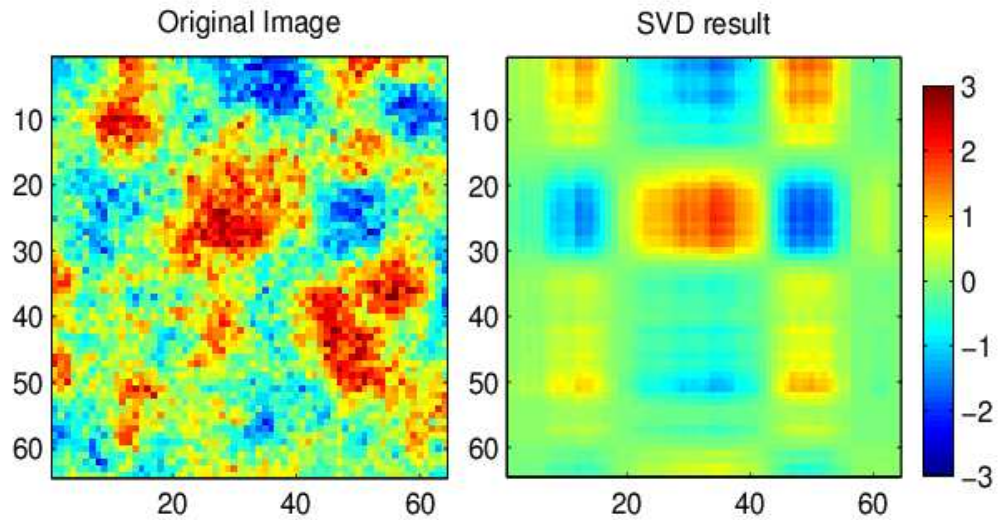


Figure 3.7: Singular value decomposition compression result for image compression. Compression ratio 0.01, 2-norm error 62.6898.

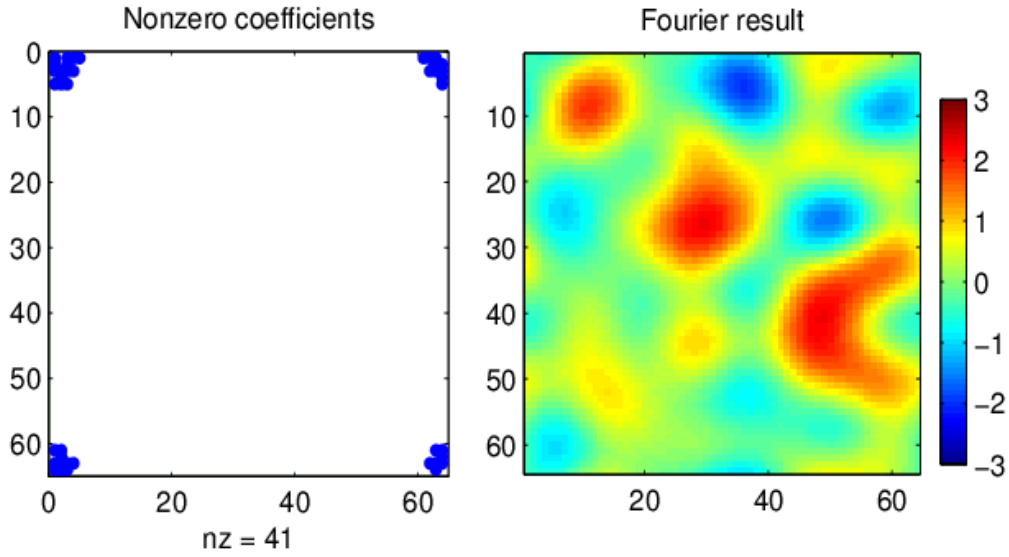


Figure 3.8: Fourier transform compression result for image compression. Compression ratio = 0.01, 2-norm error = 49.0244.

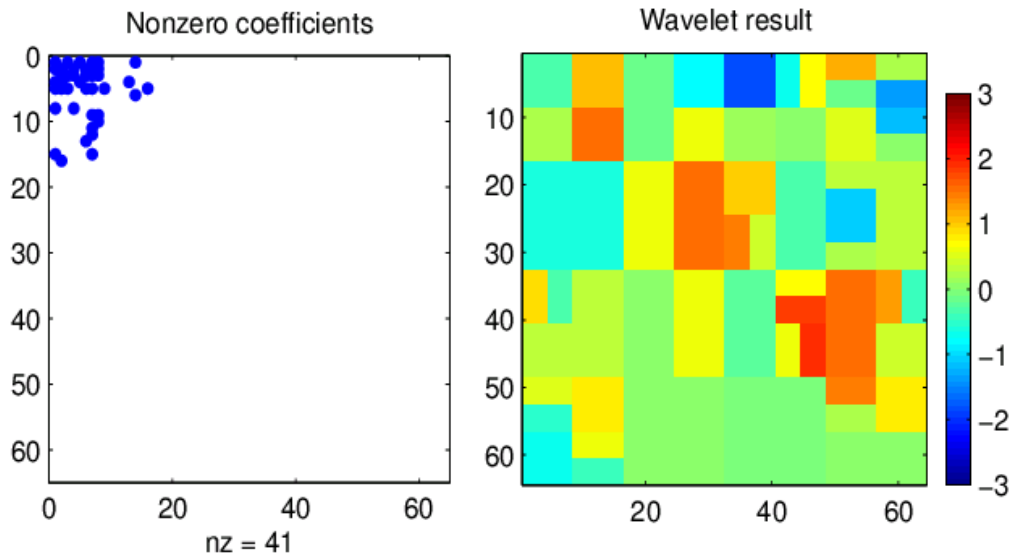


Figure 3.9: Wavelet analysis compression result for image compression. Compression ratio = 0.01, 2-norm error = 50.2461.

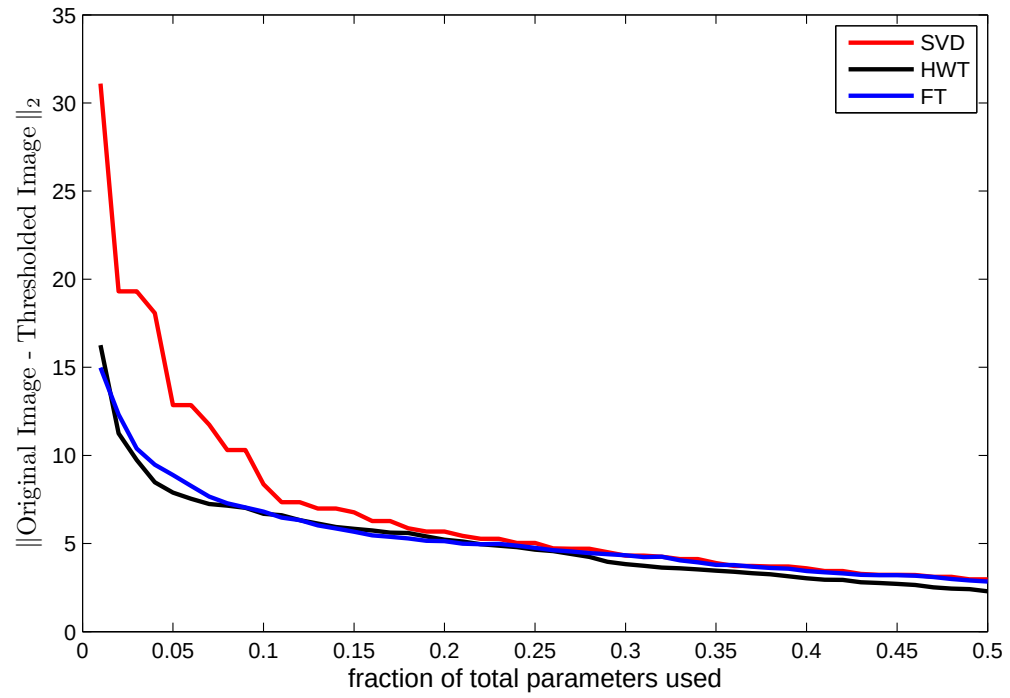


Figure 3.10: Comparison of 2-norm error magnitudes for SVD, HWT and FT compression of a Gaussian distribution.

Channelized reservoir The second example we show is based on a channelized reservoir model. This second example has a much more structured distribution than the Gaussian example. We performed the same series of compression experiments as performed on the Gaussian case. Figures 3.11 to 3.13 document the results on using 20% of the total number of parameters, Figures 3.11 to 3.13 to 5% and Figures 3.17 to 3.19 to 1% parameters. We observe that for a compression ratio of 0.20 and higher the Haar Wavelet compression gives an exact reproduction of the original, to the level of accuracy that machine error allows. This can also be observed from Figure 3.20 which compares the error for all three methods at different levels of compression of the channelized distribution. We see that the performance of the FT and the SVD are similar to each other, both showing higher errors as compared to the HWT for those compression ratios. As the image is compressed more, the performance of the HWT deteriorates, and using 5% of the original coefficients or less, we see that the FT performance is marginally better than that of the HWT.

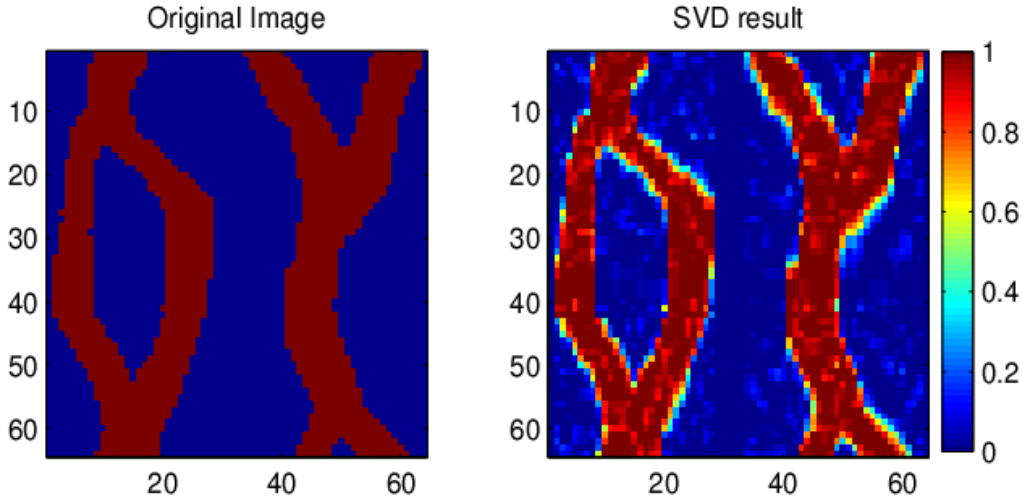


Figure 3.11: Original data distribution and singular value decomposition compression result for image compression. Compression ratio 0.2, 2-norm error = 6.08.

In summary, in most cases studied we observed that the SVD is least effective, followed by the FT and the HWT. The *performance* or *effectiveness* of the compression is dependent on the metric used to compare the compressed reproduction with the original image. In our case, we use the 2-norm difference between the two images for comparison. In other applications some other metric may prove to be a better measure of effectiveness. Based

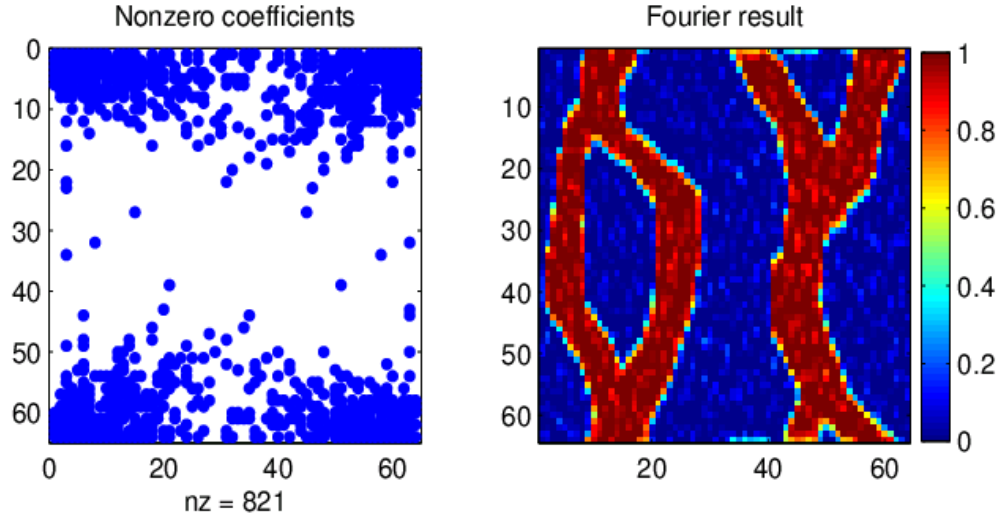


Figure 3.12: Fourier transform compression result for image compression. Compression ratio = 0.2, 2-norm error = 9.22.

on the two examples discussed we can also conclude that the compression performance depends on the nature of the image to be compressed (for example smooth/discontinuous, multiscale/homogeneous). We observed that the performance of the different techniques varied between the Gaussian and the more structured, channelized reservoir case.

Hence we see that Haar wavelets are effective tools for upscaling [68], and that they capture details at different frequencies at an appropriate scale. As described in Appendix A.3, thresholding is the technique used for parameter reduction in image (or signal) compression applications of wavelets. Thresholding is thus performed on the wavelet coefficients corresponding to pixels of the image (or signal). In effect, through thresholding, the low contrast details of the image (or signal) are removed while fine scale details are preserved in areas of higher contrast [69]. Thus, when reverse transformation is performed, we obtain a multiresolution reconstruction of the original object with fine details in areas of high contrast and smoother descriptions elsewhere. A measure of *compressibility* of an object with respect to a certain reparameterization technique is based on the maximum percentage of parameters that can be ignored while maintaining a certain level of *accuracy* for the reproduced image vis-a-vis the original. The time-frequency localization and multiresolution nature of wavelet allows them to zoom-in or zoom-out as required (see Section 2.3.1 for details).

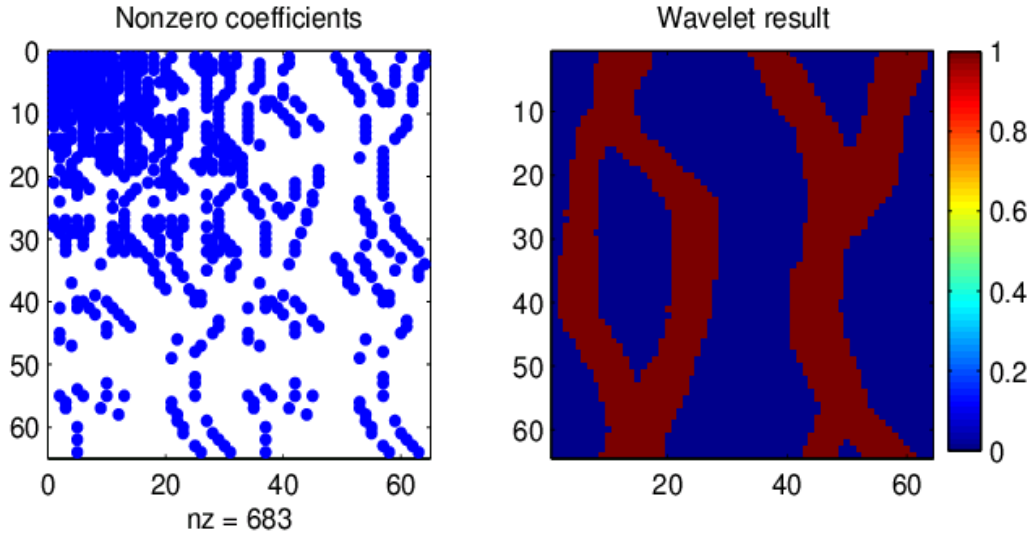


Figure 3.13: Wavelet analysis compression result for image compression. Compression ratio = 0.20, 2-norm error = 5.0E-14.

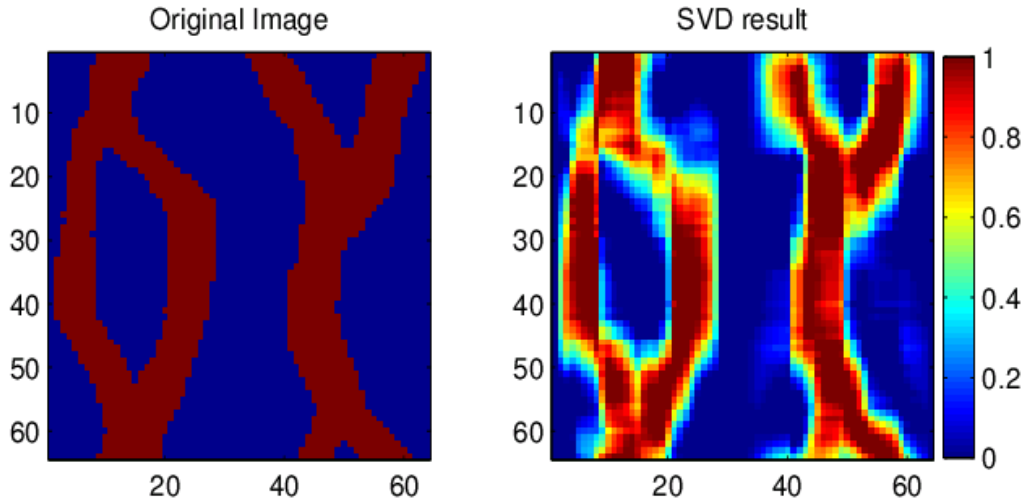


Figure 3.14: Singular value decomposition compression result for image compression. Compression ratio 0.05, 2-norm error 17.18.

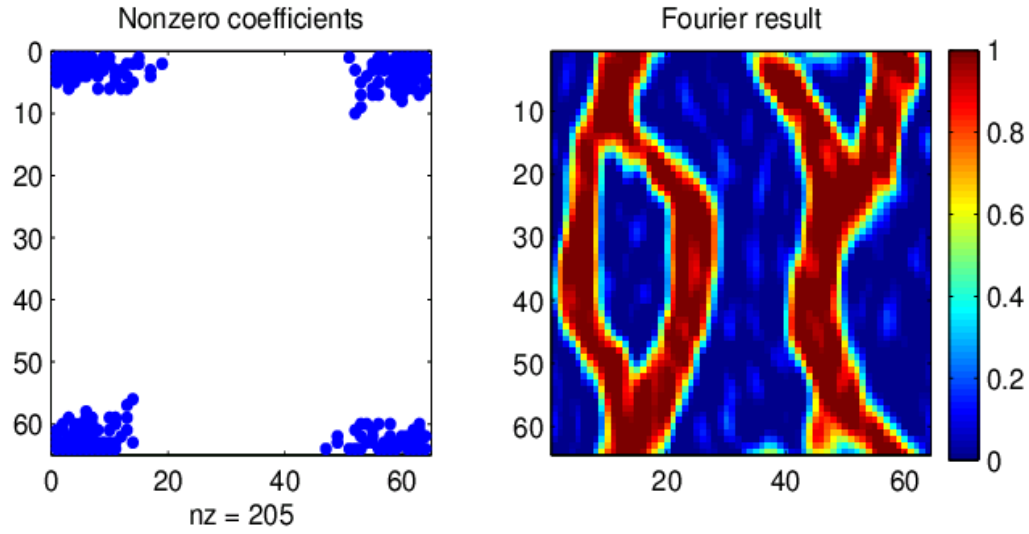


Figure 3.15: Fourier transform compression result for image compression. Compression ratio = 0.05, 2-norm error = 12.34.

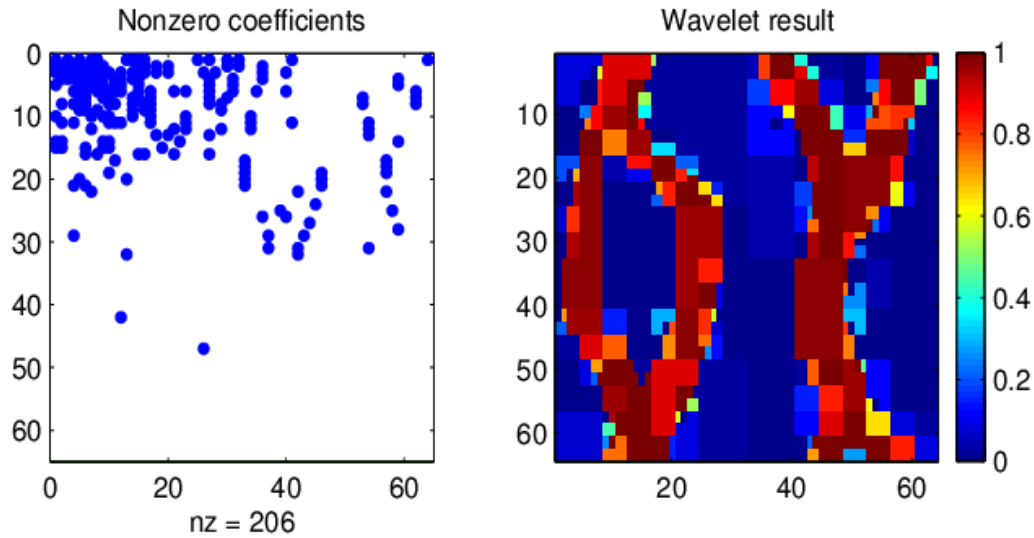


Figure 3.16: Wavelet analysis compression result for image compression. Compression ratio = 0.05, 2-norm error = 8.51.

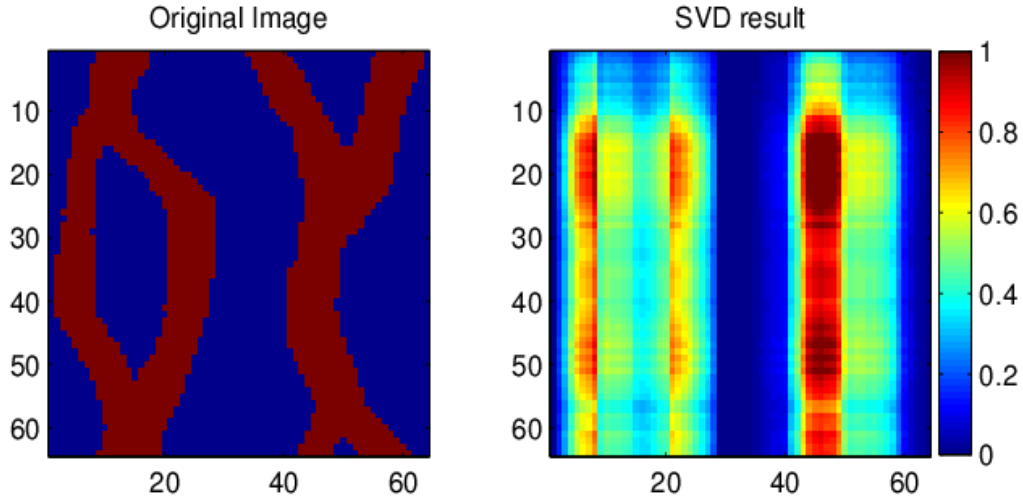


Figure 3.17: Singular value decomposition compression result for image compression. Compression ratio 0.01, 2-norm error 31.81.

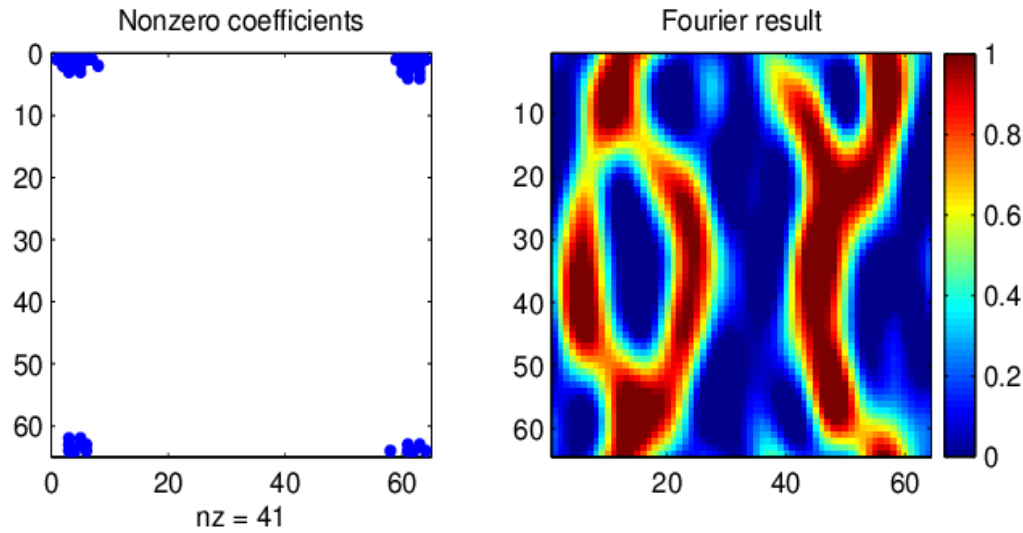


Figure 3.18: Fourier transform compression result for image compression. Compression ratio = 0.01, 2-norm error = 17.41.

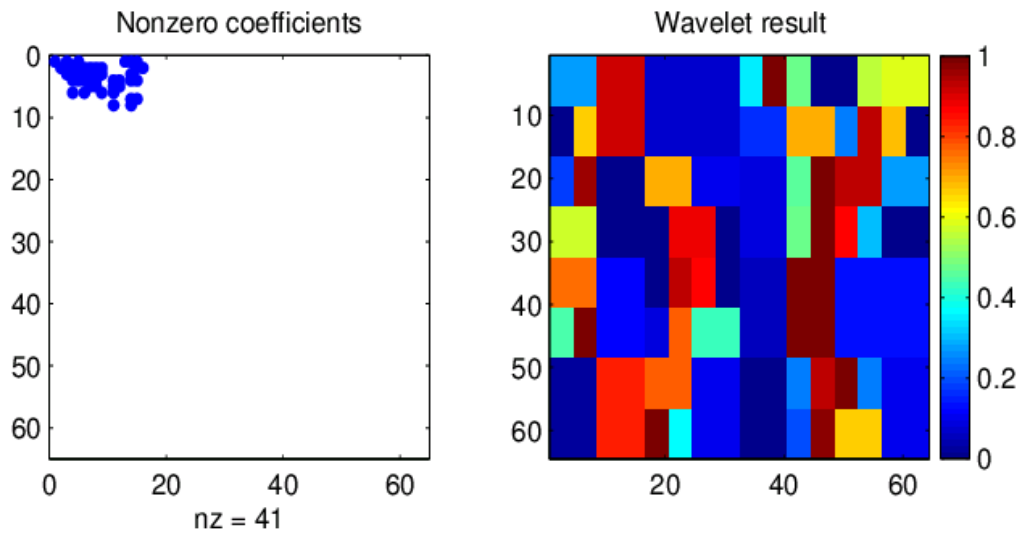


Figure 3.19: Wavelet analysis compression result for image compression. Compression ratio = 0.01, 2-norm error = 18.73.

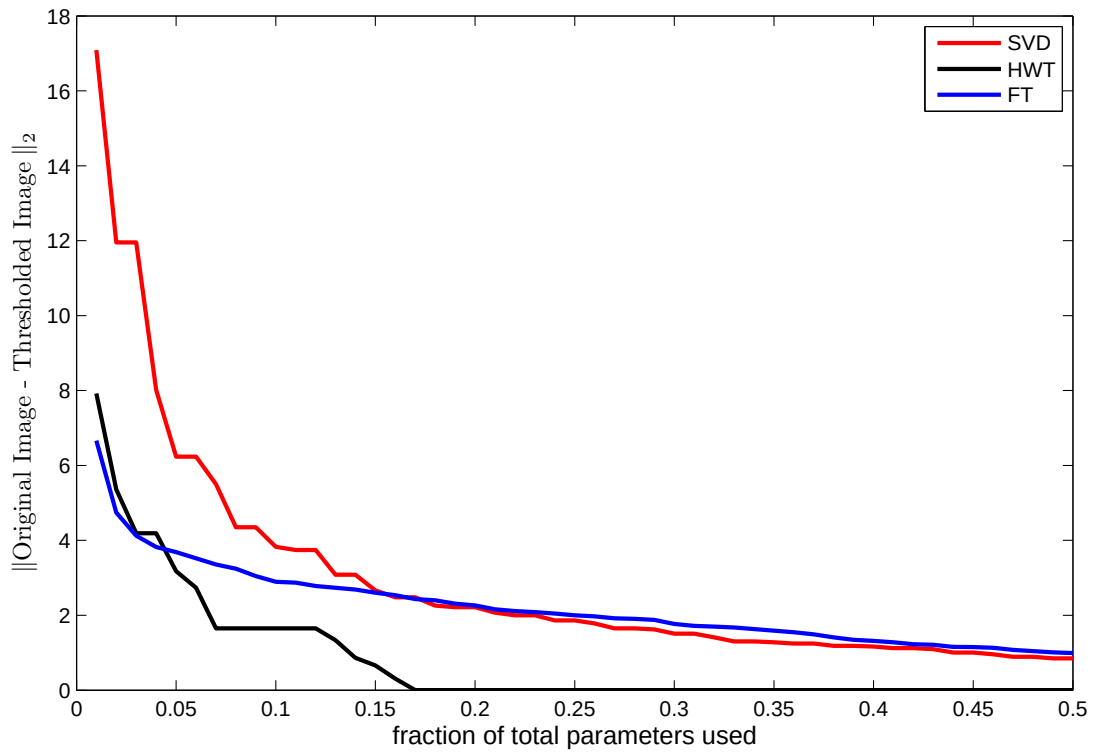


Figure 3.20: Comparison of 2-norm error magnitudes for SVD, HWT and FT compression of a channel distribution.

3.1.3 Exploring Reservoir Models in Wavelet Space

Section 3.1.2 described the compression properties of the Haar Wavelet Transform as compared to SVD and FT for *static* upscaling, that is, upscaling based solely on the features of the image. This is because in the context of image compression, the objective is to obtain a good reproduction of the visual aspects of an image, which is in turn related to shapes and contrast in the original image. However, for reservoir modeling, the focus is more on retaining the flow and production properties rather than just the structural features of the model. Of course, it should be noted here that the production profiles from a reservoir are in turn dependent on the key underlying geologic features through physical laws and flow equations.

Since the measure of effective reservoir model upscaling (or compression) is fluid flow, thresholding in this case is based not on the magnitudes of the wavelet coefficients corresponding to the permeability field, but on the magnitude of *production data sensitivities* that corresponds to each wavelet coefficient. This is the key difference between simple image compression applications of wavelets and the current context of reservoir parameter analysis and estimation.

Sensitivity coefficients are explained in greater detail in Section 3.3. In brief, the value of each sensitivity coefficient quantifies the significance of the corresponding wavelet coefficient of the permeability field to production history data. Thus we see that in our case, the reservoir parameter distribution is thresholded not on the basis of the values of its wavelet transform but on the basis of the absolute value of the corresponding sensitivities. Hence, we can expect that the resulting reproduction of the reservoir parameter field will preferentially retain the reservoir characteristics that yield a production profile that is close to the production history of the original distribution. Also, a key feature of wavelet-based thresholding is that the reproduced field will be constrained to block averages and contrasts of the permeability distribution at different resolutions, and not the individual gridblock values. One direct result of this is that we can expect to see fine details reproduced in areas close to wells and only block averages elsewhere. Since this idea of thresholding based on sensitivity to production data is central to our data-integration algorithm, we will explain it further with an example.

Consider an example reservoir model with permeability distribution and other properties as described in Appendix B.1. We have production history from the four wells in this reservoir model - three producers and one injector. The objective of this experiment is so

explore how the production history of a reservoir model is affected by wavelet thresholding based on sensitivities. As such, we perform sensitivity calculations for the wavelet transform of this reference permeability field, with respect to the historical WCT and BHP data. Based on the magnitudes of these sensitivity coefficients, the wavelet transform of the reference field is thresholded, retaining only a certain percentage of the original number of parameters. In this case, since the reservoir dimensions are 32×32 gridblocks, we have in total 1024 permeability parameters.

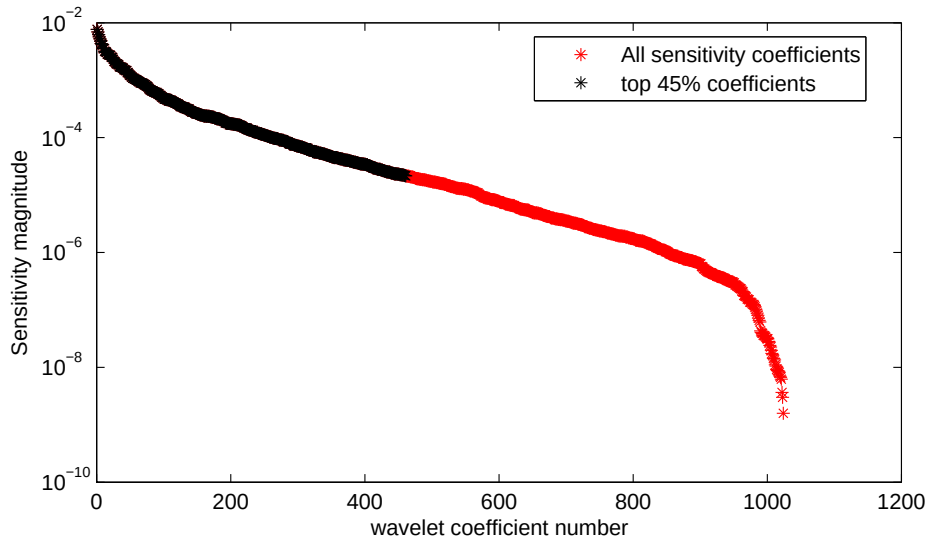


Figure 3.21: Sorted sensitivity magnitudes showing 45% of the highest valued coefficients being retained.

We start by choosing 45% of these original parameters to describe the permeability distribution. Figure 3.21 shows the sorted sensitivity coefficients, highlighting the top 45% in magnitude. Figure 3.22 maps with dots the locations of the corresponding wavelet coefficients of the permeability field. Using this truncated set of wavelet coefficients, setting the rest to zero, we perform an inverse wavelet transform. The permeability field obtained upon this inversion is depicted in Figure 3.23. Comparing this permeability distribution with the reference distribution, we see that fine details are retained in locations surrounding the wells, and block averages appear in areas away from the wells. At the next stage, we start again with the reference permeability distribution but this time we retain only the top 35% coefficients (see Figures 3.24 and 3.25). The permeability field associated with this

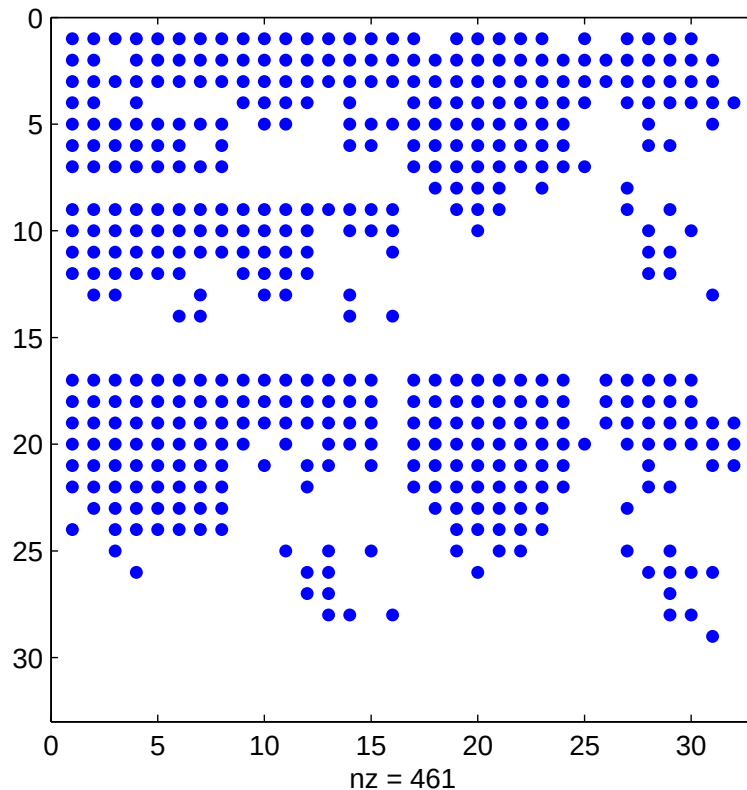


Figure 3.22: Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.

subset of wavelet coefficients is shown in Figure 3.26. Comparing this with the reference distribution and the 45% thresholded result (Figure 3.23) we see that fewer permeability field details are retained, though these details still lie in the area surrounding the wells.

The same procedure for thresholding is repeated while reducing the percentage of wavelet parameters included in the description. We obtain permeability distributions using 25% (see Figures 3.27 to 3.29), 15% (Figures 3.30 to 3.32) and 5% (Figures 3.33 to 3.35) of the reference wavelet coefficient set for the inversion. Figure 3.35 shows the permeability distribution that would be obtained by using only about 52 out of the original 1024 parameters. We see that details are still retained in the area between wells Producer 1 and Injector.

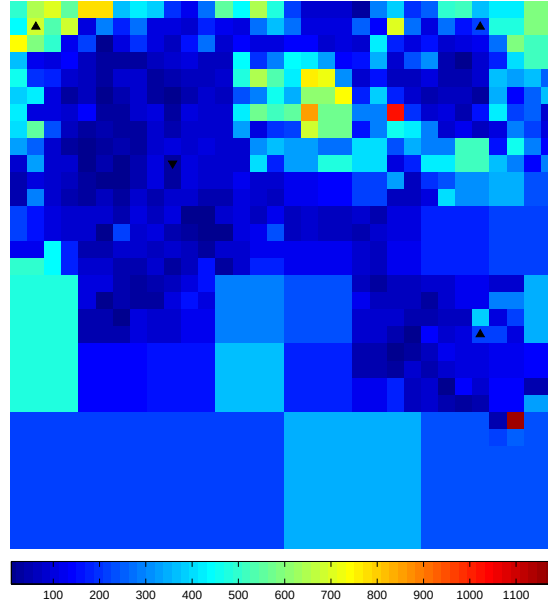


Figure 3.23: Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.

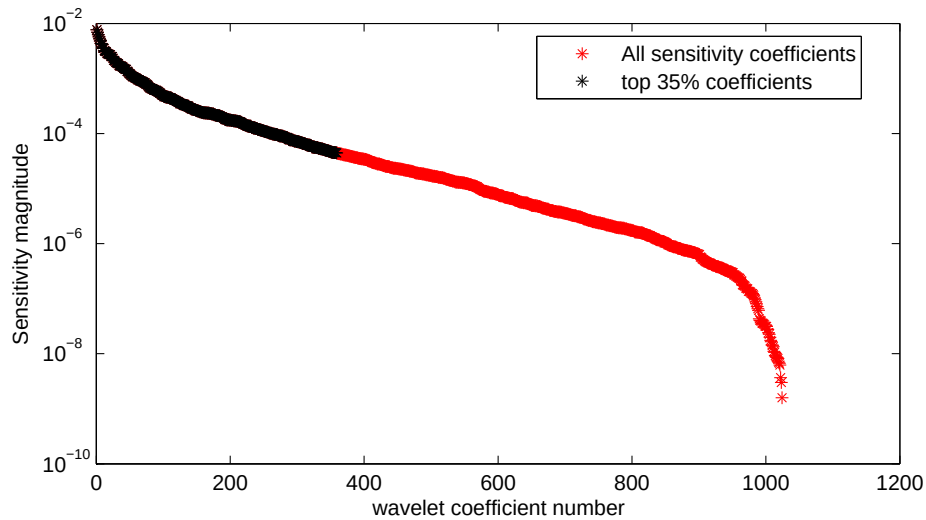


Figure 3.24: Sorted sensitivity magnitudes showing 35% of the highest valued coefficients being retained.

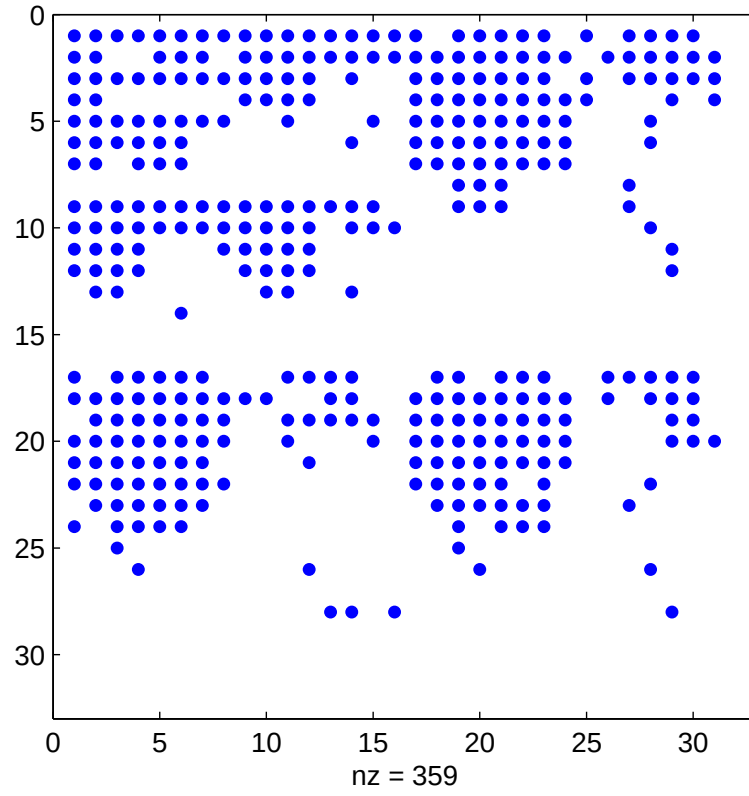


Figure 3.25: Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.

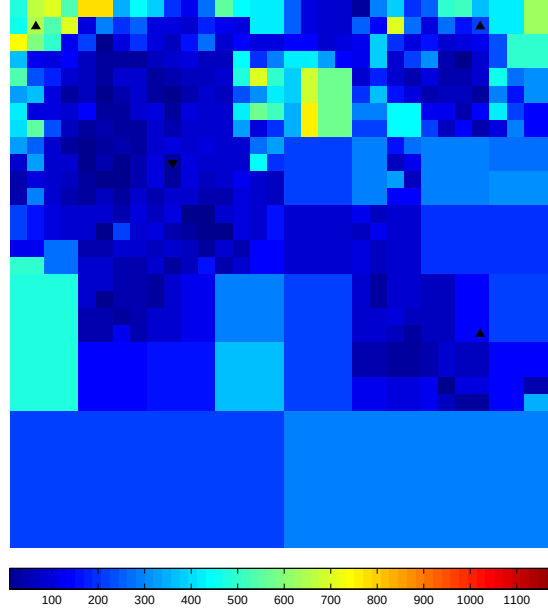


Figure 3.26: Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.

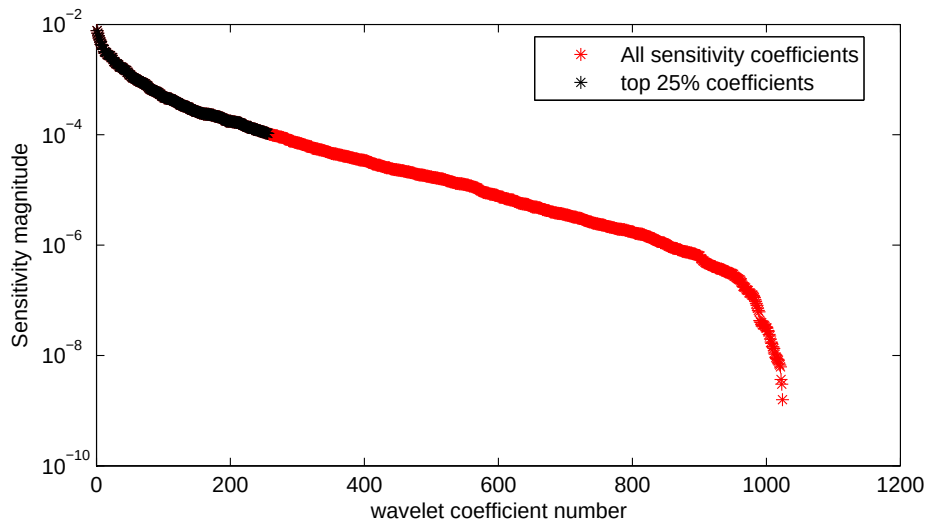


Figure 3.27: Sorted sensitivity magnitudes showing 25% of the highest valued coefficients being retained.

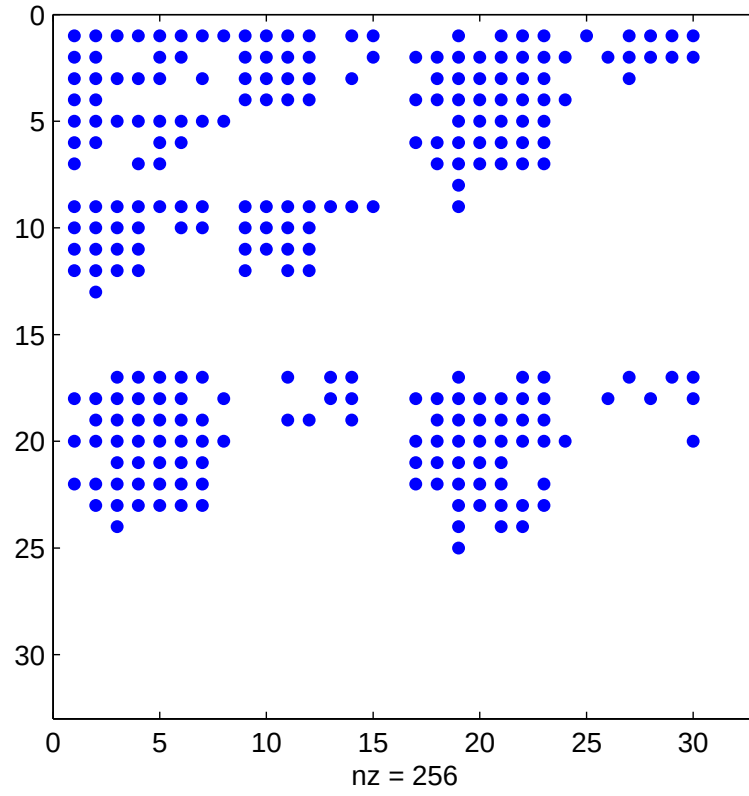


Figure 3.28: Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.

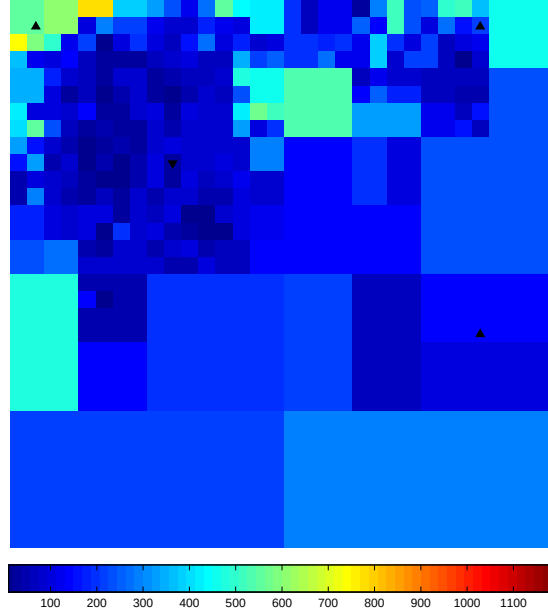


Figure 3.29: Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.

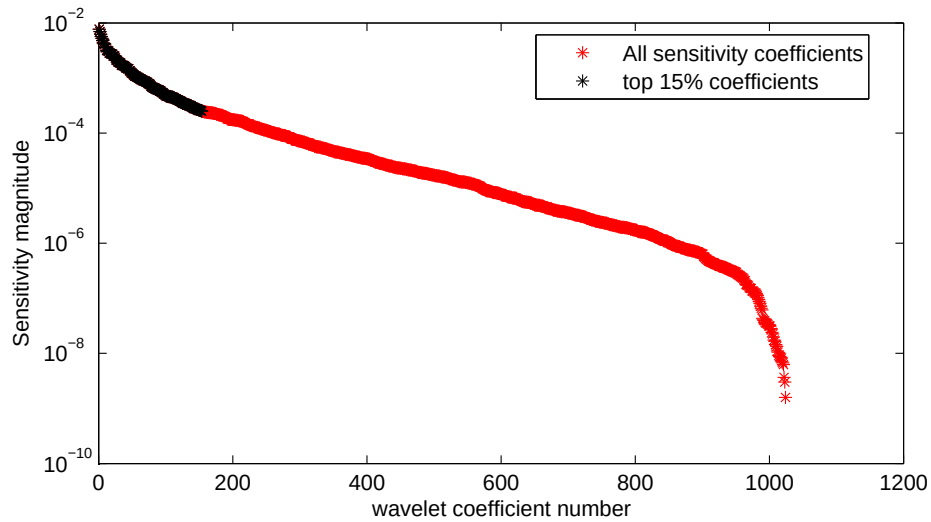


Figure 3.30: Sorted sensitivity magnitudes showing 15% of the highest valued coefficients being retained.

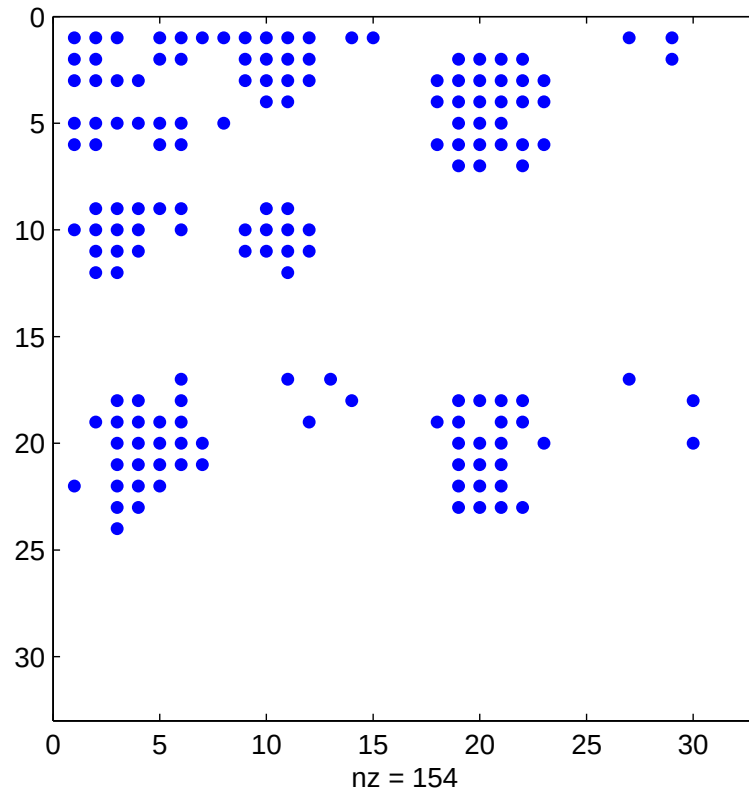


Figure 3.31: Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.

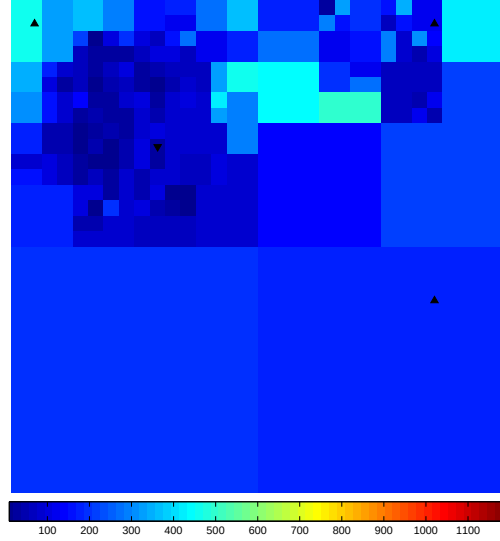


Figure 3.32: Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.

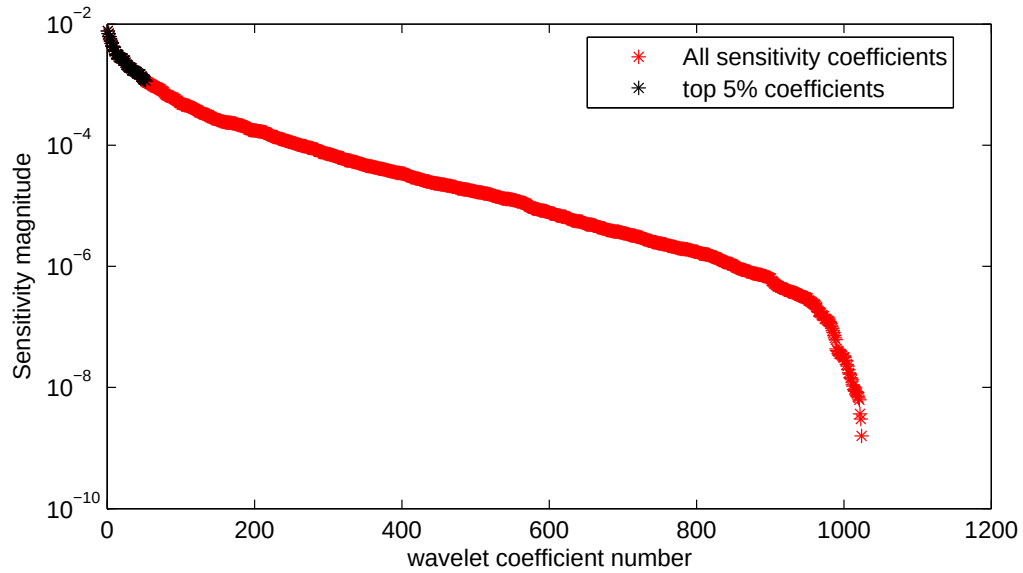


Figure 3.33: Sorted sensitivity magnitudes showing 5% of the highest valued coefficients being retained.

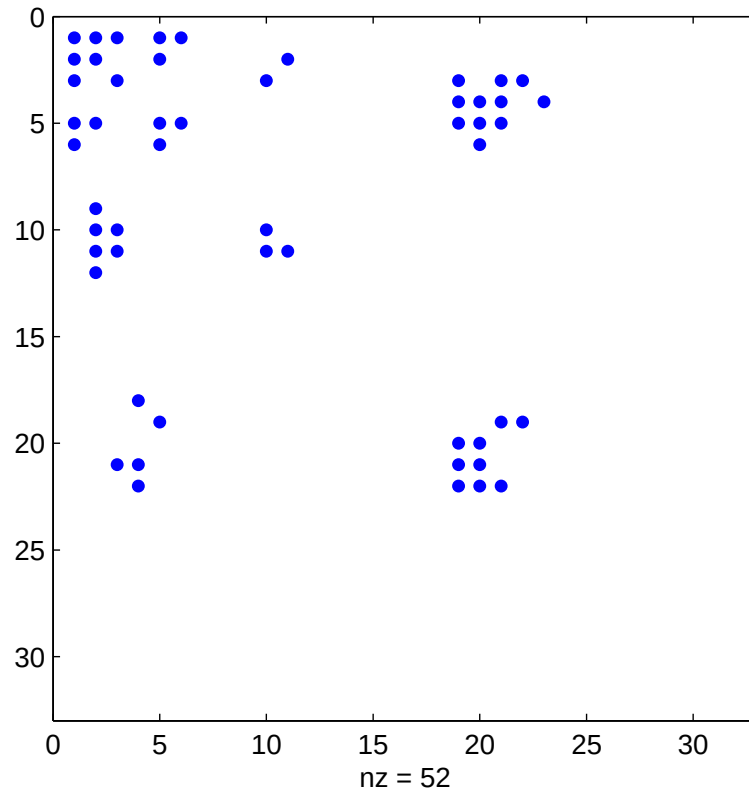


Figure 3.34: Sensitivity coefficient distribution in wavelet space showing the coefficients that are retained for production history match.

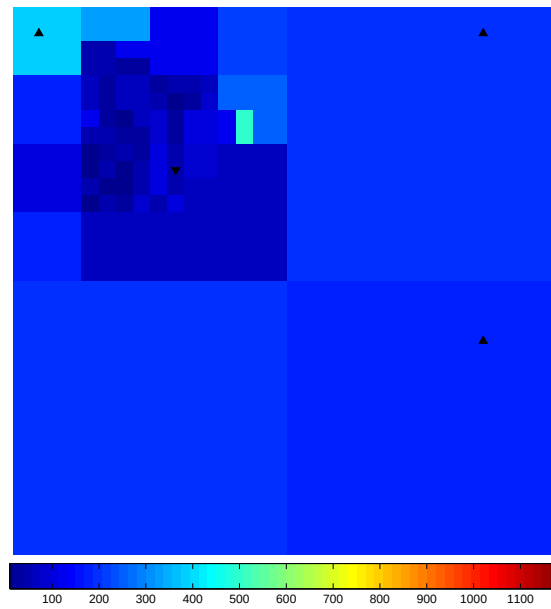


Figure 3.35: Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.

This thresholding exercise shows how a reservoir parameter distribution would change with the reduction of number of parameters used to describe it. As mentioned, wavelet parameters are retained on the basis of their significance to the production output from the reservoir model. Hence, the real test of thresholding or compression in this case would be based on the impact on the production profile obtained by simulation on each thresholded model. These simulations were performed and the results compared with the reference production history from each well, as shown in Figures 3.36 through 3.39. For all the wells, one common observation is that the production profiles of the thresholded models follows the trends of the historical profiles closely. However, as expected, we see that as we reduce the number of parameters being used to describe the reservoir model, the deviation from historical profiles becomes greater.

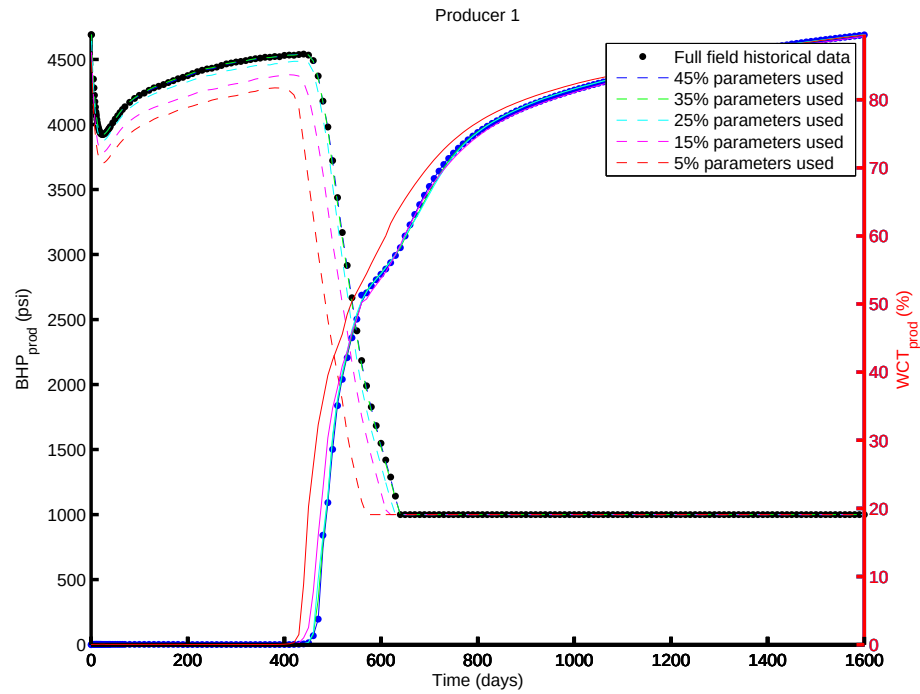


Figure 3.36: Producer 1 BHP and WCT results after thresholding compared with the historical production data.

This exercise was repeated on a number of different types and sizes of reservoir models, number of wells and amounts of production history. Based on our observations we can conclude that:

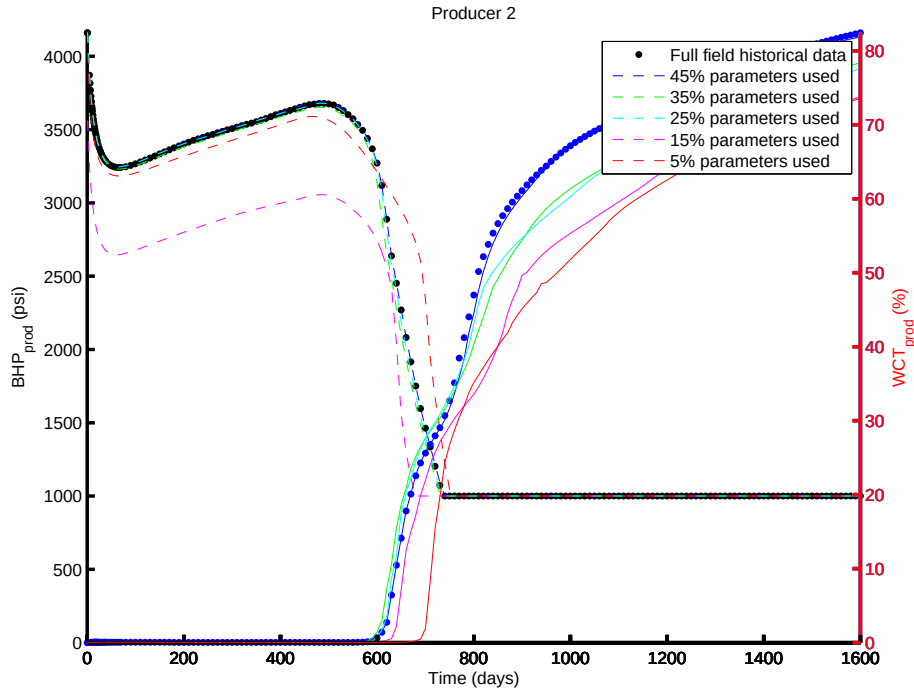


Figure 3.37: Producer 2 BHP and WCT results after thresholding compared with the historical production data.

- Parameter reduction can be performed on reservoir models based on sensitivity to historical production data.
- Major trends in the production profile are retained for compression ratios much smaller than 50%.
- At some level of thresholding of the wavelet coefficients, the permeability field obtained by an inverse transform is unable to adequately resolve the model and fails to constrain to reference production data.
- The more production data we have from a reservoir, the greater number of wavelet coefficients are required to constrain the model to them [1].

As a result, we can determine a level of threshold that corresponds to the minimum number of wavelet parameters required for an ‘adequate’ production data match. What counts as an ‘adequate’ match is dependent on the particular application. As a suggestion, an adequate match could be based on the uncertainty associated with the measured

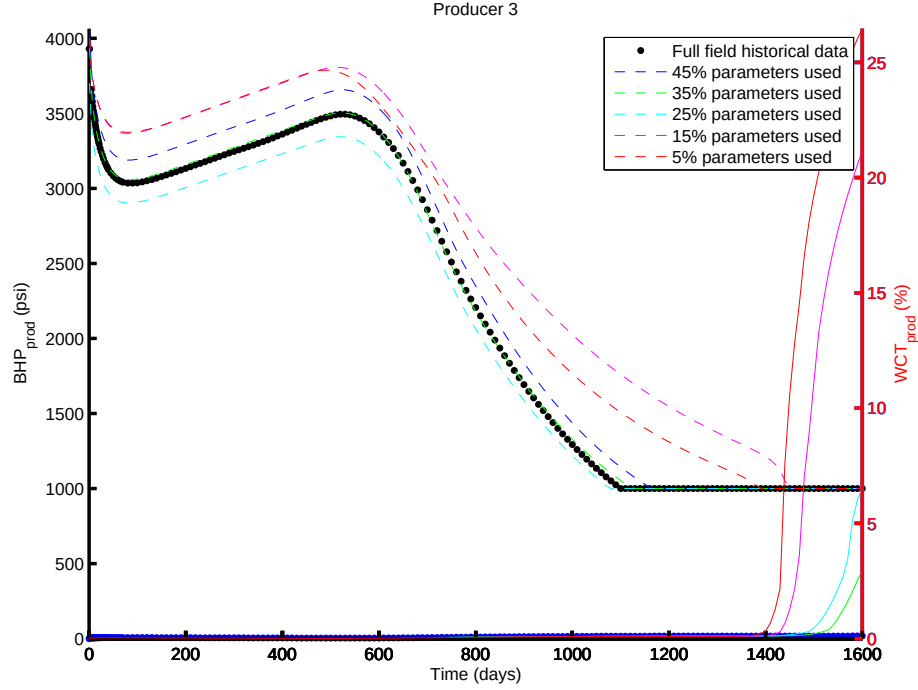


Figure 3.38: Producer 3 BHP and WCT results after thresholding compared with the historical production data.

production data and it could be quantified as a weighted norm of the mismatch.

3.2 Haar Wavelet Implementation Methodologies

There are two different ways of implementing the Haar wavelet transform in two dimensions referred to as Standard and Nonstandard (see Appendix A.1). These two different implementations of the two-dimensional Haar wavelet have been compared extensively in the literature in terms of their image compression properties [70]. However, in the current application, we used two-dimensional Haar wavelets for multiresolution analysis for the purpose of parameter reduction and estimation. Thus, in order to compare and contrast the relative merits of the Standard and Nonstandard implementations for parameter analysis, both techniques were tested using several test cases, one of which is shown here as example reservoir G1a (permeability distribution and well locations shown Appendix B.1).

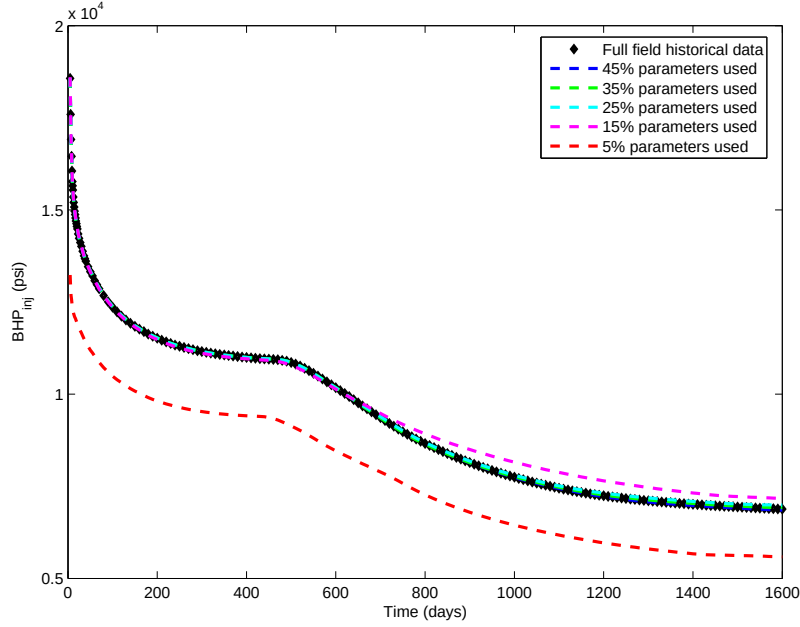


Figure 3.39: Injector BHP results after thresholding compared with the historical production data.

3.2.1 Standard and Nonstandard Wavelet Decomposition

Starting from a single history-matched prior, we perform the Haar wavelet transform, using both the Standard and Nonstandard implementation methods. This yields two distinct sets of Haar wavelet coefficients, each a different linear transform of the original permeability field. Hence, we now have two multiresolution Haar wavelet descriptions of the initial reservoir permeability model which are different in the averaging and differencing bases used in their implementations. Sensitivity coefficients are calculated with respect to each set of wavelet coefficients derived from the Nonstandard (Figure 3.40) and Standard (Figure 3.41) implementations separately. These are derivatives of the pressure and watercut profiles with respect to wavelet coefficients obtained using the Standard implementation and the Nonstandard implementation separately. Given that the two implementations use different bases for averaging the permeability field, the production data can be expected to show a different sensitivity distribution for each case. These sensitivity coefficients are normalized, sorted with respect to magnitude and plotted in Figure 3.42.

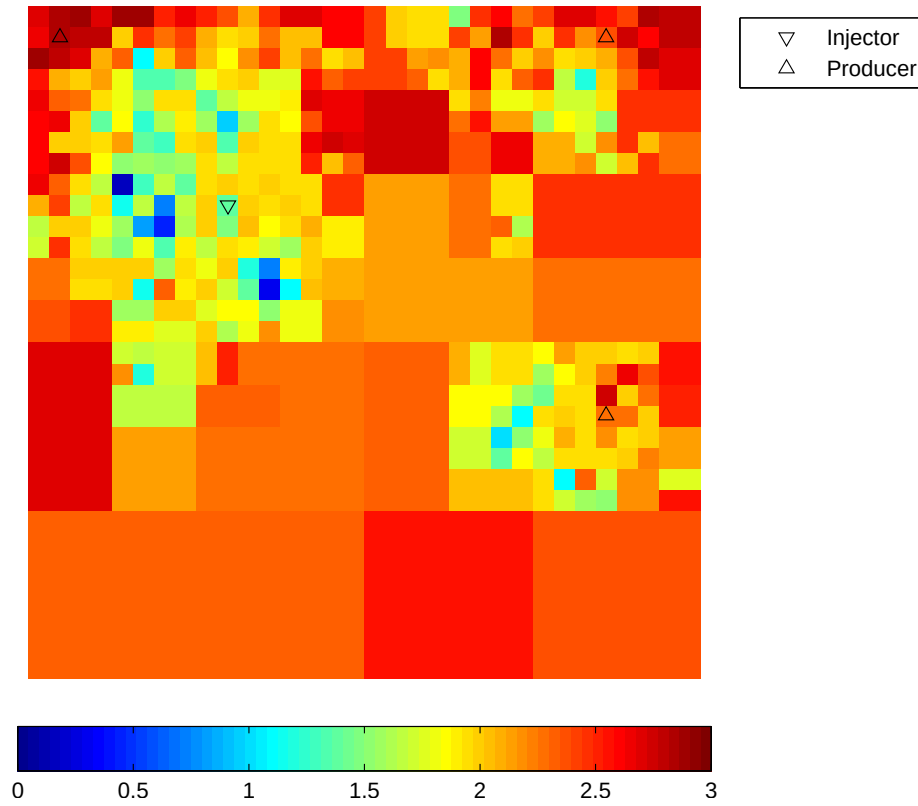


Figure 3.40: Thresholded log permeability distribution based on sensitivity to production data using Nonstandard implementation.

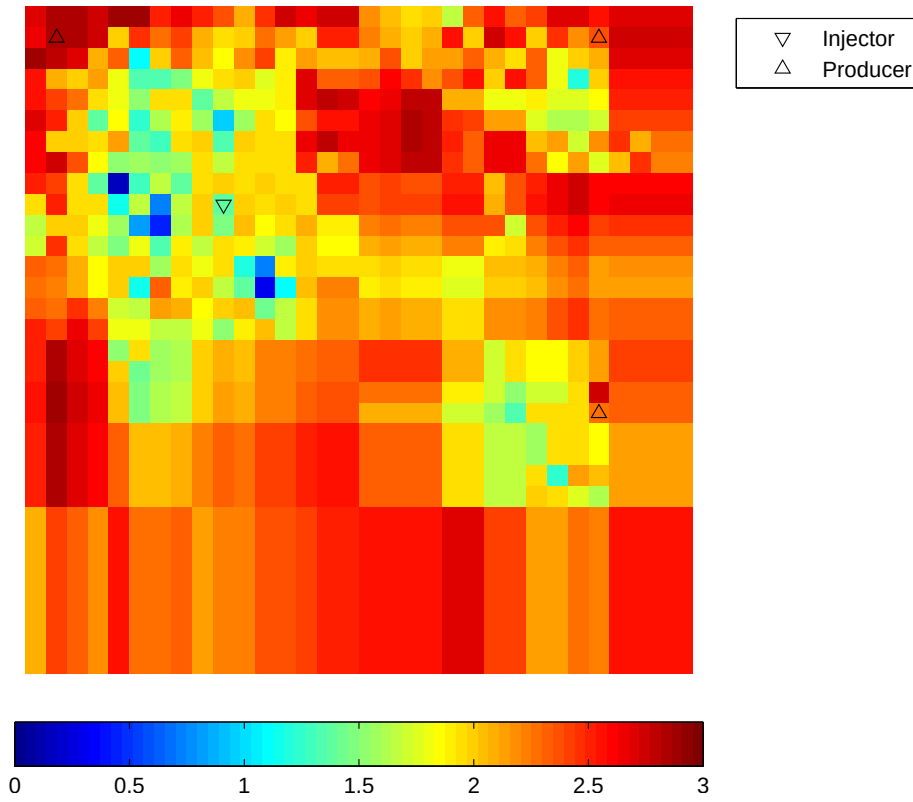


Figure 3.41: Thresholded log permeability distribution based on sensitivity to production data using Standard implementation.

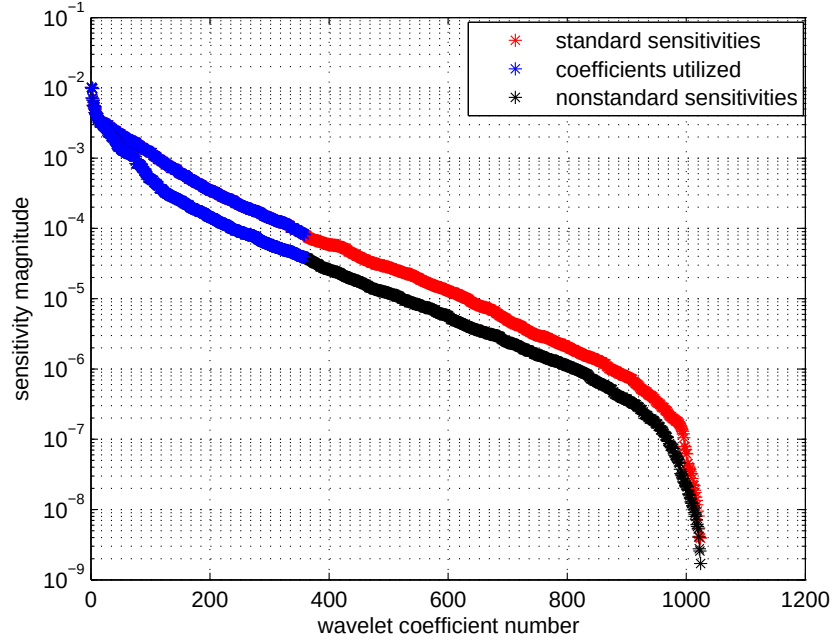


Figure 3.42: Standard and Nonstandard sensitivity coefficients to production data sorted in decreasing order.

From Figure 3.42 we observe that the sensitivity coefficients corresponding to the Nonstandard method fall more sharply in magnitude as compared to those derived using the Standard method. Thus the relative sensitivity of the Nonstandard coefficients is concentrated in a smaller number of coefficients, whereas for the case of Standard coefficients, the sensitivity coefficient magnitudes are more evenly divided among the coefficients. The implication of this concentration in the Nonstandard case is that fewer coefficients can be used to represent the permeability field while retaining the production history. For the Standard implementation, a greater number of wavelet coefficients would be required to constrain the realization to production data with a similar error threshold. From Figure 3.42 we can also conclude that for the same magnitude of sensitivity threshold, the number of wavelet coefficients included in the Standard method would be higher than that required in the Nonstandard method. Consequently, if the same level of compression was used, the Nonstandard method would do a better job at retaining a realization that corresponded to the original production history data as compared to the Standard method.

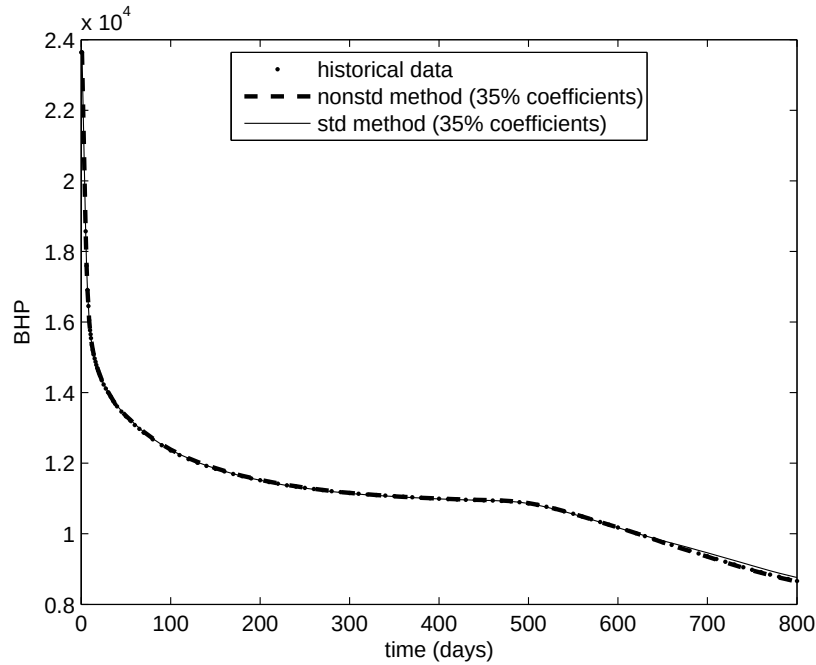


Figure 3.43: Injector BHP comparison of Standard and Nonstandard implementation results with respect to historical production data.

In order to check whether this assertion is valid, we ran flow simulations on realizations obtained by thresholding (see Appendix A.1 for a definition of thresholding) or compressing the corresponding wavelet coefficients obtained using the two wavelet implementations. In both cases, we retained 35% of the highest sensitivity wavelet coefficients and inverted to obtain the corresponding permeability fields. These permeability fields, smoothed used the Nonstandard and Standard implementations are shown in Figure 3.40 and Figure 3.41 respectively.

Performing flow simulation on these two results, we obtained production profiles that we compared with the historical production profiles. We observed that as expected, the Nonstandard method, with its rapidly dropping sensitivities, indeed gives a better overall (norm of the difference) match to the historical data as compared to the Standard method. As a sample of the results Figure 3.43 to Figure 3.46 plot the BHP and WCT data for all the wells to which the reservoir models were constrained. We see that using the same number of parameters in both cases, the Nonstandard method gives a closer match to the

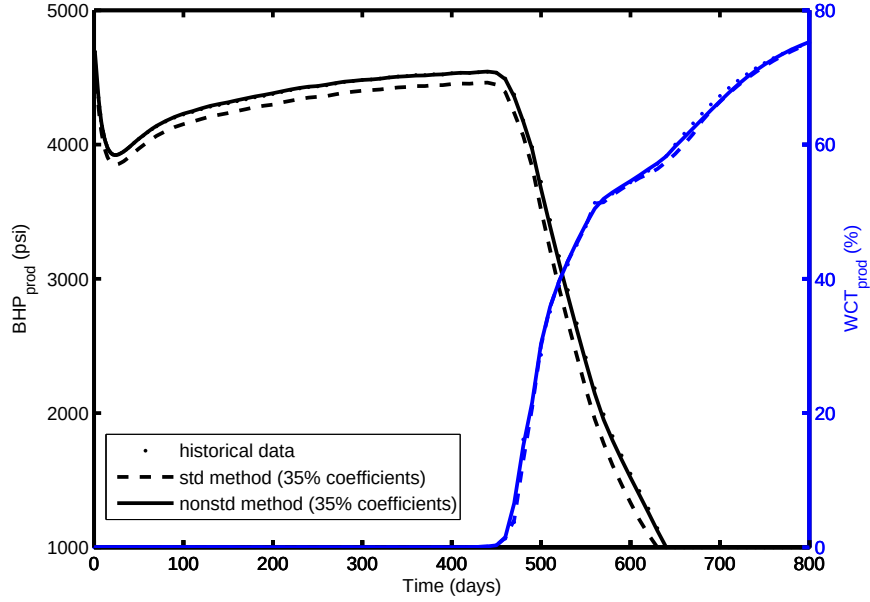


Figure 3.44: Producer 1 WCT and BHP comparison of Standard and Nonstandard implementation results with respect to historical production data.

true production history than does the Standard method. As such, we can conclude that for the given case, the Nonstandard Haar wavelet implementation gives better results in terms of reduction of the number of parameters than the Standard method. This result extends the observations of Stromme and McGregor in their 1997 paper [70] in which they concluded that the Nonstandard implementation gives better results than the Standard method in the field of image compression. Despite its apparently greater efficiency, it is important to note that the Nonstandard approach is less broadly applicable, as it may only be used for square systems.

3.3 Sensitivity Calculations for Reservoir Parameters

The functional relationship between reservoir and fluid parameters, and hydrocarbon production is very complex. The relationship is based on a number of different sets of equations and is highly nonlinear. The fundamental physical laws governing fluid flow in porous media include:

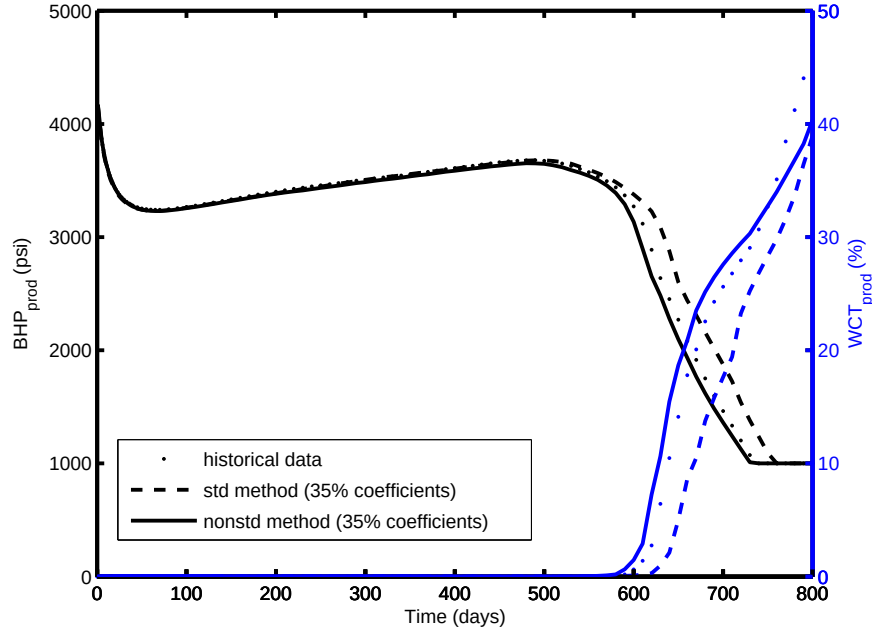


Figure 3.45: Producer2 BHP and WCT comparison of Standard and Nonstandard implementation results with respect to historical production data.

- Mass conservation or material balance
- Energy conservation
- Darcy's law for flow through porous media
- Equation of state
- Relative permeability and capillary pressure

Due to the complexity and nonlinearity of the function connecting these physical laws with the reservoir properties, it is not possible to construct an analytical form of the solution. As such, numerical methods are employed to solve these systems of equations.

In the case of petroleum reservoirs, the function is the set of physical equations governing flow and the output is the production data (*pressure, water and oil production, water saturation, etc.*). The system parameters include porosity and permeability at each gridblock and fluid properties etc. In this study, the system parameters were limited to

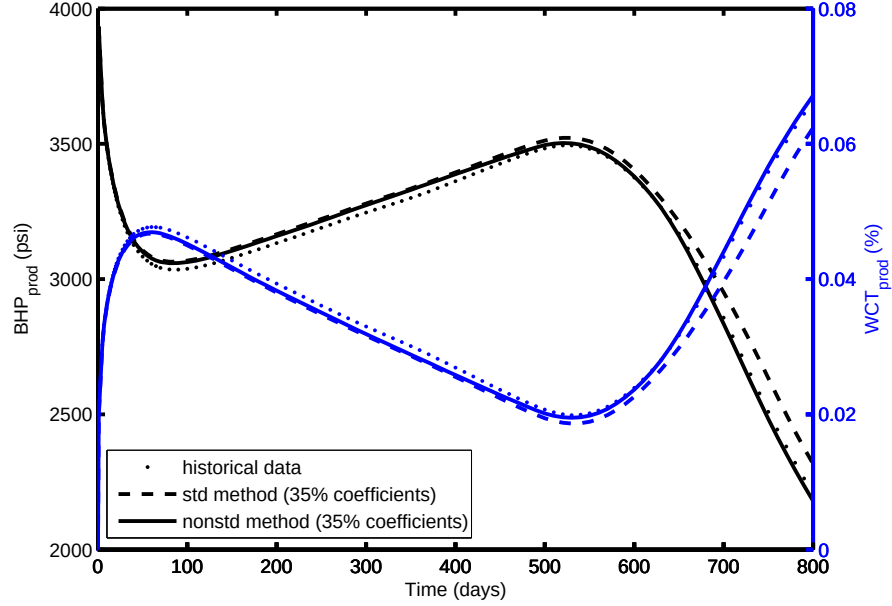


Figure 3.46: Producer3 BHP and WCT comparison of Standard and Nonstandard implementation results with respect to historical production data.

permeability at each gridblock and the production data were considered to be bottom hole pressure (BHP) and watercut (WCT). WCT is defined as:

$$wc = \frac{q_w}{q_w + q_o} \quad (3.1)$$

Thus in the context of a discrete reservoir, the sensitivity can be defined as the derivative of the production data (pressure and watercut) with respect to permeability in each grid block. However, since the discrete reservoir system is complex and does not have an analytical solution, we are limited to work with a numerical approximation of the actual partial derivative values. There are a few different ways of calculating these sensitivities, two of which we will now discuss.

Substitution Method Given a reservoir simulation function we make a simulation run with $\vec{\alpha} = \vec{\alpha}_0$ and obtain $\vec{u}_0 = \vec{u}(\vec{\alpha}_0)$. We perturb a single parameter $\alpha_{0,i}$, to yield $\alpha_i = \alpha_{0,i} + \delta\alpha_i$ where $\delta\alpha_i$ is small, and repeat the simulation process to obtain $\vec{u}_i^*$. The expression

for how the production profile changes with a change in a single reservoir parameter is thus given by:

$$\frac{\delta \vec{u}}{\delta \alpha_i} \big|_{\vec{\alpha}=\vec{\alpha}_0} = \frac{\vec{u}_i^* - \vec{u}_0}{\delta \alpha_i} \quad (3.2)$$

This gives the value of the sensitivity coefficient for that parameter. This procedure can be repeated for each parameter to obtain the corresponding sensitivity value. Hence for N_{par} parameters this procedure requires $N_{par} + 1$ simulation runs. Thus we see that this procedure can be expensive for large N_{par} . However, this method is straightforward and intuitive and can be used in conjunction with any simulator, without requiring it to have in-built sensitivity calculations.

Modified Generalized Pulse Spectrum Method This method is much more efficient than the substitution methods, though it requires a more complicated code to run. The algorithm can be described as follows:

$$J^{(k+1)} S^{(k+1)} = -D^{(k+1)} S^{(k)} - Y^{(k+1)} \quad (3.3)$$

where

$$S^{(k)} = \frac{\partial \vec{u}^{(k)}}{\partial \vec{\alpha}} \quad (3.4)$$

is the sensitivity coefficient matrix,

$$J^{(k+1)} = \frac{\partial \vec{f}^{(k+1)}}{\partial \vec{u}^{(k+1)}} \quad (3.5)$$

is the Jacobian matrix (which can be obtained from the simulator),

$$D^{(k+1)} = \frac{\partial \vec{f}^{(k+1)}}{\partial \vec{u}^{(k)}} \quad (3.6)$$

is a very sparse block-diagonal matrix that is easy to calculate, and

$$Y^{(k+1)} = \frac{\partial \vec{f}^{(k+1)}}{\partial \vec{\alpha}} \quad (3.7)$$

is also a sparse matrix with a pattern similar to the Jacobian.

Theoretically, permeability is a Jeffreys parameter [44], as it can take values between zero

and infinity. The proper way of evaluating contrasts and averages etc. of such parameters is to work with their logarithm. Taking the logarithm of this set of Jeffreys parameters yields Gaussian parameters that may range anywhere from $-\infty$ to ∞ . This representation is more convenient to use, and the corresponding sensitivity coefficients can easily be modified as:

$$\frac{\partial u}{\partial \ln k} = \frac{\partial u}{\partial k} \frac{\partial k}{\partial \ln k} = k \frac{\partial u}{\partial k}. \quad (3.8)$$

The techniques for sensitivity calculation are described in greater detail in [13, 18].

3.3.1 Wavelet Reparameterization

An important part of reservoir model description as discussed in Section 3.1, is the choice of parameterization. The calculation of sensitivities discussed here is based on derivatives calculated with respect to pixel or gridblock values of permeability. However, some key strengths of the data integration algorithm (multiresolution analysis, parameter reduction) are based on using the wavelet transform of the reservoir permeability distribution for estimation purposes. Thus, we need to transform the sensitivity values to wavelet space as well.

As described in Chapter 2 the discrete wavelet transform is a linear transform of the underlying permeability parameters. If $\vec{k}$ is the original permeability distribution and W is the wavelet transformation matrix, we can express this linear transform in thus:

$$\vec{c} = W \cdot \vec{k} \quad (3.9)$$

with $\vec{c}$ representing the wavelet coefficient set. The wavelet transformation matrix W is orthogonal and hence:

$$W \cdot W^T = I \quad (3.10)$$

multiplying both sides by W^{-1} , we get:

$$W^T = W^{-1} \quad (3.11)$$

As such, we can express the reverse transform as follows:

$$\vec{k} = W^T \cdot \vec{c} \quad (3.12)$$

Based on this linear relationship between the reservoir parameter $\vec{k}$ and its wavelet transform $\vec{c}$, given the sensitivity coefficient value with respect to $\vec{k}$ we can compute the sensitivity coefficient with respect to $\vec{c}$, using the chain rule. Thus:

$$S_{ck} = \Sigma \vec{S}_{kj} \frac{\delta k_j}{\delta c_k} = S_k \cdot W^T \quad (3.13)$$

Thus we have a straightforward way of calculating sensitivity coefficients with respect to Haar wavelets given the sensitivity coefficients with respect to reservoir permeabilities.

3.4 Chapter Summary

In Chapter 3, we looked at different parameterization techniques. In particular, we compared the image compression properties of SVD, FT and HWT with the help of some examples. We saw that the relative ability of these mathematical tools to compress an image efficiently depends on the image itself – on its structural and continuity properties, and is also dependent on the metric used to measure compression performance. For our reservoir characterization application we chose the Haar wavelets since they not only have good image compression properties, but also have added advantages such as multiresolution analysis and time frequency localization, giving us more degrees of freedom and control in describing the problem.

Production data integration forms the first step of the wavelet-based data integration algorithm. We discussed various aspects of the application of wavelets for production data integration and parameter partitioning for reservoir model. A detailed study of the impact of thresholding on reservoir model representation and production history match was described in detail using an example reservoir with Gaussian distributed permeabilities. As we reduce the number of parameters used to describe the permeability distribution for the model, we see that the production history is not significantly impacted up to some threshold of compression. At this point it should be noted that while in image compression the key is to capture the main visual features of the image, for our application in reservoir modeling and production data integration, we set criteria that would help us retain wavelet parameters based on their sensitivity to the production data output. The thresholding example indicates that for the type of history-matched reservoir considered, there exists a subset of the total number of wavelet parameters that has high significance to the production history match. There are other parameters in the superset of parameters that might have low or zero

impact on the production history output from that reservoir. From the example studied, not only do we recognize the existence of such a subset, we are also able to identify it using the concept of sensitivity coefficients. This concept is central to this thesis and is described using a Venn diagram in Figure 3.47.

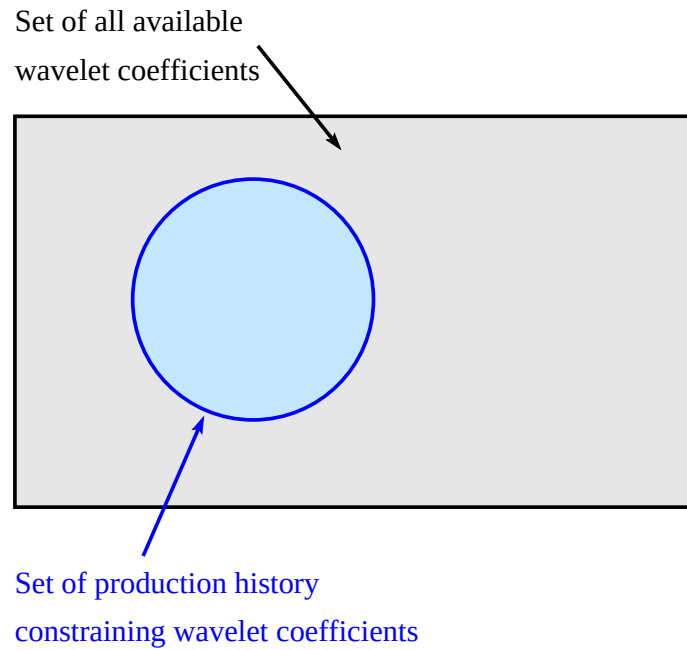


Figure 3.47: Venn diagram showing the complete set of wavelet coefficients corresponding to a permeability field, highlighting the fact that there exists a subset that constrains the model to production data.

The two-dimensional Haar wavelet transform can be implemented using the Standard or the Nonstandard method. These two implementations are compared with respect to their ability to reduce the number of parameters required for a production history match. It is observed that not unlike in image analysis applications, the Nonstandard implementation has better compression properties when it comes to describing reservoir models as well. The compression properties of a parameterization methodology is related to the shape of the sorted sensitivity distribution. If the magnitudes of sensitivities fall sharply in the distribution, compression is expected to be better, since that would imply that a majority of the information is stored in a small fraction of the total parameters. Section 3.3 explored how sensitivity coefficients are used for the integration of production data, the method of calculation and the adaptation of sensitivity calculations to wavelet space.

Chapter 4

Production Data Integration and Parameter Partitioning

4.1 Parameter Estimation

History matching is the first step in the data integration algorithm developed in this study. In most of the cases discussed here, we started from a realization that is assumed to be history matched, though it may or may not conform to the other types of data available for the reservoir, for example, geostatistical information. This initial history-matched model might be the result of manual history matching or an assisted or automatic history matching procedure. The wavelet sensitivity analysis was then applied in order to integrate the other data sources. Alternatively, the wavelet theory developed (see Chapter 2) can be used to perform an efficient initial history match. That is, the calculation of wavelet sensitivity coefficients aids in reducing coefficients for the purpose of integrating production history data [13, 14] and also to partition the parameter set to determine the subsets required to integrate other forms of data described in this work [1, 2, 3].

Thus, for the sake of completeness, in this section, we present a procedure for history matching using wavelet coefficients of the reservoir parameters. This procedure is based on methodologies described by Lu [13, 14]. This automatic history matching technique uses wavelet-based gradient methods that are described in Chapters 2 and 3. A subset of the wavelet coefficients are perturbed in order to obtain a permeability model that matches the production profile. As a measure of importance to production profile, sensitivity coefficients are calculated at each time step of the simulation run for wavelet coefficients corresponding

to the permeability. The next stage is to determine those wavelet coefficients that are most significant to the *overall* production profile, that is, for all time steps. The wavelet coefficients corresponding to the permeability distribution that have high sensitivity to production data are at the scales and spatial locations that are *resolved* by the available data. The coefficients with low sensitivity can be ignored or set to zero without significantly affecting the history match (see Section A.3). Thus we see that the actual history-matching procedure along with determining the sensitivity thresholds form the first two important steps in the data integration methodology. These two essential steps are defined in this current chapter.

4.1.1 History Matching Algorithm

The Levenberg-Marquardt method may be used for gradient-based optimization for performing a history match for the reservoir model. We describe the algorithm here in terms of the reservoir modeling problem.

4.1.2 Gauss-Newton Method for Parameter Estimation

The Newton method and its variation, the Gauss-Newton method are both gradient-based methods of optimization.

$$H_n \nabla \vec{\alpha} = -\vec{F}_n, \quad (4.1)$$

$$\alpha_{n+1} = \vec{\alpha}_n + \nabla \vec{\alpha}_n \quad (4.2)$$

$$\vec{F}_n = \nabla E_n = G_n^T C_D^{-1} (d_n^{\vec{cal}} - d_n^{\vec{obs}}) + C_M^{-1} (\vec{\alpha}_n - \vec{\alpha}^{pri}) \quad (4.3)$$

and

$$H_n = \nabla \vec{F}_n = G_n^T C_D^{-1} G_n + C_M^{-1} + \nabla G_n^T C_D^{-1} (d_n^{\vec{obs}} - d_n^{\vec{cal}}) \quad (4.4)$$

$$G_n = \frac{\nabla d_n^{\vec{cal}}}{\nabla \vec{\alpha}} \quad (4.5)$$

$$\vec{\alpha}_{n+1} = \vec{\alpha}_n - \mu_n H_n^{-1} \nabla E_n \quad (4.6)$$

The Gauss-Newton Hessian matrix is:

$$H_n^{GN} = G_n^T C_D^{-1} G_n + C_M^{-1} \quad (4.7)$$

Thus each iteration can be written in form of the following update:

$$\vec{\alpha}_{n+1} = \vec{\alpha}_n - \mu_n H_n^{-1} \nabla E_n \quad (4.8)$$

The Levenberg-Marquardt variation of the update defines the search direction in the following manner:

$$(H_n^{GN} + \nu_n I) \nabla \vec{\alpha}_n = -\vec{F}_n, \quad (4.9)$$

ν_n being a nonnegative scalar number.

The history matching algorithm works is based on the Levenberg-Marquardt optimization algorithm as described in Chapter 2. The procedure is:

- Evaluate C_M and C_d . Determine d_{obs} . Determine $\alpha_{initial}$.
- In iteration n :
 1. Run sensitivity calculation using α_{n-1} to evaluate d_{calc} and G_n (sensitivity coefficients).
 2. Evaluate F_n and H_{GN} .
 3. Evaluate $\nabla(\alpha)$ and update $\alpha_n = \alpha_{n-1} + \nabla(\alpha)$.
 4. $n = n + 1$. Repeat, until converged.

4.2 Sensitivity Thresholding Schemes

There are different ways in which the wavelet parameters can be decoupled and isolated as being significant to a particular data type. Of these data types, the fluid flow information is the most complex and expensive to integrate. Thus, in order to determine the wavelet parameters significant to production history data into we use sensitivity coefficients (Section 3.3), which are essentially derivatives of the production data with respect to the reservoir parameters. In the following section, we use an example reservoir model HM1 of Gaussian permeabilities with grid size 16×16 to describe some attributes of this sensitivity map.

4.2.1 Sensitivity Coefficient Values as a Function of Time Step Number

Sensitivity values are calculated for each wavelet coefficient parameter at every time step and they vary with the time step of the simulation run. To demonstrate this, we plot

sensitivities of pressure and watercut from the producers and of pressure from the injector for a few different wavelet coefficient parameters as a function of time (Figure 4.2 to 4.4). Figure 4.1 shows the location of the different wavelet coefficients in the wavelet coefficient map $\mathbf{w}$. As described in Section 2.3 the location of a wavelet coefficient in this map signifies not only its position corresponding to real (permeability) space, it is also representative of its scale or resolution.

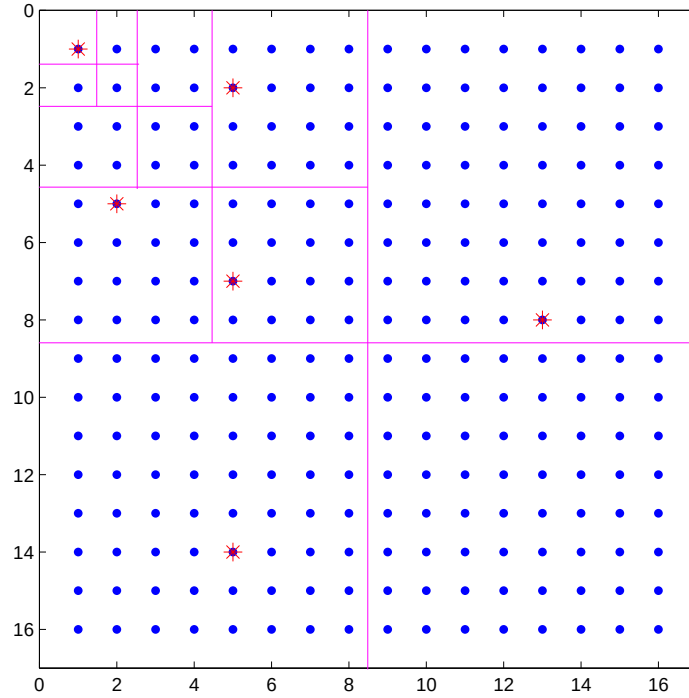


Figure 4.1: Sensitivity map in wavelet space. Blue dots represent the complete set of wavelet coefficients. Red stars represent the subset of wavelet coefficients for which the sensitivity to BHP and WCT are plotted with time in Figures 3.40 through 3.42

As we can see from Figure 4.1, the wavelet coefficient parameters considered in this example are chosen such that they span various spatial locations as well and different scales of parameterization of the reservoir model. These coefficients - w1 through w6 - correspond to the elements $w(1,1)$, $w(5,2)$, $w(2,5)$, $w(7,5)$, $w(14,5)$ and $w(8,13)$ respectively, where w is the wavelet coefficient matrix of size 16×16 . The wavelet coefficient parameters considered in this example are chosen to span various spatial locations and also different

scales or resolutions of the reservoir model. All plots and discussions in this section are based on absolute values or magnitudes of the wavelet coefficient sensitivities, and their mathematical signs are ignored.

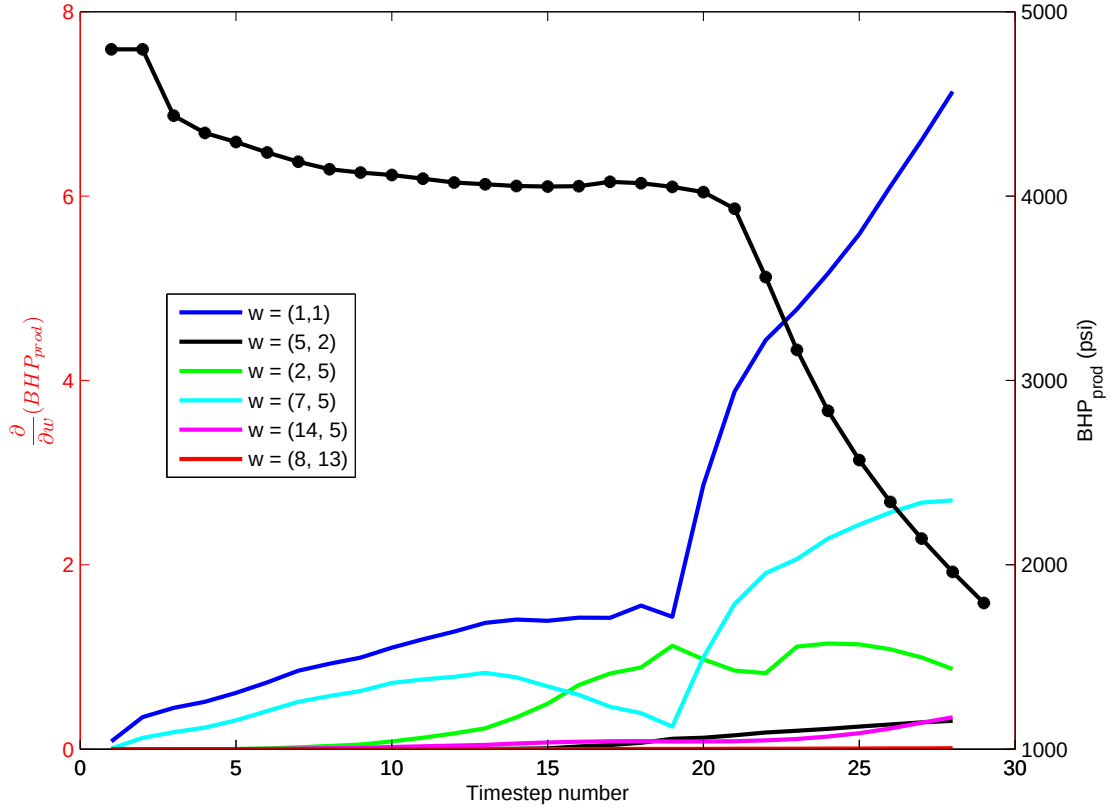


Figure 4.2: Producer BHP sensitivity coefficient profile with time also showing the evolution of producer BHP (as closed circles).

Figure 4.2 through 4.4 plot the sensitivity coefficient magnitudes on the y-axis and the timestep number corresponding to the production data on the x-axis. Figure 4.2 and Figure 4.3 show how the sensitivities to the producer and injector BHP of different wavelet coefficients change over time. Element w_1 corresponds to the top left wavelet coefficient in the wavelet coefficient matrix w , and represents the overall permeability average over the entire permeability field. We see that BHP is highly sensitive to changes in large scale

average values of permeability for this reservoir model. The sensitivity coefficient magnitude also goes up with time, implying that at later times these particular wavelet coefficients are more sensitive to BHP data. The coefficients w_2 , w_3 and w_4 also represent contrast at a high scale, and they are highly sensitive to the BHP information. Sensitivity of BHP to finer scale contrasts (w_5 and w_6) is much lower, and depends more on their spatial location and local support. On the same figure we also show a plot of the BHP data from the producer and injector respectively. We cannot make any clear inferences regarding a relationship between the variation of BHP value and sensitivity magnitude with time.

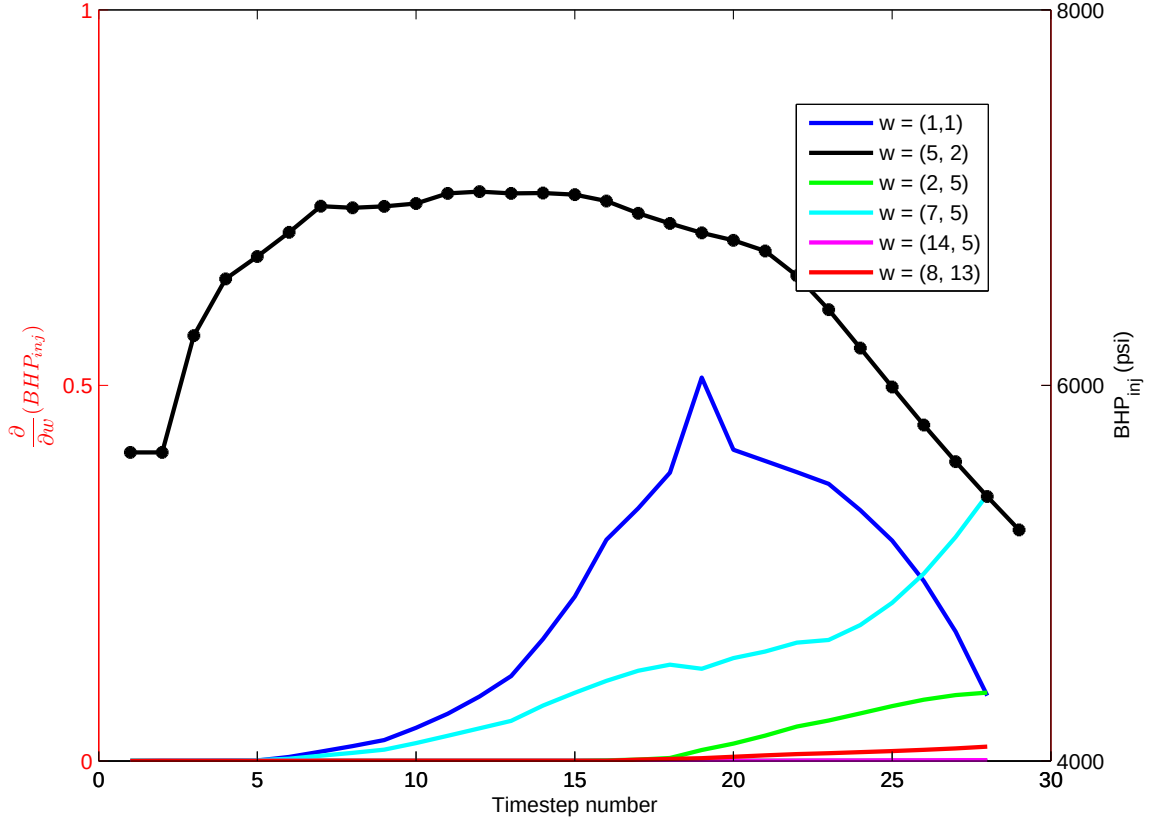


Figure 4.3: Injector BHP sensitivity coefficient profile with time also showing the evolution of injector BHP (as closed circles).

Figure 4.4 shows sensitivities of the wavelet coefficients to the producer WCT. WCT sensitivities show sharper fluctuations with time, whereas we saw in Figure 4.2 that BHP sensitivities vary more gradually in comparison. An interesting point to note here is that

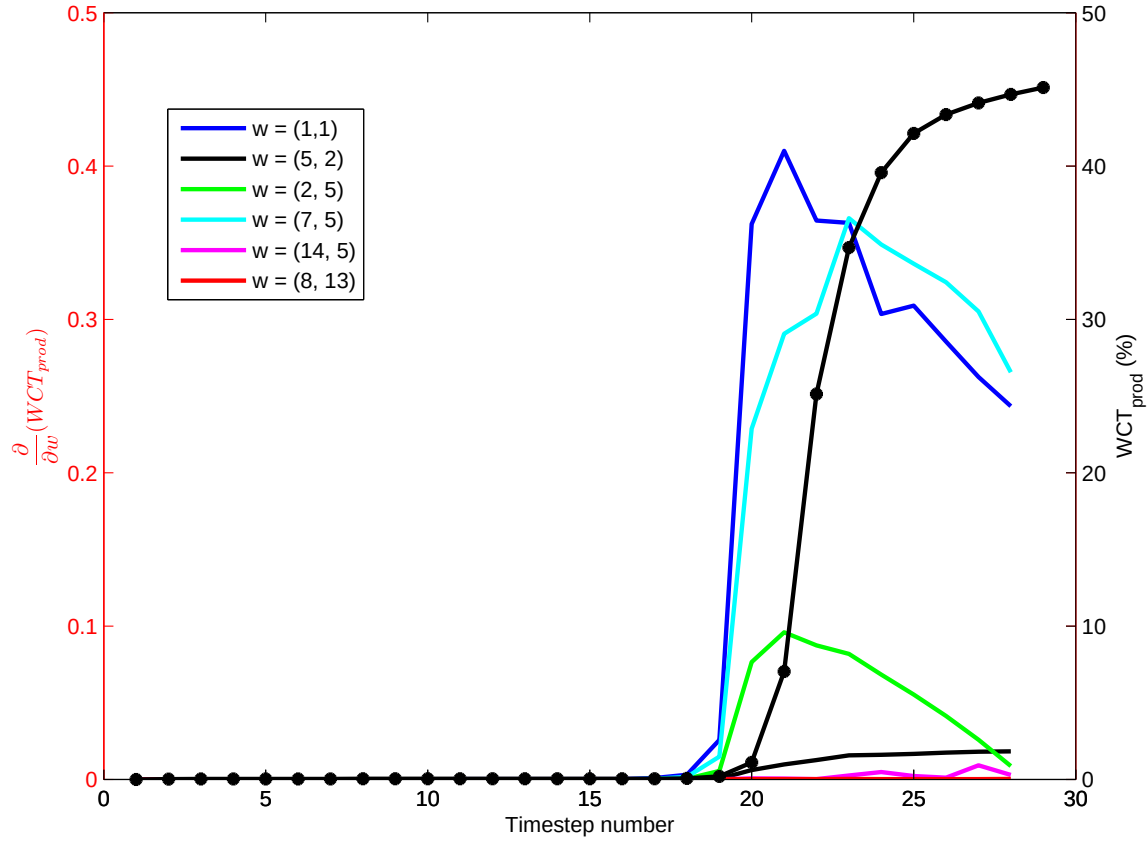


Figure 4.4: Producer WCT sensitivity coefficient profile with time also showing the evolution of producer WCT (as closed circles).

the sensitivities for coefficients - especially w_1 , w_3 and w_4 - shoot up from a near zero value just before water breakthrough occurs at the producer well. This implies that these wavelet coefficients, which were unimportant to production history match up to this time step, suddenly gain significance as the water front approaches them. As can be seen from their profile, the sensitivity magnitudes eventually begin to decline. Over time, these sensitivity magnitudes are expected to reach near-zero values again, as the front passes over them and moves on. This brings us to an important issue regarding the use of sensitivity coefficients over time as a means to capture the significance of wavelet parameters to production data.

4.2.2 Effect of Thresholding Technique

From the discussion in the previous section, we saw that the sensitivity coefficient magnitude for any single wavelet coefficient parameter varies with time, in some cases depending on the actual production data profile. However, for our data integration algorithm, we require a single index of ‘sensitivity’ to describe the importance of a wavelet parameter for all time steps. Thus, there is a need to assimilate sensitivity information over time into a single sensitivity map. In this regard, we studied the use of two different methodologies. We explain these two methodologies with the help of the following example.

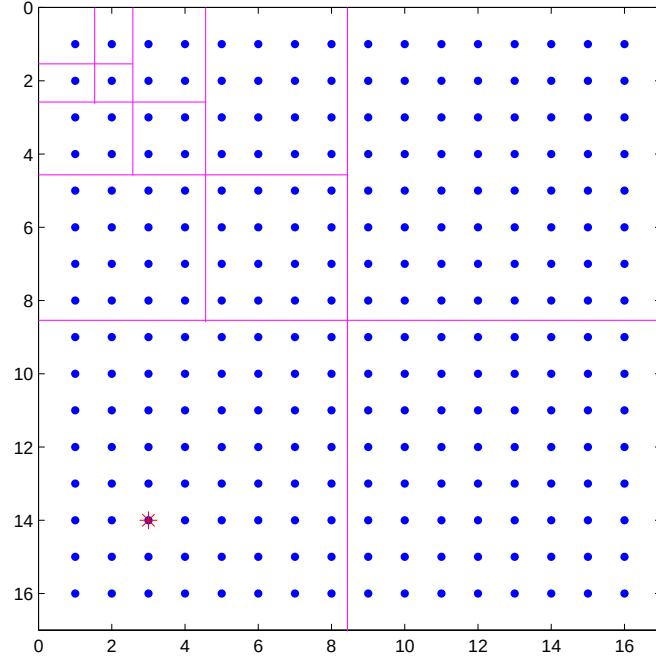


Figure 4.5: Sensitivity map in wavelet space. Blue dots represent the complete set of wavelet coefficients. Red stars represent corresponds to the location of wavelet coefficient $w(14,3)$ for which the sensitivity to BHP and WCT are plotted with time in Figures 4.6 and 4.8.

For the reservoir model HM1, consider the sensitivity profile with respect to time for injector BHP, and producer BHP and WCT for a certain wavelet coefficient v (where $v = w(14, 3)$). Figure 4.5 shows the location of this wavelet coefficient in the wavelet coefficient matrix. This coefficient is a fine-scale descriptor of the model, and is spatially located in the vicinity of a well. In Figures 4.6 to 4.8 are plotted the sensitivity coefficient magnitudes for this particular wavelet coefficient with respect to producer BHP and WCT and injector BHP

respectively. We observe that the absolute magnitudes of sensitivity values rise from low initial values as the simulation proceeds, though this rise is not monotonic. In some cases, for example, sensitivity WCT at the producer in Figure (4.7), the sensitivity magnitude rises sharply to a peak high value of 0.0576 as water breakthrough occurs and then eventually falls to lower values. Similarly, the sensitivity of this wavelet coefficient to BHP at the injector also rises slowly and reaches a peak towards the end of the simulation time scale Figure (4.8).

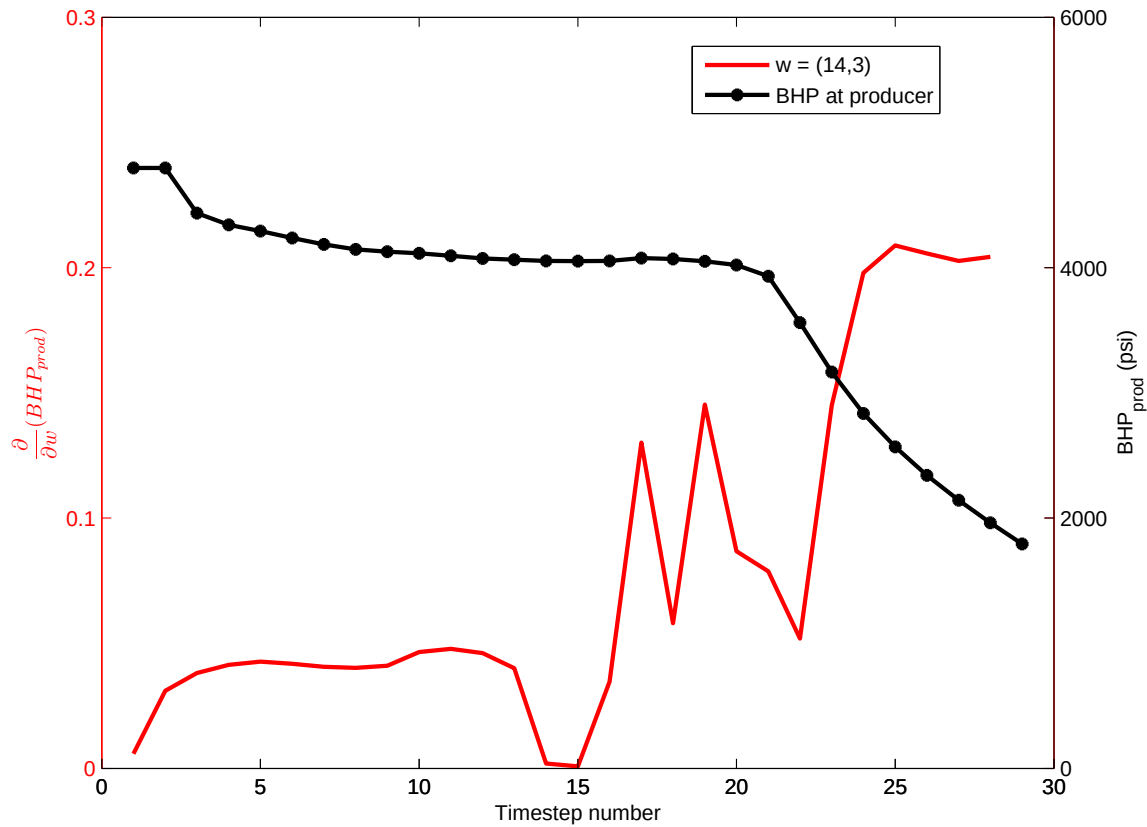


Figure 4.6: Producer BHP sensitivity coefficient profile with time also showing the evolution of producer BHP.

Now consider two different methodologies to assimilate these time-variant sensitivity coefficient maps into a single map. The objective is to determine if a particular sensitivity coefficient is significant to matching the production history information or not. This process of selecting a subset of wavelet coefficients based on sensitivity values to a particular data

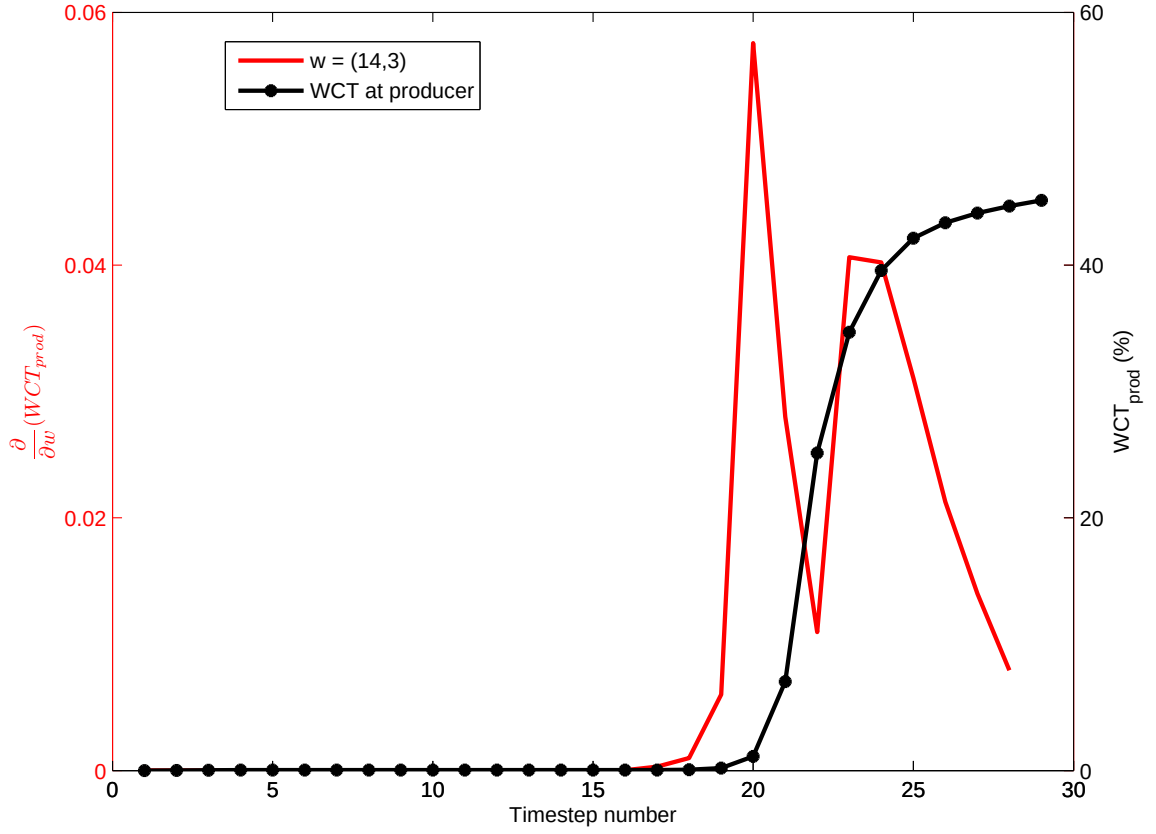


Figure 4.7: Producer WCT sensitivity coefficient profile with time also showing the evolution of producer WCT.

type is called thresholding (see Appendix A.3).

Methodology 1 - Area under the curve One method to threshold sensitivity coefficients over time would be to first determine the area under the sensitivity coefficient curve corresponding to each wavelet coefficient. For the wavelet coefficient $w(14,3)$, the areas under the curves for the corresponding sensitivity coefficients to the set of production data available are shown in Figure 4.9 to Figure 4.11. In this case, sensitivity coefficients are sorted according to highest area under the curve over time. The wavelet coefficients corresponding to a fixed number of the highest sensitivity magnitudes are retained for production history match and the rest are kept aside for including other data types.

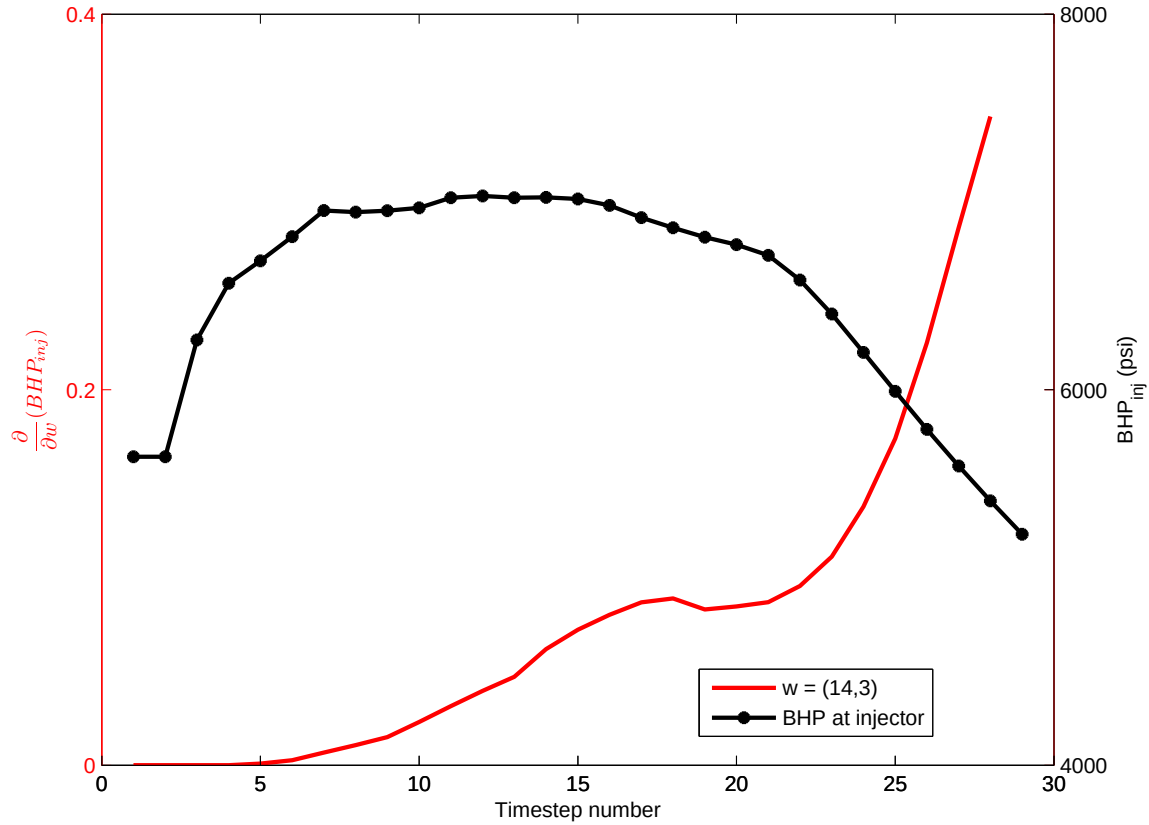


Figure 4.8: Injector BHP sensitivity coefficient profile with time also showing the evolution of injector BHP.

Methodology 2 - Cutoff method As seen in Figures 4.6 to 4.8, some sensitivity coefficients rise sharply and then decline, all over a very short interval of simulation time. These wavelet coefficients show a high sensitivity to production data, but the high value lasts only for a short span of time. As a result, if we used the area under the curve method for thresholding, these coefficients would be eliminated as being of low importance to production data. However, as we can see from Figures 4.6 through 4.8, these coefficients do indeed play an important role in determining production output from the simulation model, though indeed for a few time steps. In Methodology 2 thresholding is based on a fixed cutoff for sensitivity magnitude. All sensitivity values for all time are checked against a cutoff minimum value. Wavelet coefficients that correspond to all sensitivities that rise above the cutoff, even if for just a single time step, are retained for production history match and the

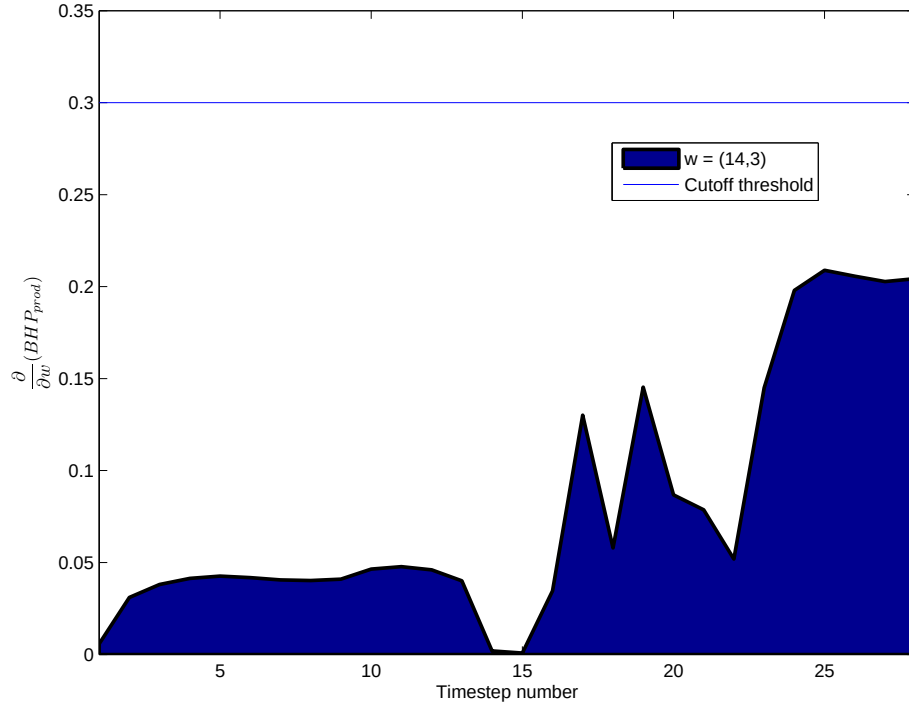


Figure 4.9: Area under the curve and cutoff limit for producer BHP sensitivity to wavelet coefficient $w(14,3)$ with time.

rest are kept aside for including other data types.

We evaluated the effectiveness of both these methods for thresholding by testing a particular wavelet coefficient, $w(14, 3)$. In this context, effectiveness is based on the ability of the thresholded reservoir model to capture essential reservoir features so that a simulation run would yield production profiles that are closest to the original full/unthresholded model. Figures 4.9 to 4.11 show the sensitivity magnitude vs. time corresponding to wavelet coefficient $w(14, 3)$. These figures show the area under the curve and the cutoff, both set so that the final number of wavelet coefficients retained in both cases is the same (equal to 23% of the total number). For sensitivity to producer BHP, Figure 4.9, we see that the cutoff value is never reached, although the area under the curve is substantial. In Figure 4.10 we notice that the sensitivity of coefficient $w(14, 3)$ to injector BHP just makes the cutoff limit at the end time steps. Sensitivity to producer WCT rises sharply (Figure 4.11), just making the cutoff and then declining rapidly, to give a low value for area under the curve. We see that the maximum value of sensitivity is for this profile is 0.0576. The normalized area under

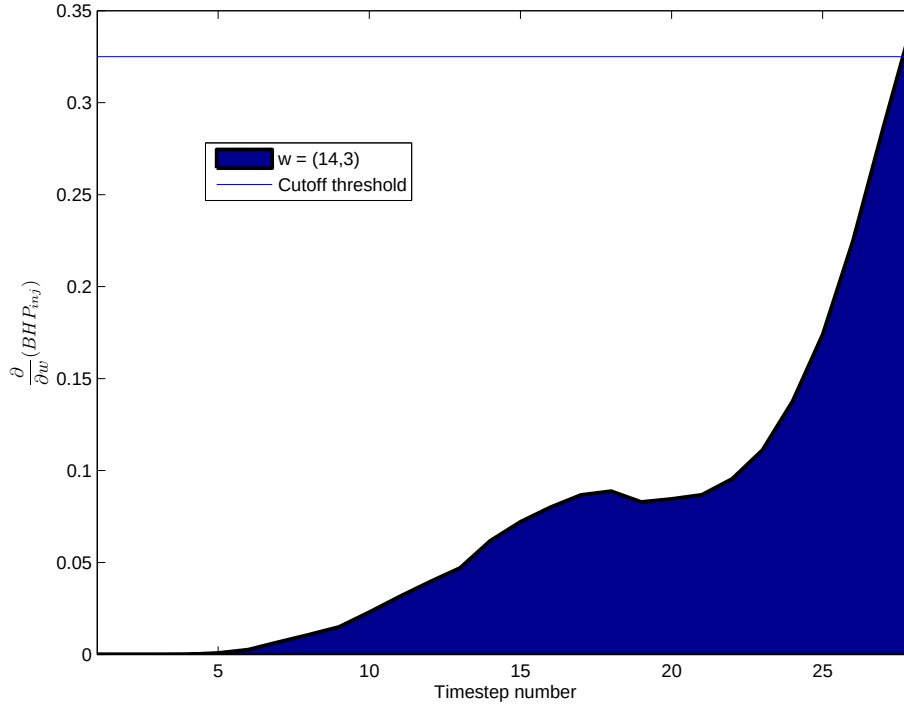


Figure 4.10: Area under the curve and cutoff limit for injector BHP sensitivity to wavelet coefficient $w(14,3)$ with time.

the curve for this case is 0.0021, and the minimum value of area for which a coefficient is retained in this case is 0.0102. Thus under Methodology 1, this wavelet coefficient is not counted as being significant for production data match. However, for Methodology 2 (minimum cutoff method), the minimum cutoff is set at 0.0565, and since 0.0576 is greater than 0.0565, this wavelet coefficient passes the test and is retained.

Thus we notice that the cutoff methodology for thresholding enables the retention of wavelet coefficients whose sensitivity to production data is high though for short time intervals. Using the area under the curve method, we would effectively give higher preference to retaining wavelet coefficients that might have a steady low value of sensitivity for the entire history. Theoretically, both these methods have some advantages and disadvantages. However we used this example with the wavelet coefficient $w(14,3)$ to determine which method of thresholding is better suited for our purpose. Based on Methodologies 1 and 2 we obtain sensitivity maps shown in Figure 4.12 and Figure 4.13 respectively. As can be seen for area under the curve method (Figure 4.12), coefficient $w(14,3)$ is retained in one out of the

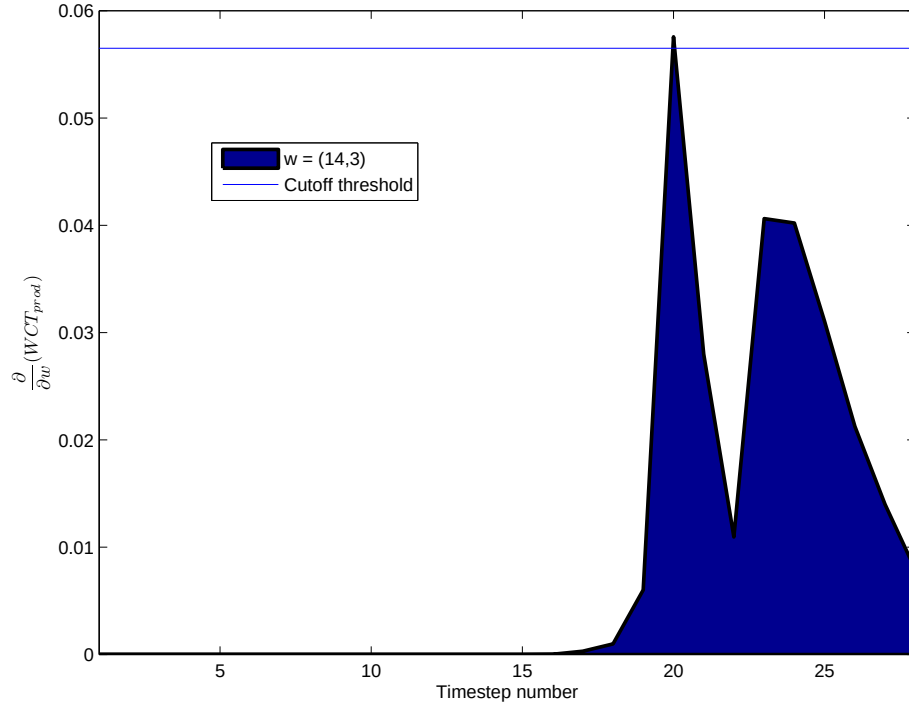


Figure 4.11: Area under the curve and cutoff limit for producer WCT sensitivity to wavelet coefficient $w(14,3)$ with time.

four maps, whereas for the cutoff method (Figure 4.13), it is retained in three out of four maps. The total number of wavelet coefficients used in both cases was kept equal for a fair comparison.

In order to compare the effectiveness of the two methodologies, we performed flow simulation on the two thresholded permeability distributions obtained using each type of thresholding. The results from these simulation runs are plotted in Figures 4.14 through 4.16. We observe that we get a good match to the historical data using the area-under-the-curve method of thresholding, while using the same number of parameters under the minimum cutoff scheme gives a poorer match. Thus we can conclude that for the case in question (reservoir HM1) the area-under-the-curve method of thresholding is better able to capture the essence of the time-varying sensitivity coefficients. However, it is still possible that in some other case, the minimum cutoff method might be more effective. As such, to get the highest degree of parameter compression while retaining a data match, it is advisable to make a preliminary trial of both these methods of thresholding.

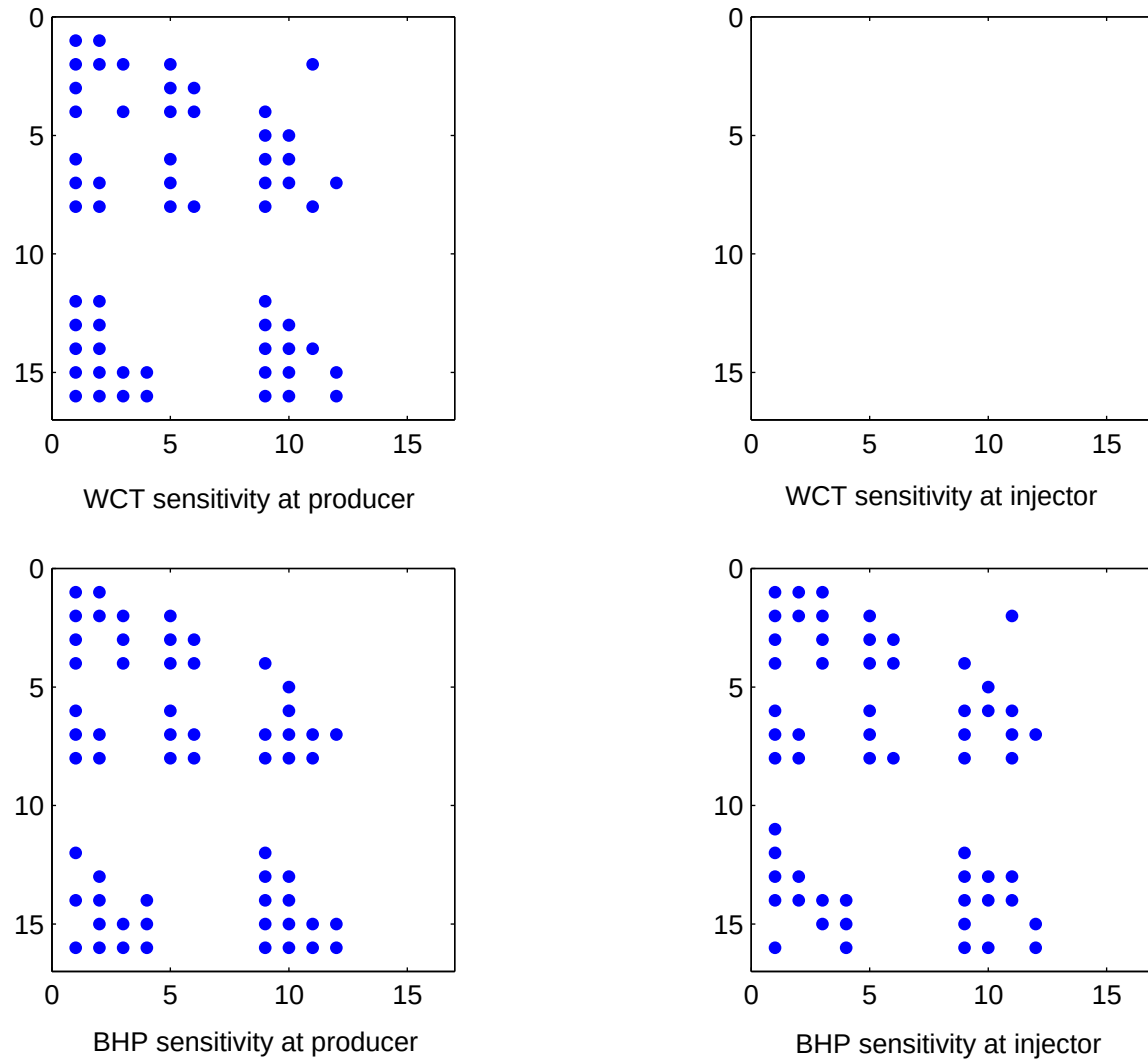


Figure 4.12: Nonzero sensitivity maps using methodology 1 (area-under-the-curve) for thresholding.

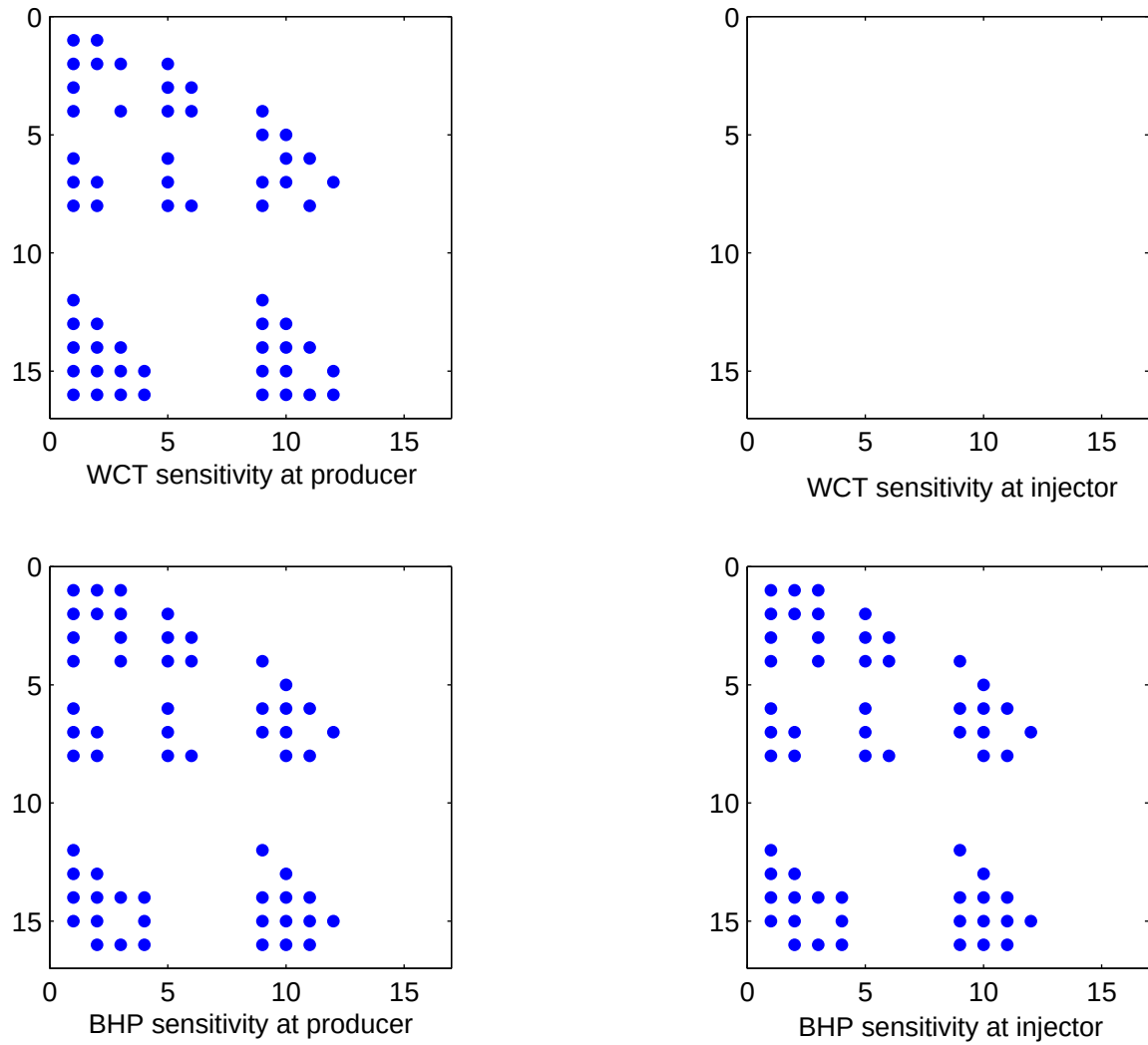


Figure 4.13: Nonzero sensitivity maps using methodology 2 (minimum cutoff) for thresholding.

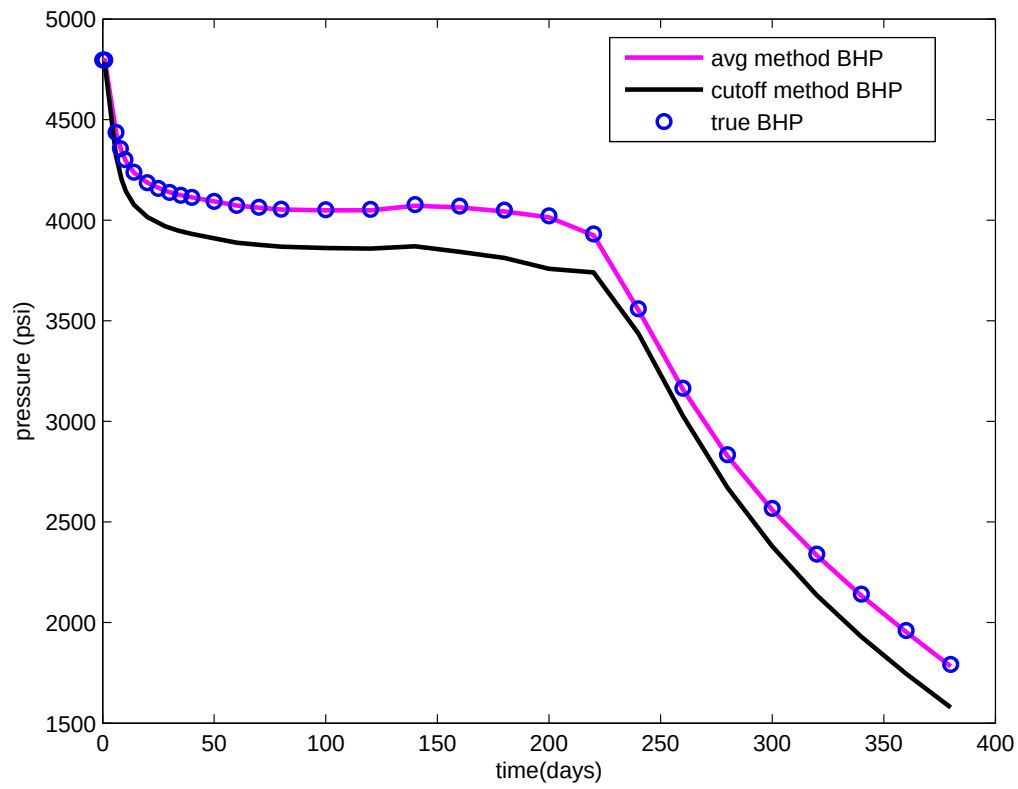


Figure 4.14: Producer BHP with time for reservoir HM1, showing the production history data along with results from the two thresholding techniques.

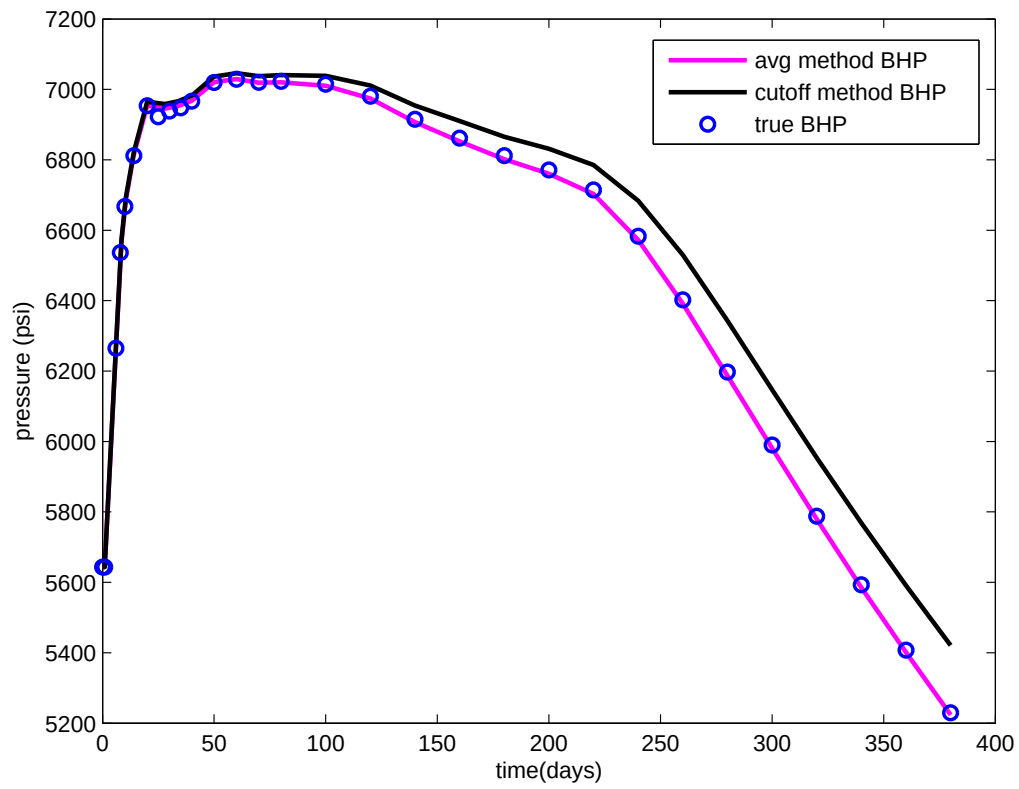


Figure 4.15: Injector BHP with time for reservoir HM1, showing the production history data along with results from the two thresholding techniques.

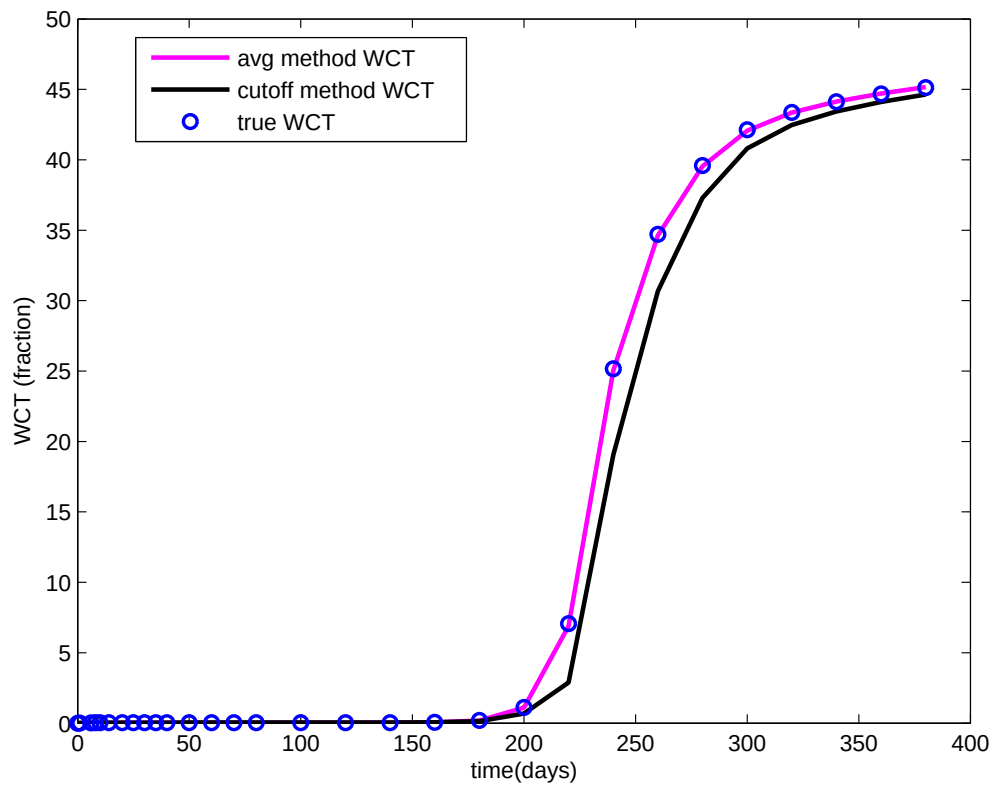


Figure 4.16: Producer WCT with time for reservoir HM1, showing the production history data along with results from the two thresholding techniques.

4.2.3 Well by Well Thresholding

So far we presented thresholding techniques that set cutoffs for the integration of production history data based at the field scale. In other words, all wells were given equal importance (or weight) and the sensitivity calculated is for the entire period of production from all wells with respect to the wavelet coefficients of the model permeability distribution. However, we can constrain the permeability field to production data from each well separately. We demonstrate the key issues and advantages of well-by-well data integration using an example reservoir G1b.

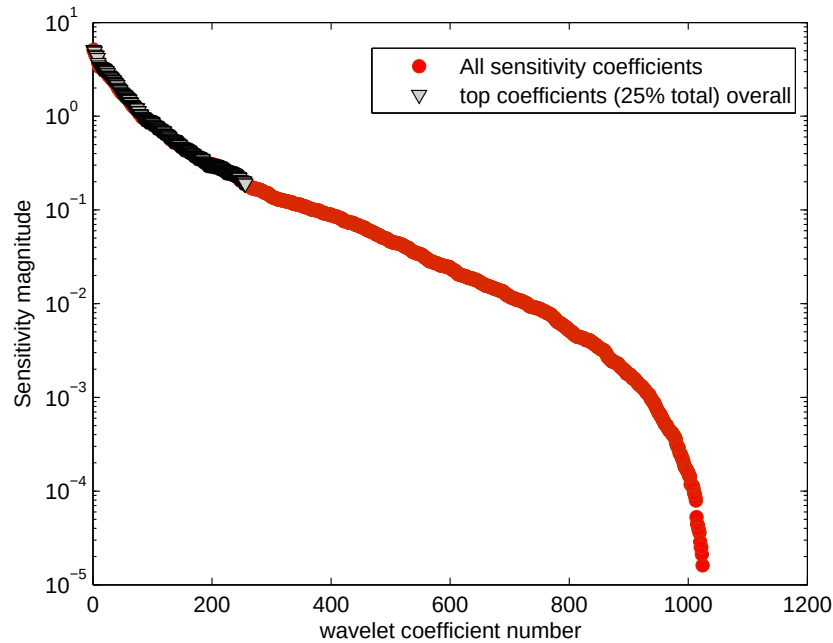


Figure 4.17: Reservoir G1b - sensitivity coefficients of all production data with respect to wavelet parameters, sorted in descending order, highlighting in black the top 25% sensitivity coefficients in magnitude.

The reference permeability field for the field G1b is shown in Appendix B.1. This reservoir is identical in parameter distribution as well as well locations as reservoir G1 (see Appendix B.1), and differs only in the production scenario. The object is to find the minimum number of wavelet coefficient parameters required to constrain the model to production history data that consist of the well BHP and WCT from the three producing

wells and the BHP from the single injection well.

We start the analysis by computing the sensitivity of the full field production data (all wells included) with respect to the full set of wavelet coefficients obtained by a Nonstandard Haar wavelet transform of the permeability field. These coefficients are plotted in Figure 4.17 in descending order. We see that the sensitivity of the production data to wavelet coefficients of the permeability field varies over many orders of magnitude.

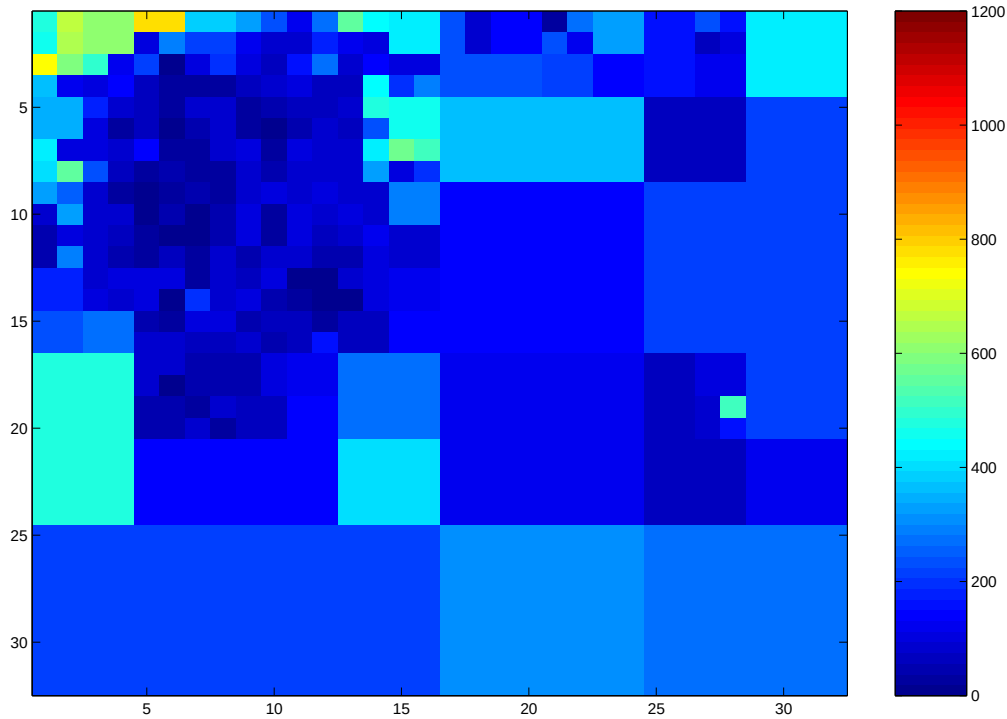


Figure 4.18: Reservoir G1b - thresholded permeability field (md) using the top 25% wavelet coefficients of the permeability field that are highly sensitive to the overall field production history.

Given the sensitivity coefficient distribution as shown in Figure 4.17, we choose the top 25% in magnitude and fix the values of the corresponding wavelet coefficients of the permeability field, while setting other rest of the 75% to zero (or don't-care) values. The inversion of this subset of the wavelet coefficients yields a smoothened and compressed permeability distribution as shown in Figure 4.18. This permeability field is obtained as a

result of using only those 256 wavelet parameters (25% of the original 1024 parameters) that are most crucial to production history match. As such, we expect that for the given level of parameter-reduction, the reduced field should provide a good match to the reference production data for the full field (all wells considered together). In order to check this hypothesis, we ran flow simulation on this permeability field with the same rock and fluid properties as the reference, and the same production scenario. Some of the results from this run are plotted along with the reference production history in Figures 4.19 through 4.22.

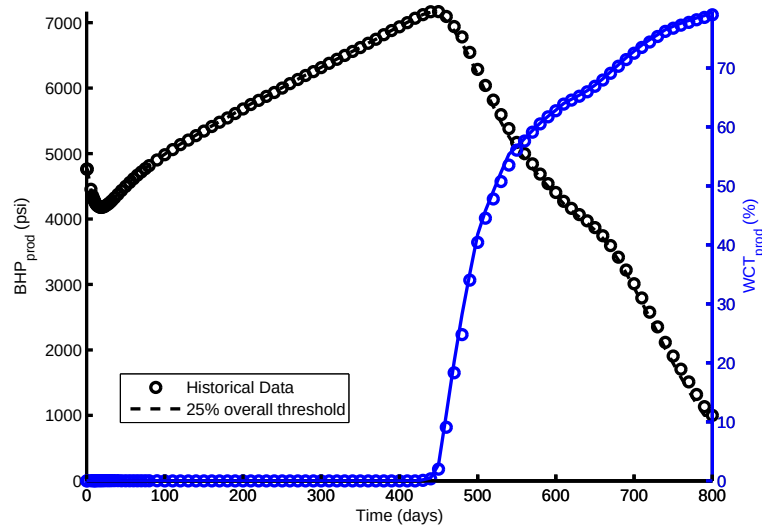


Figure 4.19: Producer 1 - production data match for permeability field shown in Figure 4.18.

We obtained a very good match for BHP and WCT for most of the wells using only a fraction of the total number of permeability parameters. Figures 4.19 through 4.22 show the production data match for WCT and BHP for Producer 1 (Well 1) using only 25% of the initial parameters. In Figure 4.20 we observe that we are unable to match the BHP or WCT for Producer 2 (Well 2) using these top 25% wavelet coefficients. We also checked and confirmed that if we were to use a greater fraction of wavelet coefficients we would be able to match all production history from all the wells. It was observed that retaining a minimum of 35% of the highest sensitivity wavelet parameters would yield a good match for production from all the wells.

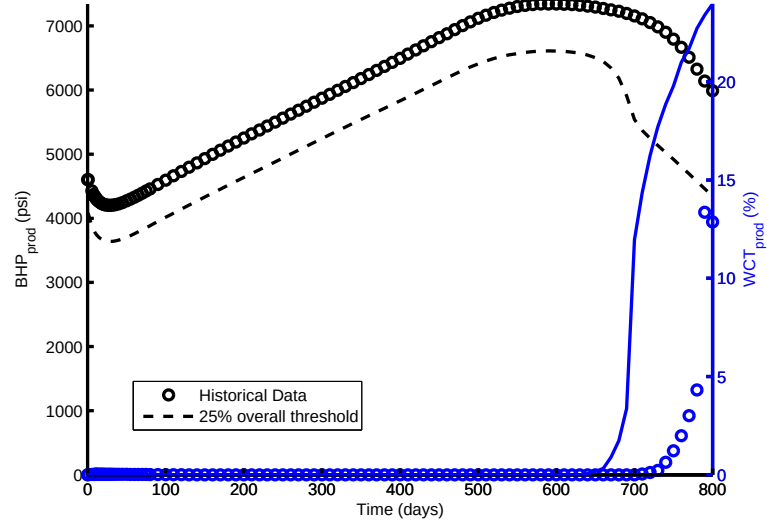


Figure 4.20: Producer 2 - production data match for permeability field shown in Figure 4.18.

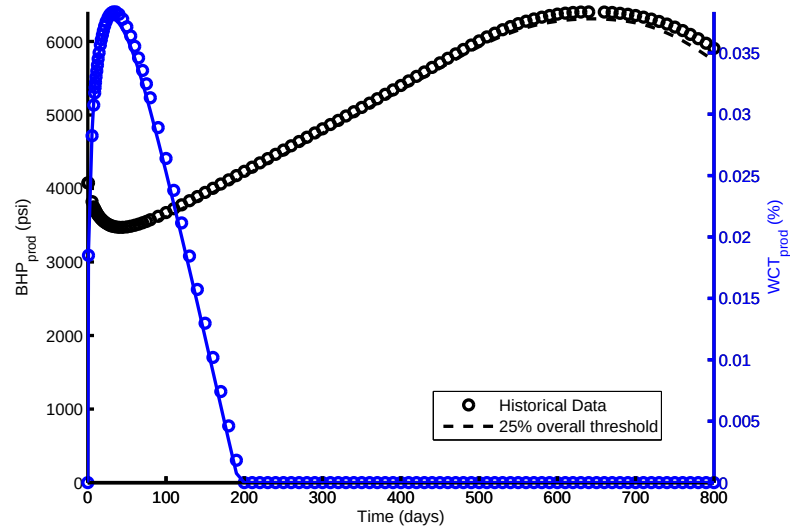


Figure 4.21: Producer 3 - production data match for permeability field shown in Figure 4.18.

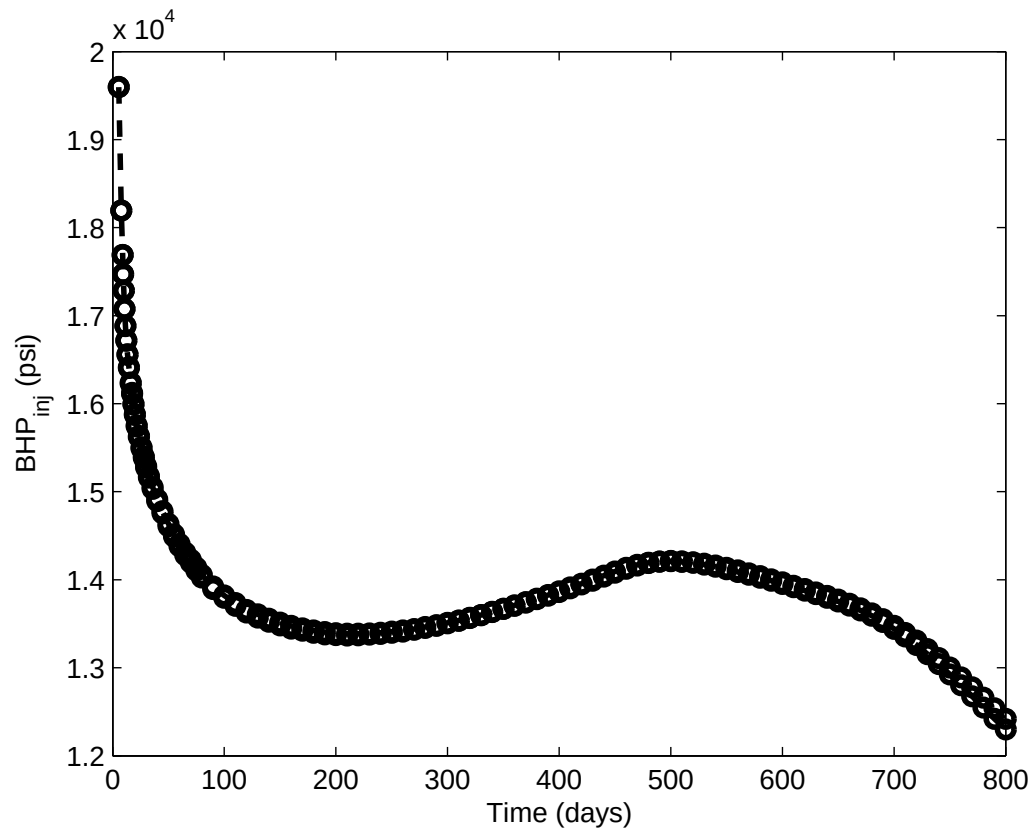


Figure 4.22: Injector - production data match for permeability field shown in Figure 4.18.

As we saw from this example, it is possible that in trying to constrain wavelet coefficients to best match the overall field production history, the match for one or more individual wells might not be preserved. We saw that using 25% of the wavelet coefficients corresponding to the top *overall* highest magnitude sensitivities was able to constrain the model to all but one well (Producer 2). This suggested the possibility that in constraining to the top 25% *overall* highest sensitivity coefficients, we might be constraining preferentially to the production history from some wells and not others (Producer 2 for example). Since we have sensitivity information to production data from each well individually, we can fix that potential problem by constraining to a fixed proportion of high sensitivity wavelet coefficients for *each* well individually. Figure 4.23 shows the sorted sensitivity magnitudes for each well, and we observe that these plots differ from each other and also from the *overall* sensitivity plot with regards to the rate at which the sensitivity magnitudes decline. To give equal weight to each well in terms of production data constraint, we fix the top 12.5% sensitivity wavelet coefficients for each well. Figure 4.24 highlights the location of these wavelet coefficients by well on the overall sensitivity map. Figure 4.25 shows the resulting permeability model if the field was constrained to production data from a single well at a time. We observe that some of the parameters constrain production history more than one well at a time (see Figure 4.27). Due to this overlap, when we combine these sets of parameters, the total number of parameters considered is still 25%. However, based on the *overall* sensitivity magnitudes, these coefficients are no longer the top 25%, but a different subset as seen in Figure 4.26. Figure 4.28 shows the permeability field obtained as a result of constraining to this subset of parameters and Figures 4.29 through 4.32 plot the resulting production data match. We observe that while the match for Producer 2 has improved (as compared to Figure 4.20), the match to Producer 1 has become worse (compared to Figure 4.19). This observation leads to us to believe that Producers 1 and 2 may need a higher proportion of wavelet coefficients to constrain to production data as compared to Producer 3 and the injector.

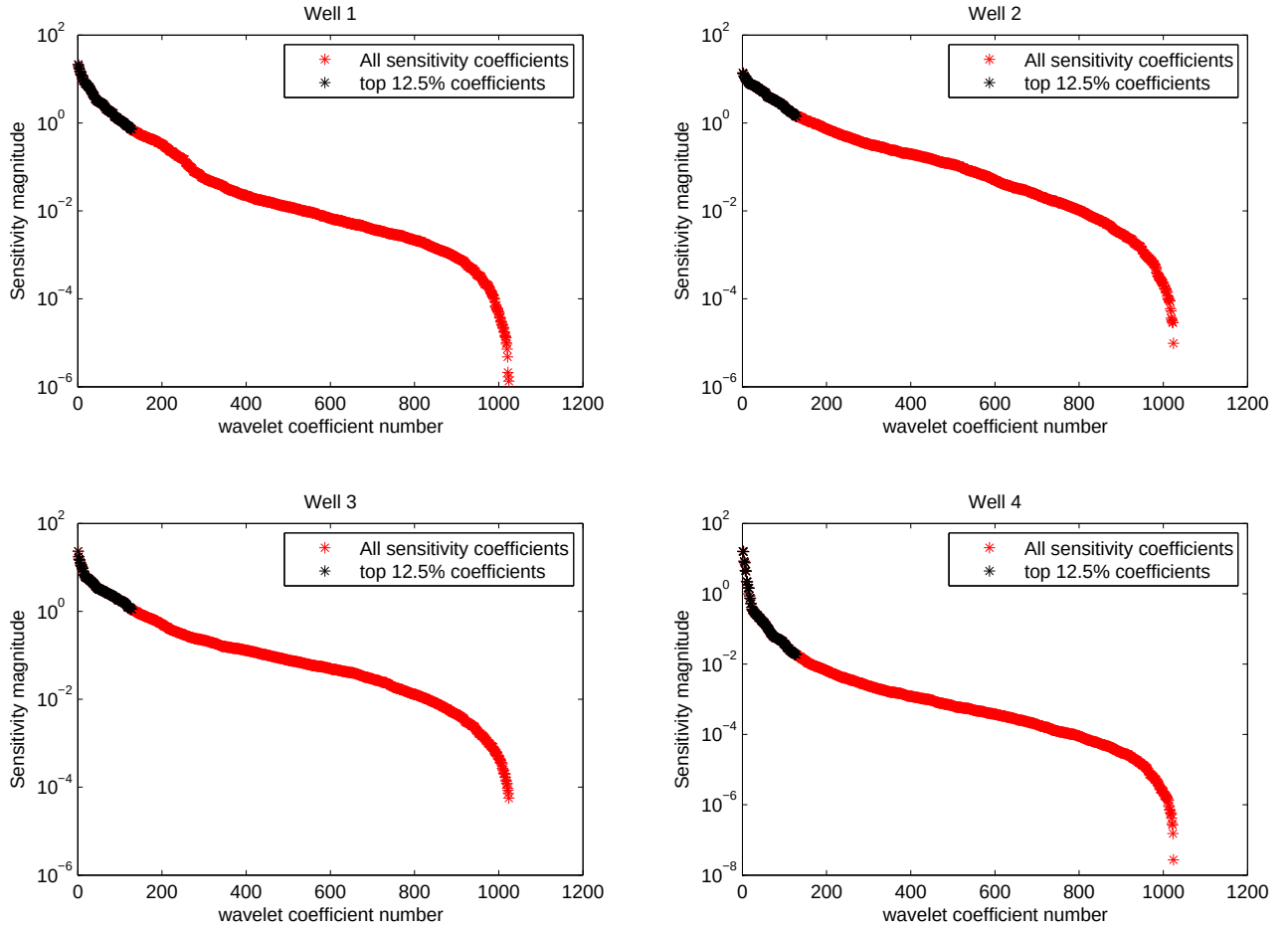


Figure 4.23: Sorted sensitivity coefficients by well, highlighting in black the percentage of coefficients constraining data from each well.

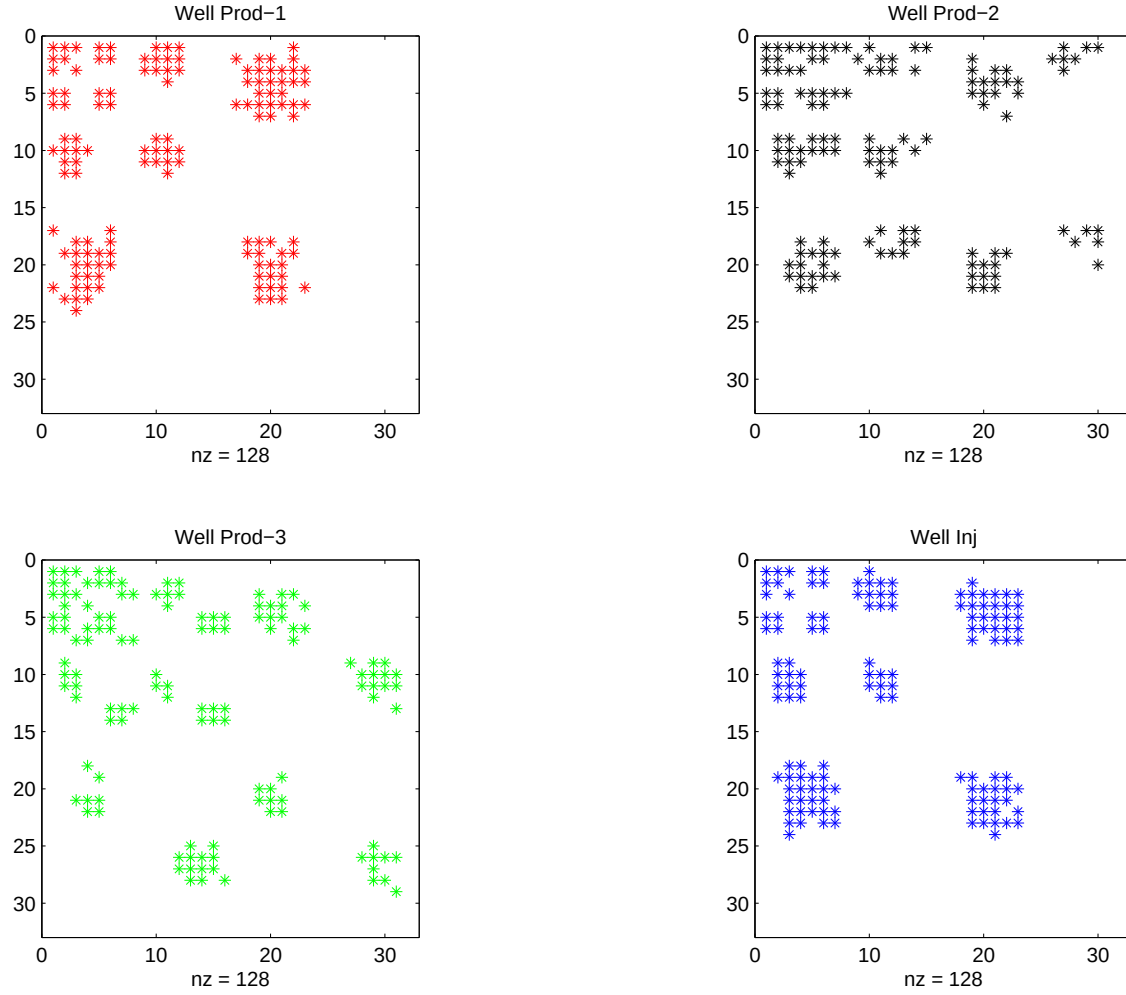


Figure 4.24: Sensitivity coefficient maps by well, showing the subset of coefficients constraining data from each well.

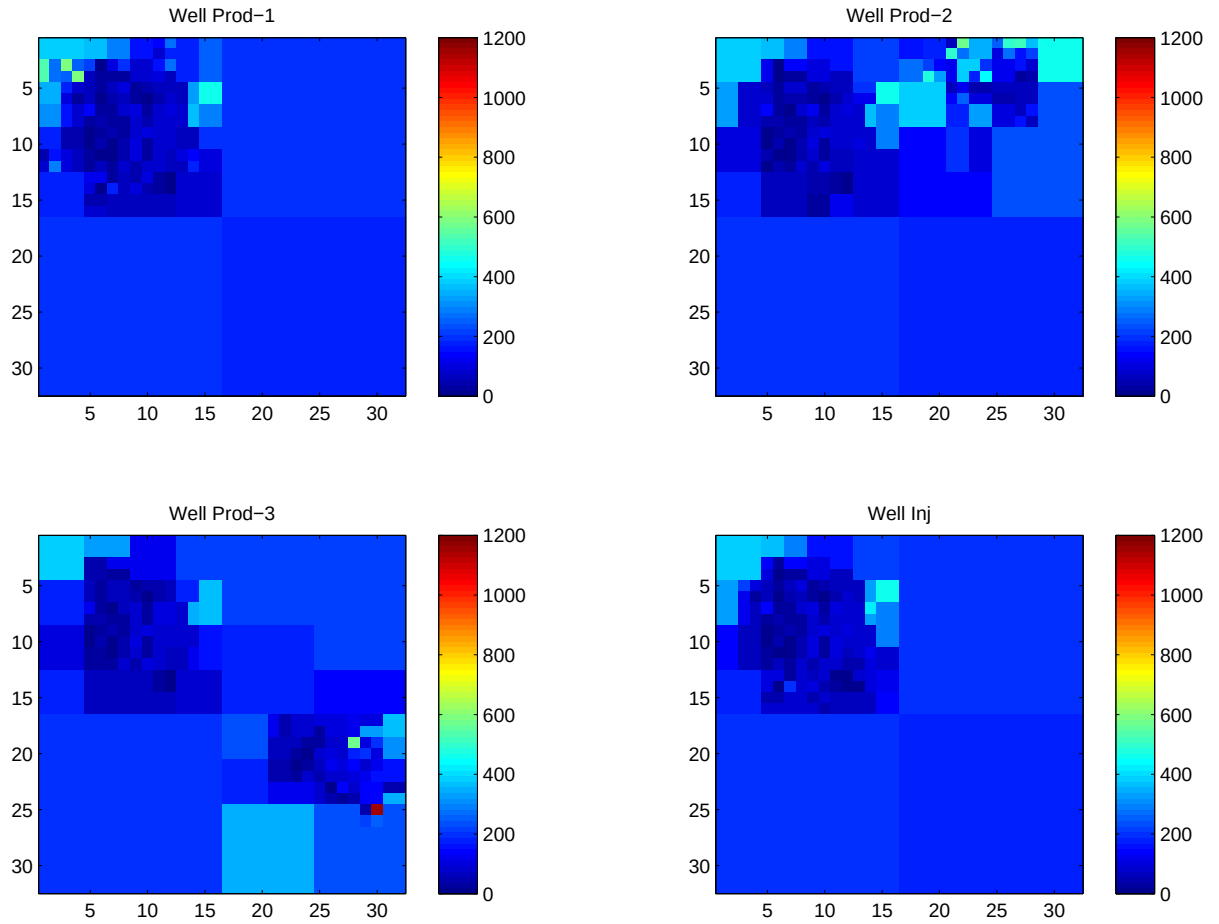


Figure 4.25: Permeability distribution (md) corresponding to thresholding separately for each individual well as shown is Figure 4.24.

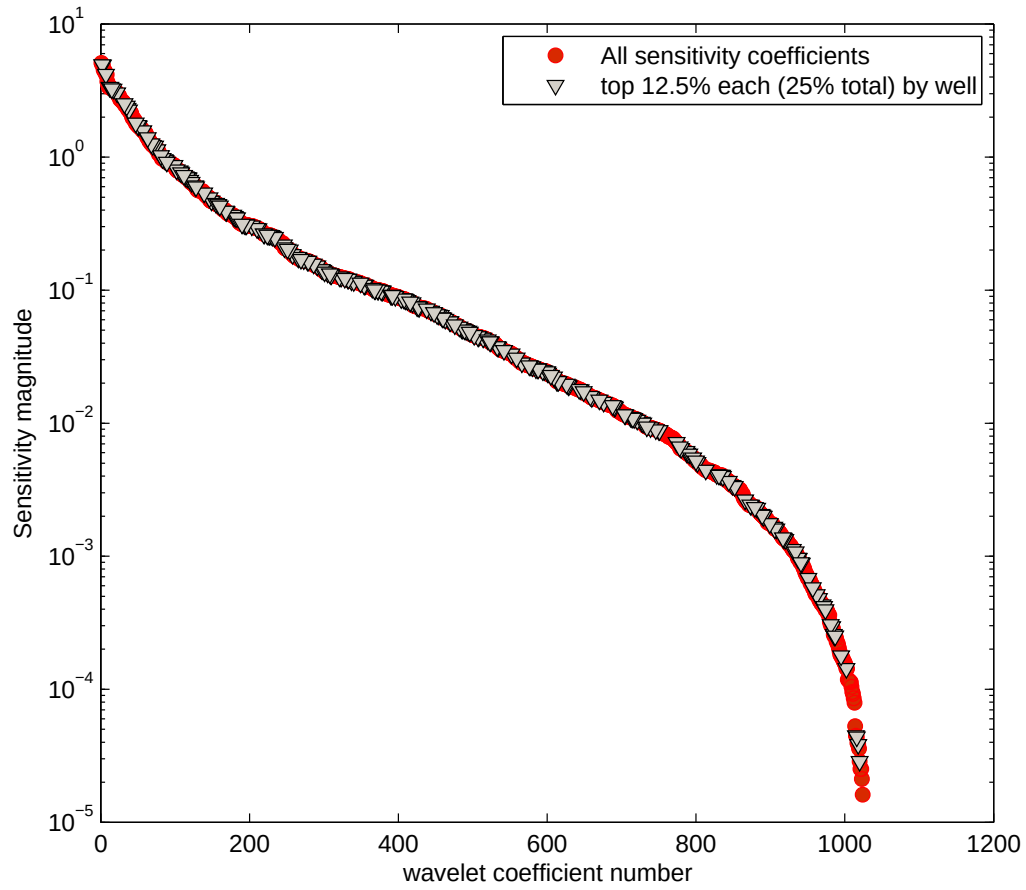


Figure 4.26: Reservoir G1b - sensitivity coefficients of all production data with respect to wavelet parameters, sorted in descending order, highlighting in black the top 12.5% sensitivity coefficients in magnitude.

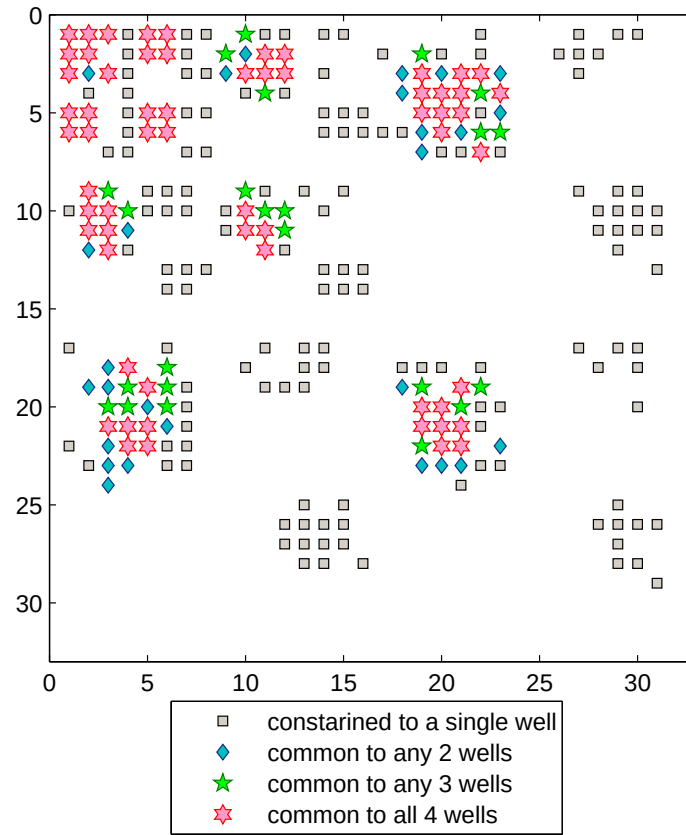


Figure 4.27: Reservoir G1a Sensitivity Coefficient map showing location of subsets of highest sensitivity wavelet coefficients with respect to production from each well.

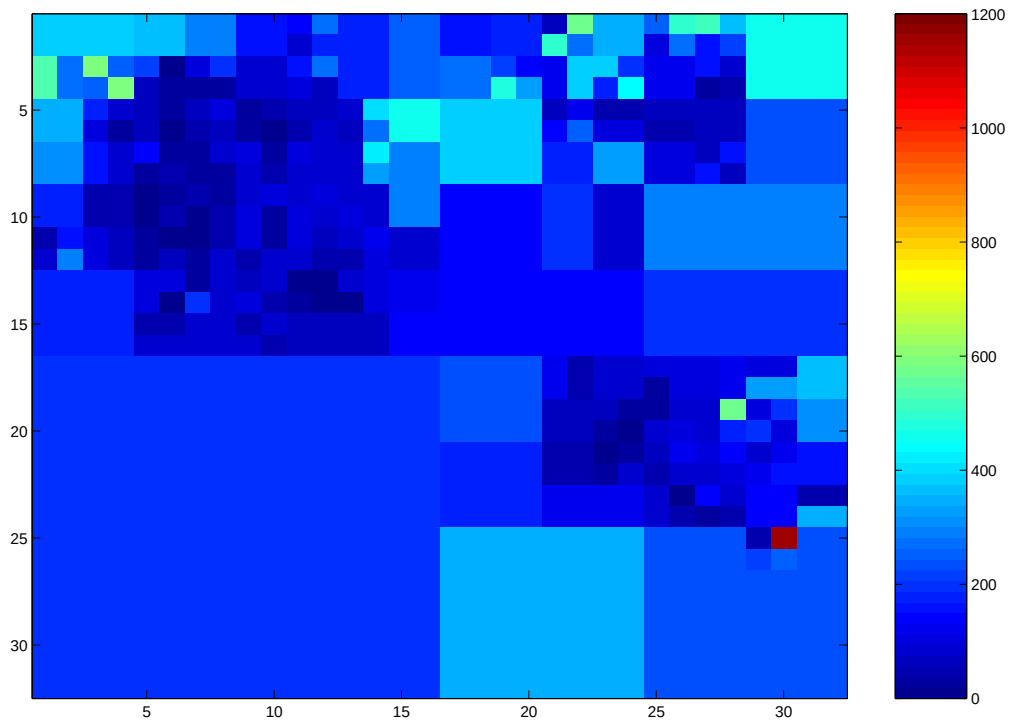


Figure 4.28: Reservoir G1b - thresholded permeability field (md) using the top 12.5% wavelet coefficients of the permeability field that are highly sensitive to the overall field production history.

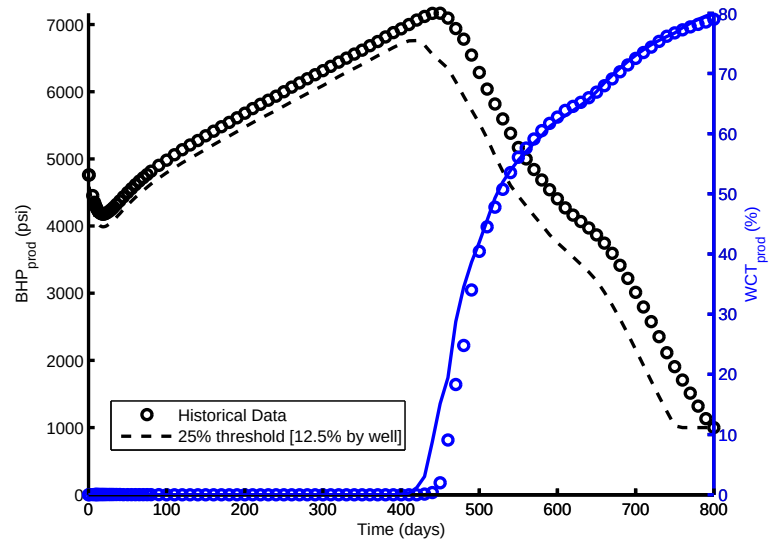


Figure 4.29: Producer 1 - production data match for permeability field shown in Figure 4.28.

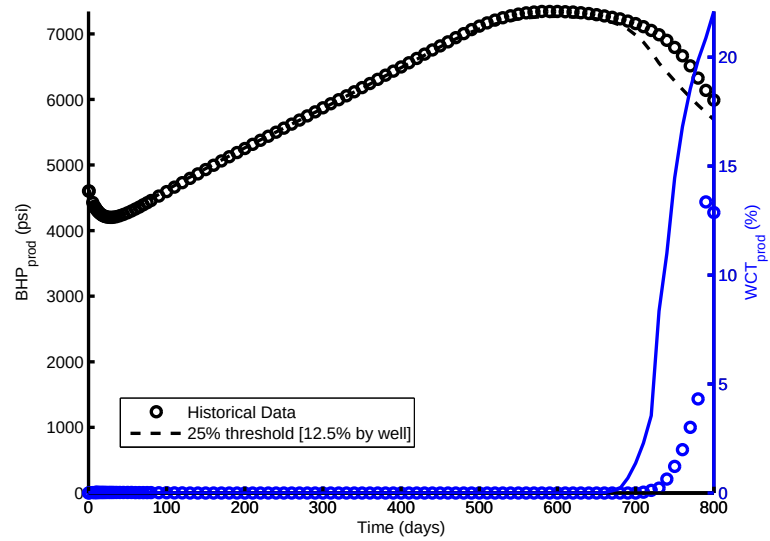


Figure 4.30: Producer 2 - production data match for permeability field shown in Figure 4.28.

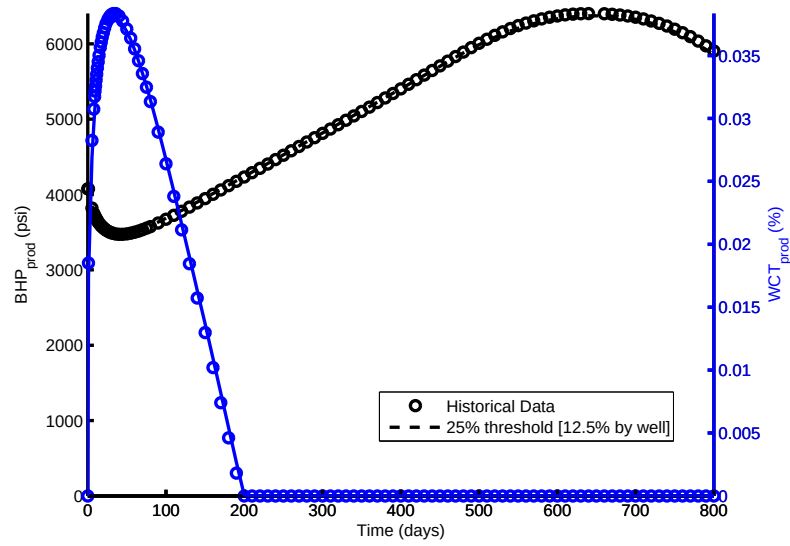


Figure 4.31: Producer 3 - production data match for permeability field shown in Figure 4.28.

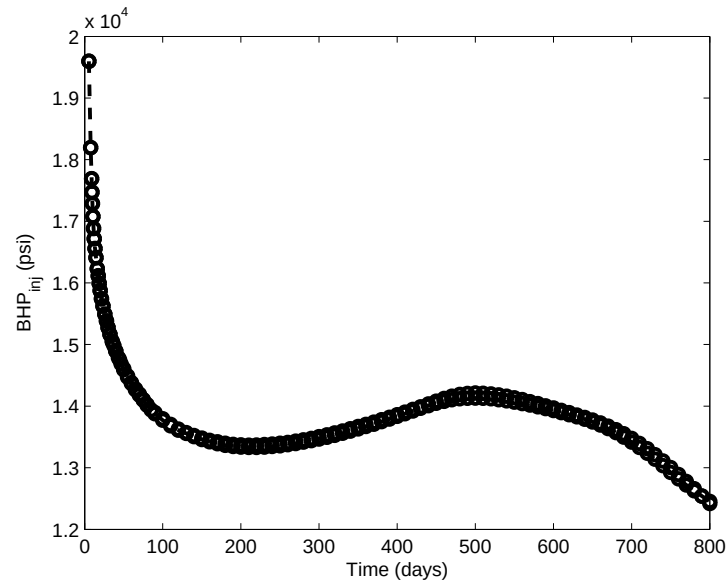


Figure 4.32: Injector - production data match for permeability field shown in Figure 4.28.

This brings us back to our observations about the difference in the distributions of sensitivity magnitudes for each individual well. Figure 4.33 shows the sensitivities of WCT and BHP data from individual wells to all the wavelet coefficient parameters. Based on these individual distributions, we can optimize the maximum number of high sensitivity coefficients we choose for each well in order to match the production history for all wells individually while keeping the total number of nonzero coefficients used at 25%. The optimized percentages obtained were 16% for Well 1, 19.8% for Well 2, 1% for Well 3 and 6.8% for Well 4. It is observed that the number of coefficients required to match production data from wells 1 and 2 is much higher than the number required to match production output from wells 3 and 4. It is interesting to note here are that the sensitivity magnitudes for Well 4 (the injector) fall off much faster than the other wells. Also, both wells 3 and 4 predominantly have BHP history only, whereas wells 1 and 2 have significant WCT history as well. For these two reasons amongst others, we can explain why wells 1 and 2 require a greater proportion of wavelet coefficients to match production data than wells 3 and 4.

By applying the individual thresholds based on each well separately, we can determine the portion of the permeability model influenced by each individual well. These plots are shown in Figure 4.35, and we can see that the portions of the permeability field as well as the level of resolution of detail depend on the number of wavelet coefficients that are used to constrain to production data from each well.

As can be expected after analyzing Figure 4.33, the 25% of the sensitivity coefficients required to constrain the production history well by well are not the same as the coefficients with the highest magnitude of sensitivity (as in Figure 4.17 corresponding to the overall field match). Instead, we see that for the case in which sensitivity thresholds are set individually for each well, the actual coefficients may span a broader zone of overall sensitivity magnitudes (see Figure 4.36). Based on this selection of wavelet coefficients we can construct a permeability field as depicted in Figure 4.38. Flow simulation with this reduced permeability field gives BHP and WCT data, and we plot these data along with the reference history in Figure 4.39 and Figure 4.42. Thus we can conclude that if the overall thresholding technique had been used for partitioning the wavelet coefficient set important for a history match, we would have required to constrain more than 25% of the wavelet coefficients. However, using a well-by-well thresholding technique, we see that the final choice of coefficients turns out to be different from the top 25% in sensitivity magnitude overall, and we are able to achieve a production history match in as few as 25% coefficients.

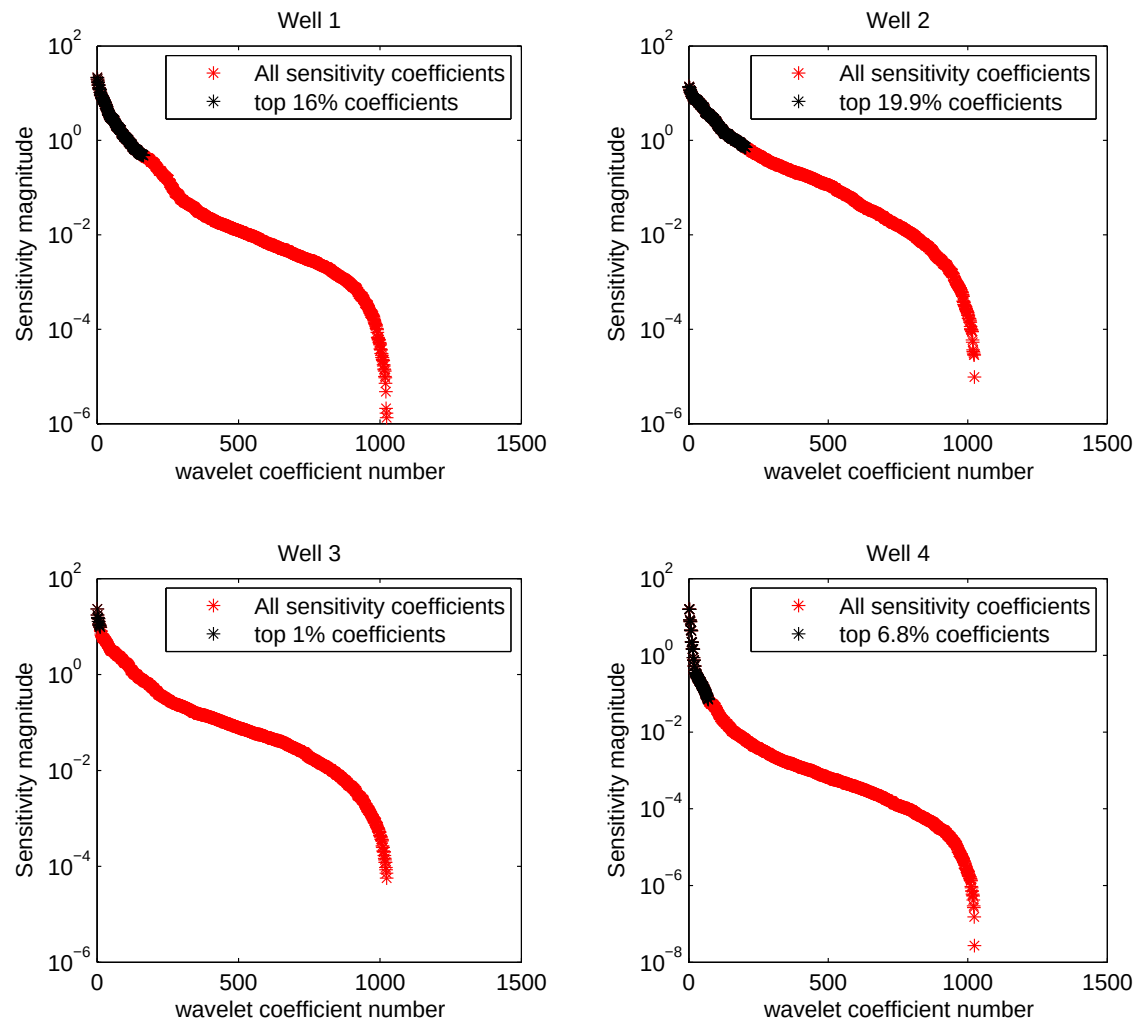


Figure 4.33: Sorted sensitivity coefficients by well, highlighting in black the percentage of coefficients constraining data from each well.

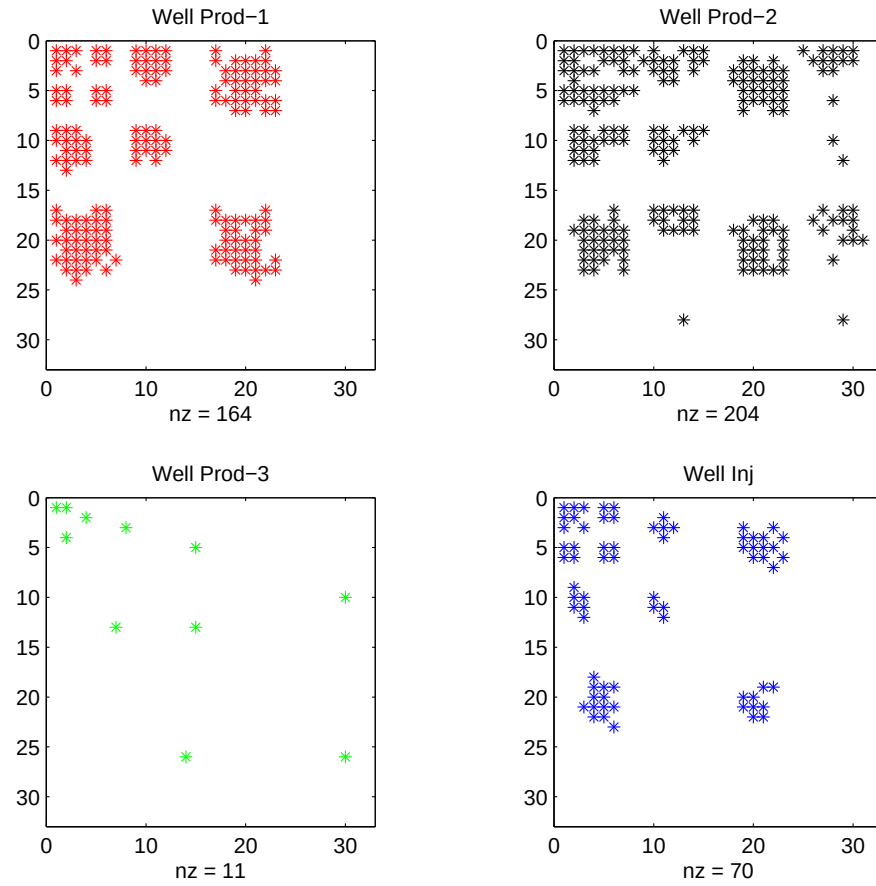


Figure 4.34: Sensitivity coefficient maps by well, showing the subset of coefficients constraining data from each well.

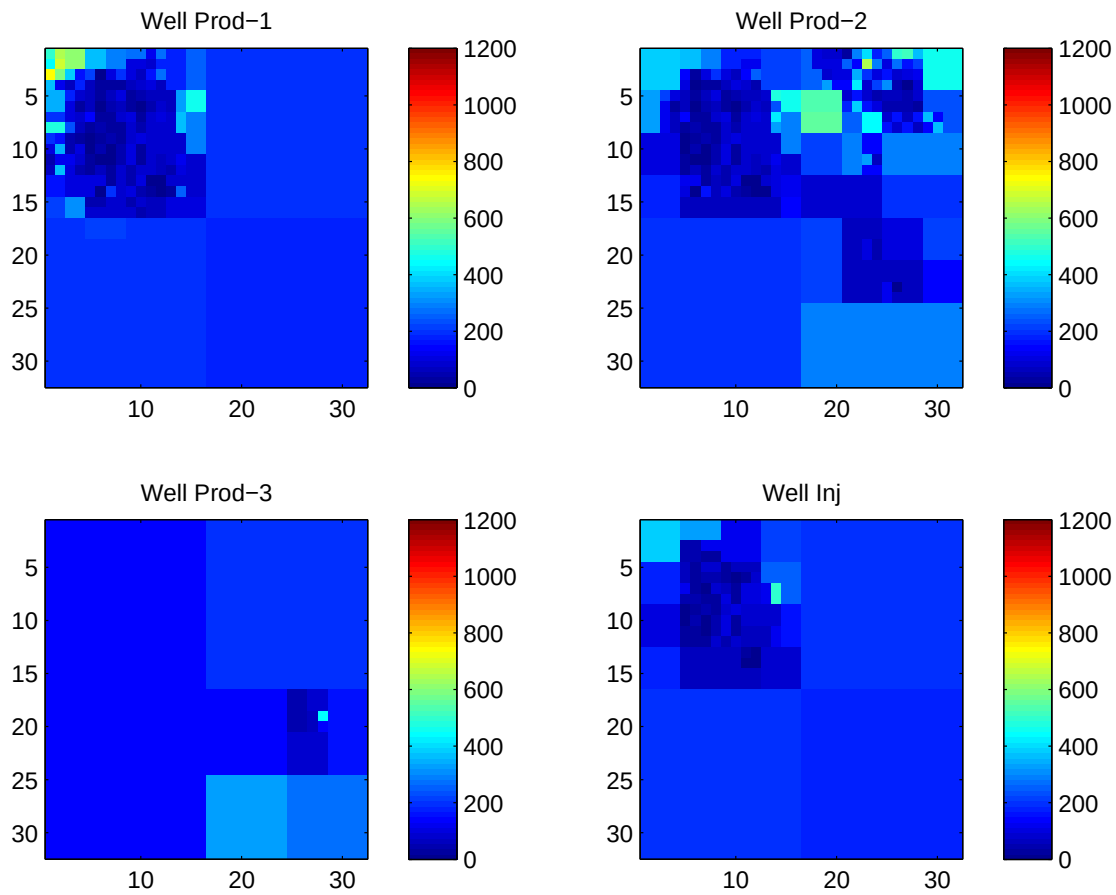


Figure 4.35: Permeability distribution (md) corresponding to thresholding separately for each individual well as shown is Figure 4.34.

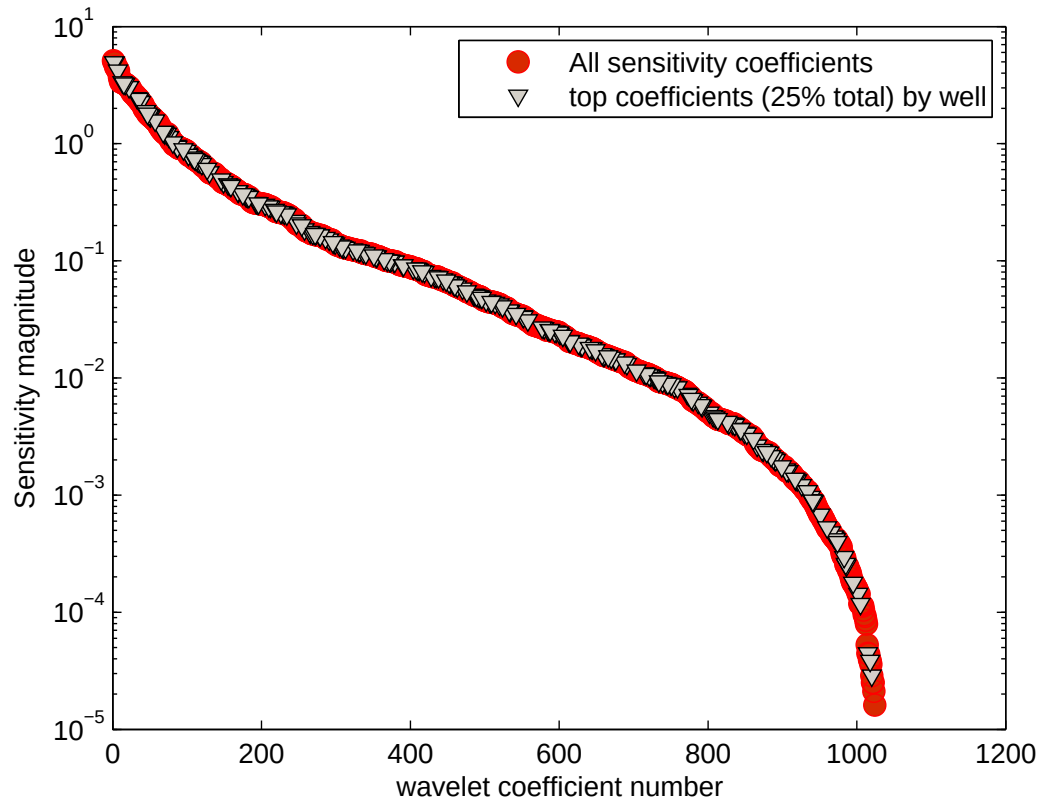


Figure 4.36: Overall sensitivity coefficient magnitudes sorted in descending order, highlighting how the coefficients chosen by well correspond to the overall sensitivity distribution.

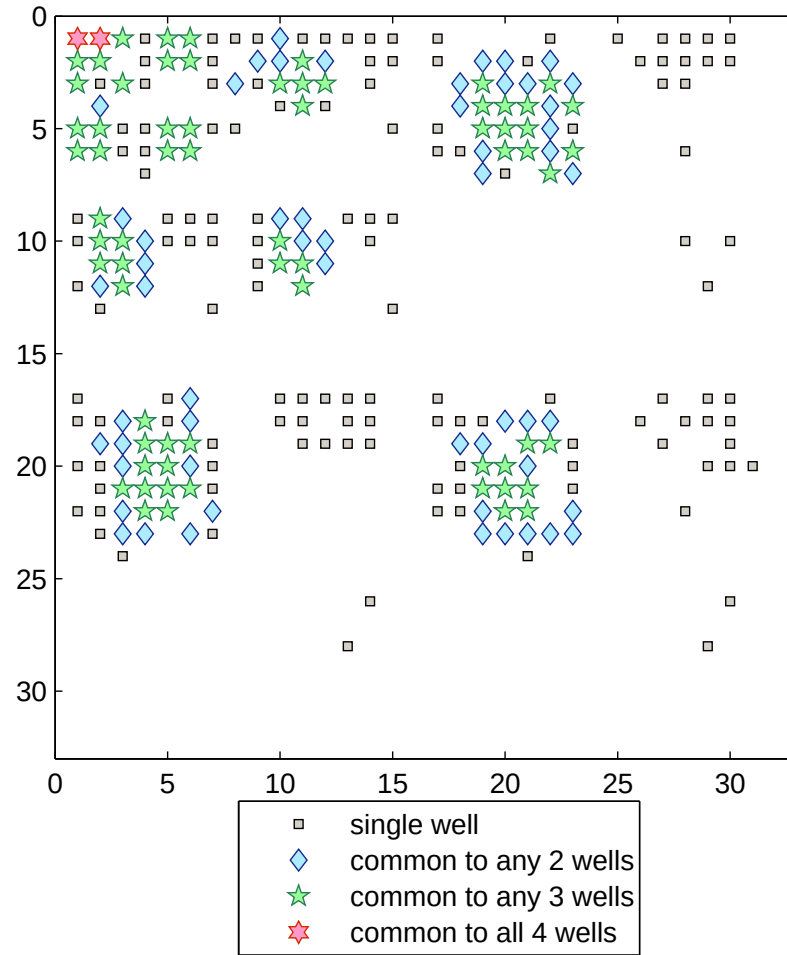


Figure 4.37: Reservoir G1a - Sensitivity coefficient map showing location of subsets of highest sensitivity wavelet coefficients with respect to production from each well.

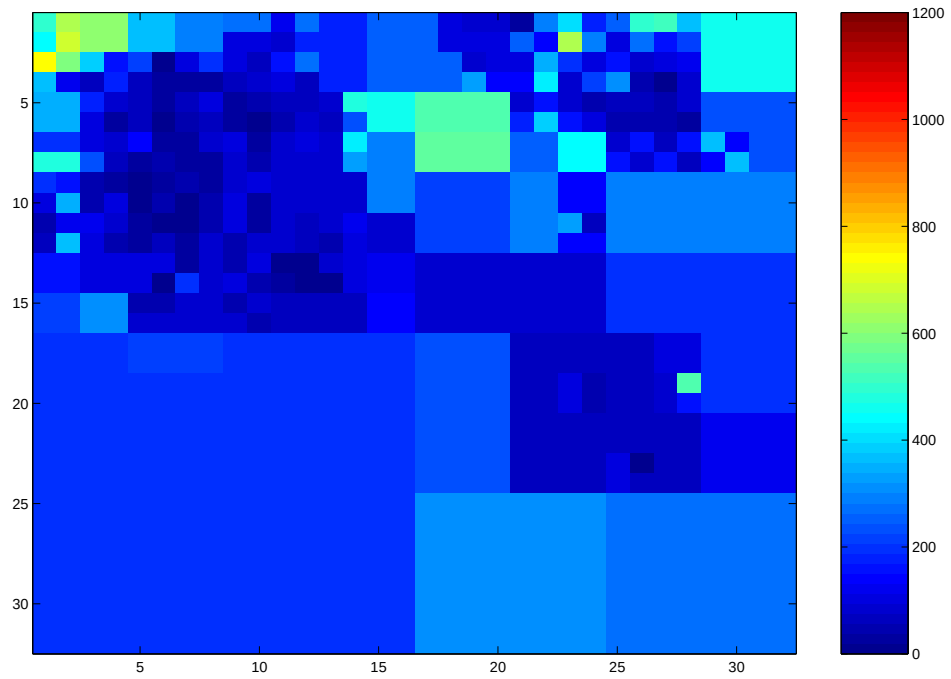


Figure 4.38: Reservoir G1b - Thresholded permeability (md) using individual well thresholds set at [16% 19.9% 1.0% 6.8%] for each well respectively.

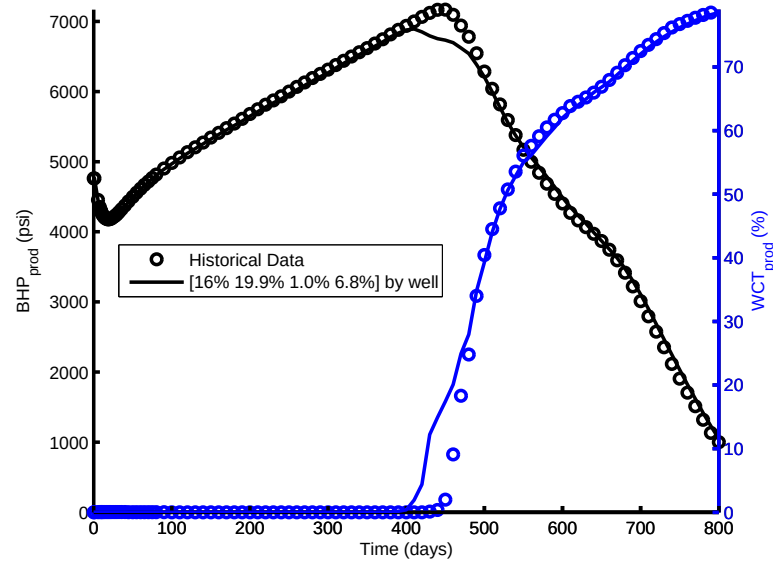


Figure 4.39: Producer 1 BHP and WCT production history match for thresholded permeability distribution as shown in Figure 4.38.

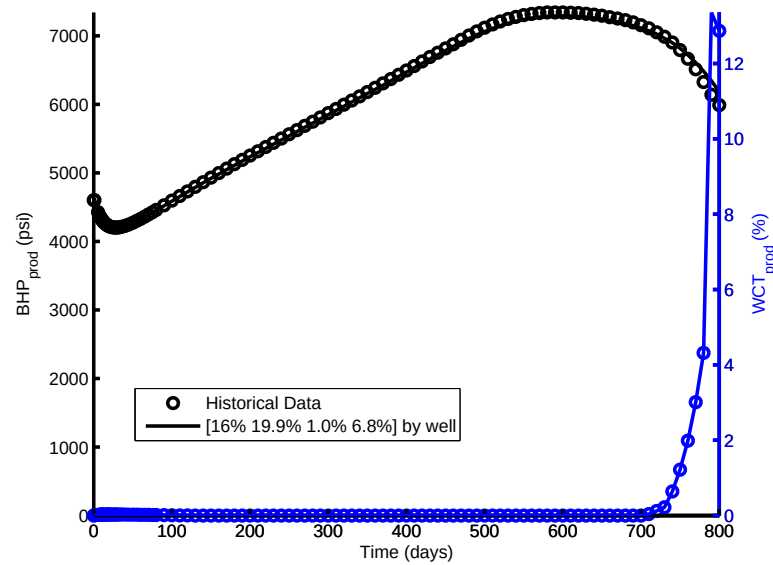


Figure 4.40: Producer 2 BHP and WCT production history match for thresholded permeability distribution as shown in Figure 4.38.

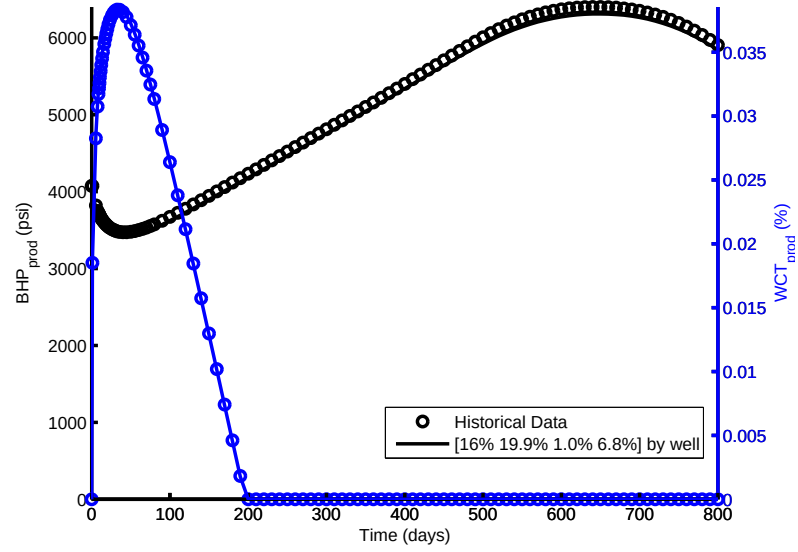


Figure 4.41: Producer 3 BHP and WCT production history match for thresholded permeability distribution as shown in Figure 4.38.

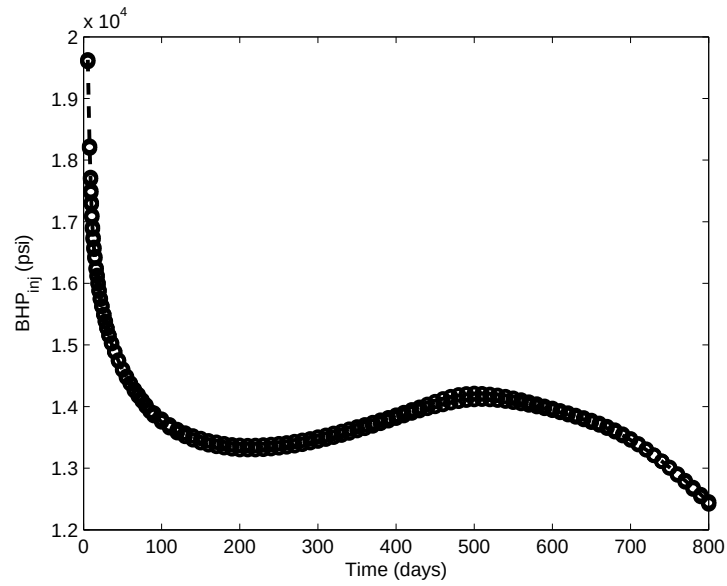


Figure 4.42: Injector BHP production history match for thresholded permeability distribution as shown in Figure 4.38.

4.2.4 Thresholding Based on Data Type

Just as we are able to partition the wavelet coefficient set based on sensitivity to production data from different wells separately, we can also partition the set based on the type of data. As we know, pressure and watercut data are based on different types of differential equations (elliptic and hyperbolic respectively) and as such their dependences on the underlying parameters (permeability, porosity) are different. In general, pressure data are more tightly coupled with property averages, while water production is more a function of local variations of properties. This key insight is often made use of in history-matching work-flows, in which the pressure profile is adjusted first using certain reservoir parameters (usually reservoir volume through field average porosities and permeabilities) and subsequently other parameters are perturbed in order to obtain a watercut match.

In the framework of wavelet-based data integration, this procedure of sequential integration of BHP and WCT data can be performed in a straightforward and elegant manner. The sensitivity of the BHP and WCT data to the wavelet coefficients of permeability can be evaluated separately. For example Reservoir G1b (Figure B.1), the wavelet coefficients most sensitive to BHP data (top 20% in magnitude) and those most sensitive to WCT data (top 13% magnitude) are plotted in Figure 4.43. We see that subsets of wavelet coefficients have high sensitivity to BHP and WCT data respectively, while the intersection of these two sets constrains both BHP and WCT data simultaneously. The subsets shown are composed of the minimum number of parameters required for an acceptable match to production history. Due to an overlap of 11% of these coefficients, the overall number required to match both BHP and WCT is further reduced from the previous result of 25% to as few as 22% of the total number of parameters.

Figure 4.45 shows the permeability distributions that would be obtained by inverse wavelet transform after a reduction of parameters based on BHP and WCT alone. As discussed, the permeability field constrained to BHP data alone is composed using 20% of the wavelet parameters most sensitive to BHP, and the permeability field constrained to WCT data alone is composed using 13% of the original parameters. In terms of history matching or sequential data integration, the parameters sensitive to BHP can first be adjusted to achieve the best match for BHP, followed by an adjustment of the parameters sensitive to WCT. The common parameters can then be adjusted to obtain an overall match to all production data available. Figures 4.48 through 4.51 show historical production data for producing wells in example reservoir G1a, and we can see that constraining the reservoir

model to BHP data only gives an initial match for the well BHP and not to WCT data. Starting from this data match, we constrain to an additional 2% of the parameters that are important for WCT data (corresponding to the blue triangles in Figure 4.43) without modifying the parameters already constrained to match BHP. The resulting permeability field is thus constrained to both BHP and WCT production history data. Thus we see that there exist a lot of different options in terms of the sequence and methodology of production data integration using the wavelet reparameterization. In the example cases studied, production data was integrated well by well or by BHP and WCT data sequentially. This flexibility is especially important for the modeling of large reservoirs with a huge number of wells and greater uncertainty of parameters.

4.2.5 Grayscale-based Thresholding

There is a third thresholding technique that uses a probabilistic framework for determining the subset of wavelet coefficients that need to be fixed in order to maintain a match with the production history data. This method is referred to as grayscaling and has been described in [3] as probabilistic history matching. In the grayscaling method, for every resulting reservoir model a different set of wavelet coefficients could potentially be used to constrain to production data, thereby making a wider sweep of the uncertainty space than what was possible with a deterministic partition. The key difference between grayscaling and the two methods described in Section 4.2 is that grayscaling sets a soft threshold on the sensitivity coefficients, allowing them to be fixed or perturbed probabilistically, whereas the other two methods set a hard threshold, deterministically fixing the coefficients that will be perturbed or kept constant. This methodology of thresholding is explained in greater detail in Section 5.1.2.

4.3 Three-Dimensional Data Integration

There is a need to make the data-integration process more robust and compatible with existing commercial reservoir simulators. As such, the algorithm would be able to work with any type of production scenarios that can be simulated using a commercial simulator. This has the added advantage of making it possible to manipulate the calculation of sensitivity coefficients and of making the algorithm ready to use in industry. Another step in this direction would be to extend the two-dimensional wavelet toolbox to three dimensions, thus

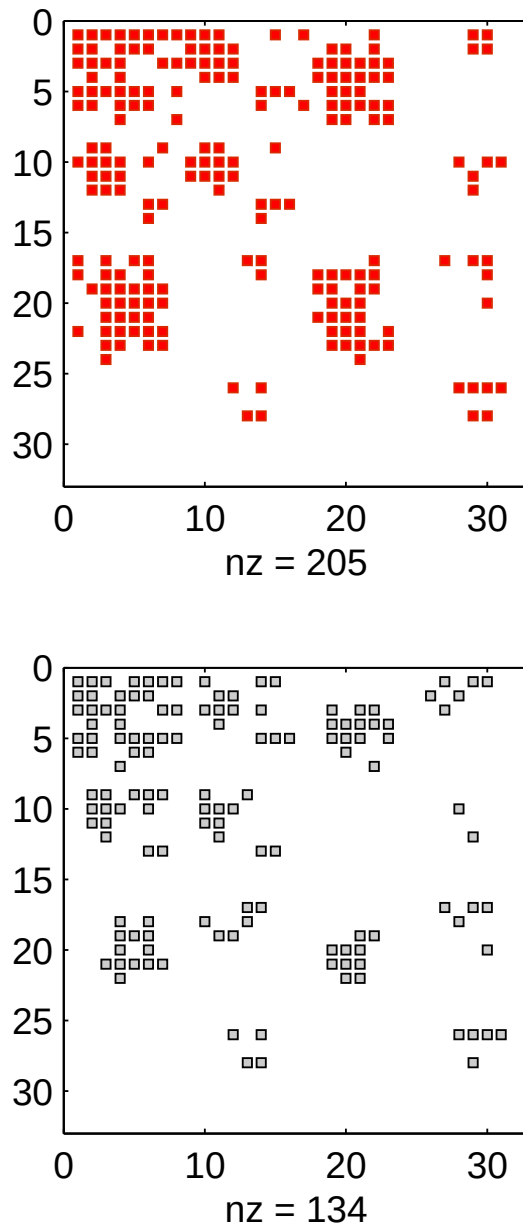


Figure 4.43: Sensitivity coefficient maps showing location of subsets of highest sensitivity wavelet coefficients with respect to BHP data (top) and WCT data (bottom).

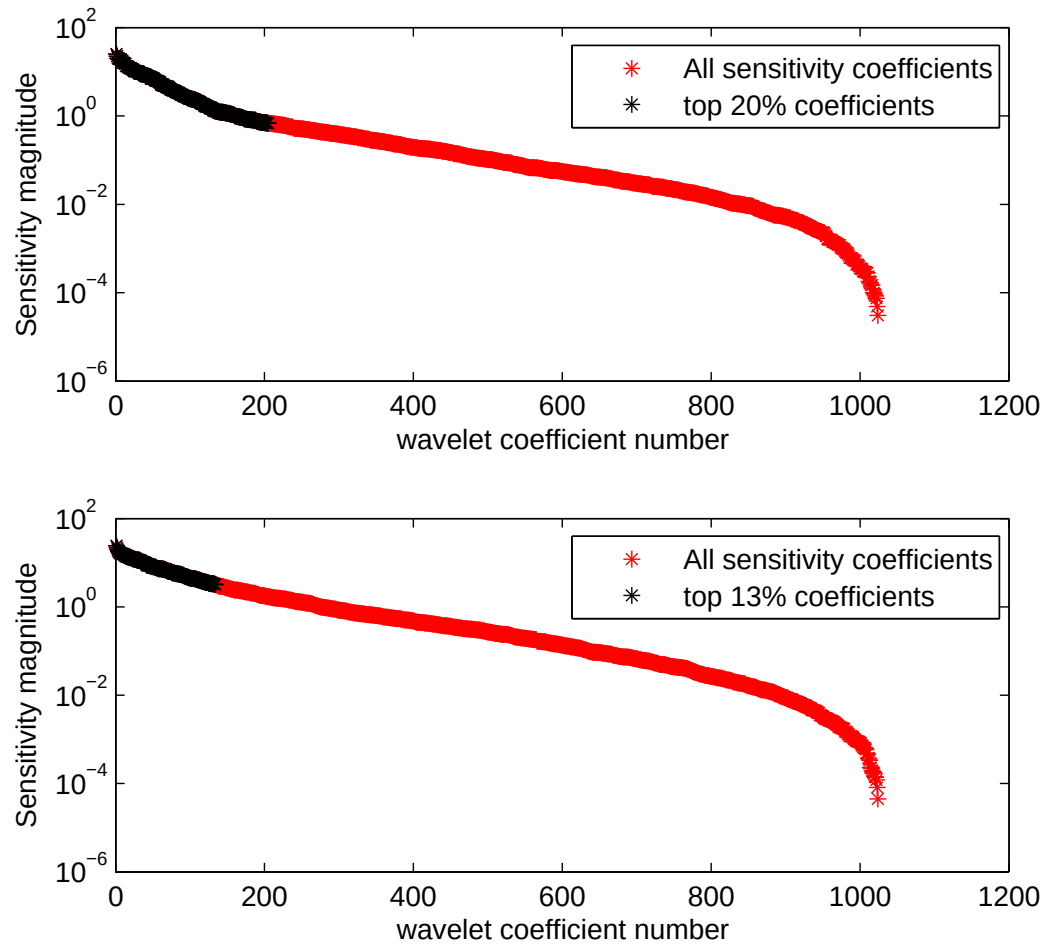


Figure 4.44: BHP (top) and WCT (bottom) sensitivity coefficient magnitudes sorted in descending order, highlighting the coefficients retained during the thresholding process.

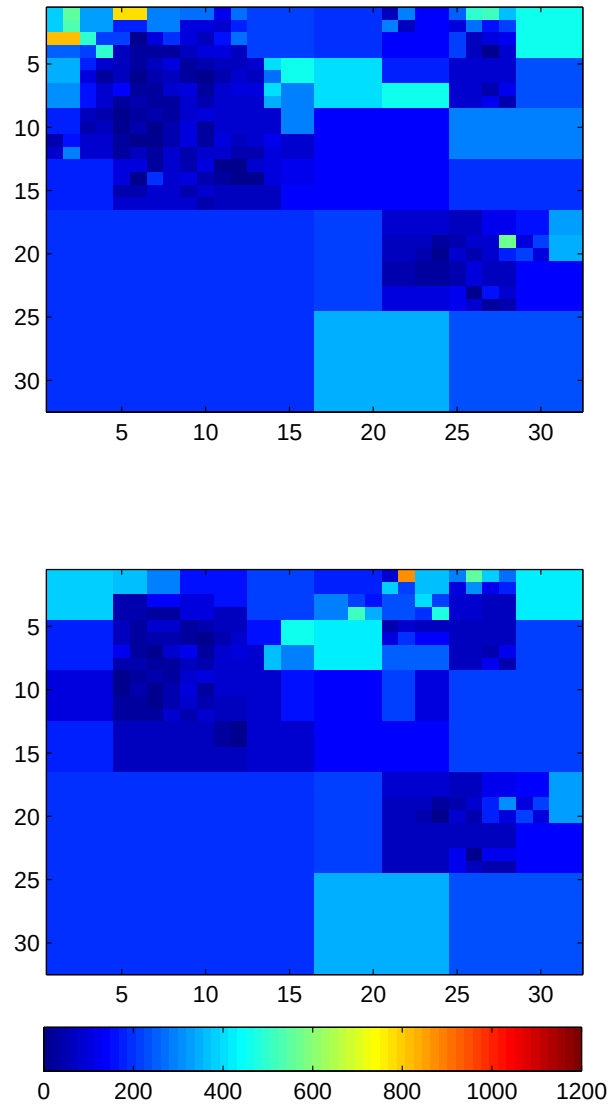


Figure 4.45: Permeability distributions (md) obtained by thresholding based individually on BHP data (top) and WCT data (bottom).

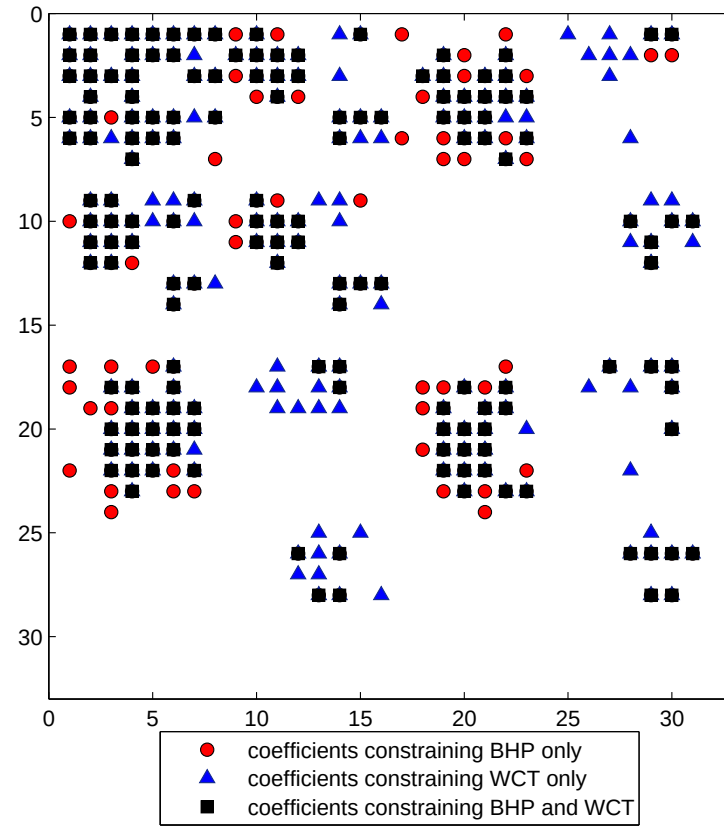


Figure 4.46: Reservoir G1b location of subsets of highest sensitivity wavelet coefficients with respect to BHP and WCT production profiles.

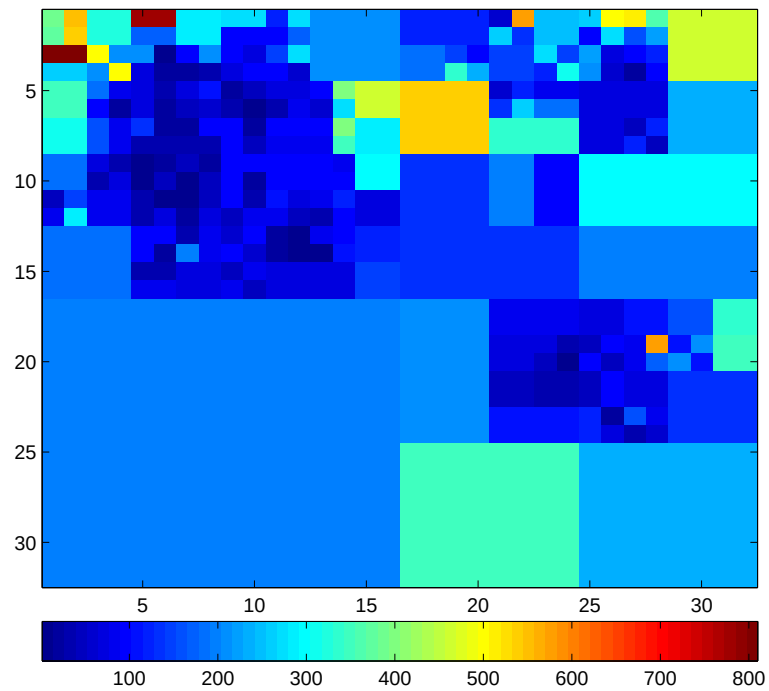


Figure 4.47: Permeability distribution (md) corresponding to thresholding separately for each individual well as shown is Figure 4.45.

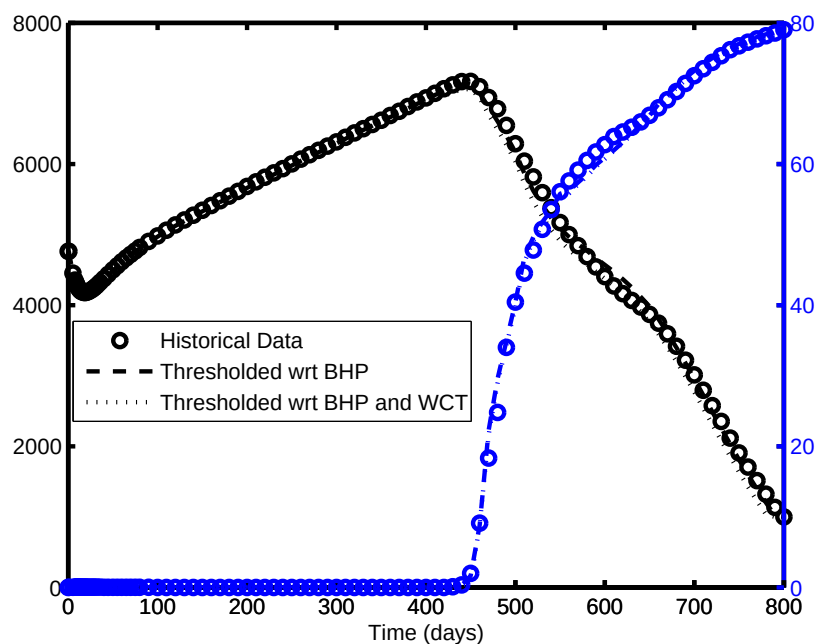


Figure 4.48: Producer 1 - production data match for permeability field shown in Figure 4.47.

making it possible to generate three-dimensional, history-matched, geologically constrained reservoir models.

The algorithm described in our earlier papers [1, 2, 3] used the two-dimensional Non-standard implementation of Haar wavelets (see Appendix A.1). The underlying basis function for the Nonstandard Haar wavelet implementation is square thus the methodology based on it was limited in scope to the description of reservoir property distributions for which the number of gridblocks in the x coordinate equaled those in the y coordinate. As such, the algorithm was limited to the analysis of two-dimensional square-shaped reservoir models. However, real life history matching and reservoir simulation is seldom confined to two-dimensional reservoirs. The data-integration workflow as it was implemented in [1, 2, 3] is generally applicable for data-integration in three-dimensional models as well. In order to extend the algorithm to work in three dimensions, the discretized Haar wavelet toolbox was extended for three-dimensional analysis using the generalized Standard implementation [71, 72, 73, 70] as described in Appendix A.1. In a 1998 paper Jansen [9] used

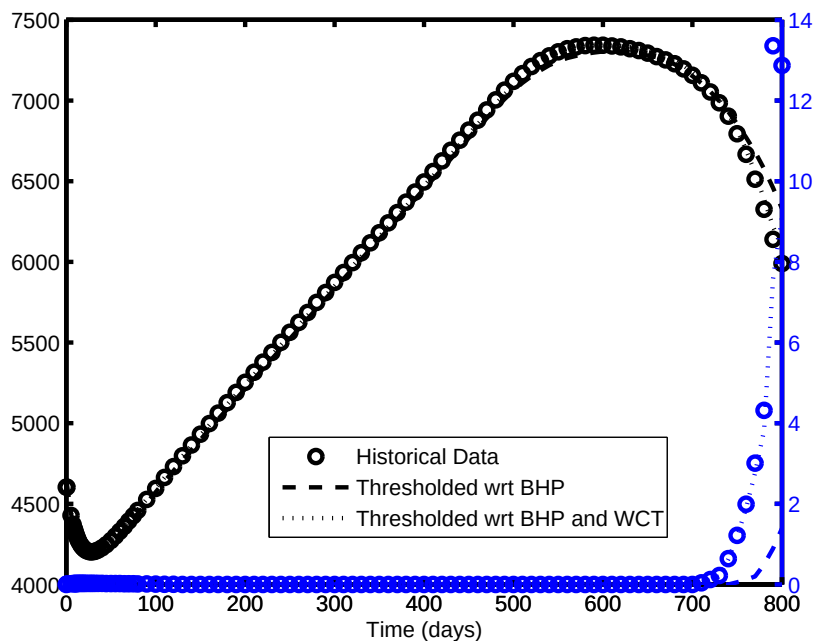


Figure 4.49: Producer 2 - production data match for permeability field shown in Figure 4.47.

three-dimensional wavelet transforms for the purpose of upscaling. Jansen used Standard Haar wavelet functions to threshold the fine resolution coefficients in order to obtain a uniformly upscaled three-dimensional reservoir model.

In order to demonstrate this increase in scope of the wavelet-based data-integration algorithm, we applied this technique to a few example reservoir models. One of the models that the three-dimensional data-integration methodology was applied to was Reservoir 2A. This reservoir model is of size $32 \times 32 \times 8$ and other details of this reservoir are given in Section B.3 of the Appendix.

Wavelet coefficients were computed for the three-dimensional Gaussian permeability field as shown in Figure B.8 of Appendix B.3. Using this Haar wavelet reparameterization of the permeability field, sensitivity coefficients of the production data at each time step were calculated for a production history of 1000 days. The production data considered were BHP and WCT data from two producing wells and injection BHP from the single injector well. Thus for each time step of the simulation we now have three BHP sensitivity

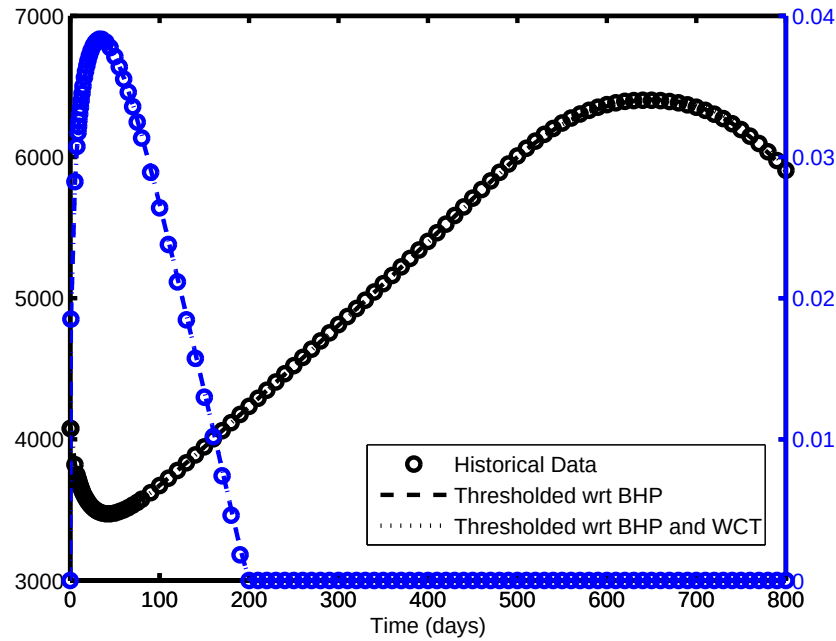


Figure 4.50: Producer 3 - production data match for permeability field shown in Figure 4.47.

maps for all three wells, and two WCT sensitivity maps for the two producing wells. The sensitivity maps are averaged over time using the area-under-the-curve technique (Section 4.2.2). These sensitivity derivatives represent the derivative of BHP and WCT separately at each particular wells with respect to all wavelet coefficients. In order to get a match for the entire field, we then combined these sensitivity maps after weighting appropriately for data type and variance. These sensitivity coefficients are depicted as the red curve in Figure 4.52 after sorting according to decreasing absolute magnitude. Using this sorted vector of sensitivities, we optimized on the smallest subset of permeability wavelet coefficients required in order to match overall field production history. We discovered that using only as few as 35% of the highest sensitivity wavelet coefficients yields permeability field that shows well matched production history curves for the 1000 days history match period. These 35% coefficients are labelled in Figure 4.52 as the blue data points.

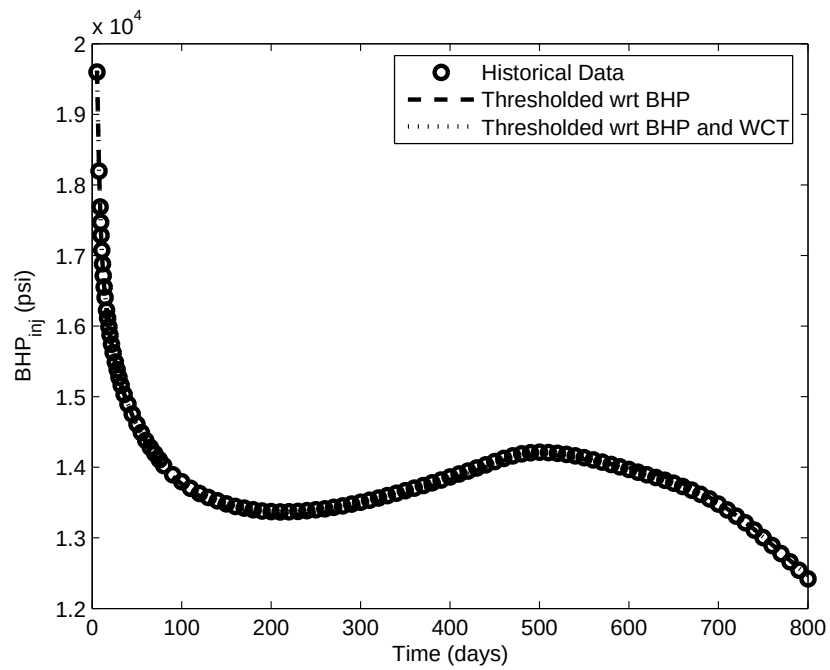


Figure 4.51: Injector - production data match for permeability field shown in Figure 4.47.

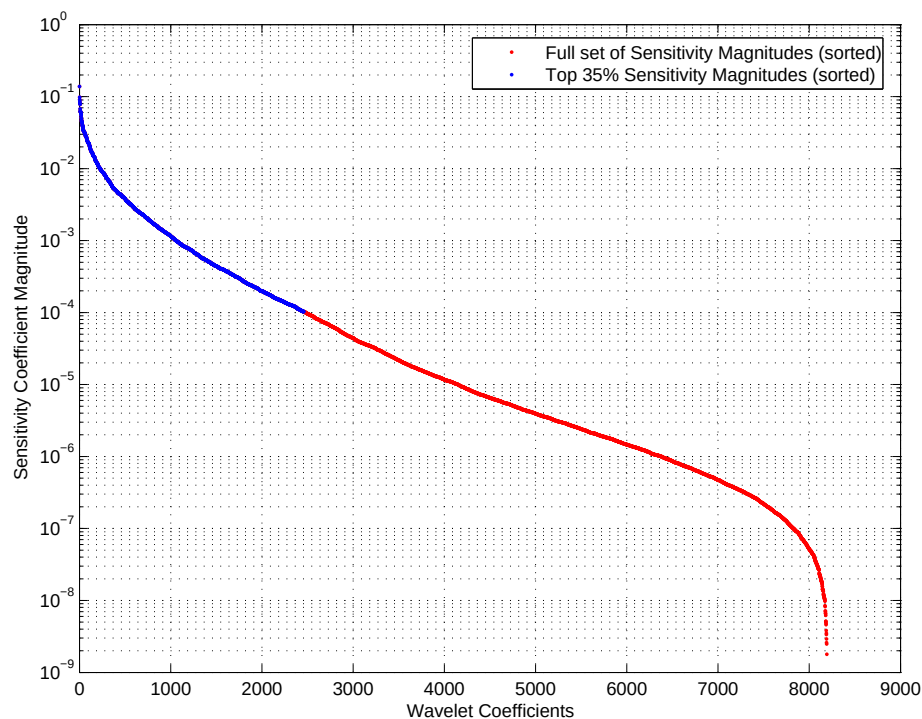


Figure 4.52: Sensitivity coefficient magnitudes sorted by absolute value for Reservoir 3A.

4.4 Chapter Summary

In Chapter 3 we saw how production data from a reservoir model are dependent on the thresholding of wavelet coefficients of reservoir parameters. We saw that a handful of wavelet coefficients are sufficient to capture the key features of the reservoir model that influence production. Chapter 4 looked at how this parameter subset can be optimized for the purpose of history matching, starting from a *prior* model using the Gauss-Newton method.

The efficient integration of reservoir data is based on our ability to partition the sets of wavelet coefficients that constrain different types of data at different resolutions. In particular, once a history match is performed, if we fix the wavelet coefficients that are most sensitive to production data, we can subsequently integrate other sources of data into the model. However, production sensitivity coefficients are obtained at each time step of the simulation and we see that they vary in magnitude over these time steps. In order to determine the coefficients that have the highest *overall* sensitivity, we need an averaging

technique for sensitivities over time. Different techniques were developed for the purpose of partitioning the set of parameters based on sensitivity coefficient magnitudes. The two methods that are described and compared with the help of an example are:

1. Area under the curve method.
2. Minimum cutoff method.

In the set partitioning process, Method 1 favors coefficients that have high average sensitivity magnitudes for many time steps, whereas Method 2 leans towards coefficients that have sensitivity magnitudes which reach a certain high (cut-off) value even for a short period of the simulation run. In the case studied, we saw that for the same proportion of parameters used, Method 1 was better able to capture the production history. However, in general, the possibility exists that there might be cases which are better described using Method 2, or some combination of Method 1 and Method 2.

Given sensitivity coefficient values for different production data (pressure, water cut) for each well, we can identify parameters that are significant for production history match for each well and data type individually or in combination. If these sets of parameters are sufficiently disjoint, we can modify each them independently, and thereby match production history well-by-well or for example for BHP and water cut in succession. In the example case studied, we found that a different proportion of parameters are required to match to production history from each well, and we have the freedom to constrain these parameter sets in any order. We also described with the help of an example how we can constrain the reservoir model to BHP and water cut data in that order. These techniques offer a higher degree of flexibility for the data integration methodology, while at the same time reducing the total number of parameters required for optimization by better identifying only the most significant ones.

In Section 4.3 we showed how the algorithm can be extended for application to three-dimensional reservoir models. This extension is based on the development of a three-dimensional wavelet transform using the Standard implementation (Section 3.2) of Haar wavelets. The algorithm was also extended to work with a commercial simulator, which enables the incorporation of complex production scenarios and well trajectories thus making it more applicable to real field cases.

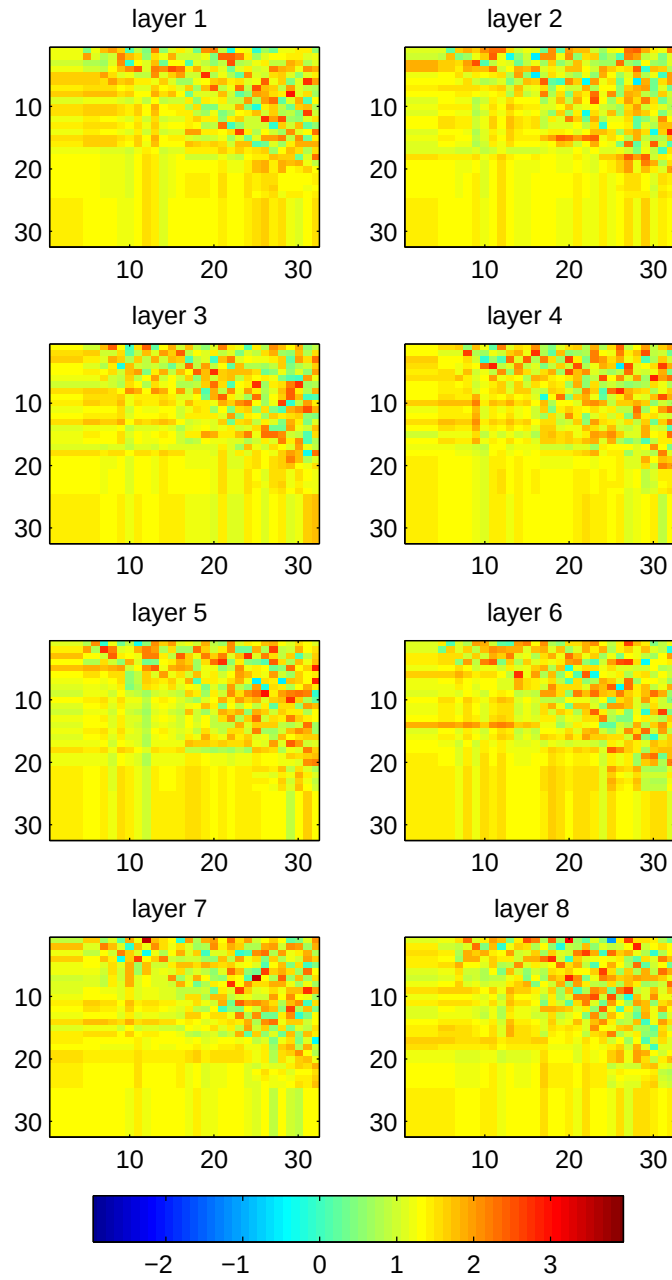


Figure 4.53: Log permeability distribution by layers for layers 1 through 8 for Reservoir 3A computed using 35% of the wavelet coefficients with the highest sensitivity to production data (B.3).

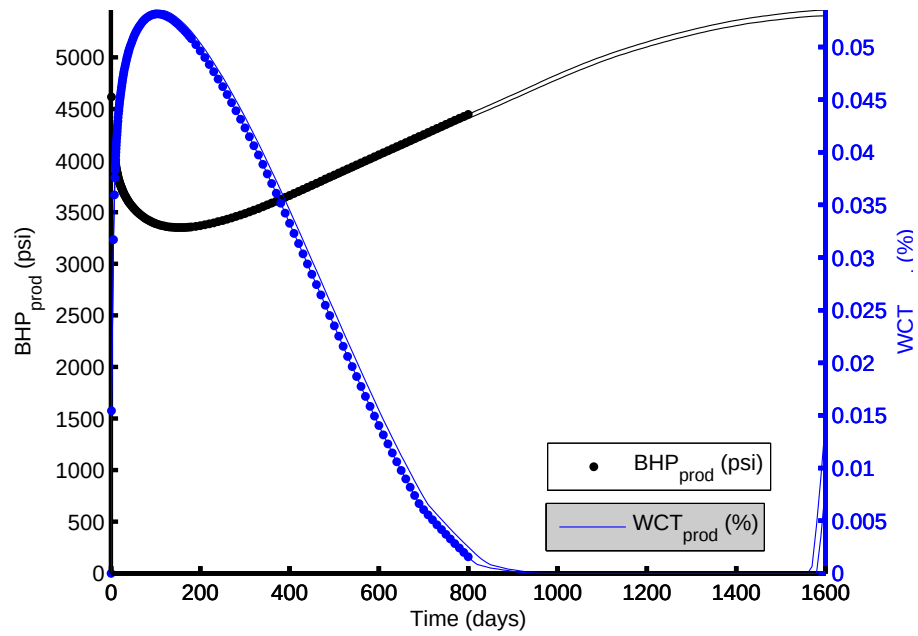


Figure 4.54: WCT (%) and BHP (psi) with time for production from oil producing well Prod 1.

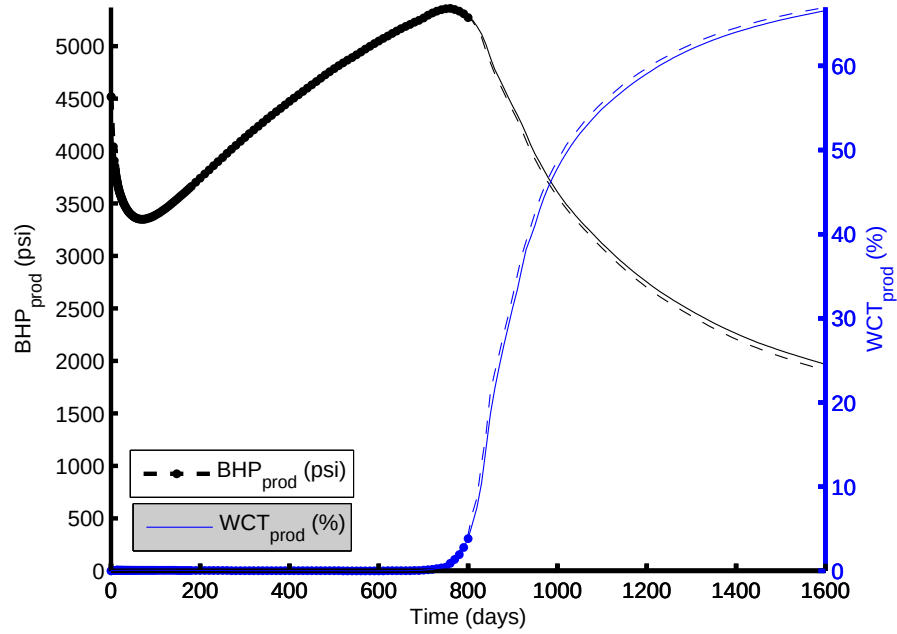


Figure 4.55: WCT (%) and BHP (psi) with time for production from oil producing well Prod 2.

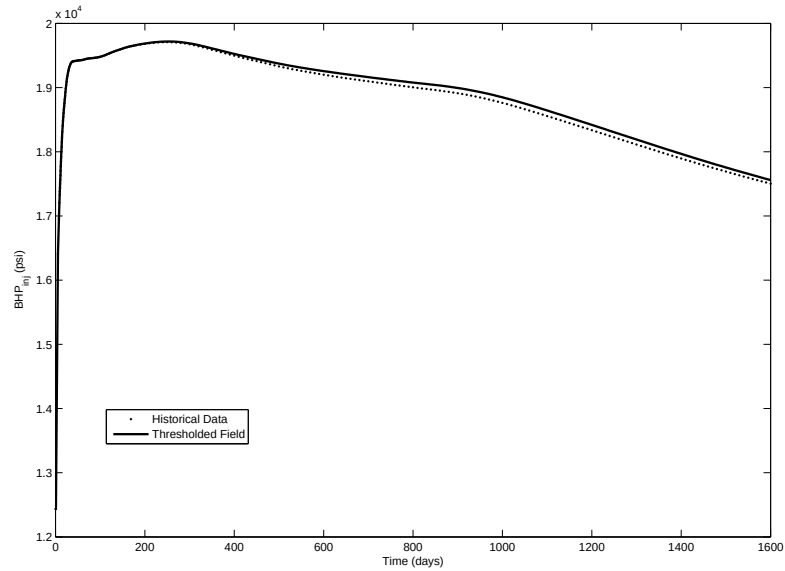


Figure 4.56: BHP (psi) with time for production from water injection well INJ.

Chapter 5

Geostatistical Data Integration and Extensions

So far we looked at the process of production-data integration and different methods of identifying the subset of wavelet parameters that are sufficient to constrain to historical data. The next stage is the integration of geological information into the reservoir model. This sequential integration of different data into the model is made possible as a result of the partitioning or decoupling of the wavelet parameters. The values of the wavelet coefficients identified as being significant for *dynamic* production data are kept constant while a different set of wavelet coefficients are perturbed for the integration of *static* geologic information.

5.1 Wavelet Decoupling and Geostatistical Data Integration

Consider a prior reservoir model (of Gaussian permeabilities, say) consisting solely of hard data and geostatistical information in the form of a Gaussian histogram (parameterized by mean and variance), and a variogram of the parameter. Consider, also, a history-matched reservoir model that is not constrained to the statistical parameters of geology (histogram and variance). This study showed how the latter model can be constrained to geostatistical information without losing the history match. In other words, the algorithm shows that by holding a subset of the wavelet coefficients of the model fixed, and modifying the rest to honor the geostatistical data, we can obtain, stochastically, reservoir models that honor both geological and production history data. This can be done as many times as the number of reservoir models we would like to generate corresponding to the given data, without redoing

the history match.

The partitioning of the sets of wavelet coefficients is based on the values of sensitivity coefficients. Changing a wavelet coefficient with high sensitivity to production data will lead to a greater deviation in the simulated production data as compared to changing one with a lower sensitivity. We showed in Chapter 4 that a threshold can be set on the minimum number of wavelet coefficients required to be held constant to provide a satisfactory history match. This set forms the history-matching wavelet coefficients that correspond to production data information. The complement of this set, i.e. the less sensitive (to production data) wavelet coefficients may now be modified without significantly affecting the history match. To incorporate geostatistical information into the model, what remains to be done is to find a way of modifying these free wavelet coefficients.

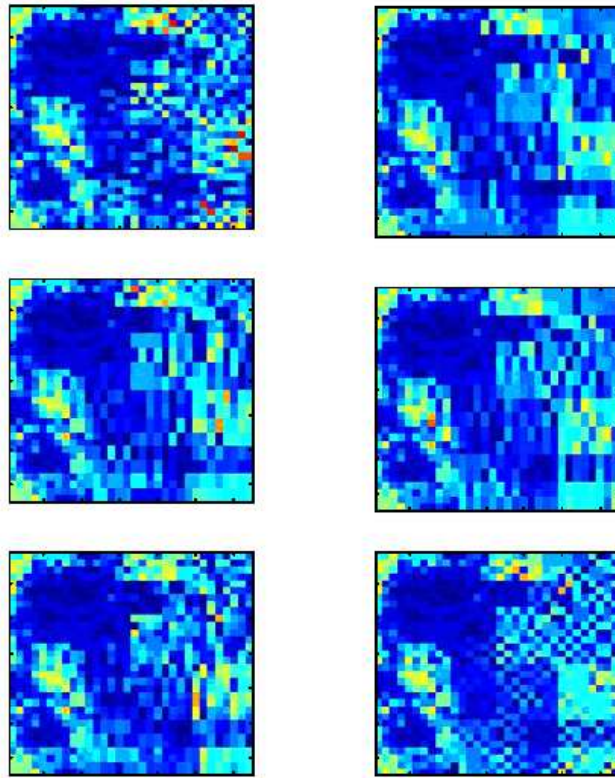


Figure 5.1: Permeability distributions with oriented artifacts caused by modifying sets of wavelet coefficients constraining only the corresponding orientations.

Two different methods were devised for the integration of geological information into the reservoir. The first method uses simulated annealing in different ways to constrain the reservoir model to geostatistical information. The second method is a valid for the special case of a Gaussian distribution of permeabilities in the reservoir model. This method uses special statistical properties associated with Gaussian random variables in order to simplify the optimization procedure to a noniterative technique. These two methods are described here with the help of examples.

5.1.1 Simulated Annealing

This first method used for the integration of geological information is based on an undirected iterative optimization technique (simulated annealing, [62]) and seeks to match the variogram of the model to the prescribed model variogram. A detailed description of the algorithm is given in Chapter 2. The simulated annealing algorithm visits each of the free wavelet coefficient nodes and perturbs the magnitude of the coefficient. The objective function here is defined as a norm difference between the current variogram and the target variogram. Since the geostatistical constraints are on the permeabilities values themselves, the wavelet coefficients need to be inverted back to permeability values in order to compute the objective function. Based on the change in objective function the perturbation is either retained or ignored. A second node is then picked and perturbed in a similar fashion. Some reservoir results using this algorithm are described in detail in [1].

However, this process had an issue with potentially generating artifacts based on the method of traversal of the wavelet coefficients [1, 2, 3]. This problem occurred because it is only a smaller subset of the free wavelet coefficients that are required to incorporate the geostatistical constraints into the model. Moreover, different sets of wavelet coefficients constrain the reservoir model statistics in different orientations. The overall statistics of the reservoir model could be honored even if the wavelet coefficients belonging to only a particular orientation got perturbed, based on the traversal path within the simulated annealing module. This could potentially lead to the production of oriented artifacts (Figure 5.1) in the resultant reservoir model obtained whereas the reference model is thought to be isotropic. This problem was solved by using a method of random traversal that visited the nodes across all orientations and scales of wavelets coefficients in a random order, based on a random seed. The resultant reservoir models obtained using random traversal had relatively uniform statistics in all directions (Figure 5.2).

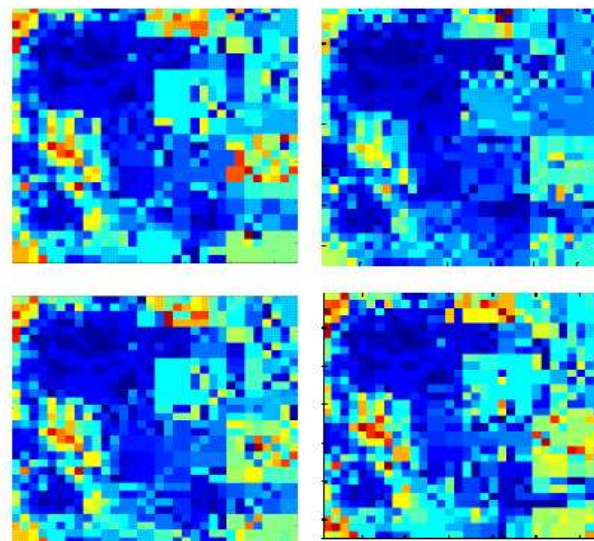


Figure 5.2: Reservoir model results obtained using random traversal to avoid oriented artifacts.

In the procedures of data integration described in Chapter 4, the production history data was the first to be integrated and fixed deterministically before moving on to the integration of other sources of data (e.g. geostatistical data). In other words, based on what is thought to be good history match, a threshold of sensitivities was determined and the corresponding wavelet coefficients were fixed. This step insures that for all the subsequent models generated by integrating geology, the history match is always almost exactly as good as that fixed at the thresholding stage. Implicitly, we are making an assumption of perfect history information and then integrating as much geology into the model as is consistent with that assumption. In realistic situations, we are never perfectly sure of the production history data. The data are susceptible to many types of errors from different sources from equipment malfunction and measurement uncertainty to random noise. That being the case, deterministically fixing a particular set of wavelet coefficients corresponding to the fixed production data would be incorrect since it also means that we are limiting the integrated geological information to be consistent with this fixed set. This is equivalent to giving too much weight to the available production data and constraining the model very strongly to match history. In order to address this issue, a different method of partitioning

the sets of wavelet coefficients was developed, referred to as *grayscale* or soft thresholding.

5.1.2 Grayscale - Probabilistic History Matching

For a more realistic picture of the uncertainty it is important to consider some degree of freedom in matching the production data. This uncertainty in production data can be integrated easily into the approach by modifying the sensitivity mask that separates the sets of wavelet coefficients constraining production data and geology. Fixing the wavelet mask constraining the history-matching wavelet coefficients is equivalent to assigning all the wavelet coefficients a probability of either zero or one to be able to be perturbed to make the model geologically consistent (Figure 5.3).

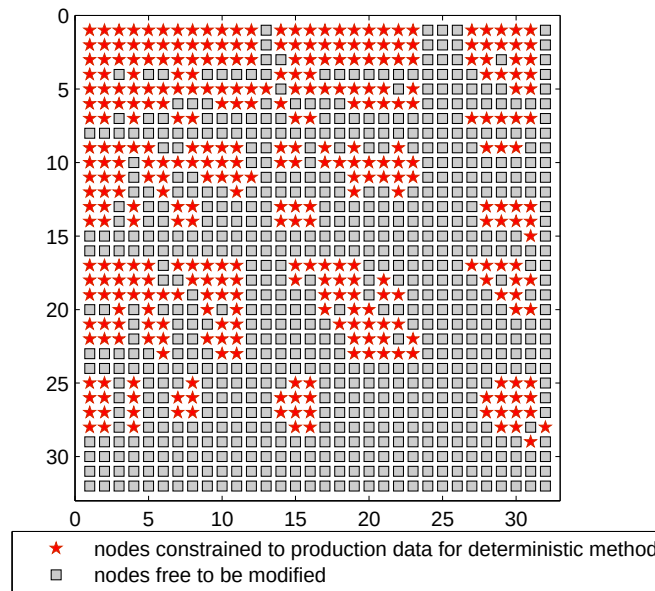


Figure 5.3: Binary wavelet mask. Probability of perturbation of ‘red’ wavelet coefficients is zero and ‘gray’ wavelet coefficients is one.

Uncertainty in production data can be included in the model by replacing this black and white probability mask with a grayscale mask such that each wavelet coefficient has a probability between zero and one to be perturbed in order to match geology (Figure 5.4). As can be seen from Figure 5.4, most of the coefficients that earlier had probability zero of being modified (Figure 5.3) still have a very low probability and most of the coefficients

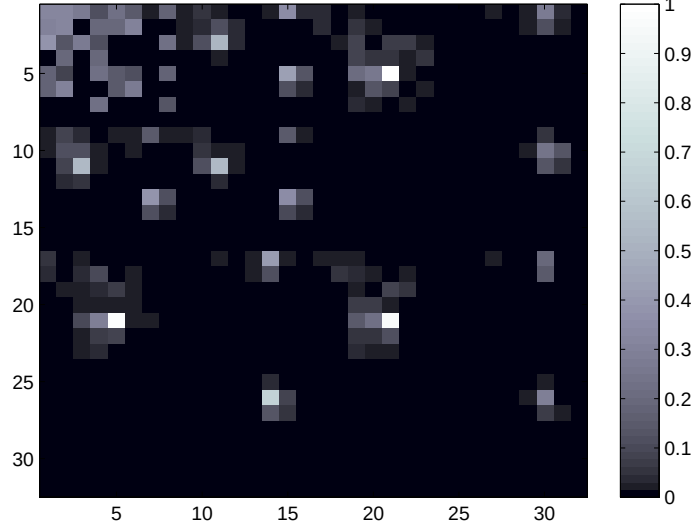


Figure 5.4: Grayscale wavelet mask. Probability of keeping a wavelet coefficient fixed for history-match may lie between zero and one.

that had a probability one of being modified still have a very high probability. In the grayscale sensitivity mask however, there exist wavelet coefficients that have intermediate probabilities of being perturbed to match geology. Thus, using this methodology there is a chance of constraining different wavelet coefficients to history as well as geology for each resulting reservoir model generated.

The grayscale approach to constraining wavelet coefficients mentioned above was applied to a log-permeability distribution as shown in Appendix B.1. Starting from a reference reservoir permeability model, we calculated sensitivity coefficients in wavelet space and evaluated and fixed those coefficients that the production history is most dependent on (that is, probability of perturbation is zero for these coefficients). Keeping the values of these coefficients fixed and setting the rest of the coefficients values to zero, we performed an inverse wavelet transform to get a permeability model, as depicted in Figure 5.5. This permeability distribution, derived from the history-matched model, shows the parameters to which the production history is most sensitive - that is, permeability details are retained in the regions close to wells, whereas further away from the wells, we see that large block averages of the parameter are constrained by production history. The

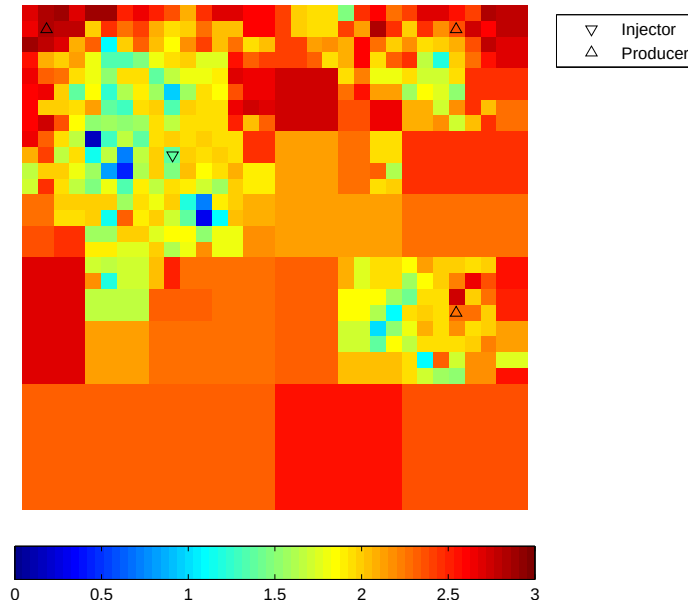


Figure 5.5: Thresholded permeability distribution (log md) based on sensitivity to production data using Nonstandard implementation (refer to Section 3.2).

nonzero wavelet coefficients corresponding to this reservoir model became the input to our geostatistics-integration algorithm, which modified these zero-valued coefficients to obtain a better representation of geology in our final reservoir models. Figure 5.6 shows the number of times nodes were visited by the random traversal of the simulated annealing algorithm. It also shows how this traversal compares to deterministic sensitivity mask. We observed that some of the nodes that would have not been visited given a hard threshold get perturbed using the grayscale mask, whereas there are others that would have been visited, but are left unchanged in the grayscale run. In general, the nodes that were ‘fixed’ by the deterministic method were visited a fewer number of times than other nodes. Figure 5.7 shows the equivalent grayscale ‘mask’ for this particular run, along with the deterministic mask. However, it should be noted here that the grayscale mask is different for each run of the algorithm, being chosen probabilistically. Some of the permeability distribution results using a grayscale sensitivity map are shown in Figure 5.8. The variogram was used as an objective function and Figure 5.9 shows the resulting match.

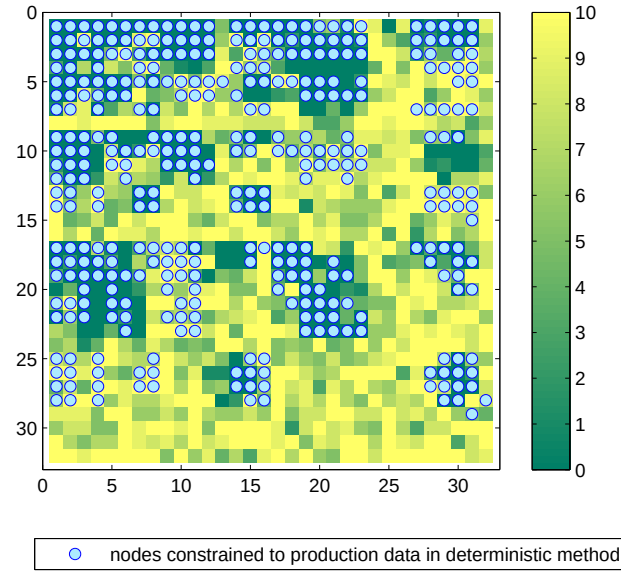


Figure 5.6: Random traversal showing number of visits to a particular wavelet coefficient node using the grayscaling method along with the nodes constrained to production data in the deterministic method.

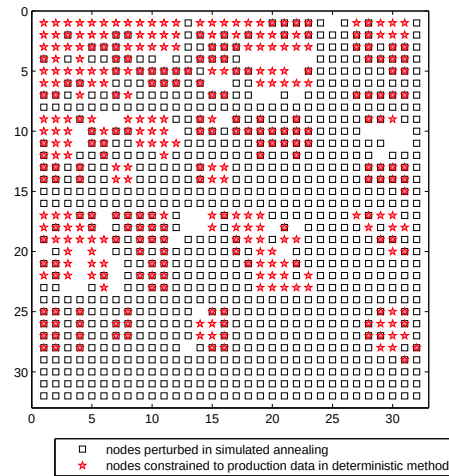


Figure 5.7: Random traversal showing the perturbed and unperturbed wavelet coefficient node using the grayscaling method.

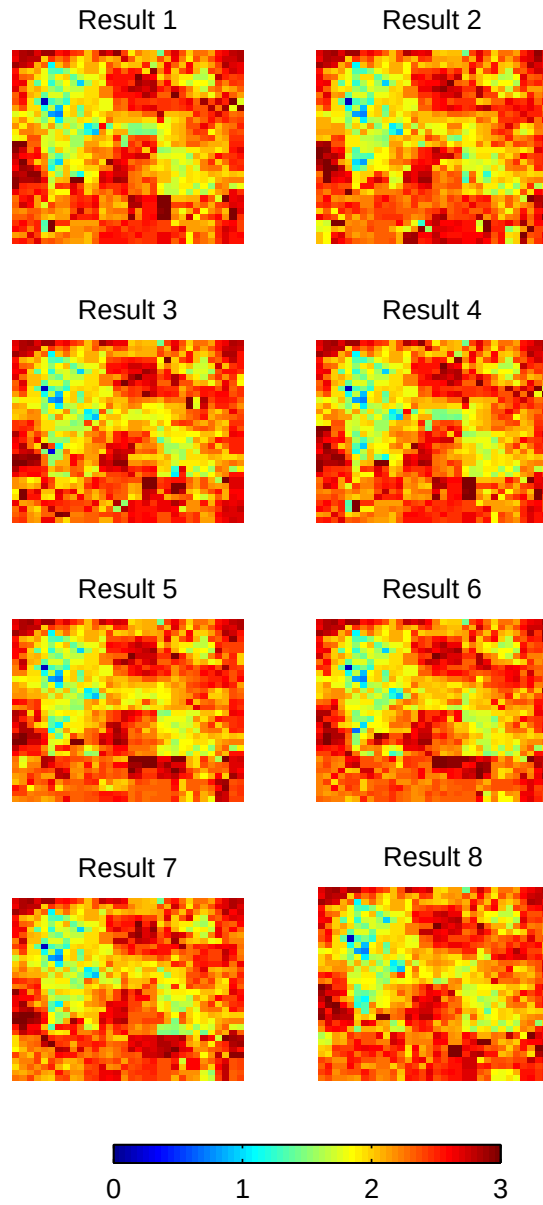


Figure 5.8: Reservoir model result (log-permeabilities in md) using grayscale sensitivity coefficients.

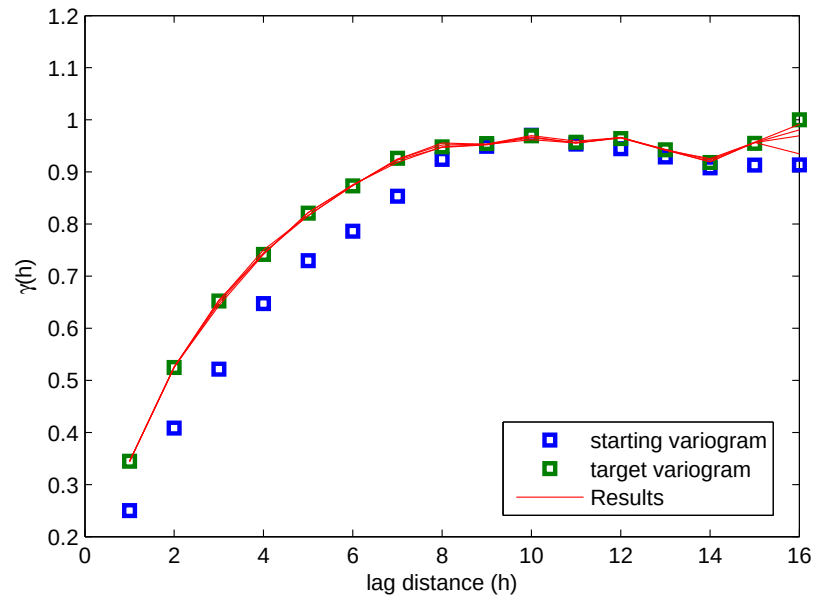


Figure 5.9: Variograms for the prior and history-matched model and variogram results for permeability fields obtained after optimization.

We modified a subset of wavelet coefficients in order to match the geostatistical information of a reservoir model subsequent to history match. In order to prove that this modification did not disturb the production history profiles, we need to compare production data simulated using the resulting reservoir models with the reference production history. The BHP and WCT plots for the three producers and one injector for all the results along with the reference are shown in Figure 5.10 through Figure 5.13. We see that even after modifying a subset of the parameters, the simulated production is still close to the original production history data. Being less constrained than the previous models, these results are expected to show more variability in their prediction of future production. This is more realistic than assuming that the history data is perfect, which would give an unrealistically low value of uncertainty. Figure 5.14 shows a variance map of the log permeability results using the grayscaling methodology. We see that the variance is low in regions of high certainty - areas around the wells - whereas there is higher variance in area that are not resolved by the BHP and WCT information from the wells.

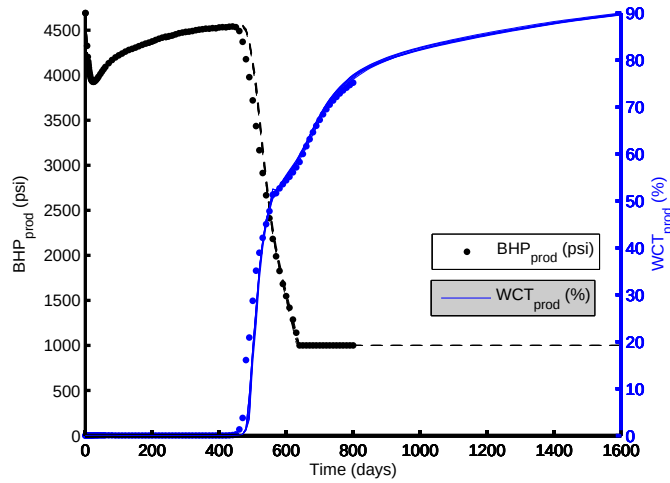


Figure 5.10: Producer 1 - production data match for permeability field shown in Figure 5.8.

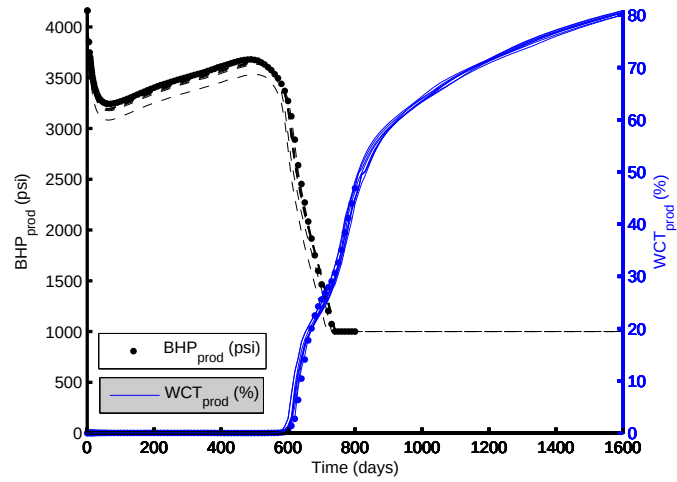


Figure 5.11: Producer 2 - production data match for permeability field shown in Figure 5.8.

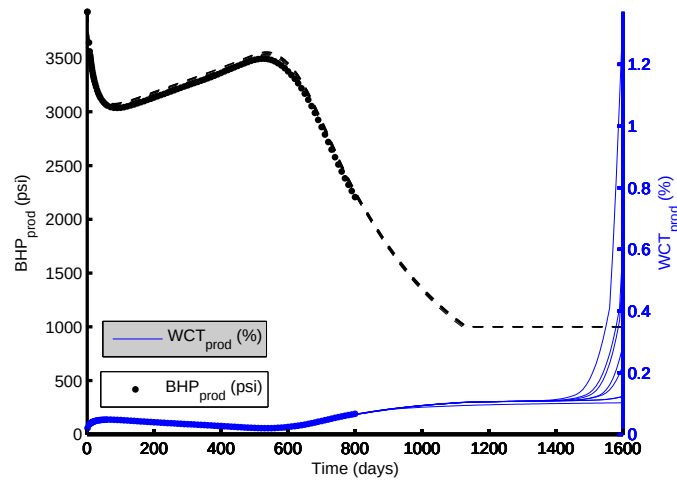


Figure 5.12: Producer 3 - production data match for permeability field shown in Figure 5.8.

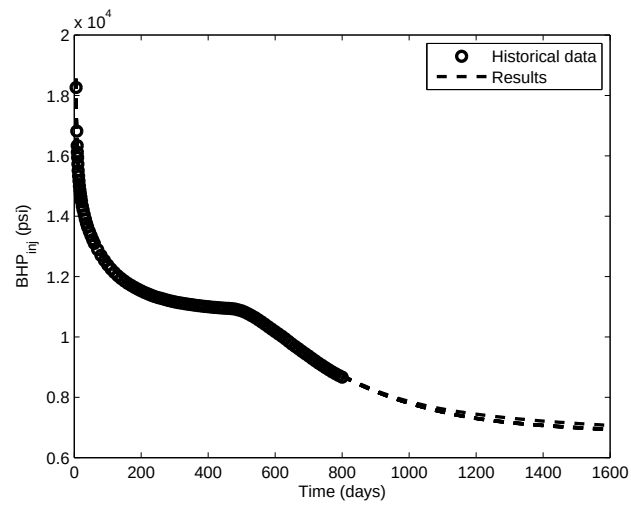


Figure 5.13: Injector - production data match for permeability field shown in Figure 5.8.

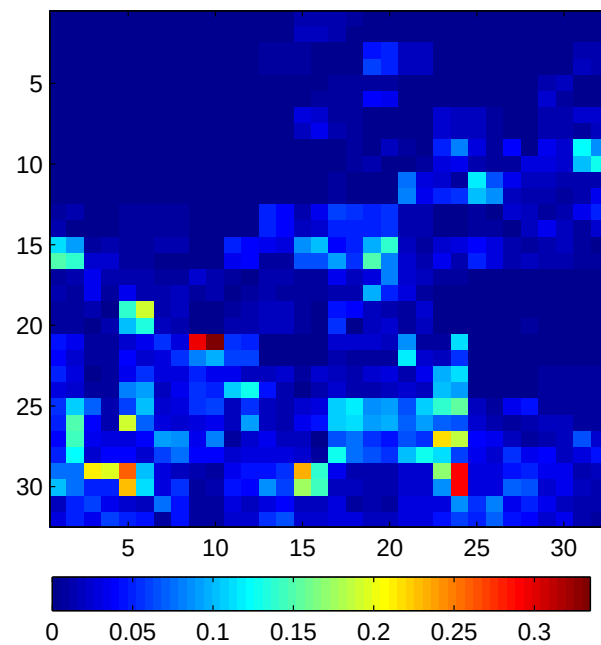


Figure 5.14: Variance between the reference and resulting log permeability distributions.

The method of simulated annealing in wavelet space is based on honoring statistical constraints (mean, variance and variogram) in permeability space. Therefore this method requires frequent conversions from the permeability grid to the corresponding wavelet coefficient grid and back. Given that the wavelet transform is a linear operation, the cost of frequent inversions is not very high. However, the wavelet coefficients are perturbed randomly and at each iteration the objective function (that is, the variogram) needs to be computed in order to check if that particular perturbation was successful. This procedure was found to be wasteful, since more than half the perturbations turned out to be unsuccessful. As such a better way of constraining to geostatistical parameters was sought. Given the fact that the Haar wavelet coefficients of a parameter distribution are mere linear combinations of the parameters themselves, a more efficient, noniterative technique for geostatistical data integration was developed as an alternative to SA. This second approach is described in the next section.

5.1.3 Analytical Development for Gaussian Distribution of Parameters

From the formulation of the Haar wavelet coefficients (see Chapter 2) we see that different sets of wavelet coefficients are essentially different linear combinations of the original parameters (log-permeabilities, in our case). This development is to demonstrate how it is possible to evaluate the statistics of the sets of wavelet coefficients, given the corresponding statistics of the original parameters.

Suppose that we have a random function, $V(\mathbf{x})$ composed of a (stationary) spatial distribution of random variables. Assume that the random variable V (in our case, the logarithms of permeabilities) is a Gaussian random variable with mean = m . The variance is:

$$\sigma^2 = E\{V^2\} - m^2 \quad (5.1)$$

and the semivariogram is $\gamma\{\mathbf{h}\}$.

Now consider linear combinations of these random variables, such as W , where ω_i are weights corresponding to each V ,

$$W = \sum_{i=1}^n \omega_i V_i. \quad (5.2)$$

Note that a linear combination of Gaussian random variables yields a Gaussian random

variable.

$$E\{\mathbf{W}\} = \mathbf{E} \left\{ \sum_{i=1}^n \omega_i \mathbf{V}_i \right\} = \sum_{i=1}^n \omega_i \mathbf{E}\{\mathbf{V}_i\} \quad (5.3)$$

$$Var\{\mathbf{W}\} = \mathbf{Var} \left\{ \sum_{i=1}^n \omega_i \mathbf{V}_i \right\} = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \mathbf{Cov}\{\mathbf{V}_i, \mathbf{V}_j\} \quad (5.4)$$

Consider,

$$Cov\{V_1, V_2\} = E\{(V_i - E\{V_i\})(V_2 - E\{V_2\})\} \quad (5.5)$$

Because the random function is stationary, we have:

$$E\{V_1\} = E\{V_2\} = m. \quad (5.6)$$

Moreover, we consider an isotropic field. For this case, for a distance $\mathbf{h}$ as separation between the two random variables $V_1 = V(\mathbf{x}_1)$ and $V_2 = V(\mathbf{x}_2)$, that is for, $|\mathbf{x}_1 - \mathbf{x}_2| = \mathbf{h}$, we get:

$$\begin{aligned} C_v(\mathbf{h}) &= Cov\{V(\mathbf{x}), V(\mathbf{x} + \mathbf{h})\} = \mathbf{C}_v\{\mathbf{0}\} - \gamma(\mathbf{h}) \\ &= \sigma^2 - \gamma(\mathbf{h}). \end{aligned} \quad (5.7)$$

Now, from Equations 5.4 and 5.7, we can say that:

$$\begin{aligned} Var\{\mathbf{W}\} &= Var \left\{ \sum_{i=1}^n \omega_i V_i \right\} = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j Cov\{V_i, V_j\} \\ &= \sigma^2 \sum_{i=1}^n \sum_{j=1}^n \omega_j \omega_j \mathbf{I}_{i,j} - \sum_{i=1}^n \sum_{j=1}^n \gamma(\mathbf{h}_{i,j}). \end{aligned} \quad (5.8)$$

Further, for two different sets of points denoted by V^1 and V^2 , we can calculate the covariance between their linear combinations using

$$Cov \left\{ \sum_{i=1}^n \omega_i V_i^1, \sum_{j=1}^m \omega_j V_j^2 \right\} = \sum_{i=1}^n \sum_{j=1}^m \omega_i \omega_j Cov\{V_i^1, V_j^2\}. \quad (5.9)$$

These equations suggest that there exists a way of calculating the statistics of a linear combination of random variables, given the statistics of the random variables themselves.

That is, we see that the mean of a linear combination of random variables is a linear combination of the means of the random variables; the variance and covariance of the linear combination depends on the corresponding variance and covariance (or semivariogram) of the random variable. Also note that each set of wavelet coefficients is a linear combination of the reservoir parameters (log-permeabilities) that are random variables. The weights (ω_i) associated with each linear combination are given by the wavelet function used (Haar wavelet function in our case). Thus, if we are given the mean, variance and variogram of the reservoir parameter (log-permeability) we can, under the assumption of Gaussianity, compute the mean, variance and variogram for each of the sets of wavelet coefficients separately.

From the arguments made here, we see that with the given statistical information about the parameter, we can compute the corresponding statistics (mean, variance and variogram) of each wavelet coefficient set at all the different scales. We also know the history-matching wavelet coefficients from the sensitivity evaluations. The problem is thus reduced to reassigning the free wavelet coefficients in each set, keeping the history matching coefficients constant, in such a manner that the overall statistics for that set corresponds to the one computed. This will ensure that after wavelet back-transformation, the permeability distribution obtained will have the prescribed geostatistical properties.

Using the fact that the sets of wavelet coefficients are linear combinations of Gaussian parameters (log permeability) we can compute the statistical properties of each set (mean, variance and variogram). The analysis of the sensitivity coefficients yields the set of wavelet coefficient that need to be fixed in order to fix the production history of the model. For each set of wavelet coefficients we can then perform Sequential Gaussian Simulation (using *sgsim*, see [59]) using the fixed wavelet coefficients as hard data and constraining to the computed statistics for that particular set. Thus, we perform *sgsim* in wavelet space in order to reevaluate the free coefficients with statistic that guarantee that wavelet inversion will yield a distribution that is constrained by the reference geostatistical data. The fact that we fix the wavelet coefficients corresponding to production data as hard data in the simulations ensures that the history of the field is also preserved upon wavelet inversion. The result of performing *sgsim* in wavelet space yields, upon wavelet inversion, the log of a permeability distribution that is constrained to both the geological as well as the production data. Figure 5.15 describes the methodology in the form of a flow chart.

Using different random seeds for the sequential Gaussian simulation of wavelet coefficients will yield different permeability fields, all of which will be constrained to all sources

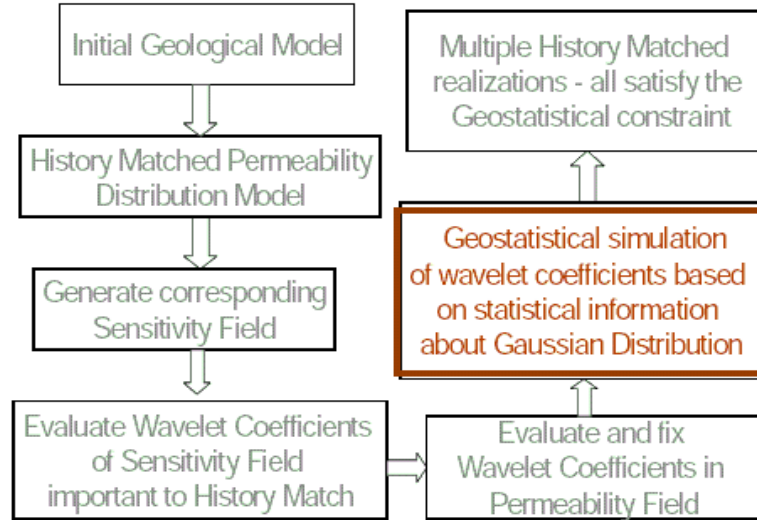


Figure 5.15: Data integration: Methodology for Multivariate Gaussian permeability distributions.

of data for the reservoir. Interestingly, since the simulations are independent from one set of wavelet coefficients to another, a combination of wavelet coefficients from across the cases with different random seeds will yield yet more permeability field models.

Note also that sequential Gaussian simulation visits each node using a random path based on a random seed. This eliminates the chance of traversal-based artifacts appearing in the results as shown in Figure 5.1. The noniterative algorithm proposed in this study was applied to a number of example reservoir models. One of these examples is described here.

Reservoir G1 Wavelet *sgsim* was applied to the reservoir model G1 shown in Appendix B.1. As seen before, this example consists of a reference two-dimensional Gaussian permeability distribution that matches the production data from four wells. Starting with a history-matched model that is not constrained to geological parameters (Figure 5.5) and using an iterative algorithm (simulated annealing), we showed that a number of history-matched, variogram constrained algorithms can be generated.

We applied the new noniterative algorithm to the same example for comparison. Similar to the previous method, at the first stage, history-constraining wavelet coefficients were determined using a sensitivity coefficient mask. The mean, variance and variogram of the wavelet coefficient sets were then computed using the property that they are linear combinations of permeabilities. Sequential Gaussian simulation (*sgsim*) was performed in wavelet space using these statistics, constraining to the fixed history-matching parameters as hard data. By changing the random seed used in *sgsim*, many different reservoir models can be obtained by inversion. All of these match the both production data and geological constraints. Only a few seconds of CPU time is required to generate each new model. Wavelet inversion of these wavelet coefficients gave a permeability reservoir model (Figure 5.16) that matched both production data as well as geostatistical constraints (mean, variance, hard data and semivariogram). The semivariogram match is shown in Figure 5.17. Figures 5.18 through Figure 5.21 show how the production data from resulting permeability models compares with the true production history and with each other in prediction mode. From these figures we notice that while the production history match is good, the resulting reservoir models show widely varying production profiles in prediction mode. This is an expression of the inherent uncertainty involved in resolving the reservoir parameters given the limited amount of information. Figure 5.22 shows how the algorithm keeps wavelet coefficients corresponding to the production data constant while modifying the rest in order to match geostatistical constraints. The two distributions (reference and result) are distinct from each other in the areas where there is no production data to constrain the reservoir models (see Figure 5.23). In the regions around the wells the new result is similar to the history-matched model, whereas far away from the wells, these realizations are quite different. This is also apparent from a plot of the variance map for the different resulting permeability distributions (see Figure 5.24). This is a more accurate model of our uncertainty about the reservoir in the regions where we have limited information. These would have a significant impact on infill drilling results as can be seen from plots of cumulative oil and water production from an infill well drilled after the production history period (see Figure 5.25).

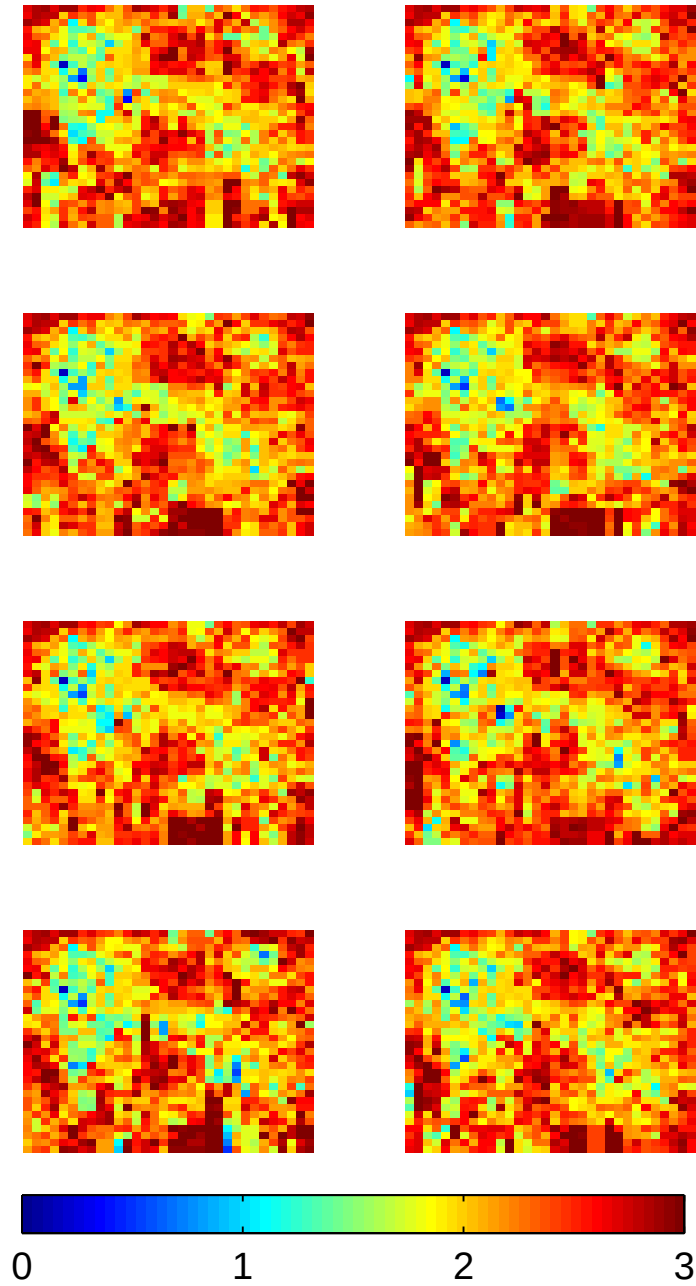


Figure 5.16: Reservoir model result (log-permeabilities in md) using wavelet based *sgsim*.

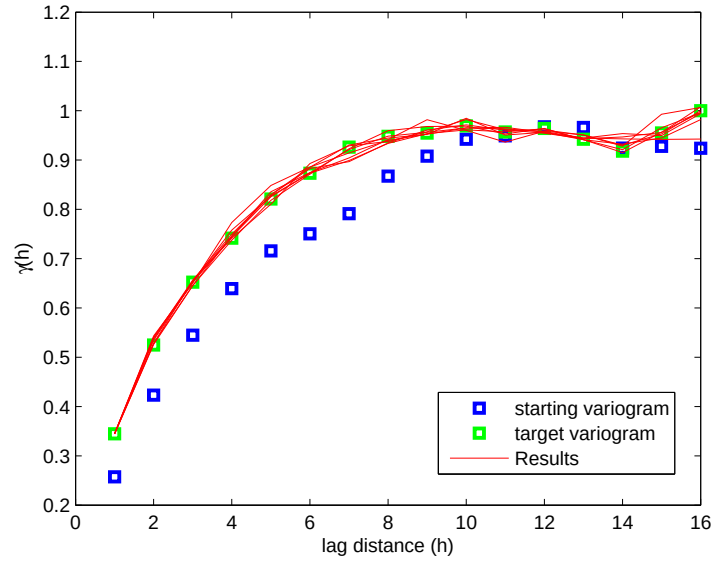


Figure 5.17: Variograms for the prior and history-matched model and variogram results for permeability fields obtained after optimization.

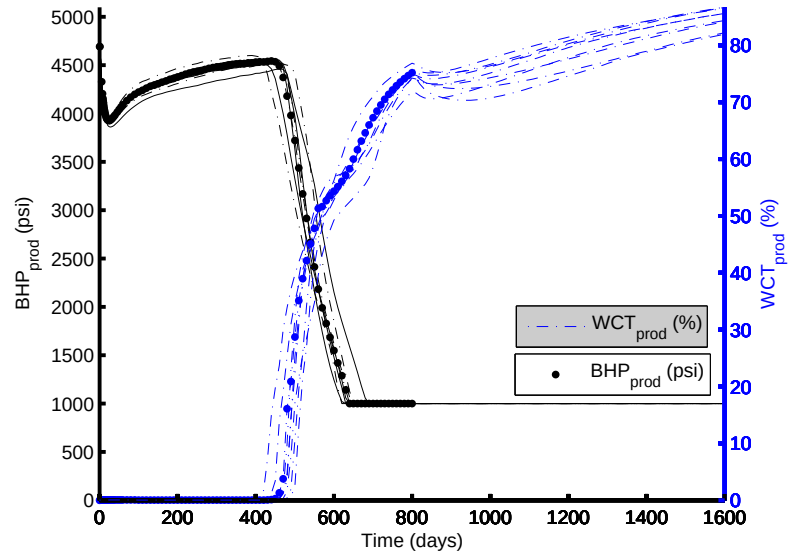


Figure 5.18: Producer 1 - production data match for permeability field shown in Figure 5.16.

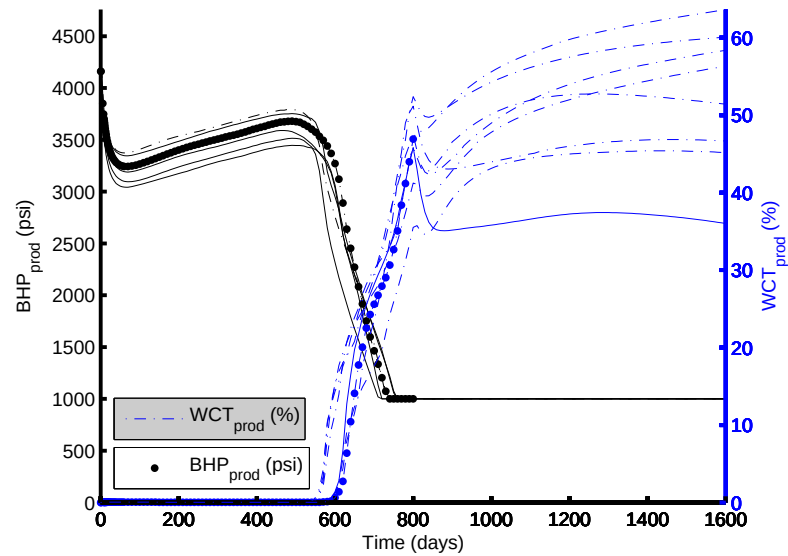


Figure 5.19: Producer 2 - production data match for permeability field shown in Figure 5.16.

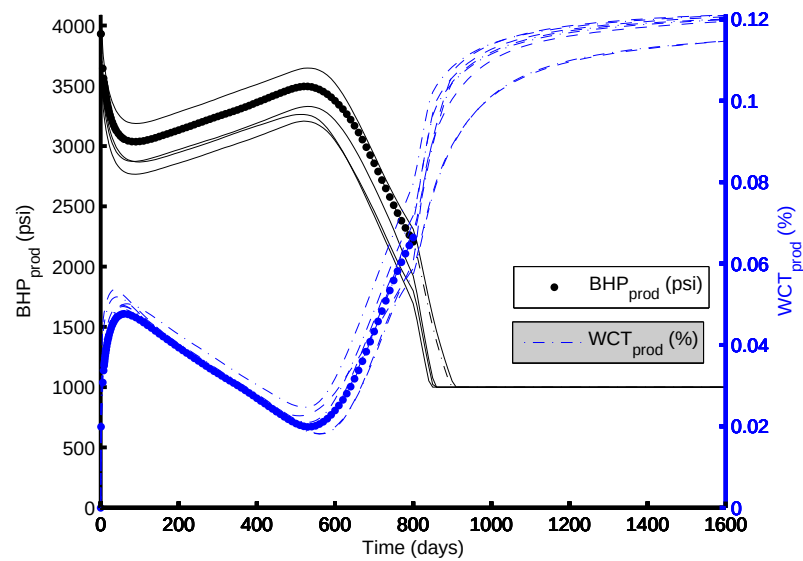


Figure 5.20: Producer 3 - production data match for permeability field shown in Figure 5.16.

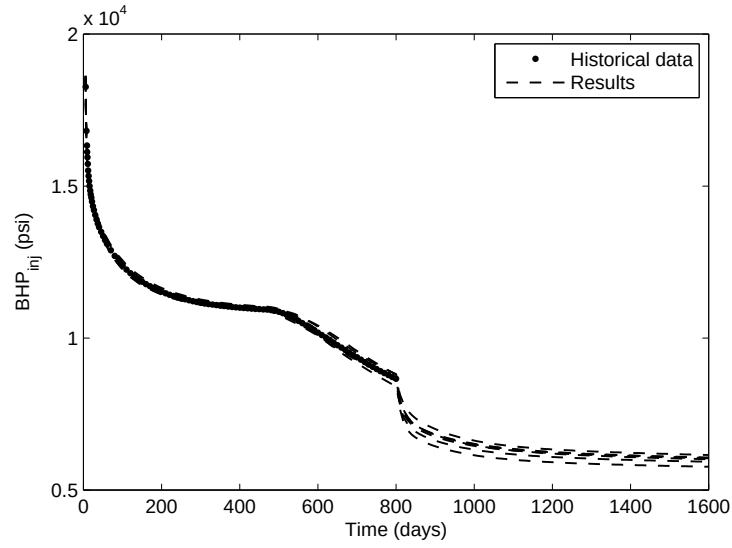


Figure 5.21: Injector - production data match for permeability field shown in Figure 5.16.

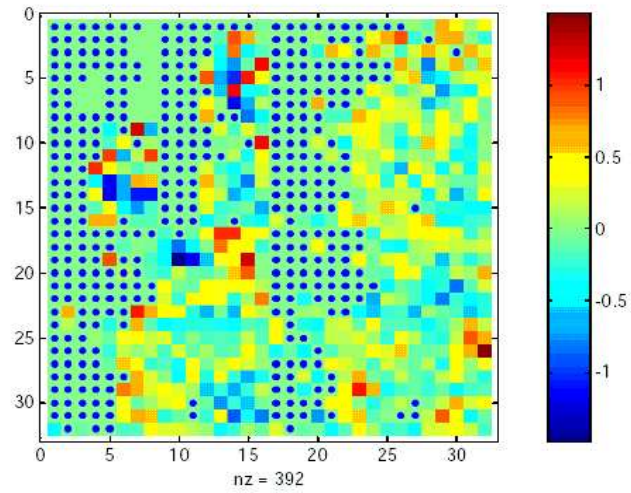


Figure 5.22: Difference between wavelet coefficients of reference permeability distribution and Result 1, showing also the wavelet mask.

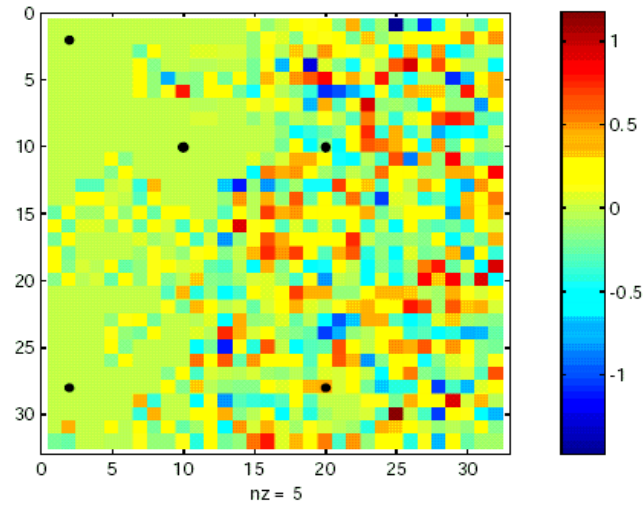


Figure 5.23: Difference between history-matched permeability distribution and Result 1.

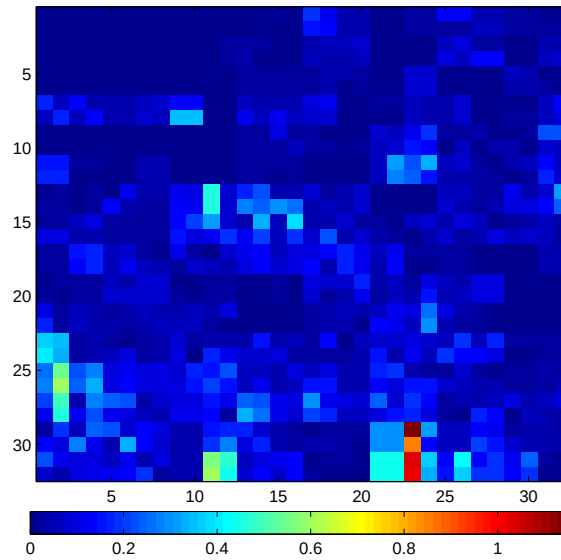


Figure 5.24: Variance between the reference and resulting log permeability distributions.

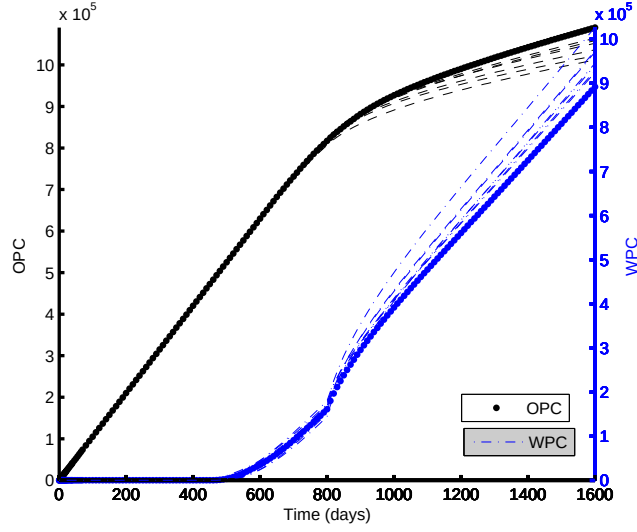


Figure 5.25: Cumulative production data match for permeability field shown in Figure 5.16.

Reservoir Model 2B As an example of the application of the method to three-dimensional problems, we demonstrate the data-integration algorithm on another example Reservoir 2B, which is of size $16 \times 64 \times 2$.

Wavelet coefficients were computed for the three-dimensional Gaussian permeability field (Reservoir 2B). This reservoir is explained in detail in Appendix B.4. Using this Haar wavelet reparameterization of the permeability field, sensitivity coefficients of the production data at each time step were calculated for a production history of 800 days. The production data considered were BHP and WCT data from one producing well and injection BHP from the single injector well, see Figure B.12. Thus for each time step of the simulation we now had two BHP sensitivity maps for the two wells, and one WCT sensitivity map for the producing well. The sensitivity maps were averaged over time using the area-under-the-curve technique (see Section 4.2.2). These sensitivity magnitudes represent the derivative of BHP and WCT separately at each particular well with respect to all wavelet coefficient parameters. In order to get a match for the entire field, we combined these sensitivity maps after weighting appropriately for data type and variance. These sensitivity coefficients are depicted as the red curve in Figure 5.26 after sorting according to decreasing absolute magnitude. Using this sorted vector of sensitivities, we optimized on the smallest subset of permeability wavelet coefficients required in order to match overall field production

history. We discovered that using 45% of the highest sensitivity wavelet coefficients yields a permeability field that satisfactorily matched production history curves for the 800 day history match period. These 45% coefficients are labeled in Figure 5.26 as the blue data points.

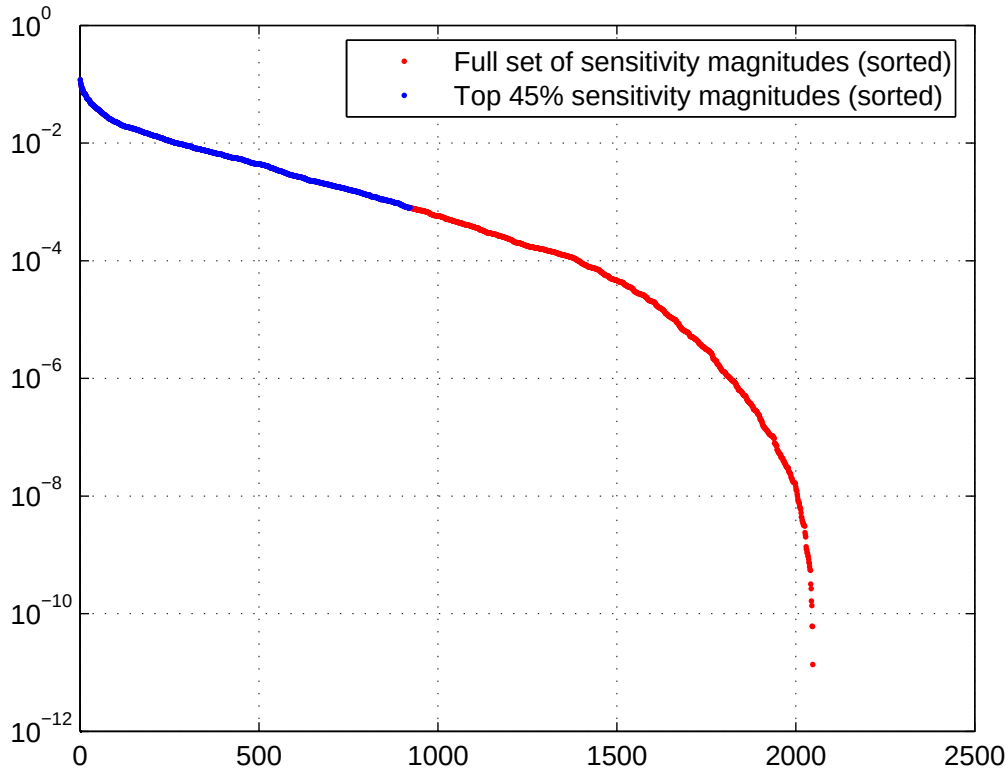


Figure 5.26: Reservoir 2B: Sorted sensitivity coefficients.

The result of a thresholding to 45% of the original parameters, while setting the rest to zero, is the smoothened permeability field shown in Figure 5.27. The next step was to integrate geostatistical information into this thresholded model. This information consisted of the prior variogram [59, 60] and histogram of the reservoir permeabilities. The integration was done using the optimization technique of simulated annealing [62] with the norm of the difference of the variograms as part of the objective function. This process is fast and efficient since instead of modifying individual pixels to get a variogram match, we are

modifying a subset of the wavelet coefficients instead. We get a new result every time a new set of random perturbations are performed by the simulated annealing algorithm. Two of the permeability distribution results obtained by the integration of the variogram are plotted in Figure 5.28 and Figure 5.29. The variograms of the truth, initial and resulting permeability distributions are shown in Figure 5.30. We see that starting from a smooth variogram (black curve) the permeability field changes such that the two resulting fields have variograms (thin red lines) that match the target variogram (thick red line). In order to check that the reservoir model is still constrained to the production history, we plot the production profiles from the two results along with the production history from the reference case in Figures 5.31 through 5.33. We see that the pressure and watercut data from the producer and injector are still constrained to the initial data up to a period of 800 days of history.

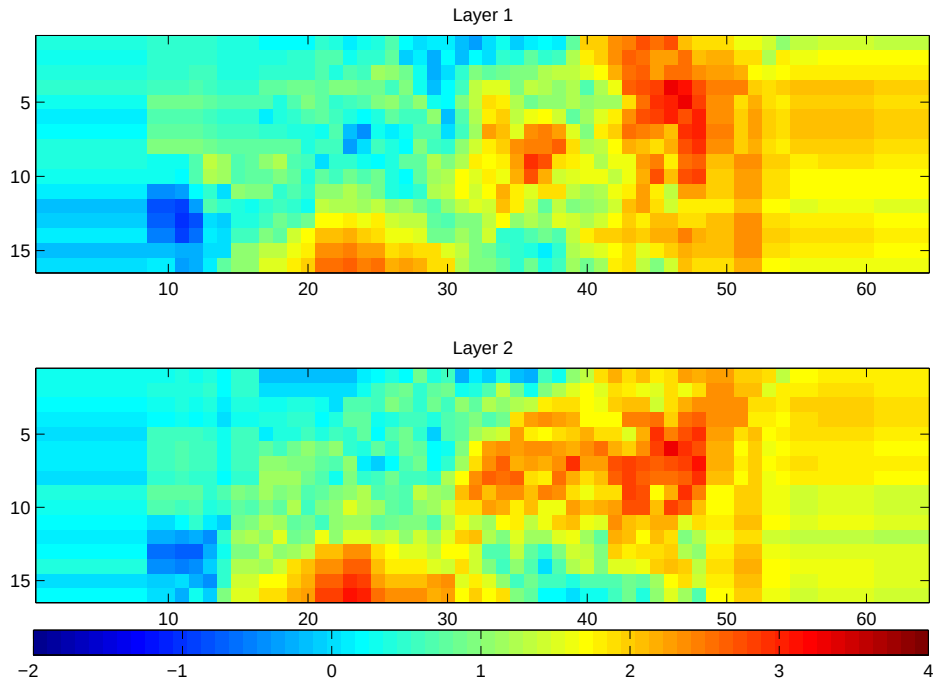


Figure 5.27: Reservoir 2B: Thresholded permeabilities.

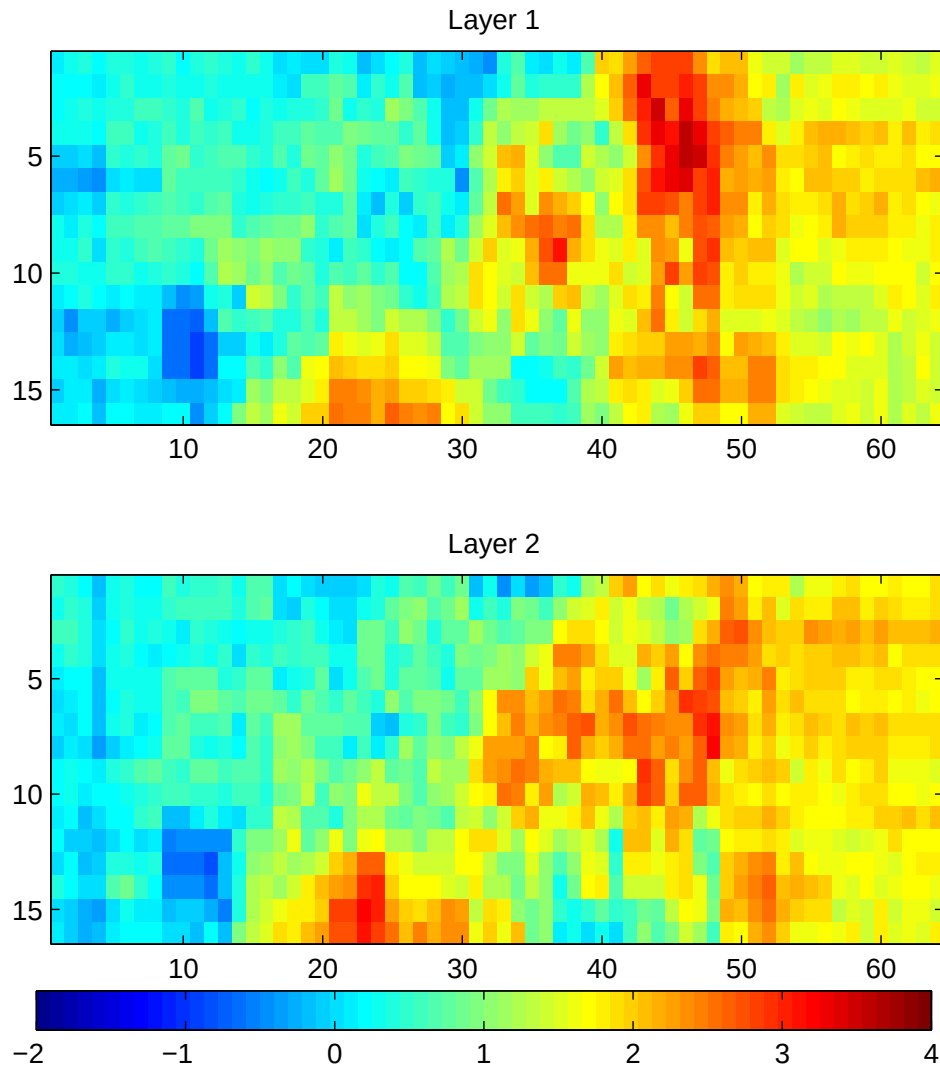


Figure 5.28: Reservoir 2B: Result 1.

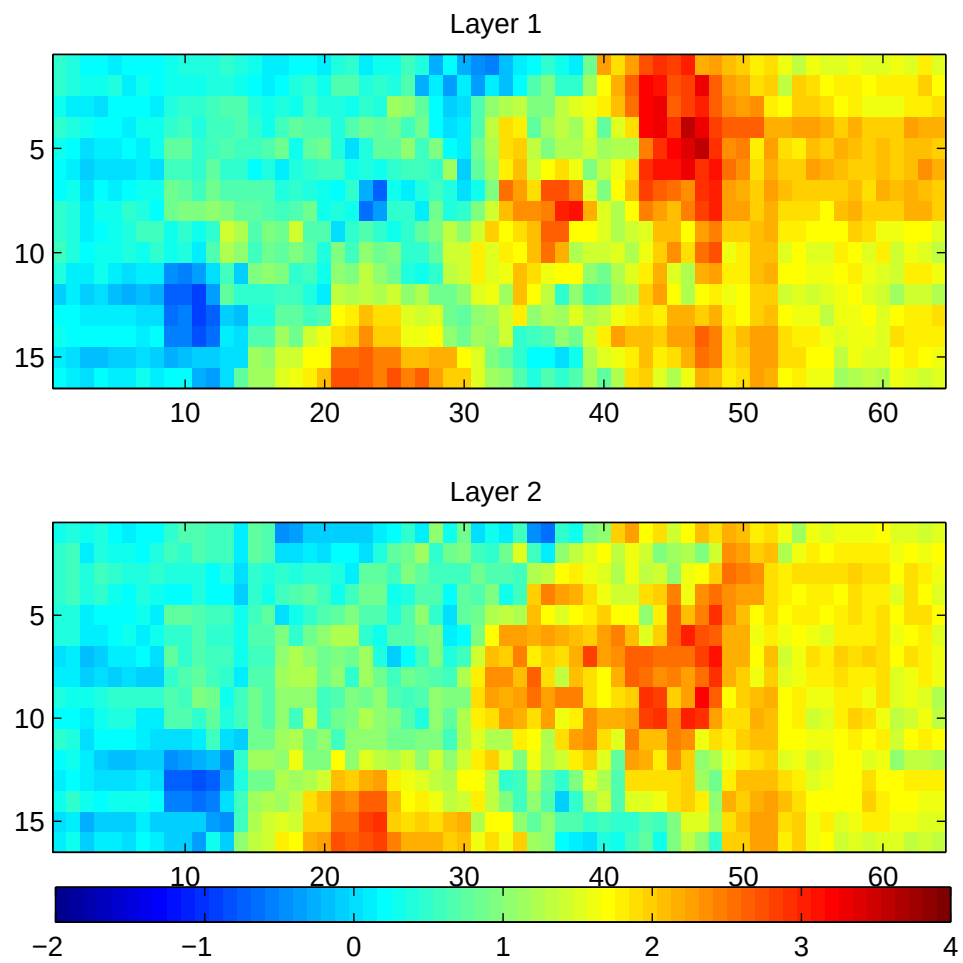


Figure 5.29: Reservoir 2B: Result 2.

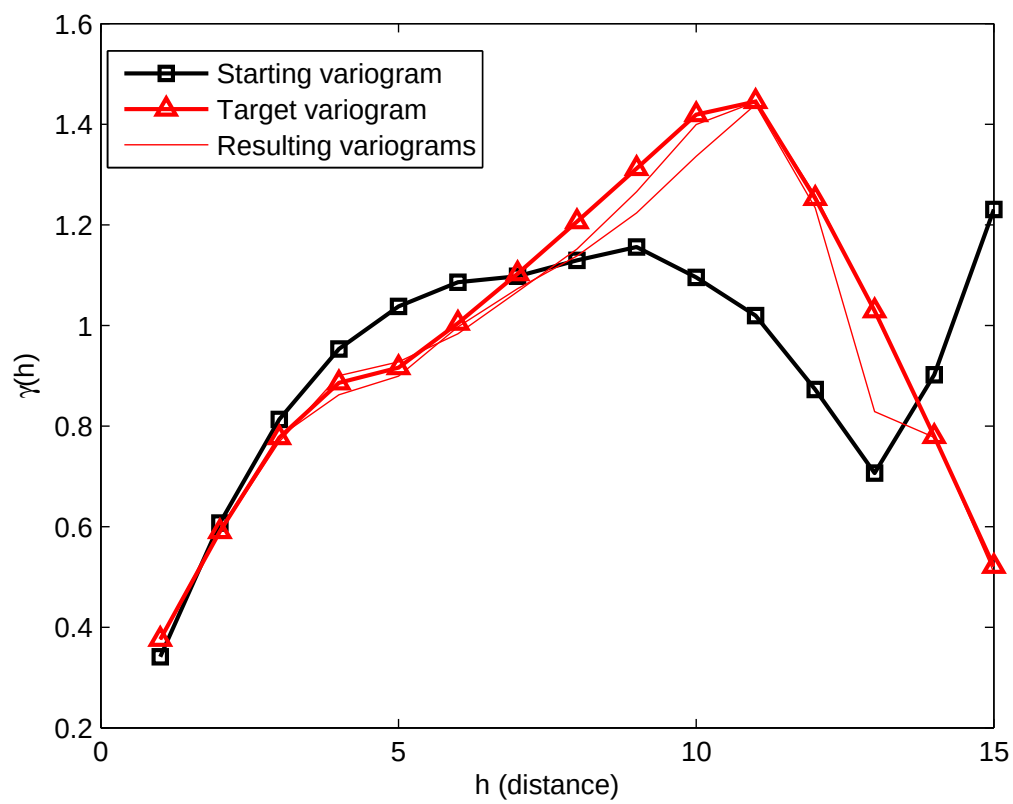


Figure 5.30: Reservoir 2B: Variograms.

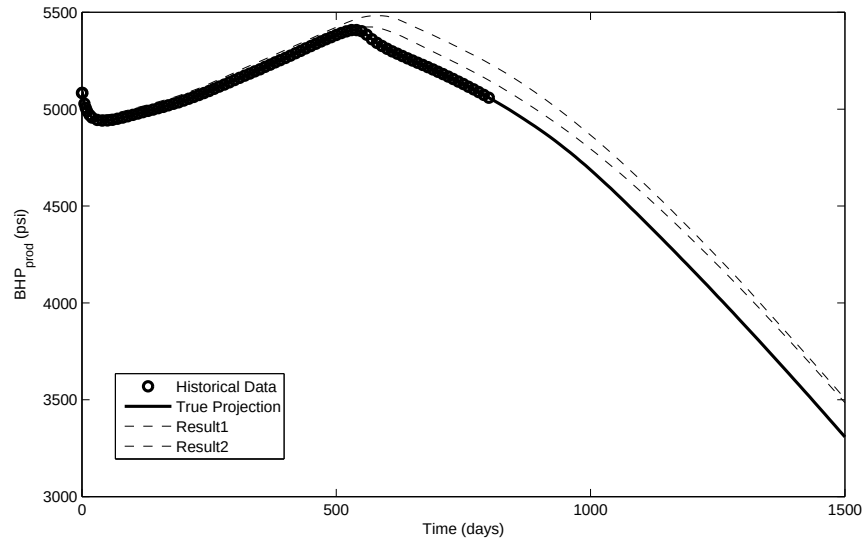


Figure 5.31: Reservoir 2B: Prod1 BHP history data and projections.

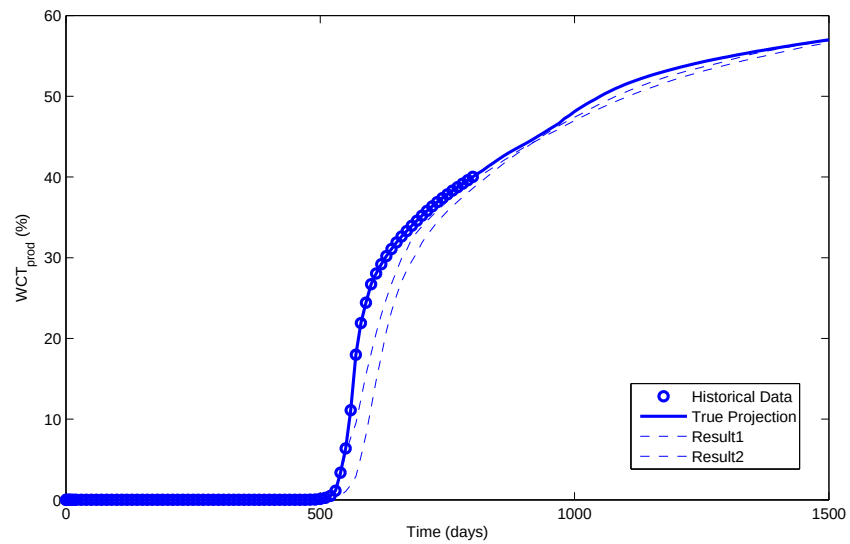


Figure 5.32: Reservoir 2B: Prod1 WCT history data and projections.

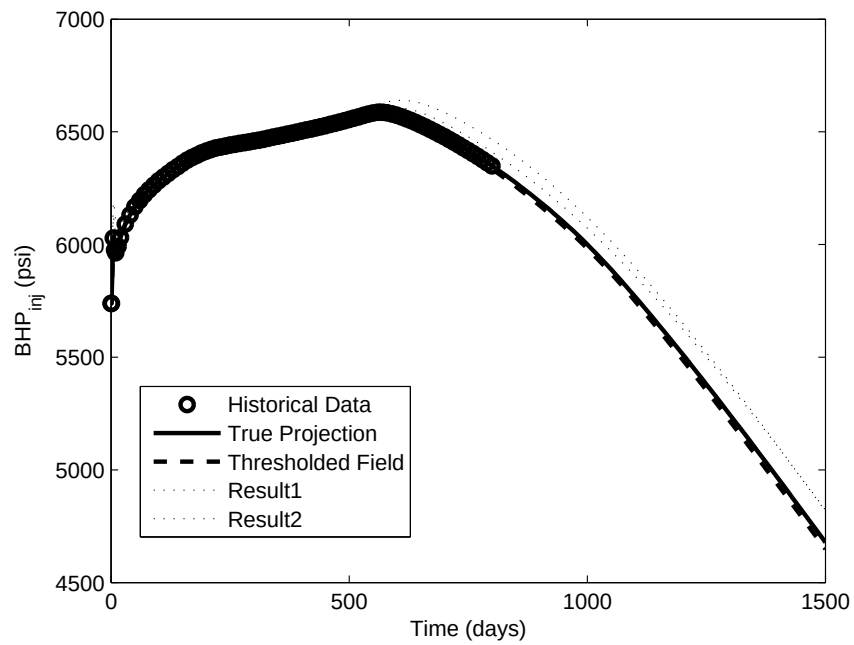


Figure 5.33: Reservoir 2B: Injector BHP history data and projections.

We can conclude that the permeability distributions shown in Figure 5.28 and Figure 5.29 both match the available geostatistical and production history data, and thus are equiprobable models for the reservoir, given the available information. In similar fashion, any number of such reservoir model results can be generated at very low computational cost without having to repeat the history matching procedure. Figure 5.34 shows the difference between truth case (Figure B.12) and Result 2 (Figure 5.29).

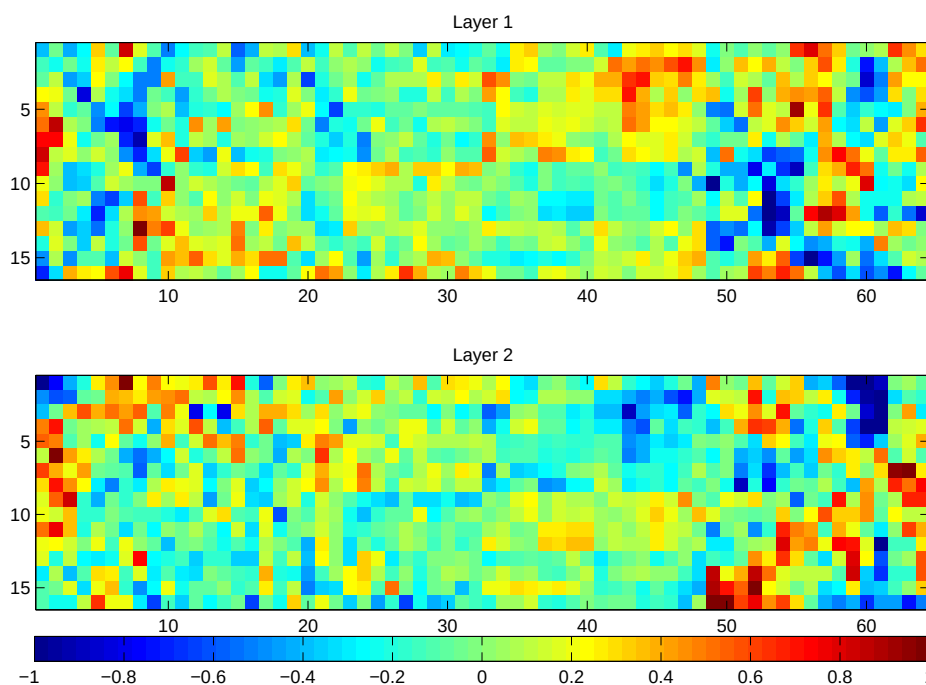


Figure 5.34: Reservoir 2B: Difference between truth case (see Appendix B.4) and Result 2 (Figure 5.29).

5.2 Logarithm Permeability Model

Consider an $n \times n$ reservoir permeability distribution that has already been matched to history. Theoretically, permeability is a Jeffreys parameter [44] and it can take values between zero and infinity. The proper way of evaluating contrasts and averages etc. of such parameters is to work with their logarithm. Taking the logarithm of this set of Jeffreys parameter yields Gaussian parameters that may range anywhere from $-\infty$ to ∞ . This

formulation, besides being appropriate for computations involving the permeability parameter, also lends itself very well to the Haar Wavelet Transformation. This is because if we use permeability values directly for wavelet analysis, we find that it is possible that some combinations of evaluated wavelet coefficients, upon inversion may yield negative values. It is hard to condition the sets of wavelet coefficients so that they would yield only positive values upon inversion. Using logarithms of permeabilities for wavelet analysis ensures that the wavelet inversion yields values that are all valid (being within the range of $-\infty$ to ∞).

Given the fact that the Haar wavelet coefficients are linear combinations of the log permeabilities, we can compute the statistical parameters describing these coefficient sets using the formulation shown in the following section. The log-permeability distribution is represented by a random function $V(\mathbf{x})$ such that each location $\mathbf{x}$, of the distribution is composed of a random variables $V(\mathbf{x})$.

5.3 Changing Geological Scenario after History Match

Given a prior and a subsequent history match, it is possible that at a later time, we might obtain more information and hence our perception about the reservoir geology might change. If existing techniques are used, we would be forced to perform a new history match starting from the new geological scenario. However, using the wavelet based data integration method, if we can successfully decouple the production-data-constraining and the geological parameter-constraining wavelet coefficients sets, we can vary them independently of the other. This implies that keeping the subset of history matching wavelet coefficients fixed, it is possible to modify the remaining coefficients in order to constrain to the new geological scenario prescribed. We used this technique in order to integrate an anisotropic variogram into an isotropic prior history-matched model. Repeating the process starting from a different random seed leads to different results. Six of the permeability model results of this data integration procedure are shown in Figure 5.36.

Figure 5.37 shows the variogram match obtained using simulated annealing as the optimization technique. As can be seen (Figure 5.38), there is a great degree of variance in the log-permeability results obtained. This variance is relatively low in the vicinity of the wells and much higher in the more loosely constrained areas away from the wells.

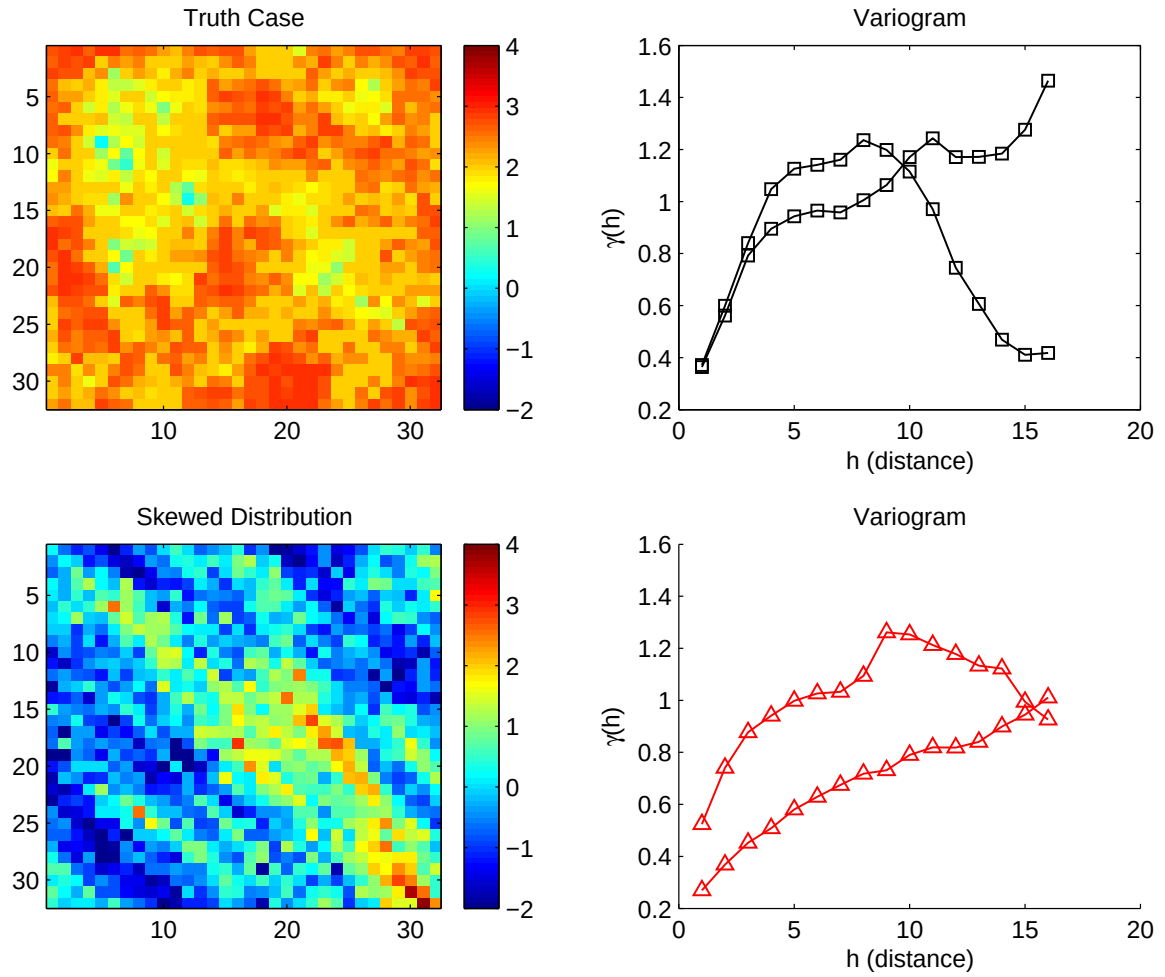


Figure 5.35: Initial (isotropic) and prior (anisotropic) log permeability fields along with corresponding variograms in (1,1,0) and (-1,1,0) directions.

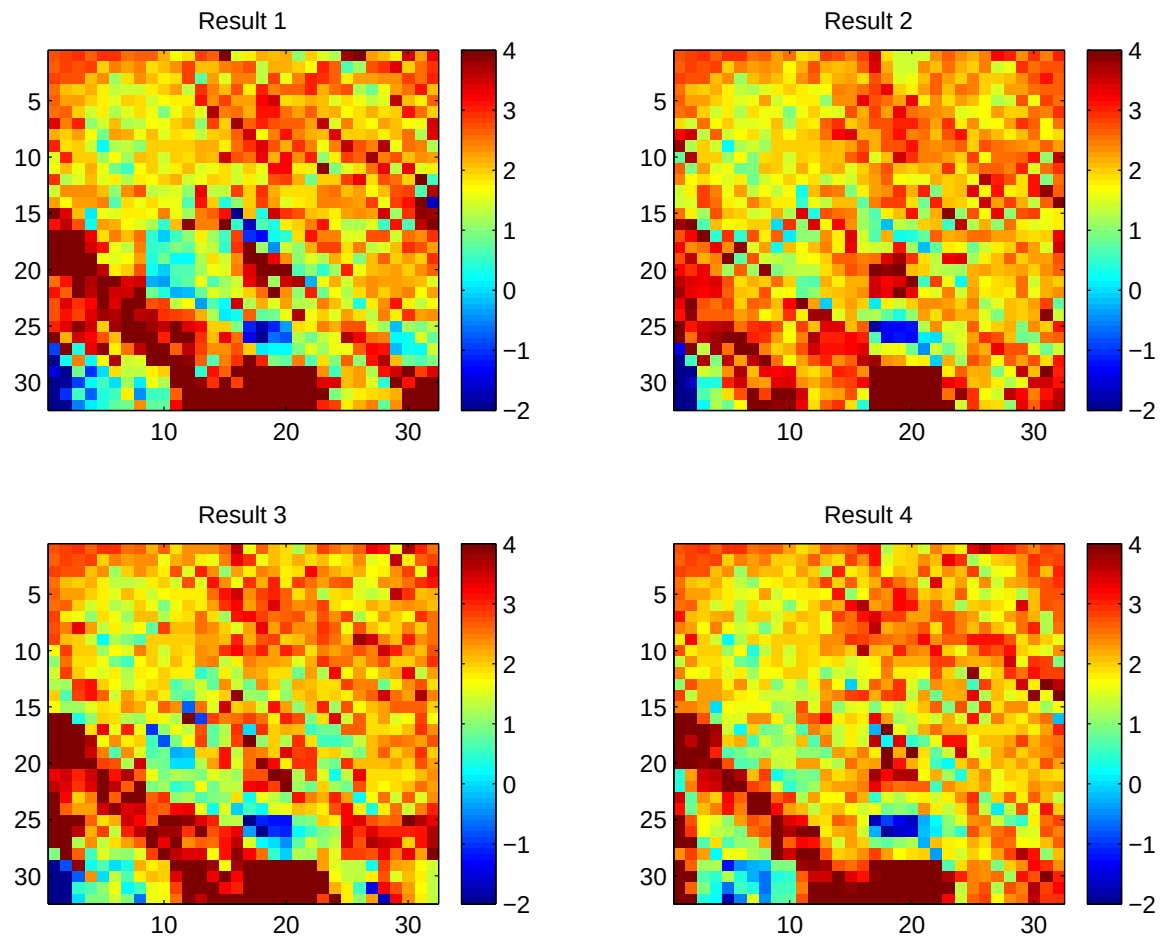


Figure 5.36: Log permeability field results for integration of anisotropic variogram in a history matched model with isotropic prior.

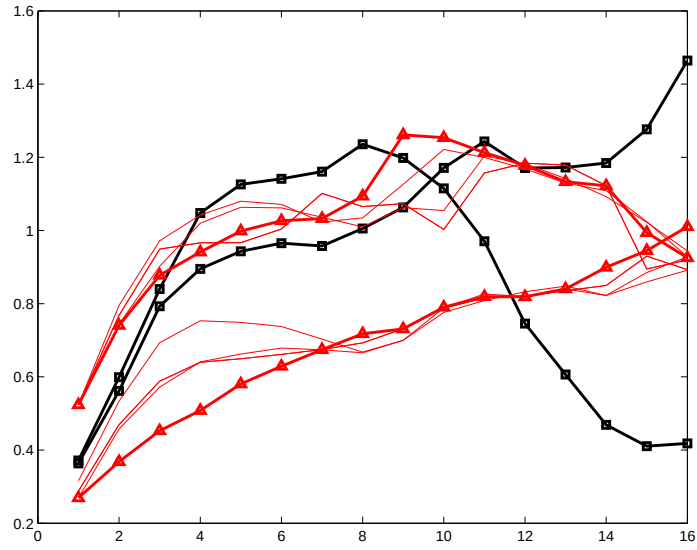


Figure 5.37: Variogram match results for integration of anisotropic variogram in a history matched model with isotropic prior. Black curves show the initial variogram and red curves show the target variogram and the matches obtained.

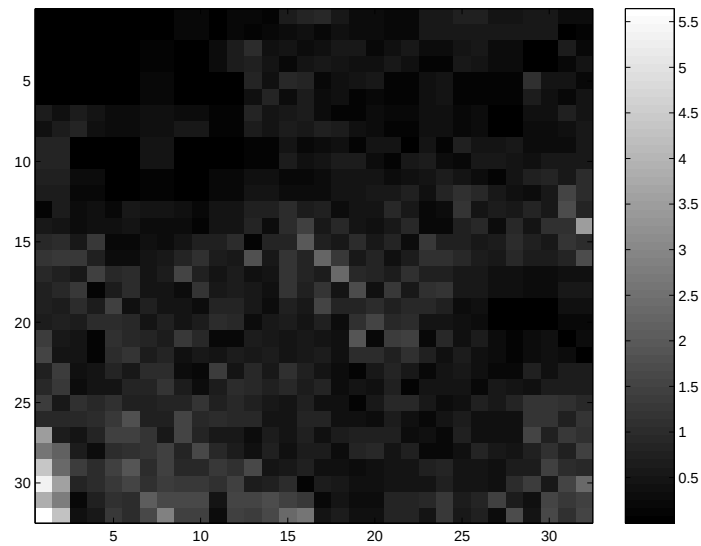


Figure 5.38: Standard deviation map of log-permeability results.

Using Simulated Annealing with variogram optimization

The method outlined in Section 5.1.1 tries to constrain the permeability model to a given variogram. Figure 5.36 shows the permeability model results of the optimization routine for matching the variogram. We see very clearly that there are some extremely high permeability stripes in the bottom left corner of the model, which the algorithm has incorporated in order to match the variogram. This is a result of the fact that the variogram, as described by [59] is an average property and that no constraint is applied on the histogram of the final permeability distribution. Each value of $\gamma(h)$ is derived as an ‘average’ using all sets of points a distance h apart. The effect of applying this variogram constraint in isolation is that in order to compensate for a different underlying variogram model in other regions, the algorithm puts a high degree of continuity in the bottom left corner, such that the overall variogram calculation will give a match to the target variogram.

However, this problem can be solved by applying a constraint on the permeability histogram in order to ensure a smooth and well distributed permeability model and to avoid artifacts observed in the unconstrained optimization. Note here that the optimization routine makes direct changes only to the wavelet coefficients of the log-permeability field and not to the permeabilities directly. Hence, to ensure that the log-permeability distribution is Gaussian and bounded, we need to ensure two things. First, that the perturbations of the wavelet coefficients yield log-normal permeabilities at each iteration. This condition is easy to incorporate. We saw in Section 5.1.3 that for Gaussian random functions, we can calculate the statistics for the corresponding wavelets coefficients. As such, by drawing from this subset of possible values of the wavelet coefficients, we ensure that the permeability field is Gaussian. The second thing we need to ensure is that the permeability field is bounded. In order to ensure the bounded nature of the permeability field, we apply constraints on the variance of the parameter distribution.

5.4 Downscaling

Upscaling using Haar wavelets is the process of obtaining a coarse scale reproduction of an image by setting the corresponding fine-scale wavelet coefficients to zero. This process of upscaling using Haar wavelets and other mathematical tools has been described with the help of some examples in Section 3.1.2. *Downscaling* is the reverse process of upscaling which includes the addition of fine-scale details to a coarse scale image. In the framework

of Haar wavelets, downscaling would involve adding fine scale coefficients to a coarse scale wavelet coefficient description of the initial image. These fine scale coefficients may be constrained to honor fine scale statistical properties, as is described here.

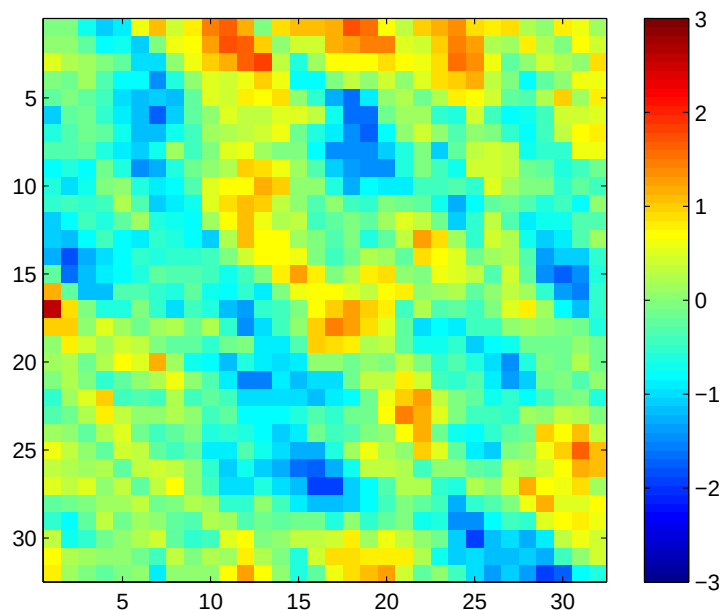


Figure 5.39: Coarse scale log-permeability distribution.

Consider an anisotropic, coarse scale permeability field as shown in Figure 5.39. This 32×32 Gaussian distribution contains 1024 gridblocks in total. Say we would like to obtain a fine scale description of this reservoir model, constrained to the block averages of the coarse model as well as to a fine scale variogram. This can easily be done using the properties of the Haar wavelet transform. As described in Section A.3, two-dimensional Haar wavelet coefficients represent the corresponding image in terms of averages and contrast at different resolutions. Hence, if we were to fix the wavelet coefficients corresponding to the average values of the reference distribution at a certain scale, we would in effect be fixing the block averages of the reproduced image, without constraining the individual pixels. Based on this observation, we can generate a fine scale reproduction of a coarse distribution by specifying wavelet coefficients corresponding to block average values of the permeability and adding *new* fine scale coefficient sets to form a finer description of the reservoir in wavelet space.

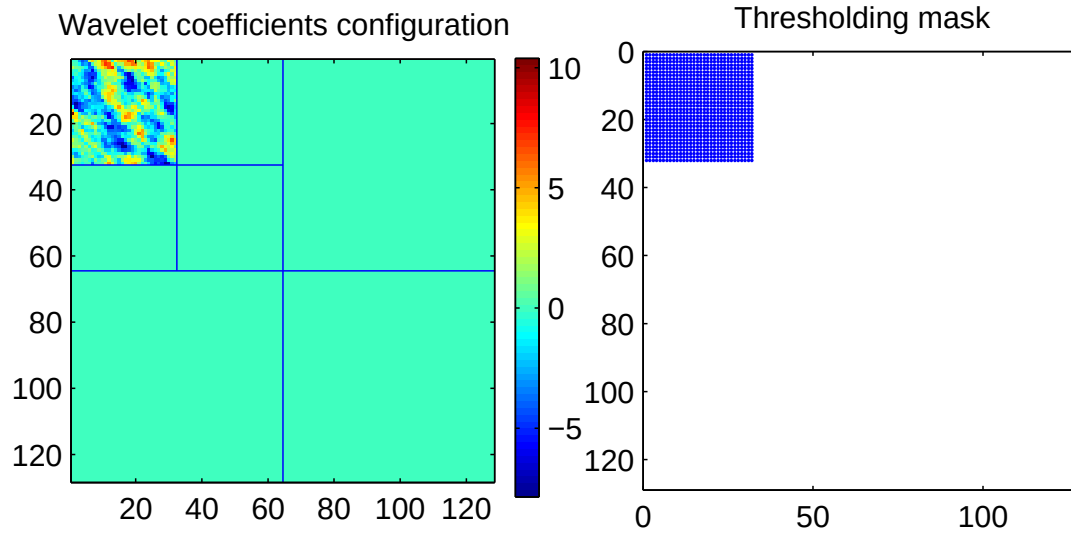


Figure 5.40: Permeability distribution substituted as a subset of a larger wavelet coefficient set along with wavelet mask for simulated annealing.

This process is depicted in Figure 5.40. Figure 5.40 (left) shows a wavelet coefficient set that is constructed using the coarse *reservoir permeability* model (with scaled values to represent the transform) as the set of average *wavelet coefficients* at the appropriate resolution. In other words, the initial coarse reservoir description can be thought of as a subset of a much larger set of wavelet coefficients, with sets of fine scale coefficients added based on the final degree of refinement desired. At initialization, all the fine scale coefficients are set to zero values.

As an exercise, if we were to perform an inverse wavelet transform on coefficients shown in Figure 5.40 (left), we would obtain a permeability distribution of size 128×128 , containing block averages of size 4×4 that correspond to the original coarse distribution. This *refined* distribution however has no finer resolution or permeability details than the original distribution. Moreover we would like to constrain the final permeability distribution to a fine scale variogram. For this purpose, we perform optimization (using simulated annealing as discussed in Section 5.1.1) in order to determine the values of the fine scale wavelet coefficients added that would yield a final fine scale covariance permeability structure as

described by the variogram. The wavelet mask used to constrain the final distribution to block averages is shown in Figure 5.40 (right).

The result of this optimization gives a full set of wavelet coefficient values at all desired scales (see Figure 5.41). Inverse wavelet transform on the coefficients shown in Figure 5.41 yields the final scale permeability distribution depicted in Figure 5.42. This fine scale distribution (of size 128×128) is the *downscaled* version of the coarse permeabilities in Figure 5.39. The downscaled distribution matches the fine scale variogram (see Figure 5.43) and by the construction of the solution set, it is also constrained to the initial permeabilities as block averages.

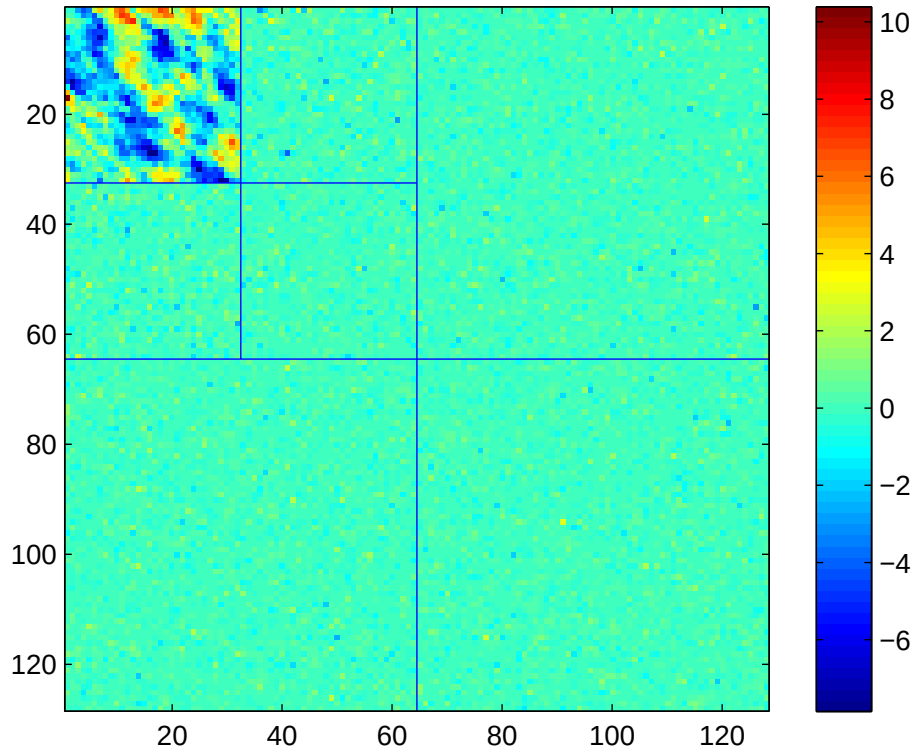


Figure 5.41: Complete wavelet coefficient set after downsampling using simulated annealing algorithm.

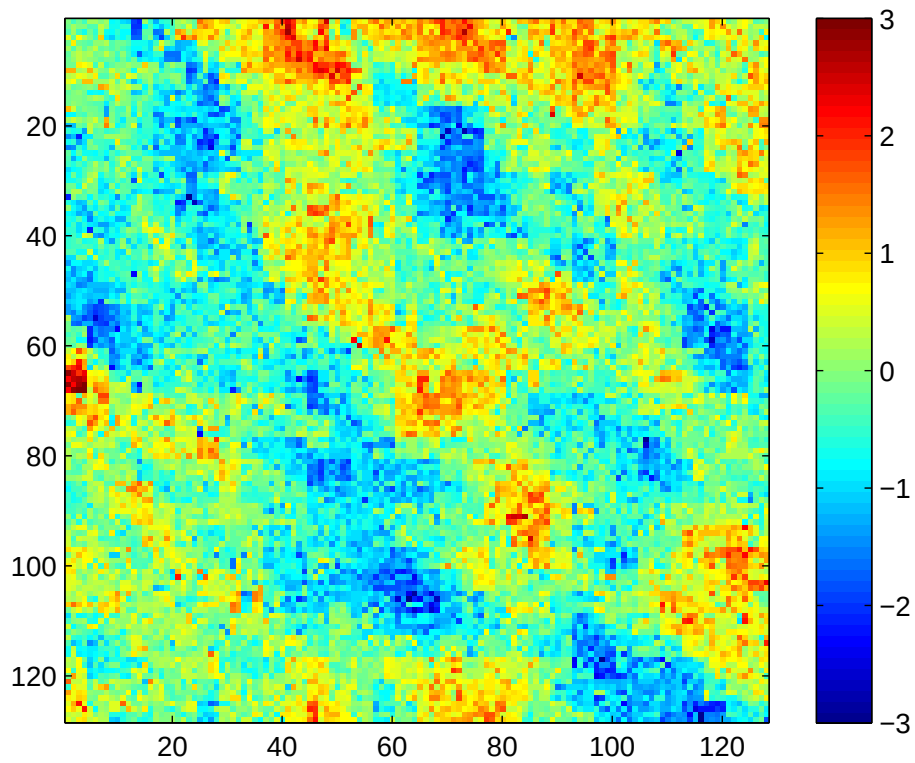


Figure 5.42: Downscaled log-permeability distribution obtained by inverse wavelet transform of full set of wavelet coefficients as shown in Figure 5.41.

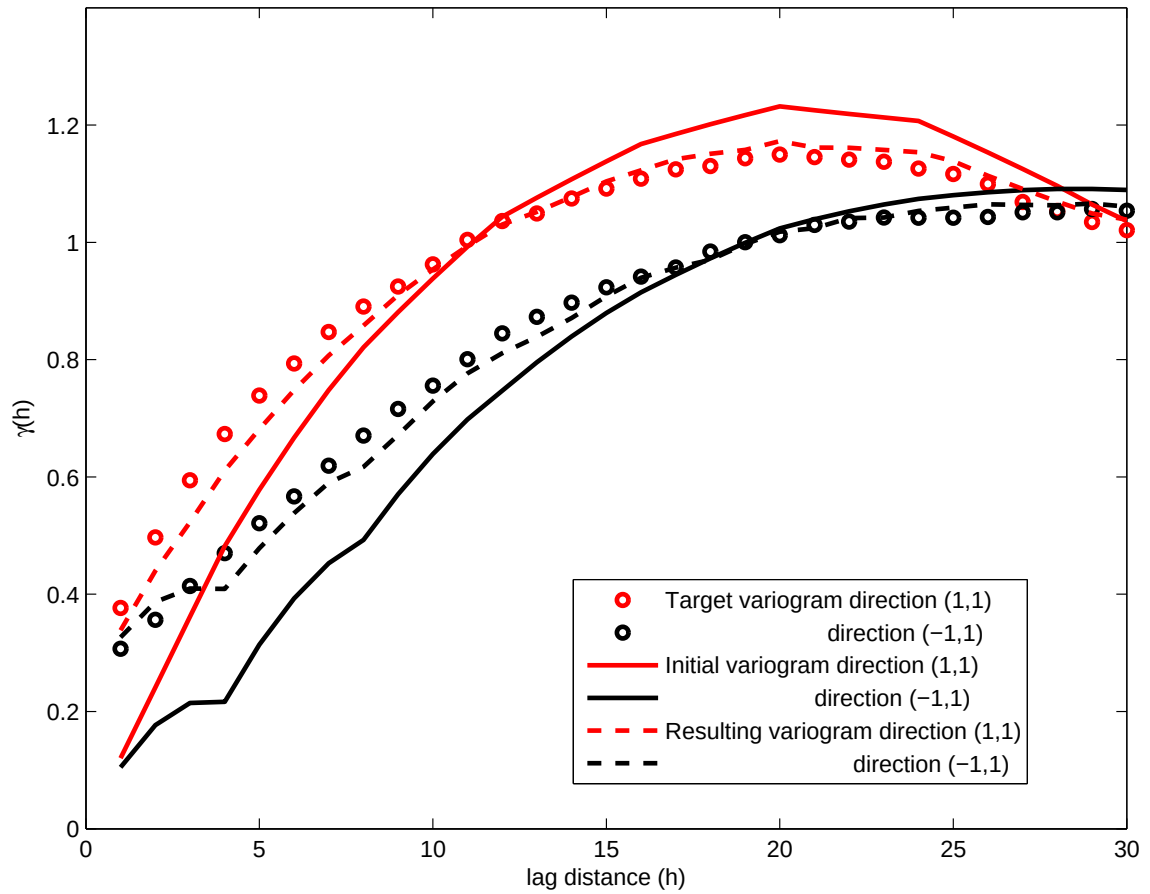


Figure 5.43: Variograms for initial coarse scale permeability distribution, target fine scale variogram model and final variogram match after downscaling.

5.5 Chapter Summary

Chapters 3 and 4 covered aspects of production data integration and partitioning of the wavelet coefficient set. In this chapter, we covered the next stage of the algorithm - the integration of geostatistical data and the generation of multiple equiprobable reservoir models. In Figure 3.47 we saw a Venn diagram representation of the complete set of wavelet coefficients highlighting the subset that is important to production history match. In Chapter 5 for the case of an example Gaussian permeability model it was shown that there exists another set of wavelet coefficients that is significant to the geostatistical properties of the distribution. Thus the Venn diagram in Figure 3.47 can be redrawn with this second set as Figure 5.44. The degree of overlap of these two sets is of course dependent on the type of reservoir model under study (Gaussian/channelized etc) and the amount of production history data available. From the point of view of sequential integration of data, in the best case these sets will be disjoint and in the worst case they will overlap completely. From the point of view of analyzing the degree of coupling of production history and geology, both these scenarios provide valuable information.

For the integration of geostatistical information, the following methods were described and explained with the help of some example applications:

1. Simulated annealing of wavelet parameters, using the norm difference of variograms as objective function.
 - Fixed threshold
 - Grayscale threshold
2. Analytical method for Gaussian fields using *sgsim* [59] in wavelet space.

The grayscaling method explores the concept of a using soft thresholding. In this method, set partitioning is done stochastically, the probability of including a particular wavelet coefficient for production history match being proportional to the magnitude of its sensitivity. As a result, for each optimization run, a different set of coefficients represent production history, thereby adding a degree of uncertainty in the production data constraint. For the case of Gaussian distributed parameters, using the fact that the Haar wavelet transform is a linear transform of the original parameters, we can make special inferences about the statistical properties of the wavelet coefficients obtained. As a result, we can generate the set of wavelet coefficients constraining the model to the geostatistics by using sequential

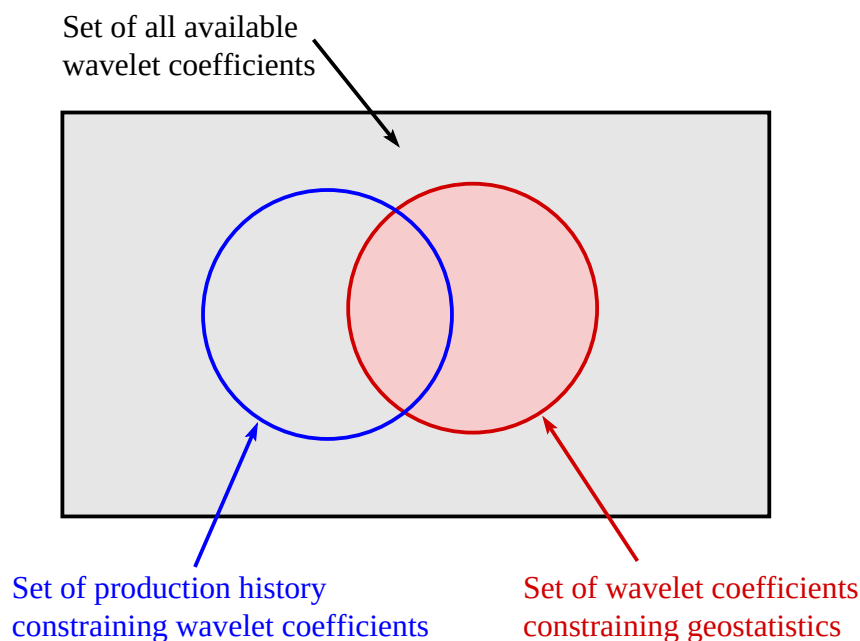


Figure 5.44: Venn diagram showing the total available space of wavelet coefficients for a reservoir model, highlighting the fact that there exists a subset that constrains the model to production data and one that constrains to the geostatistical properties of the property distribution.

Gaussian simulation (*sgsim*) in wavelet space. This development offers a huge saving in the optimization cost over iterative techniques such as simulated annealing.

We also see an application in which a history-matched model based on an incorrect (isotropic) *prior* is later constrained to the correct (anisotropic) *prior* geological scenario. The ability of the algorithm to modify the *prior* while maintaining the constraint on production history, which would be of great use in practical applications, is limited by degree of decoupling of the production data and geology. If the coupling was strong, or in other words, if the two subsets in Figure 5.44 were highly overlapping, the best way to change to *prior* would be to redo the history match with the correct *prior* model.

Downscaling is described as the process of generating a fine scale reservoir model given a coarse scale description of it. An elegant method of wavelet-based downscaling is proposed, that constrains to the coarse distribution as linear block averages while satisfying a fine scale variogram model. This method is stochastic, yielding many different fine-scale reservoir models all satisfying the same block average and variogram constraints.

Chapter 6

Discussion and Future Directions

A methodology of generating multiple history-matched, geologically-constrained reservoir models was developed. This methodology takes advantage of the fact that wavelet coefficients are linear combinations of the actual reservoir parameter (log-permeability). As a result, the statistics (mean, variance and variogram) of the wavelet coefficients can be computed from the corresponding statistics of the reference. Given the statistics for each wavelet set and using history constraints as hard data, we can use *sgsim* in wavelet space to generate equiprobable reservoir models. This is a more intuitive approach for reproducing the statistics of a distribution than the iterative procedure of simulated annealing we used earlier. Applying the *sgsim* approach we generated a number of reservoir models using only about 5% of the CPU time required for the simulated annealing approach. This is because the realizations are generated by drawing from the theoretically computed statistics directly. The simulated annealing approach was based on random perturbation followed by wavelet inversion and checking the objective function in order to match the variogram. Using grayscale sensitivity fields aids in better capturing the uncertainties of production data and provides a more complete set of possible alternative reservoir models given the data.

This study concluded that this multiresolution wavelet analysis leads to effective partitioning of parameters, thereby making possible their sequential integration into the reservoir model. This is a powerful tool since it allows for the generation of multiple history-matched models without repeating the history matching procedure. The approach can also adjust the data match well by well or by pressure, watercut and geostatistical data sequentially. This methodology yields multiple reservoir models all constrained to the same set of data,

thereby allowing the inclusion of uncertainty in prediction runs. The wavelet transform is a linear transform, not adding to the overall computational cost in any significant manner. The multiresolution properties of wavelets not only allow for a substantial reduction in parameters, they also ensure that the integration of data takes place only at the appropriate scale without overconstraining the model. The scope of the algorithm has been increased with the use of a commercial simulator for the gradient calculations and extension to three-dimensional models with the possibility of complex production profiles.

6.1 Directions for Further Study

6.1.1 Multipoint Statistics

The approach has so far been developed for and tested mainly on Gaussian fields, using the variogram as a the means of integrating geological information into the reservoir model. However, as mentioned earlier, the approach itself is modular in nature and hence the possibility exists of replacing variogram-reproduction with more complicated geostatistical constraints in order to honor reservoir geology. An algorithm that integrates production data with complex geological scenarios would be of great value since the history matching of such complicated reservoir models poses a unique challenge.

In the work done here, the histogram and variogram were used as the geological parameters that are used to integrate ‘geology’ into the porosity and permeability reservoir models. Since the property distribution models were assumed to be Gaussian, we used sequential Gaussian simulation (*sgsim*) for generating realizations. The histogram and variogram, however, describe merely one- and two-point statistics respectively for a random variable. As such, they fail to capture more complicated, multiple-point structural complexities (for example, meandering channels). It is true however, that reservoir models can rarely be fully modeled using only Gaussian random functions. In order to capture complicated geology we need to turn to multiple-point statistics.

The data integration algorithm described in the previous chapter is general in nature and is not dependent on the use of the particular statistical parameter for the integration of geological information. The variogram is but one geological parameter that quantifies the spatial distribution of random functions in Gaussian fields. As such, the data-integration methodology can be adapted to use multipoint information.

Currently, there exist two established approaches integrate complicated geological features: pixel-based [74, 75, 76, 77, 78] and object-based ([79, 80, 81]). Techniques resembling image analysis and pattern matching ([82]) have also been applied to the solution of this problem of including large scale structural features in a reservoir model. There also exist some wavelet-like averaging functions for pattern characterization (*filtersim* [83]).

Snesim algorithm: Pixel-based [77] One of the first algorithms developed to account for multiple point statistics, **snesim** [76] stands for ‘single normal equation simulation’. This approach is based on the use of a *training image*. A training image is essentially a conceptual visual description of the geology of a reservoir and may be obtained from a geologist’s impression of the depositional environment or from reservoir analogs. The training image need not be conditioned to local data and so it can not be used for flow simulation directly. What is required is a reservoir description derived from this initial concept that is also conditioned to local data (from well logs, well tests and seismic data etc.). This is achieved by scanning the training image and storing the corresponding training multiple-point statistics, and then incorporating these in conditional realizations using Bayesian methods. These algorithms have had reasonable success in shape reproduction but fail for more complicated features with sharp corners.

Object-based approach [79, 80, 81] This approach is based on the use of well defined geologically-sound geometric shapes (or *objects*). These *objects* are then arranged or ‘dropped’ on a background facies, in order to generate reservoir models. One significant advantage of the object based approach is excellent shape-reproduction. However, constrained to work with a preselected set of shapes, they lack flexibility and are hard to condition to well-logs, well-test data [84] or partially interpreted geobodies.

Feature-based approach In order to match geostatistical data, Arpat [82] used a feature-based (*f-snesim*) approach that is the middle ground between pixel-based and object-based methods. Here, a ‘feature’ is defined as as a three-dimensional configuration of pixels that identifies a meaningful piece or part of a geological shape known to exist in a reservoir. The key advantage of this method over object based methods is that the ‘features’ derived from the training image are clustered into collections that reduce the dimensionality of the problem. This reduced training image is then used to generate geological realizations using **snesim**. This approach is faster gives better feature-shape reproduction than pixel-based approaches, comparable to object-based methods.

Filter-based approach Zhang [83] developed a technique **filtersim** that generates reservoir models by patching patterns that are classified using *filters*. These *filters* are weight measures over templates to generate weighted averages of patterns from the training image (called *scores*). *Scores* are then classified into separate bins, with each bin corresponding to features that are similar. By drawing randomly from each bin, it is possible to generate stochastic reservoir models, constrained to hard data.

We can see that there are several different approaches currently being employed to generate reservoir models with complex geologies. Most of the methods enable conditioning to local well data and seismic data etc. A technique that is capable of integrating production data into a reservoir model describing complex geology would be a very useful. This is more important for these complex reservoir models since history matching such reservoirs while preserving geology is a much harder task than history matching of models developed from stationary random functions. Wavelet functions are often used for edge detection in image-analysis and so can be useful in summarising relevant data from reservoir training images.

On analysis, we see that the wavelet transform of a channel reservoir (Figure 6.1.1) does indeed capture the edges of geologic features. We also see that most of the algorithms for generating reservoir models with complex geologies work on multiple grids - from coarse ones to fine grids. Wavelets are inherently suited for multiscale or multigrid analysis of the training image (see Section 3.1.2). Thus, there is a possibility of developing a new, wavelet-based technique, or modifications of existing techniques for stochastically integrating of multiple-point statistics in reservoir models.

6.1.2 Integration of Well Test and Seismic Data

Multiresolution analysis is a fundamental property of wavelet functions. That is, the wavelet operator as applied to a function such as a discretized reservoir model, produces a multiresolution description of it. At the same time, reservoir data, (such as seismic, well test, core data) provide information about the reservoir at different resolutions. Thus, it is possible to integrate each type of data with different supports, directly at the appropriate scale. In other words, each type of data can be used to constrain only the corresponding scale of wavelet coefficients. This will significantly reduce the dimensionality of the process, since

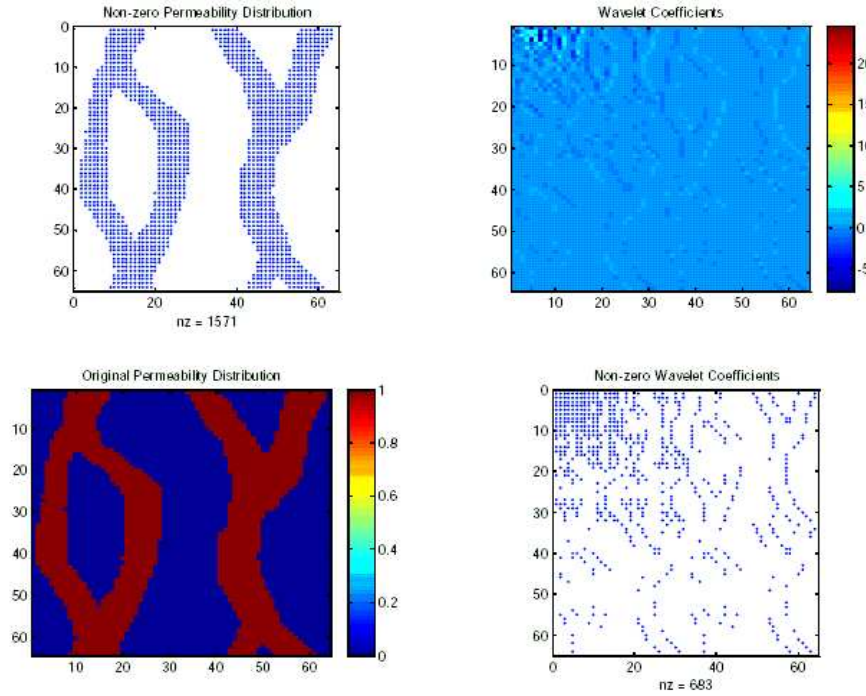


Figure 6.1: Wavelet description of a channel reservoir: (top left) Reference reservoir training image as binary field (top right) wavelet coefficients corresponding to training image (bottom left) Reference reservoir training image as continuous field (bottom right) Showing the non-zero wavelet coefficients out of all wavelet coefficients on top right .

only a small subset of the wavelet coefficients will need to be perturbed in order to integrate data at informs that scale. There lies tremendous potential for using multiscale description of wavelets for the integration of data other than well data and production data that was considered so far.

Appendix A

Reparameterization Techniques

A.1 Wavelets

Wavelets are a powerful mathematical tool for the decomposition and manipulation of functions. They allow, for example, a hierarchical/ multiscale representation of n -dimensional real functions $f : \mathbb{R}^n \rightarrow \mathbb{R}$ that are square-integrable, i.e.

$$\int_{\mathbb{R}^n} |f(x)|^2 dx < \infty,$$

as a linear combination of *orthogonal* basis functions with finite support that are derived by *dilations* and *translations* of a characterizing *scaling function* $\phi(t)$ and *wavelet* function $\psi(t)$.

The field of wavelets has made tremendous strides since the time it was first popularized in the early nineties when it was also given a firm mathematical foundation within the framework of a *multiresolution analysis* [36, 37, 35] of functions. Wavelets are particularly useful tools for data analysis because they provide far superior *time-frequency localization* than other methods such as Fourier transforms, and can be implemented efficiently in practice. Further, wavelets provide an equivalent representation of a data set with the property that a significant number of “wavelet” coefficients with low values can be easily omitted to obtain a much more compact representation of the data at the expense of only a slight loss in accuracy. Thus wavelets provide an effective method for approximating functions within an error that is acceptable in many real-life situations. Consequently, wavelets have found numerous applications in diverse fields such as signal processing, statistics, medical

imaging, computer graphics, data compression and denoising. There exists an extensive literature which provides a comprehensive introduction to the many aspects of wavelets, for example refer to [33].

In this research we used the Haar wavelet for the analysis of parameter distributions. We now present the mathematical and algorithmic details of Haar wavelets relevant to this work. We start by describing the wavelet transform in one dimension and then move on to the two-dimensional case.

Remark: The implementation of three-dimensional Haar wavelets is based on the recursive application of the one-dimensional decomposition (which is known as the Standard decomposition). The number of bases for each higher dimension thus increases by two. The number of bases as a function of the number of spatial dimensions (d) is given by 2^d (thus, for describing three-dimensional wavelets, eight bases are required).

A.2 One-Dimensional Haar Wavelet

In this section we will describe how a one-dimensional function can be decomposed using Haar wavelets. First we will define the appropriate Haar basis functions in one dimension.

A.2.1 One-Dimensional Haar Basis Functions

A useful abstraction is to think of one-dimensional functions (equivalently fields or images) as piecewise-constant functions on the half-open interval $[0, 1)$. Suppose that a one-value field is a function that is constant over the entire interval $[0, 1)$. We denote by V^0 the vector space of all such functions. Next, two-value functions will be assumed to consist of two constant pieces over the intervals $[0, 1/2)$ and $[1/2, 1)$, and will belong to the vector space V^1 . Extending this construction to higher j , the space V^j will include all piecewise-constant functions defined on the interval $[0, 1)$ with constant pieces over each of the 2^j equal length subintervals. By definition, every vector (image) in V^j is also contained in V^{j+1} . Thus, the spaces V^j are nested:

$$V^0 \subset V^1 \subset V^2 \subset \dots \quad (\text{A.1})$$

This nested set of spaces V^j is a fundamental component of the mathematical theory of multiresolution analysis [37].

The basis functions for the spaces V^j are called *scaling functions* and are denoted by the symbol ϕ . The *normalized* Haar basis for V^j is given by the set of scaled and translated box functions:

$$\phi_k^j(t) := 2^{j/2} \phi(2^j t - k), \quad k = 0, \dots, 2^j - 1, \quad (\text{A.2})$$

where the scaling function is given by:

$$\phi(t) := \begin{cases} 1 & \text{for } 0 \leq t < 1, \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A.3})$$

As an example, A.1 shows the four box functions forming a basis for V^2 .

Next define a new vector space W^j which consists of all functions in V^{j+1} that are orthogonal to all functions in V^j under, say, the standard inner product:

$$\langle f | g \rangle := \int_0^1 f(t)g(t)dt, \quad f, g \in V^j.$$

Hence W^j is the orthogonal complement of V^j in V^{j+1} . Informally, we can think of wavelets in W^j as a basis for the parts of a function in V^{j+1} that cannot be represented in V^j . Thus for a fixed j the nesting (A.1) can be expressed alternately as:

$$V^j = V^0 \oplus W^0 \oplus \dots \oplus W^{j-1}. \quad (\text{A.4})$$

where $\oplus$ denotes an orthogonal sum of functions from the respective spaces. A collection of linearly independent functions $\psi_k^j(t)$ spanning W^j are called *wavelets*. These basis functions have two important properties:

1. The basis functions $\psi_k^j(t)$ of W^j , together with the basis functions $\phi_k^j(t)$ of V^j form a basis of V^{j+1} .
2. Every basis function $\psi_k^j(t)$ of W^j is orthogonal to every basis function $\phi_k^j(t)$ of V^j .

The *normalized* Haar wavelet functions are given by:

$$\psi_k^j(t) := 2^{j/2} \psi(2^j t - k), \quad k = 0, \dots, 2^j - 1, \quad (\text{A.5})$$

where the wavelet function is given by:

$$\psi(t) := \begin{cases} 1 & \text{for } 0 \leq x < 1/2, \\ -1 & \text{for } 1/2 \leq x < 1, \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A.6})$$

A.2.2 Wavelet Transform and Reconstruction

Assume now that we have a function $f(t)$ defined at 2^j discrete points $t = 0, \dots, 2^j - 1$ and therefore belongs to the vector space V^j . The wavelet transform essentially decomposes the function into the sum of an average function $f_0(t) \in V^0$ and detail functions $w_i(t) \in W^i, i = 0, \dots, j - 1$ as follows:

$$f(t) = f_0(t) + w_0(t) + \dots + w_{j-1}(t), \quad (\text{A.7})$$

where

$$f_0(t) = \sum_k a_k^0 \phi_k^0(t) = a_0^0 \phi(t) \quad (\text{A.8})$$

and

$$w_i(t) = \sum_k d_k^i \psi_k^i(t) = \sum_k d_k^i 2^{i/2} \psi(2^i t - k), \quad i = 0, \dots, j - 1. \quad (\text{A.9})$$

The following two pseudocode procedures accomplish the normalized Haar wavelet decomposition by obtaining the coefficients a_0^0 and d_k^i . The procedure *Decomposition* takes the function $f(t)$ as a vector input and repeatedly calls the subroutine *DecompStep*. The output vector $f(t)$ is also of length 2^j but contains the average and “detail” coefficients in the following order

$$[a_0^0 \ d_0^0 \ d_0^1 \ d_1^1 \ \dots \ d_0^i \ \dots \ d_{2^i-1}^i \ \dots \ d_0^{j-1} \ \dots \ d_{2^{j-1}-1}^{j-1}].$$

procedure *Decomposition*

Input: Vector $(f(t) : t = 0, \dots, 2^j - 1 = h)$

$f(t) \leftarrow 2^{-j/2} f(t)$ (normalize input coefficients)

while $h > 1$ **do**

DecompStep $(f(t) : t = 0, \dots, h)$

```

     $h \leftarrow (h - 1)/2$ 
end while
end procedure

procedure DecompStep
Input: Vector  $(f(t) : t = 0, \dots, h)$ 
for  $i = 0 : (h - 1)/2$  do
     $g(i) \leftarrow (f(2i) + f(2i + 1))/\sqrt{2}$ 
     $g((h + 1)/2 + i) \leftarrow (f(2i) - f(2i + 1))/\sqrt{2}$ 
end for
 $f \leftarrow g$ 
end procedure

```

Note that the wavelet decomposition is a linear transformation. Consequently, the decomposition can be represented efficiently as a matrix operation on the input function. Due to the special structure of the wavelet transformation, it can be calculated in $O(n)$ time which makes wavelets extremely useful in real life applications. A function can be reconstructed from its wavelet decomposition simply by inverting the matrix transformation or equivalently, reversing the above operations.

A.3 Two-Dimensional Haar Wavelet

There are two ways we can use wavelets to transform the function values within a two-dimensional field or image. Each is a generalization to two dimensions of the one-dimensional wavelet transform described in Section A.2. To obtain the *Standard* decomposition [72] of an image, we first apply the one-dimensional wavelet transform to each row of function values. This operation gives us an average value along with detail coefficients for each row. Next, we treat these transformed rows as if they were themselves a field and apply the one-dimensional transform to each column. The resulting values are all detail coefficients except for a single overall average coefficient. The algorithm below computes the Standard decomposition. Figure A.1 illustrates each step of its operation.

```

procedure StandardDecomposition
Input:  $f(s, t) : s = 0, \dots, 2^j - 1; t = 0, \dots, 2^j - 1$ 

```

```

for row = 0 : 2j - 1 do
    Decomposition (f[row, 0, ..., 2j - 1])
end for
for col = 0 : 2j - 1 do
    Decomposition (f[0, ..., 2j - 1, col])
end for
end procedure

```

The second type of two-dimensional wavelet transform, called the *Nonstandard* decomposition, alternates between operations on rows and columns. First, we perform one step of horizontal pairwise averaging and differencing on the pixel values in each row of the image. Next, we apply vertical pairwise averaging and differencing to each column of the result. To complete the transformation, we repeat this process recursively only on the quadrant containing averages in both directions. Figure A.2 shows all the steps involved in the Nonstandard decomposition procedure below.

```

procedure NonstandardDecomposition Input: f(s, t) : s = 0, ..., 2j - 1; t = 0, ..., 2j - 1)
f ← 2-j f    (normalize input coefficients)
while h > 1 do
    for row = 0 : h do
        DecompositionStep (f[row, 0, ..., 2j - 1])
    end for
    for col = 0 : h do
        DecompositionStep (f[0, ..., 2j - 1, col])
    end for
    h ← h/2
end while
end procedure

```

The two methods of decomposing a two-dimensional field yield coefficients that correspond to two different sets of basis functions. The Standard decomposition of an image gives coefficients for a basis formed by the standard construction of a two-dimensional basis. Similarly, the Nonstandard decomposition gives coefficients for the nonstandard construction

of basis functions. The standard construction of a two-dimensional wavelet basis consists of all possible tensor products of one-dimensional basis functions. For example, when we start with the one-dimensional Haar basis for V^2 , we get the two-dimensional basis for V^2 shown in Figure A.1. Note that if we apply the standard construction to an orthonormal basis in one dimension, we get an orthonormal basis in two dimensions. The nonstandard construction of a two-dimensional basis proceeds by first defining a two-dimensional scaling function,

$$\phi\phi(x, y) := \phi(x)\phi(y),$$

and three wavelet functions,

$$\phi\psi(x, y) := \phi(x)\psi(y)$$

$$\psi\phi(x, y) := \psi(x)\phi(y)$$

$$\psi\psi(x, y) := \psi(x)\psi(y).$$

We now denote levels of scaling with a superscript j (as we did in the one-dimensional case) and horizontal and vertical translations with a pair of subscripts k and l . The nonstandard basis consists of a single coarse scaling function $\phi\phi_{0,0}^0(x, y) := \phi\phi(x, y)$ along with scales and translates of the three wavelet functions $\phi\psi$, $\psi\phi$ and $\psi\psi$:

$$\phi\psi_{kl}^j(s, t) := 2^j \phi\psi(2^j s - k, 2^j t - l)$$

$$\psi\phi_{kl}^j(s, t) := 2^j \psi\phi(2^j s - k, 2^j t - l)$$

$$\psi\psi_{kl}^j(s, t) := 2^j \psi\psi(2^j s - k, 2^j t - l).$$

The constant 2^j normalizes the wavelets to give an orthonormal basis. The nonstandard construction results in the basis for V^2 shown in Figure A.2. We have used both the standard and nonstandard approaches to wavelet transforms and basis functions because both have advantages. The Standard decomposition is appealing because it simply requires performing one-dimensional transforms on all rows and then on all columns. On the other hand, it is slightly more efficient to compute the Nonstandard decomposition. Another consideration is the support of each basis function, meaning the portion of each functions domain where that function is nonzero. All Nonstandard Haar basis functions have square supports, while some standard basis functions have nonsquare supports. Depending upon the application, one of these choices may be preferable to the other.

Thresholding Thresholding is the procedure of compressing a signal or a data set by the process of eliminating certain wavelet parameters that do not meet a threshold criterion of the form:

$$x = \begin{cases} y, & \text{if } |y| > T \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.10})$$

where

- x is the set of parameters retained,
- y is the set of all sensitivity parameters, and
- T is the threshold level.

The threshold is defined on the basis of the desired level of accuracy of the reproduction of a function (or of a field/image). In our case the threshold is set on the magnitude of the sensitivity of the production data to the coefficients.

A.4 Other Techniques for Data Compression

The problem of compressing data is one that has been studied extensively in many different engineering disciplines and is motivated by the very simple consideration that both the transmission and storage of data cost money. Data compression refers to the methods and technology employed to store information in a more compact form. There are two broad categories of compression – *lossless* and *lossy*, and depending on the application we use one or the other form of compression.

In our work, and in other areas such as image processing, lossy compression is of prime importance. Besides providing savings in cost and resources, lossy compression builds on the premise that most forms of information (natural or man-made) are highly redundant in their representation. And therefore for most practical purposes it is possible to reparameterize data in terms of fewer data points without any significant loss of fidelity (which is evaluated by some reasonable criterion such as least-squares error or perceptual error). Two important classes of lossy data compression algorithms are based on the *Singular Value Decomposition* (SVD) method and *Transform Compression* methods. We will briefly describe the two classes of algorithms.

A.4.1 SVD-based method

The SVD of an $n \times n$ matrix A with rank r is a canonical decomposition of the form $A = UDV^T$; where D is a diagonal $n \times n$ matrix whose diagonal elements

$$\sigma_1 \geq \dots \geq \sigma_r > \sigma_{r+1} = \dots = \sigma_n = 0$$

are called *singular values*, and U and V are orthogonal matrices consisting of the left and right singular vectors of A respectively. The singular values are in nonincreasing order and we can obtain a sequence of better and better approximations to the matrix A defined for $i = 1, \dots, r$ as:

$$A_i = U_i D_i V_i^T, \quad (\text{A.11})$$

where U_i and V_i consist of the first i columns of U and V respectively, and D_i is the $i \times i$ principal submatrix of D . For $i \geq r$ one can see that $A_i = A$. Thus we have a method of storing the matrix A in terms of the linear combination of a fewer number of eigenvectors weighted by the most significant singular values.

To apply the SVD method to image compression, consider an $n \times n$ image as the matrix A , whose $(i, j)^{th}$ element a_{ij} contains, say, the grayscale value of that pixel. Now we can generate a sequence of images which approximate the original to a closer and closer degree depending on how many singular values we consider in its decomposition. Frequently data have a low rank and furthermore, a large number of the smaller singular values are insignificant. Hence the smaller singular values can be ignored without much degradation of the data thus allowing for compression.

A.4.2 Transform Compression

Transform compression is perhaps the most popular method of lossy data compression for images, for example it is used in the *JPEG* standard. The main idea is that when we represent data in the frequency domain, via say the *Fast Fourier Transform* (FFT), then different kinds of information are now parameterized in terms of their frequency content. Specifically, low frequency components which correspond to average properties of the image usually carry more important information about the data than do the high frequency components. Thus expressing the high frequency components using 50% fewer bits might lead to only a 5% degradation in image quality.

There are a number of different transforms, such as the FFT or the Discrete Cosine Transform that are used in practice, with varying degrees of success depending on the characteristics of the image and the properties of the basis functions of the transform.

Finally, we should note that the Fourier transforms work solely in the frequency domain and the coefficients correspond to the image as a whole. In our work we have used the Wavelet Transform because it is able to provide a multiresolution representation of the frequency content of the image at different spatial scales. This frequently results in a much more compact representation of the data since it exploits spatial correlations at various scales and decomposes the image into a linear combination of (fewer) wavelet basis functions.

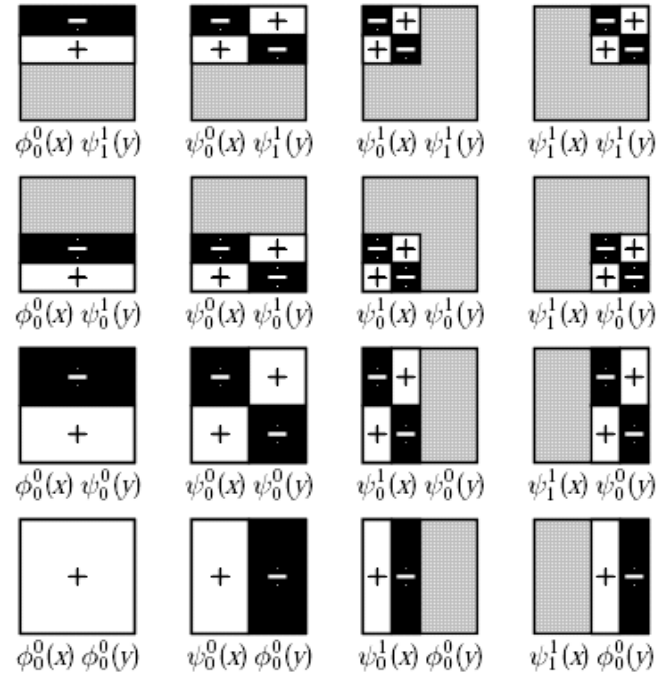


Figure A.1: Standard two-dimensional Haar wavelet basis (from [72]).

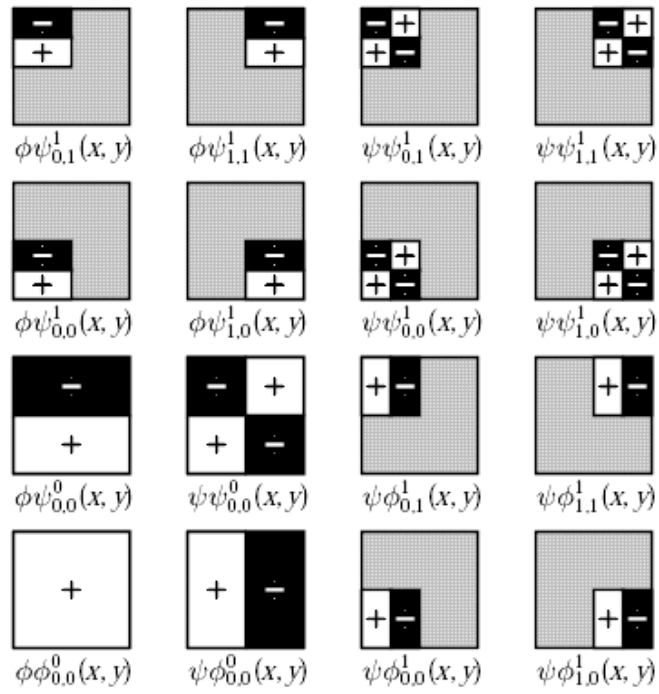


Figure A.2: Nonstandard two-dimensional Haar wavelet basis (from [72]).

Appendix B

List of Example Cases

B.1 Reservoir G1

This is a two-dimensional isotropic Gaussian permeability model of size 32 gridblocks in the x direction and 32 in the y direction as depicted in Figure B.1. The distribution was developed using *sgsim*, using the variogram as shown in Figure B.3. The simulation model is a black-oil model in Chears 2004 and the reservoir has four wells, three producers and one injector. The locations of these wells are also marked in Figure B.1.

B.2 Reservoir G1b

This example reservoir model is based on the same property distributions as Reservoir G1. The key difference is the production profiles and history. Given oil production and water injection rates, well BHP and WCT are used as input production data constraints. BHP and WCT from the four wells in this example reservoir model are depicted in Figures B.4 through B.7.

B.3 Reservoir 3A

This is a three-dimensional Gaussian model with 32 gridblocks in the x and y directions and 8 gridblocks in the z direction. The permeability field was developed using *sgsim* and is depicted layer by layer in Figure B.8. The reservoir model has three wells, two producers and one injector, and the reference BHP and WCT data from these wells are shown in

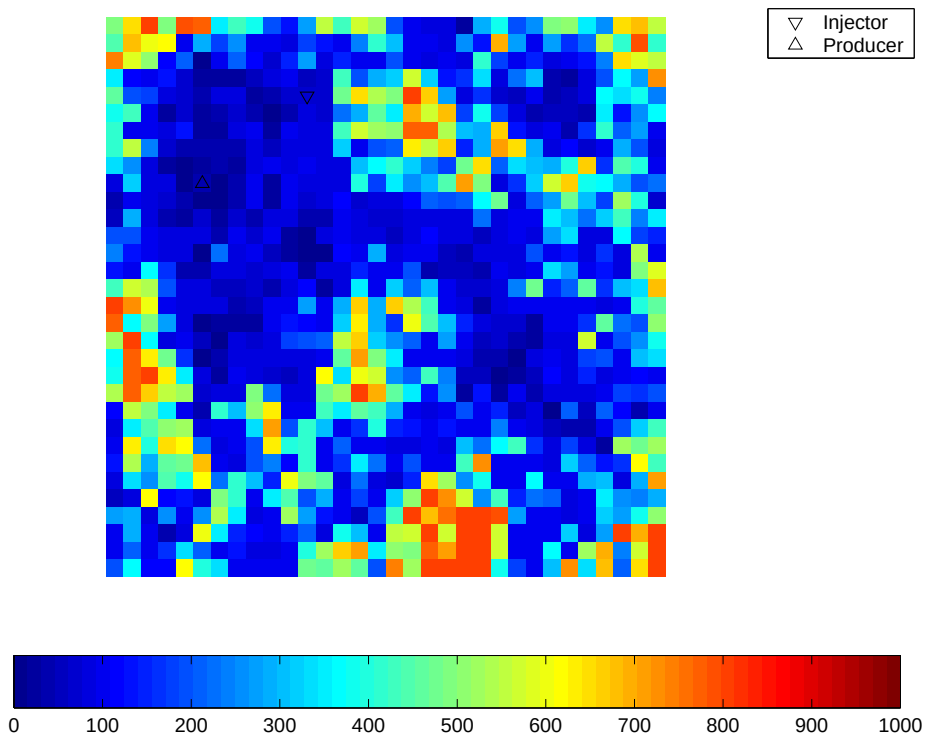


Figure B.1: Permeability distribution (in md) for Reservoir G1 with well locations.

Figures B.9 through B.11.

B.4 Case 2B

Case 2B is also a three-dimensional reservoir model of size $64 \times 16 \times 2$ gridblocks in x, y and z directions respectively. The permeability field distribution for both layers is shown in Figure B.12 along with locations of the two wells, one injector and one producer. Figure B.13 shows the variogram of the permeability distribution - the model variogram used has a very low *nugget effect* (see [59]).

The production profiles from the two wells are shown in Figures B.14 and B.15. The producer is set to a fixed OPR and the injector to fixed rate of water injection. Constraining this reservoir model to production history consists of setting OPR and WIR at the given values and comparing the BHP and WCT (at producer) to the reference data (as seen in Figures B.14 and B.15).

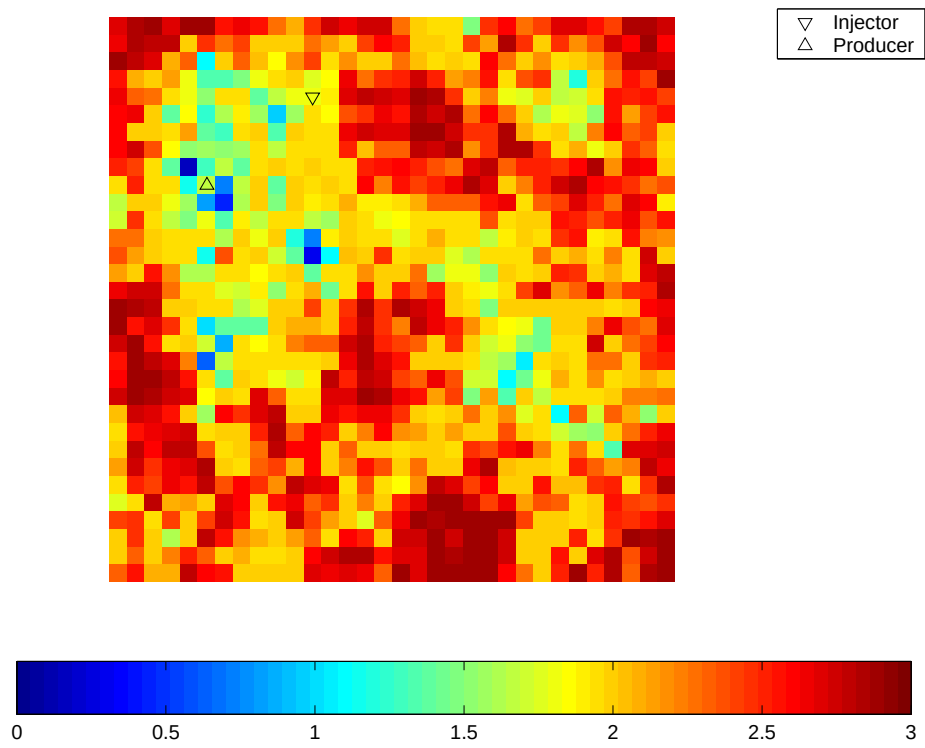


Figure B.2: Log permeability distribution (in md) for Reservoir G1 with well locations.

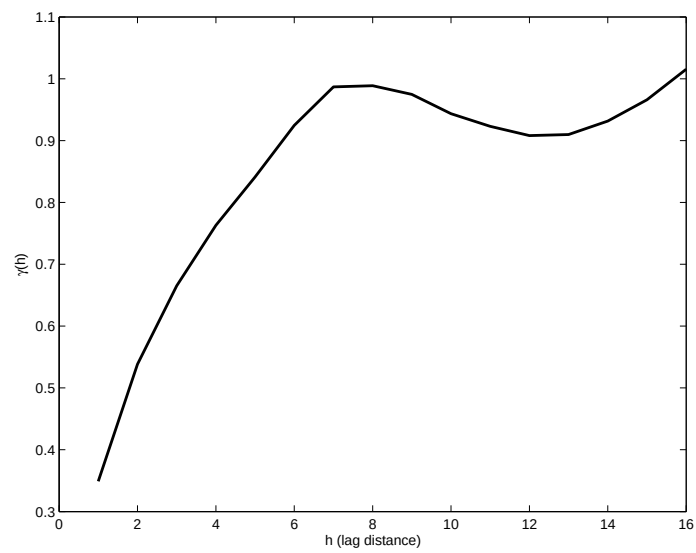


Figure B.3: Isotropic variogram for Reservoir G1.

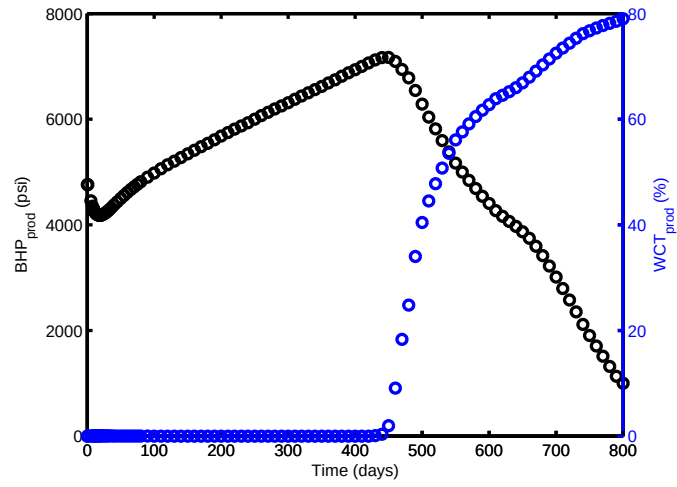


Figure B.4: Reservoir G1b: BHP and WCT data for well Prod1.

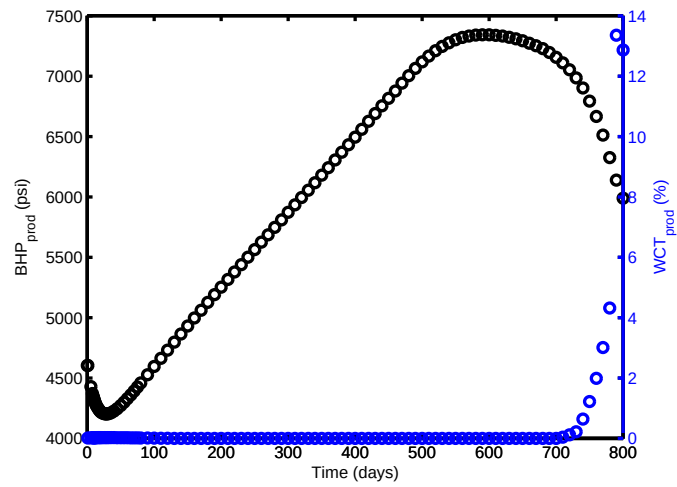


Figure B.5: Reservoir G1b: BHP and WCT data for well Prod2.

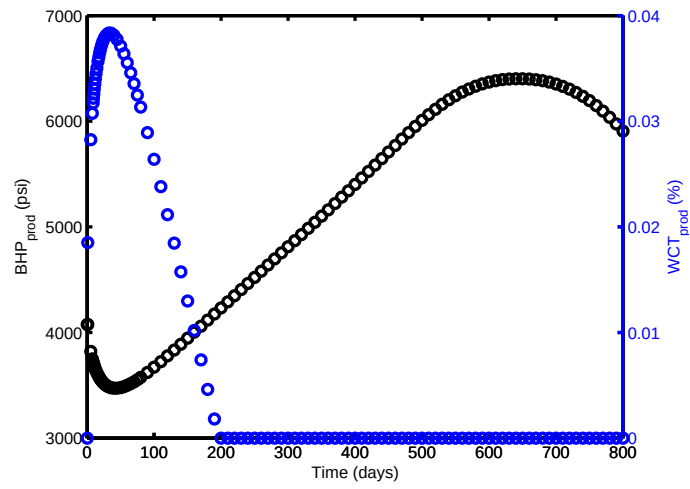


Figure B.6: Reservoir G1b: BHP and WCT data for well Prod3.

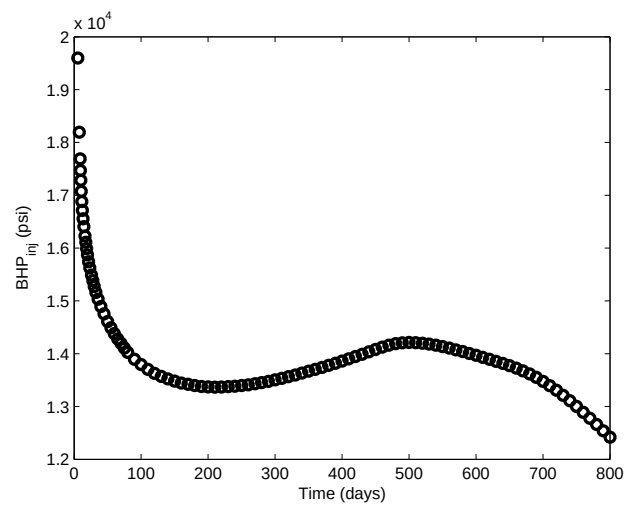


Figure B.7: Reservoir G1b: BHP and WCT data for well Inj.

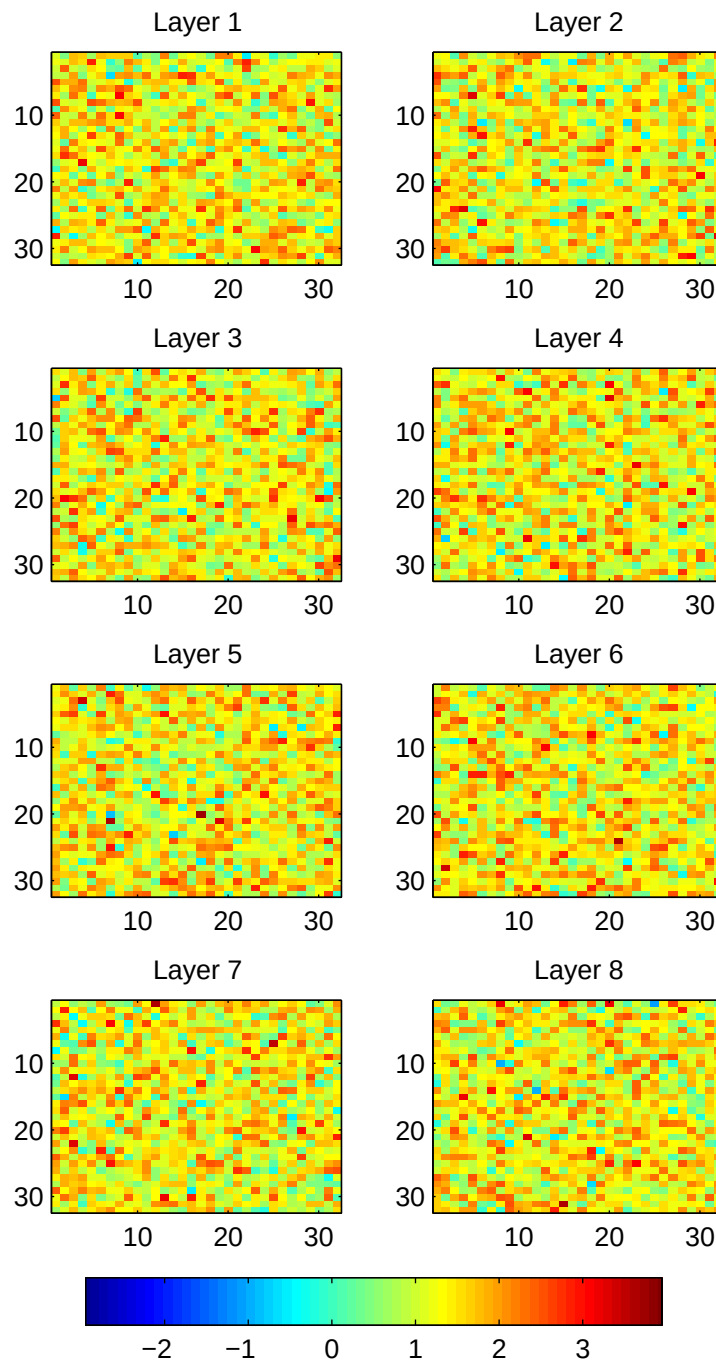


Figure B.8: Permeability distribution by layers for layers 1 through 8 for Reservoir 3A

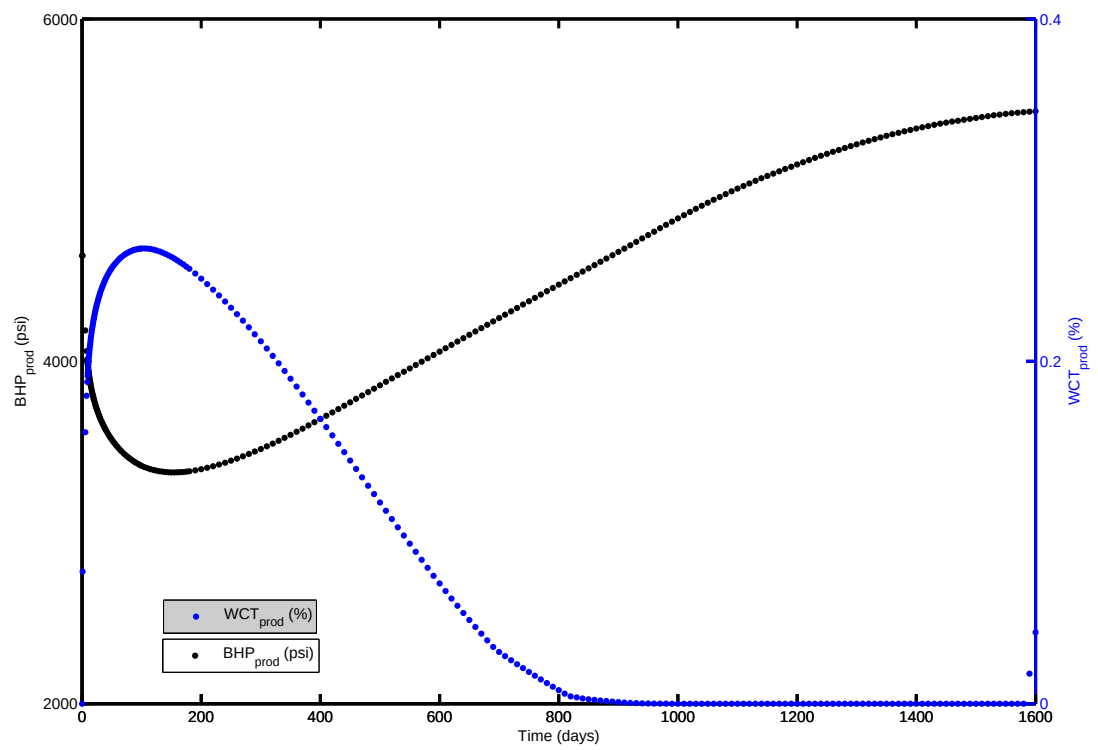


Figure B.9: WCT (%) and BHP (psi) with time for production from oil producing well Prod 1.

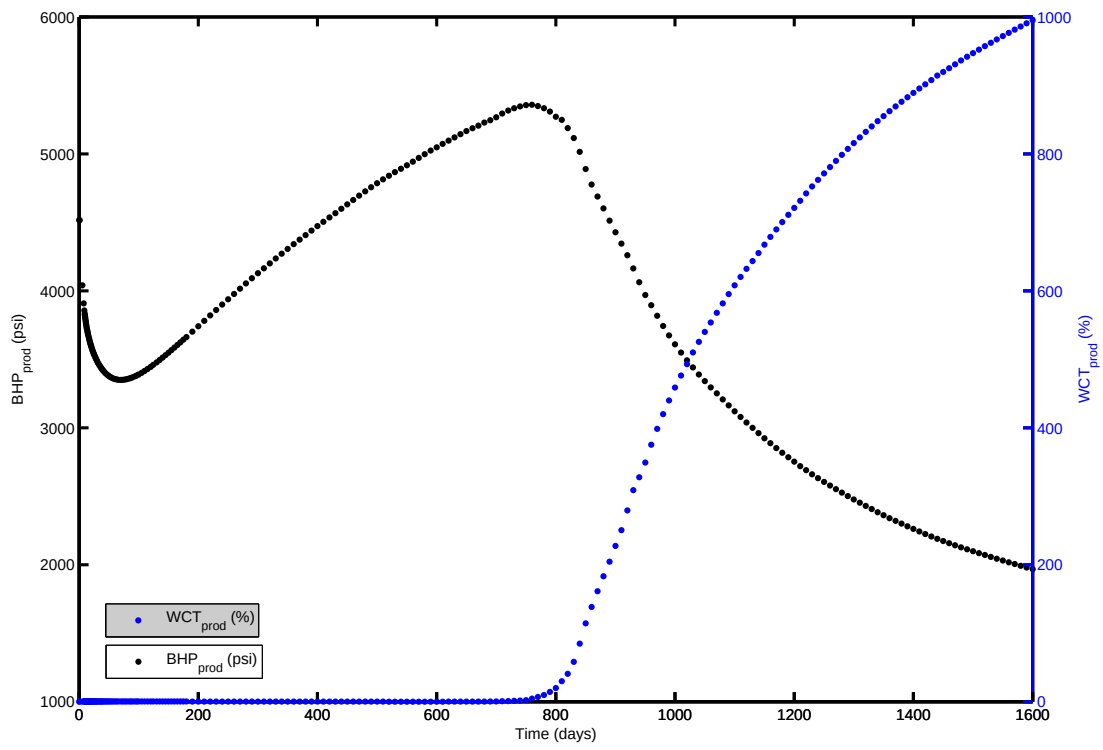


Figure B.10: WCT (%) and BHP (psi) with time for production from oil producing well Prod 2.

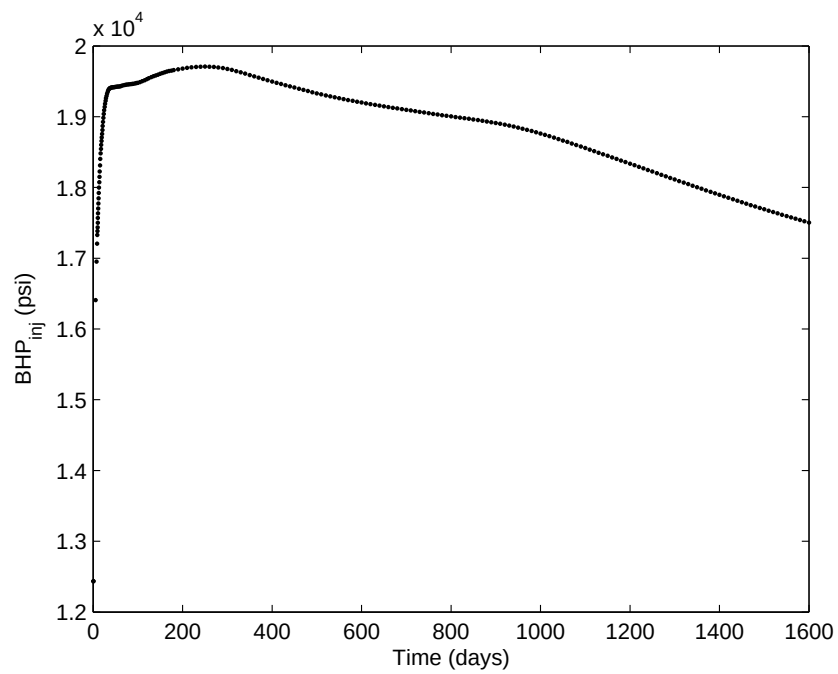


Figure B.11: BHP (psi) with time for production from water injection well INJ.

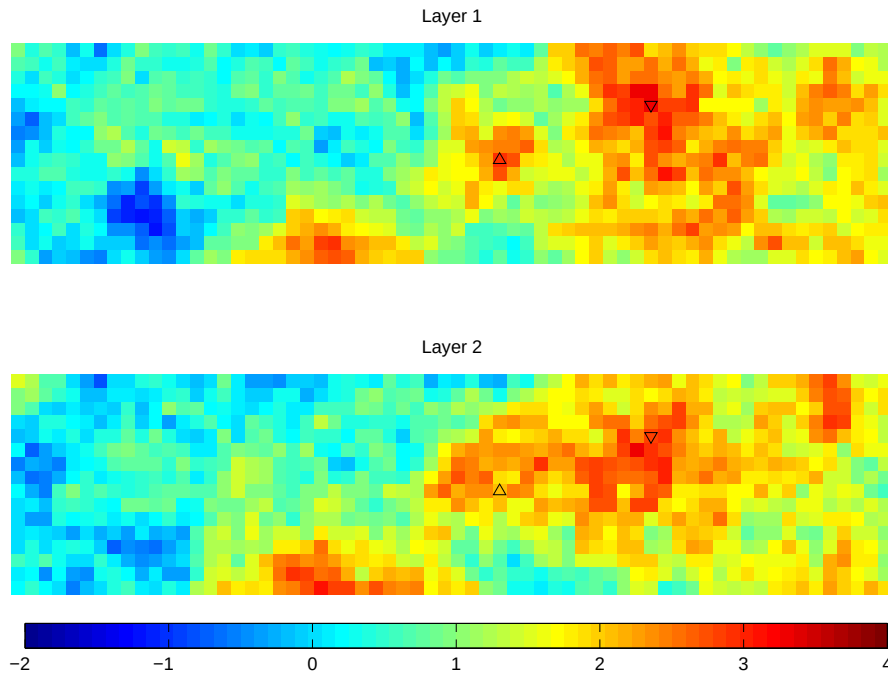


Figure B.12: Reservoir 2B: Permeability distribution by layers for layers 1 and 2.

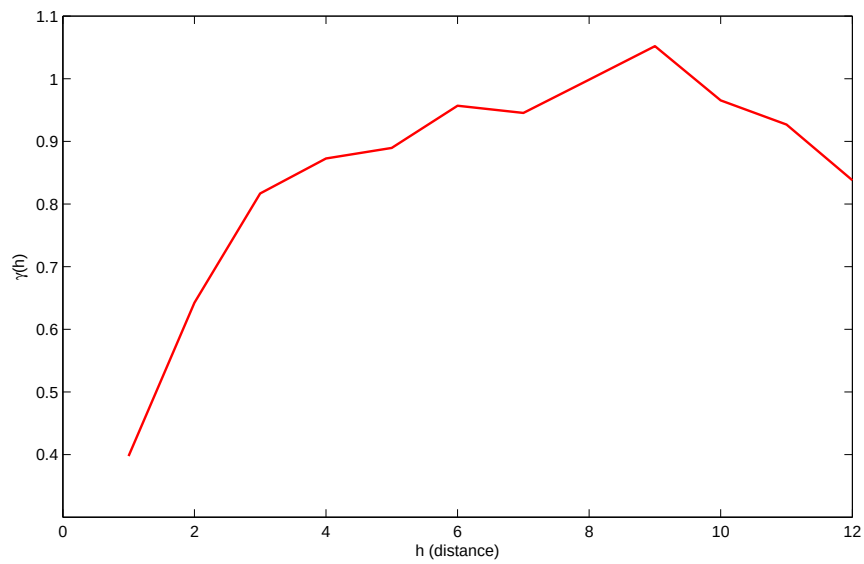


Figure B.13: Reservoir 2B: Variogram.

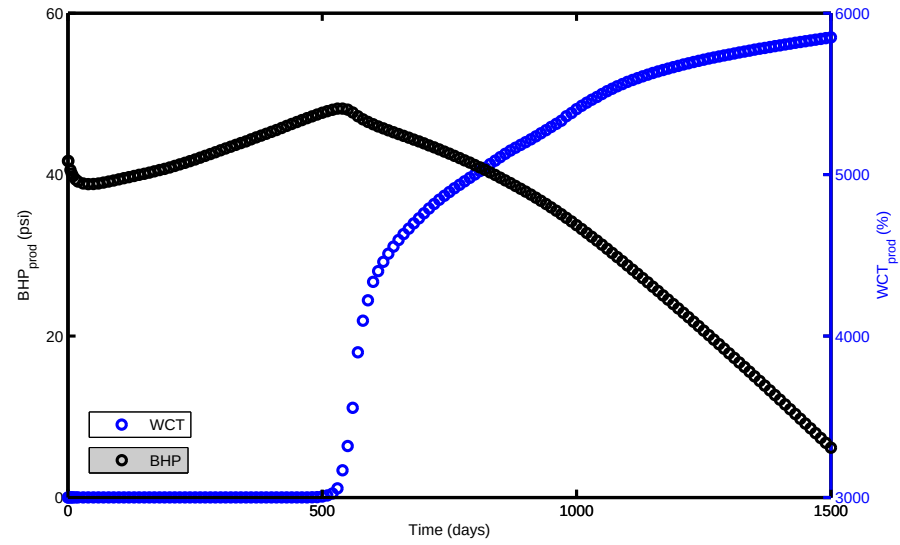


Figure B.14: Reservoir 2B: Producer BHP and WCT.

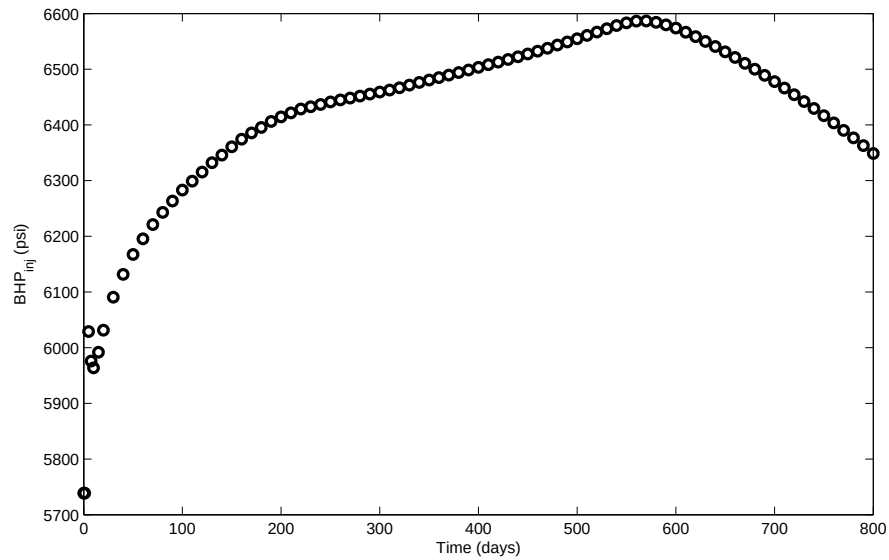


Figure B.15: Reservoir 2B: Injector BHP.

Bibliography

- [1] Sahni, I. “Multiresolution Wavelet Analysis for Improved Reservoir Description”, M.S. report, Stanford University, CA, 2003.
- [2] Sahni, I. and Horne, R.N. “Multiresolution Wavelet Analysis for Improved Reservoir Description”, SPE Reservoir Evaluation and Engineering, Feb 2005, 8(1), 53-69.
- [3] Sahni, I. and Horne, R.N. “Generating Multiple History-Matched Reservoir Model Realizations Using Wavelets”, SPE Reservoir Evaluation and Engineering, June 2006, 9(3), 217-226.
- [4] Sahni, I. and Horne, R.N., “Stochastic History Matching and Data Integration for Complex Reservoirs Using a Wavelet-Based Algorithm”, paper SPE 103107 prepared for the 2006 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 24-27 September.
- [5] Wang, Y. “Streamline Approaches for Integrating Production History with Geologic Information in Reservoir Models,” Ph.D. dissertation, Stanford University, CA, 2001.
- [6] Guan, L., Du, Y., Li, L. “Wavelets in Petroleum Industry: Past, Present and Future”, paper SPE 89952 presented at the 2004 SPE Annual Technical Conference and Exhibition, Houston, Texas, 26-29 September.
- [7] Athichanagorn, S., “Development of an Interpretation Methodology for Long-Term Pressure Data from Permanent Downhole Gauges”, PhD dissertation, Stanford University, California, 1999.

- [8] Athichanagorn, S., Horne, R.N., and Kikani, J., "Processing and Interpretation of Long-Term Data Acquired from Permanent Pressure Gauges", SPE Reservoir Evaluation and Engineering, 5(5), October 2002, 384-391.
- [9] Jansen, F.E., Kelkar, M.G. "Upscaling of Reservoir Properties Using Wavelets", paper SPE 39495 presented at the 1998 SPE India Oil and Gas Conference and Exhibition, New Delhi, India, 17-19 February.
- [10] Jansen, F.E. "Reservoir Description from Production Data", Ph.D. dissertation, The University of Tulsa, Tulsa, OK (1996).
- [11] Panda, M.N., Mosher, C.C., Chopra, A.K. "Application of Wavelet Transforms to Reservoir-Data Analysis and Scaling" SPE Journal 5 (1) (March 2000) 92.
- [12] Panda, M.N., Mosher, C.C., Chopra, A.K. "Reservoir Modeling Using Scale-Dependent Data" SPE Journal (June 2001) 157.
- [13] Lu, P. "Reservoir Parameter Estimation Using Wavelet Analysis", Ph.D. dissertation, Stanford University, California, 2001.
- [14] Lu, P., and Horne, R.N. "A Multiresolution Approach to Reservoir Parameter Estimation Using Wavelet Analysis", paper SPE62985 presented at the SPE Annual Technical Conference and Exhibition held in Dallas, TX. 1-4 October, 2000.
- [15] Anterion, F., Eymard, R., and Karcher, B., "Use of Parameter Gradients for Reservoir History Matching", paper SPE 18433 presented at the 1989 SPE Symposium on Reservoir Simulation, Houston, February 6-8, 1989.
- [16] Chu, L., Reynolds, A.C., and Oliver, D.S., "Computation of Sensitivity Coefficients for Conditioning the Permeability Field to Well-Test Pressure Data", In Situ (1995) Vol. 19, 179-223.
- [17] Landa, J.L. and Horne, R.N. "A Procedure to Integrate Well Test Data, Reservoir Performance History and 4-D Seismic Data Into a Reservoir Description", paper SPE 38653, presented at the 1997 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 5-8 October.

- [18] Landa, J.L. "Reservoir Parameter Estimation Constrained to Pressure Transients, Performance History and Distributed Saturation Data", Ph.D. dissertation, Stanford University, California, 1997.
- [19] de Marsily, G., Lavedan, G., Boucher, M., and Fasanino, G. "Interpretation of Interference Tests in a Well Field Using Geostatistical Techniques to Fit the Permeability Distribution in a Reservoir Model," *Geostatistics for Natural Resources Characterization, Part 2*, 831-849, 1984.
- [20] Datta-Gupta, A., Vasco, D.W. and Long, J.C.S. "Sensitivity and Spatial Resolution of Transient Pressure and Tracer Data for Heterogeneity Characterization", paper SPE 30589, presented at the 1995 SPE Annual Technical Conference and Exhibition, Dallas, October 22-25, 1995.
- [21] Bissel, R., "History Matching a Reservoir Model by the Positioning of Geological Objects", paper presented at the 5th European Conference on the Mathematics of Oil Recovery, Mining University, Leoben, Austria, September 3-6, 1996.
- [22] Jacquard, P. and Jain, C. "Permeability Distribution from Field Pressure Data", *SPE Journal* (December 1965) 281-294.
- [23] Carter, R.D., Pierce, A.C., Kemp, L., and Williams, D.L., "Performance Matching With Constraints", *SPE Journal* (April 1974)187-196.
- [24] Chen, W.H., Gavalas, G.R., Seinfeld, J.H., and Wasserman, M.L., "A New Algorithm for Automatic History Matching", paper SPE 4545 presented at the 1973 SPEAIME 48th Annual Fall Meeting, Las Vegas, NV, September 30 - October 3, 1973.
- [25] Chavent, G., Dupuy, M., and Lemonnier, P. "History Matching by Use of Optimal Theory," paper SPE 4627 presented at the 1973 SPE AIME 48th Annual Fall Meeting, Las Vegas, NV, September, 30 October, 3.
- [26] Watson, A.T., Seinfeld, J.H., Gavalas, G.R., and Woo, P.T., "History Matching in Two-Phase Petroleum Reservoirs", paper SPE 8250 presented at the 1979 SPE 54th Annual Technical Conference and Exhibition, Las Vegas, NV, September 23-26, 1979.

- [27] Yang, P. H. and Watson, A. T. "Automatic History Matching With Variable-Metric Methods," paper SPE 16977 presented at the 1987 SPE 62nd Annual Technical Conference and Exhibition, Dallas, TX, September, 27-30.
- [28] Bissell, R., Sharma, Y., and Killough, J. E. "History Matching Using the Method of Gradients: Two Case Studies," paper SPE 28590 presented at the 1994 SPE Annual Technical Conference and Exhibition, New Orleans, LA, September, 25-28.
- [29] Tan, T. B. and Kalogerakis, N. "A Fully Implicit, Three-Dimensional, Three-Phase Simulator with Automatic History-Matching Capability," paper SPE21205 presented at the 1991 SPE 11th Symposium on Reservoir Simulation, Anaheim, CA, February, 17-20.
- [30] Tan, T. B. "A Computationally Efficient Gauss-Newton Method for Automatic History Matching," paper SPE 29100 presented at the 1995 SPE Symposium on Reservoir Simulation, San Antonio, TX, February, 12-15.
- [31] Thiele, M.R., Batycky, R.P., and Blunt, M.J., "A Streamline-Based 3D Field-Scale Compositional Reservoir Simulator," paper SPE 38889 presented at the 1997 SPE Annual Technical Conference and Exhibition held in San Antonio, Texas, October 5-8, 1997.
- [32] Vasco, D.W., Yoon, S., and Datta-Gupta, A., "Integrating Dynamic Data into High-Resolution Reservoir Models Using Streamline-Based Analytic Sensitivity Coefficients," paper SPE 49002 presented at the 1998 SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, September 27-30, 1998.
- [33] Graps, A. "An Introduction to Wavelets," IEEE Computational Science and Engineering, vol. 02, no. 2, pp. 50-61, 1995.
- [34] Daubechies, I. 1988. "Orthonormal Bases of Compactly Supported Wavelets," Comm. On Pure and Appl. Math. 41(7): 909996.
- [35] Daubechies, I. 1992. Ten Lectures on Wavelets. CBMS-NSF Regional Conference Series in Applied Mathematics, Society for Industrial and Applied Mathematics (SIAM), 61: 115-137.

- [36] Mallat, S. "An Efficient Image Representation for Multiscale Analysis", presented in Proc. of Machine Vision Conference, Lake Tahoe, February 1987.
- [37] S. Mallat, 1999, *A Wavelet Tour of Signal Processing*, Academic Press; 2 edition, September 15.
- [38] Ogden, R.T., "Essential Wavelets for Statistical Applications and Data Analysis", Birkhauser Boston, 1997.
- [39] Foufoula-Georgiou E. and Kumar P. (eds.), *Wavelets in Geophysics*, Academic Press, Inc., 1994.
- [40] Chu, L., Schatzinger, R.A., and Tham, M.K. "Application of Wavelet Analysis to Upscaling of Rock Properties", Paper SPE 36517 presented at the 1996 SPE Annual Technical Conference and Exhibition, Denver, October 6-9, 1996.
- [41] Kikani, J. and He, M. "Multiresolution Analysis of Long-Term Pressure Transient Data Using Wavelet Methods," Paper SPE 48966 presented at the 1998 SPE Annual Technical Conference and Exhibition, New Orleans, September 27-30, 1998.
- [42] Fasanino, G., Molinard, J., and de Marsily, G. "Inverse Modeling in Gas Reservoirs," paper SPE 15592 presented at the 1986 SPE 61st Annual Technical Conference and Exhibition, New Orleans, LA, October, 5-8.
- [43] Tarantola, A. and Valette, B. "Generalized Nonlinear Inverse Problems Solved Using The Least Squares Criterion," *Reviews of Geophysics and Space Physics* (May 1982) 20, No. 2, 219-232.
- [44] Tarantola, A. "Inverse Problem Theory Methods for Data Fitting and Model Parameter Estimation" (revised edition, submitted to SIAM) 2003.
- [45] Menke, W. *Geophysical Data Analysis: Discrete Inverse Theory*, Academic Press, Inc., San Diego, CA (1989).
- [46] Parker, R., *Geophysical Inverse Theory*, Princeton University Press, Princeton, New Jersey, 1994.

- [47] Tang, Y. N. and Chen, Y. M. "Application of GPST Algorithm to History Matching of Single-Phase Simulator Models," Unsolicited paper SPE 13410 (1985).
- [48] Tang, Y.N., Chen, Y.M., Chen, W.H., and Wasserman, M.L., "Generalized Pulse-Spectrum Technique for 2-D and 2-Phase History Matching," *Applied Numerical Mathematics* (1989) Vol. 5, 529-539.
- [49] Oliver, D.S., *Incorporation of Transient Pressure Data into Reservoir Characterization, In Situ* (1994), Vol. 18, 243-275.
- [50] Oliver, D. S. "Multiple Realizations of the Permeability Field From Well Test Data," paper SPE 27970 presented at the 1994 University of Tulsa Centennial Petroleum Engineering Symposium, Tulsa, OK, August, 29-31.
- [51] Reynolds, A. C., He, N., Chu, L., and Oliver, D. S. "Reparameterization Techniques for Generating Reservoir Descriptions Conditioned to Variograms and Well Test Pressure Data," paper SPE 30558 presented at the 1995 SPE Annual Technical Conference and Exhibition, Dallas, TX, October, 22-25.
- [52] Chu, L., Komara, M., Schatzinger, R.A. "Efficient Technique for Inversion of Reservoir Properties Using Iteration Method", paper SPE 60224 *SPE Journal* 5 (1) (March 2000), 71.
- [53] He, N., Reynolds, A. C., and Oliver, D. S. "Three-Dimensional Reservoir Description from Multiwell Pressure Data," paper SPE 36509 presented at the 1996 SPE Annual Technical Conference and Exhibition, Denver, CO, October, 6-9.
- [54] Wu, Z., Reynolds, A.C. and Oliver, D.S., *Conditioning Geostatistical Models to Two-Phase Production Data*, paper SPE 49003, presented at the 1998 SPE Annual Technical Conference and Exhibition, New Orleans, September 27-30, 1998.
- [55] Ounes, A., Bréfort, B., Meunier, G., and Dupéré, S. "A New Algorithm for Automatic History Matching: Application of Simulated Annealing Method (SAM) to Reservoir Inverse Modeling," Unsolicited paper SPE 26297 (1993).
- [56] Sultan, A. J., Ounes, A., and Weiss, W. W. "Automatic History Matching for an Integrated Reservoir Description and Improving Oil Recovery," paper SPE 27712

- presented at the 1994 SPE Permian Basin Oil and Gas Recovery Conference, Midland, TX, March, 16-18.
- [57] Ounes, A., Weiss, W., and Sultan, A. J. "Parallel Reservoir Automatic History Matching Using a Network of Workstations and PVM," paper SPE 29107 presented at the 1995 SPE 13th Symposium on Reservoir Simulation, San Antonio, TX, February, 12-15.
- [58] Sen, M. K., Datta-Gupta, A., Stoffa, P. L., Lake, L. W., and Pope, G. A. "Stochastic Reservoir Modeling Using Simulated Annealing and Genetic Algorithms," paper SPE 24754 presented at the 1992 SPE Annual Technical Conference and Exhibition, Washington, DC, October, 4-7.
- [59] Deutsch, C.V. and Journel, A.G. *GSLIB: Geostatistical Software Library and Users Guide*, Oxford University Press, New York, second edition, 1998.
- [60] Isaaks, E and Srivastava M. *An Introduction to Applied Geostatistics*, Oxford University Press, New York 1989.
- [61] Zhang, F., Skjervheim J.A., Reynolds, A.C., Oliver, D.S. "Automatic History Matching in a Bayesian Framework, Example Applications", paper SPE84461 presented at the 2003 SPE Annual Technical Conference and Exhibition, Denver, Colorado, 5-8 October. 267.
- [62] Gill, P.E., Murray W., and Wright, M.H. *Practical Optimization*, Academic Press, New York, 1981.
- [63] Oliver, D.S., Reynolds, A.C., Bi, Z., Abacioglu, Y. "Integration of Production Data into Reservoir Models," *Petroleum Geoscience*, 7:65-73, 2001.
- [64] Ounes, A., Bhagavan, S., Bunge, P. H., and Travis, B. J. "Application of Simulated Annealing and Other Global Optimization Methods to Reservoir Description: Myths and Realities," paper SPE 28415 presented at the 1994 SPE 69th Annual Technical Conference and Exhibition, New Orleans, September, 25-28.
- [65] S. Kirkpatrick and C. D. Gelatt and M. P. Vecchi, "Optimization by Simulated Annealing", *Science*, Vol 220, Number 4598, pages 671-680, 1983.

- [66] V. Cerny "A Thermodynamical Approach to the Travelling Salesman Problem: an Efficient Simulation Algorithm." *Journal of Optimization Theory and Applications*, 45:41-51, 1985.
- [67] Metropolis, N., Rosenbluth, A.W., Rosenbluth, M. N., Teller, A.H. and Teller, E., "Equations of State Calculations by Fast Computing Machines," *J. Chem. Phys.* 21, 1087- 1092, 1958.
- [68] Sahimi M., Rasaei M.R., Ebrahimi F. and Haghighi M., "Upscaling of Unstable Miscible Displacements and Multiphase Flows Using Multiresolution Wavelet Tranformation", Paper SPE 93320 presented at the 2005 SPE Reservoir Simulation Symposium, Houston, Texas, January 31, 2005 to February 2, 2006.
- [69] Donoho, D. "Nonlinear Wavelet Methods for Recovery of Signals, Densities, and Spectra from Indirect and Noisy Data", *Different Perspectives on Wavelets, Proceedings of Symposia in Applied Mathematics*, 47(I). Daubechies ed. American Mathematics Society, Providence, R.I., 1993, 173-205.
- [70] Stromme, O. and McGregor, D.R. 1997. "Study of Wavelet Decompositions for Image/Video Compression by Software Codecs". *Image Processing and Its Applications*, 1997., Sixth International Conference on Image Processing and its Applications, Volume 1, 14-17 July 1997, 61-63.
- [71] Muraki, S. 1993, "Volume Data and Wavelet Transforms". *IEEE Computer Graphics and Applications*, 13(4), 50-56.
- [72] Stollnitz, E.J., DeRose, T. D. and Salesin, D.H. "Wavelets for Computer Graphics: A Primer, Part 1." *IEEE Computer Graphics and Applications*, 15(3):76-84, May 1995.
- [73] Stollnitz, E.J., DeRose, T. D. and Salesin, D.H. "Wavelets for Computer Graphics: A Primer, Part 2." *IEEE Computer Graphics and Applications*, 15(4):75-85, July 1995.
- [74] Caers, J., Srinivasan, S., Journel, A.G. "Geostatistical Quantification of Geological Information for a Fluvial-type North Sea Reservoir", paper SPE 56655 presented at the 1999 SPE Annual Technical Conference and Exhibition, Houston, Texas, 3-6 October.

- [75] Strebelle, S. "Sequential Simulation Drawing Structures from Training Images", Ph.D thesis, Department of Geological and Environmental Sciences, Stanford University, Stanford (2000).
- [76] Strebelle, S. "Conditional Simulation of Complex Geological Structures Using Multiple-Point Statistics", *Math. Geol.* (2002) 34, No. 1.
- [77] Strebelle, S., Journel A. G. "Reservoir Modeling Using Multiple-Point Statistics", paper SPE 71324 presented at the 2001 SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, 30 September - 3 October.
- [78] Strebelle, S., Payrazyan, K., Caers, J. "Modeling of Deepwater Turbidite Reservoir Conditional to Seismic Data Using Multiple-Point Geostatistics", paper SPE77425 presented at the 2002 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 29 September - 2 October.
- [79] Damsleth, E., Tjolsen, C.B., Omre, H., Haldorsen, H.H. "A Two-Stage Stochastic Model Applied to a North Sea Reservoir," *JPT* (April 1992) 402.
- [80] Deutsch, C.V., Wang, L. "Hierarchical Object-Based Geostatistical Modeling of Fluvial Reservoirs", paper SPE 36514, presented at the 1996 SPE Annual Technical Conference and Exhibition, Denver, Colorado, 6-9 October.
- [81] Haldorsen, H.H., Macdonald, C.J. "Stochastic Modeling of Underground Reservoir Facies (SMURF)", paper SPE 16751, presented at the 1987 SPE Annual Technical Conference and Exhibition, Dallas, Texas, 27-30 September.
- [82] Arpat, B., Caers, J., Strebelle, S. "Feature-Based Geostatistics: An Application to a Submarine Channel Reservoir", paper SPE77426 presented at the 2002 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 29 September - 2 October.
- [83] Zhang, T., Switzer, P., Journel, A. "Classification and Simulation of Patterns from Filters", paper presented at Seventh International Geostatistics Congress, Banff, Canada, 26 September - 1 October, 2004.

- [84] Bi, Z., Oliver, D.S., Reynolds, A.C. “Conditioning 3D Stochastic Channels to Pressure Data”, paper SPE 56682 presented at the 1999 SPE Annual Technical Conference and Exhibition, Houston, Texas, 3-6 October.