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Stochastic Fractality of the Gilondi Geothermal Reservoir

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ABSTRACT

The high-altitude geothermal reservoir of Jilondi is located 125 km east of Khorog (South-Western Pamir, Tajikistan) at an altitude of 3500 m above sea level and includes 10 wells for the use of geothermal resources. These wells were installed between 1980 and 1990 and used to measure higher temperatures than at the Icarus hydrogeological site (Western Pamirs). Detailed stratigraphic, lithographic, and geochemical data were made. However, a model containing important geophysical changes has not yet been drawn up for this reservoir. Here we consider the permeability function in the form of a random three-dimensional field and, using perturbation and averaging methods, establish the effective conductivity of filtration processes in the field. In the simplest case, it is possible to conduct a correlation analysis of the main components of the stochastic system. This takes into account the fractal nature of geothermal systems. The ultimate goal of our study is thermohydromechanical modeling of the hydrothermal circulation of the Jilondi object.

1. INTRODUCTION

The Jilondi thermal spring is located on the left bank of the Tokuzbulak River, 300 meters west of the village of the same name, at an absolute altitude of 3500 meters above sea level. Ten manifestations, located in one line, knock out small notches on the river bank, taking nitrogen and other gases with them. The temperature of the water in the manifestations varies from 35℃ to 67℃. The water with the highest temperature is used by the locals for medical purposes. Another source Tokuzbulak consists of 6 manifestations overlooking the right bank of this river, 17 km from the village, at an altitude of 3360 m above sea level. The largest outlets are two with a water temperature of 36℃. The water from the springs forms a stream connected to the river. This water is inhabited by freshwater mollusks (Mellanoides tuberculatus pamiricus sub-Lindholm), currently living only in Africa and tropical Asia. This species of mollusk in the Pamirs was considered extinct. But it turns out that it survived the harsh climate of the ice age and was discovered in the source.

In the last decade, researchers in European countries have implemented geothermal projects based on advanced geothermal system (EGS) technologies. This concept consists in increasing the permeability of the reservoir through hydraulic, thermal, and chemical influences, and then the forced circulation of natural brines in deep wells using the thermal anomaly associated with a large-scale hydrothermal system in porous and fractured rocks (Valier B. at all (2018), Tester at all (2016), Shvidler at all (2010), Gerard at all (2016), Schill at all (2017)). Unfortunately, in our conditions, there is a lack of completeness of data, which is available in the location of European scientists. In this regard, the compilation of a detailed and adequate model of the Geothermal Reservoir of Jilondi is a matter of the future. In this paper, we present an analysis of this geothermal system by mathematical methods of perturbation and averaging. The structure of the work is as follows. The second section presents the results of geothermal research in the Pamirs, carried out mainly by the efforts of Dushanbe and Tashkent geologists and their students (Akimov A.P., Semenov G.S. (1972), Gorbunov A.P. (1972), Kutas R.I. (1979), Churshina N.M., Krat V.N. (1979), Rao A. U. M., Rao G. V., Nazain H. (1976), Zuev Yu.N., Ponomarev A.A. (1984)). In the article by Kireeva T.A. et al. (2020) new data on the chemical composition of the Mineral Springs of the Pamirs (Firuza - Moscow State University, Garm-Chashma) are presented and the geological conditions of their distribution are considered. A number of works are devoted to the evaluation of the parameters of fractal porous media under conditions of uncertainty in filtration processes (Egorov A.A. at all (2020), Sakhaee-Pour A. (2016), Ilolov at all (2022), Mandelbrot B. (2002)).

The third section of the work is devoted to non-stationary filtration in a porous-fractured environment with random permeability. Note that the asymptotics of the Green function in the correlation approximation of the perturbation method was studied in the work of Shvidler M.I. (1985). In the work of Shvidler M.I. (1986), the process of filtration transport of a homogeneous liquid in an inhomogeneous composite medium composed of m phases homogeneous in physical characteristics is considered. In section 3, for a detailed description of the process in such environments, conditional functionals are practically important - the average fields for individual phases of the composite medium. The task of describing the process is reduced to the definition of globally and conditionally averaged fields or fractional order equations that bind them, to elucidate the thermohydrodynamic mechanism of mass transfer between phases in a porous medium.

The fourth section is devoted to the analysis of the stochastic differential equation with the Levy discontinuous process. Boundary and initial problems for such equations more adequately reflect the real processes occurring during hydrothermal circulation in fields. For the first time, such problems are studied in a monograph Holden H. at all (2010).

2. GEOTHERMAL RESEARCH IN THE PAMIR

The first geothermal scientific and thermal research work in the Pamirs was carried out under the program of the Pamir-Himalayan International Project in 1977 by joint efforts of geologists from Dushanbe and Tashkent. They carried out comprehensive geological and geophysical studies of the lithosphere on the profile of Kalai-Khumb - Khorog - Ishkashim (see e.g. Gorbunov A.P. (1979)). In the course of geothermal works, the thermal carrotating was carried out with thermistor sensors in the well №7 in the Icarus site and well №32 in the Kuhilal site (Fig. 1).

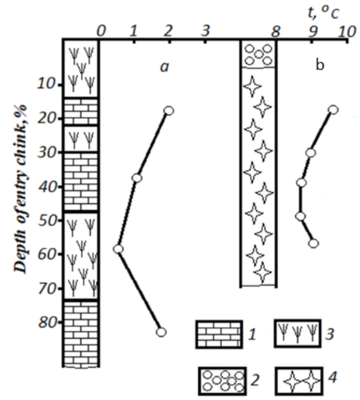

Figure 1: Thermograms from wells a) №32 (Kuhilal site), b) №7 (Icarus site), 1 - forsterite, jadeite, ecstatic rocks; 2 – marble; 3 - diluvit quaternary; 4 quartz diorites).

The depth of penetration of sensors in wells №7 and №32 is small. The obtained thermograms make it possible to record the temperature and depth of the "neutral" layer, but do not make it possible to characterize the normal hydrothermal gradients.

Well, №7 was drilled from an underground chamber. Its mouth is 300 m horizontally and 230 m vertically removed from the daytime surface, so the thermogram is free of gradient variations associated with annual fluctuations in the Earth's surface temperature. Before thermometry, the well was dormant for more than nine months, which is enough for the thermal regime in it, disturbed during drilling, to balance.

The nonlinearity of the thermogram (Fig. 2) is due to various regional and local distorting factors. This is primarily the influence of topography, the daytime surface, neotectonics movements, later glaciation cycles, and groundwater dynamics.

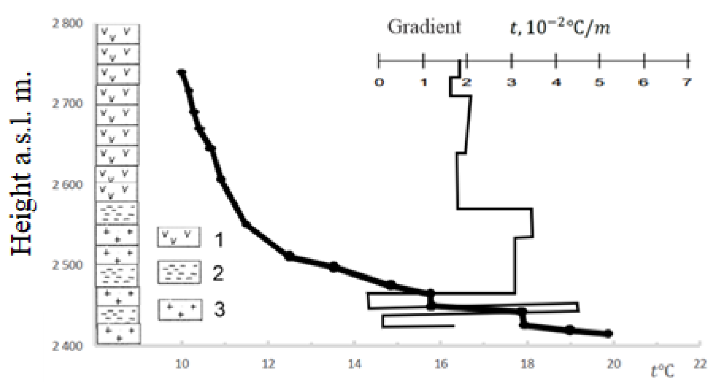

Figure 2: Thermograms from wells №7 (Icarus site), 1 - andesits; 2 – crushing zones; 3 quartz diorites.

Let's consider each of them in more detail.

2.1 Neotectonics and relief

The average rate of elevation of the Rushan Range area in the Holocene ($t=10^4$ years) is $2.5 \cdot 10^{-4}$ m/ year. 3. The same figure shows the relief correction calculated from a topographic map of a scale of 1:100,000 by the Jeffreys–Bullard method. For such elevation speed, the curve $\gamma/\gamma_H = f(h)$ shown in Figure 3 is calculated. The same figure shows the relief correction calculated from the topographic map at a scale of 1:1000000 by the method of Jeffreys Bullard. Curve 3 in Figure 3 shows the combined effect of a change in the normal geothermal gradient under the influence of relief and neotectonics.

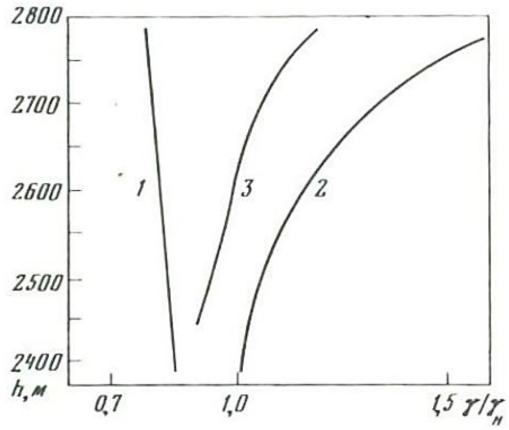

Figure 3: Reduction to geothermal gradient in Icarus site. 1-3 curves: 1 – correction for neotectonics motion; 2 – relief correction; 3 is the resulting curve correction for relief and neotectonics.

2.2 Climate correction

In the Pamirs, the most recent glaciation is the Arzyng glaciation, which took place at the end of O_4, lasted together with the late phase of the onset time of $2.5 \cdot 10^4$ years and ended $0.5 \cdot 10^4$ years ago (Gorbunov A.P. (1972)). The influence of this glaciation is expressed in the increase in the geothermal gradient to a depth of at least one and a half kilometers from the surface. Below is a climatic correction for the Icarus site (Table 4).

Table 4: Climate correction in Icarus Site.

Depth from day surface, m	10	50	100	200	500	1000	2000
$\Delta_\gamma = (\gamma_{meas} - \gamma_{norm}) \cdot 10^{-2}$°C$/M$	0,470	0,466	0.465	0.460	0.426	0.426	0.116

Thus, accounting for relief corrections, neotectonics motions, and glaciations somewhat evens out the measured geothermal gradient, but the shape of the thermogram remains concave to the depth axis, indicating hydrodynamics as the main temperature-distorting factor.

Generalized results on thermal conductivity are shown in Table 5.

Table 5: Thermal conductivity of rocks from the regions of the South-Western and Central Pamirs.

Square	Rock	Age	Thermal conductivity W/(m.K.)
Icarus	Diorites	P_3	2.99
	Quartz diorite	…	3.34
	Corneas	…	3.24
	Corneas with sulfide minerals	…	4.73
	Andesites	K_3	2.50
Kuhilal	Magnesia marble	AH-PH	5.74
	Jadeite-ecstatit rock	…	5.38
	Ecstatit-tremolite rock	…	2.95
	Marble	…	3.82
	Marble with forstreet	…	4.81

2.3 Radiochemical and seismic studies

On the profile of Kalai-Khumb - Khorog - Ishkashim, radiochemical and seismic studies were carried out that made it possible to determine the average capacities of various layers of the lithosphere and specific heat generation of rocks. Based on these data, it is possible to build a radio thermal model of the lithosphere and trace the release of radiogenic heat flux in layers in it (Table 6).

With a layer-by-layer model, the total radiogenic flux created within the lithosphere is 67.6 mW/m^2. Assume that radioactive heat sources are distributed in the mantle up to the upper limit of the nucleus, then the heat flux on the surface created by them is equal to the value of the following integral (Kutas R.N. (1979)):

$$q_0 = \frac{1}{R^2}\int_{H}^{R-d} A(x)x^2dx,$$

where is q_0 – heat flux on the surface of the Earth, R – radius of the Earth, d – depth to the upper limit of the kernel, $A(x)$ – heat source distribution function, x – depth coordinates, $x = R - h$ where h - the depth of r the point at which is defined q_0.

Table 6: Radiothermal model of the lithosphere of the Central Pamirs.

Lithosphere layer	Average power, layer, km	Specific heat generation A, μW/m^2	Radiogenic heat fluxq_0 mW/m2
"Sedimentary" layers Structural and formational floors			
a) Orogenic volcano plutonic K_3-M	3.5	0.235	8.25
b) geosynclinals (terrigen-carbonate) T_3-K_3	3.0	0.188	5.65
c) Sub-platform C_m – T_3	7.0	0.134	9.34
Consolidated crust			
a) Granit metamorphic A_r – P_{t1}	24	0.137	32.96
b) granulite-basalt layer	20		11.39
including			
in the roof		0.83	
in the sole		0.03	
Total on the earth's crust	57.5	-	67.60

Integral solution for $A(x) = A_0e^{kx}$ equals $q = \frac{1}{k}A_0e^{kh}$.

The heat flux at a point $h = 0$ (i.e., on the surface of the Earth) is calculated in this way 71.0 mW/m^2. This value can be considered close to the regional background of the Central Pamirs. Thus, the boundaries of the change in the conductive density of the heat flux determined in the Icarus site will become closer, namely

$$71.0 < q < 128.7 \text{ mW/m}^2.$$

2.4 Convective heat flow

On the territory of the Pamirs, modern geothermal activity is widely and intensively manifested. The geothermal activity of this region is subject to tectonic zonation. About 73% of the discovered sources operate in the zone of the South-Western Pamirs (Fig. 7). They are grouped mainly in the zones of deep faults, fracture, and to a lesser extent in the areas of Neogene magmatism. All the rising sources of the Pamirs carry thermal energy from the depths of the earth's crust, the concentrated power of which at various points ranges from 100 to 1000 watts. The sources of the South-Western Pamir zone have the greatest power. Along individual segments of deep fault zones, for example, Gunt-Alichursky, the specific power of heat loss reaches 8 mW/m^2.

In addition to hydrothermal, heat-intensive gases such as He and H_2 also carry heat from the interior. These gases carry a certain amount of heat, the capacity of which for the Pamirs has not yet been established, but may be close to the amount of heat carried by hydrothermal.

Gases He and H_2 have a unique high permeability, lightness, and heat capacity and probably play the role of a "seed" in the formation of porous channels in the thickness of rocks.

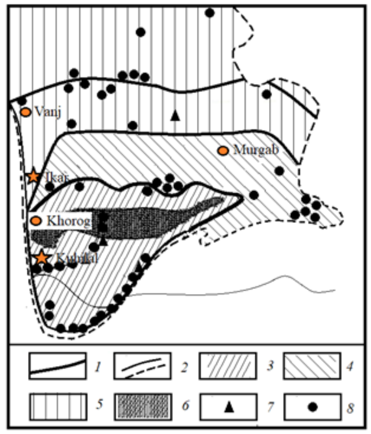

Figure 7: *1* - deep faults (interzonal); *2* - faults and fracture zones with the highest convective heat loss; *3* - zone of the greatest convective heat loss in the earth's crust (South-Western Pamir); *4* - zone of average convective heat loss in the earth's crust (South-Eastern and Central Pamirs); *5* - zone of the least convective heat loss in the earth's crust (Northern Pamir); *6 – an* area of distribution of predominantly nitrogen therms; *7* – sources; *8* - travertines

2.5 Some models of porous media reflecting the stochastic-fractal nature of the Jilondi geothermal reservoir

In the analysis of porous media, an average description of the ratio of matrix segments and porous space is often used. For example, the ratio of the volume of fluid filling the pore space to the total volume. The very concept of the density of the medium is an average indicator that gives only a rough estimate of the ratio of mass to volume. One of the key issues is the actual value of the volume of the liquid to the porous medium, and the area of interaction of the liquid to the porous medium. In turn, the estimation of the interaction area is complicated by the fractal nature of the matrix of the porous medium. The calculation and estimation of the volume and area directly depends on the accuracy of the measurement (Ilolov M. at all (2021)). We used as digital models fractal pore structures created based on the stochastic fractal "Perlin noise".

Figure 8 shows 3D matrices derived from stochastic fractals.

The matrices shown in Fig. 8, are 3-dimensional cubes in size $257 \times 257 \times 257$. The models were built with the specified binary filtering parameter $f = 500$. As a result, based on Perlin noise, the porosity value was 49.84%, and density 50.16%.

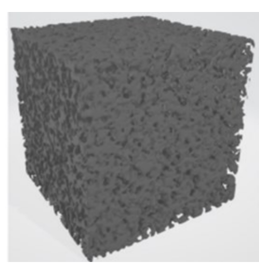

Figure 8: Matrix of Perlin noise

The porosity of the model was calculated as the ratio of the volume of pores to the volume of the body:

$$\varepsilon = \frac{V_{pore}}{V_{full}}.$$

The density of the ratio of the volume of frame elements to the dense volume of the body:

$$1 - \varepsilon = \frac{V_{carcass}}{V_{full}}.$$

By changing the parameters of noise generation and binary filtering parameters, it is possible to change the structural characteristics of the model and achieve the required porosity and permeability. The dimension of the matrix is determined by the size of the generated fractals and the dimension of the grid that forms the frame of the internal structure.

2.6 Determination of the fractal dimension of porous media

Regardless of the method of construction, all fractals have a fractal dimension, this is a certain number D, called the Hausdorff fractal dimension (Egorov A.A. at all (2020)). For size models $257 \times 257 \times 257$, D – Hausdorff fractal dimension is defined as follows.

Let's take in three-dimensional space a set of points N_0. To cover this set, it is necessary $N(\varepsilon)$ – cubes with a characteristic size ε, and $N(\varepsilon) \approx \frac{1}{\varepsilon^D}$ at $\varepsilon \to 0$ is defined by the law of similarity. For the value D have a limiting expression

$$D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log N(1/\varepsilon)}.$$

Calculations have shown that fractal dimensionality has an inverse relationship with the number of open pores. An increase in filtration leads to a decrease in the dimensionality of the fractal (Fig. 9).

Fig. 9 shows a graph of the dependence of the dispersion of fractal dimension on the filtering parameter.

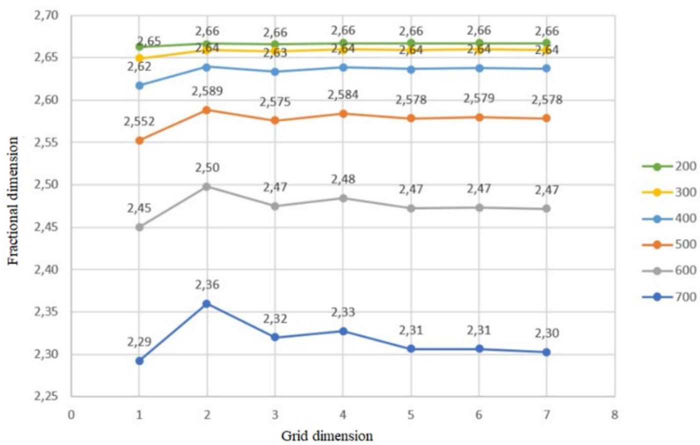

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Figure 9: Graph of the dependence of the variance on the binary filtering parameter (see Egorov A.A., 2020)

Table 10 shows the fractal dimensions of the model with a fixed filtering parameter $f = 500$.

Table 10: Fractal dimensions.

	Grid dimension						
Octave	2	3	4	5	6	7	8
1	2.552	2.546	2.552	2.547	2.544	2.555	2.559
2	2.589	2.54	2.547	2.546	2.551	2.563	2.551
3	2.575	2.53	2.526	2.543	2.558	2.548	2.569
4	2.584	2.527	2.515	2.552	2.554	2.556	2.569
5	2.578	2.537	2.521	2.55	2.553	2.551	2.566
6	2.579	2.538	2.52	2.55	2.556	2.555	2.566
7	2.578	2.537	2.521	2.551	2.556	2.556	2.566
8	2.577	2.536	2.52	2.551	2.555	2.556	2.565

3. FILTRATION FIELDS IN FRACTAL INHOMOGENEOUS MEDIA

Non-stationary filtration in a fractal medium with random permeability is considered. The corresponding process is described by the methods of the theory of stochastic differential equations with fractional derivatives both in time and in coordinates. A significant role in this is played by the asymptotic analysis of the Green function in the correlation approximation of the perturbation method.

For macroscopically significantly heterogeneous porous media, the corresponding tasks were studied in the works of Shvidler (1985), Shvidler (1986).

More specifically, we will study the process of filtration transport of a homogeneous liquid in a heterogeneous composite medium composed of m phases homogeneous in physical characteristics. The physical parameters of the phases are considered to depend only on spatial coordinates and are random fields. For detailed analysis, conditional functional-average fields for individual phases of the composite medium will be needed.

The task of describing the process is reduced to the definition of globally and conditionally averaged fields or equations that connect them, to elucidate the mechanism of mass transfer between phases.

Let $\Omega \subset \mathbb{R}^n$ – domain in n – dimensional Euclidean space $\mathbb{R}^n$. Consider in the field of Ω initial problem

$$\alpha\ {}^{c}D_t^{\alpha}u - div\ (\sigma\nabla u) = f(x,t), \quad (1)$$
$$u(x,0) = u_0(x), \qquad u_{b\Omega} = \varphi(x,t), \quad (2)$$

where is $u(x,t)$ – pressure, $v = -\sigma\nabla u$ – filtration rate vector, $\sigma(x)$ – positively defined and symmetrical conduction tensor, $\alpha(x)$ – positive random function of elastic-capacitance system of liquid-porous medium, $f(x,t)$ – density of non-random sources, $u_0(x)$ - initial and $\varphi(x,t)$ – boundary functions. Because in a composite environment $\sigma(x)$ and $\alpha(x)$ are discontinuous, then the solution of the problem (1), (2) is understood as a generalized solution satisfying the corresponding integral equations.

Let's introduce into consideration the average for the ensemble of implementations of random fields σ and α of field $u(x,t)$ и $v(x,t)$

$$U = < u >, V = < v > \quad (3)$$

and let us assume that δ – characteristic scale of heterogeneity of stochastically homogeneous fields $\sigma(x)$ and $\alpha(x)$ satisfy inequality $\delta \ll l_\Omega$, where l_Ω – scope of the domain Ω. Then the probabilistic averaging in (3) can be replaced by averaging by the volume of the domain ω_Δ, scale of which Δ satisfies inequality $\delta \ll \Delta \ll l_\Omega$.

For a detailed description, let's move on to conditional averaging of fields u and v phases and introduce random indicator functions

$$z_i(x) = \begin{cases} 1, if\ \ x \in \Omega_i \\ 0, if\ x \in \Omega \setminus \Omega_i, \end{cases} \quad (4)$$

where Ω_i – part of the domain Ω, occupied by the i-th phase. Obviously, $\sum_i z_i(x) = 1, < z_i >= \mu_i, \mu_i$ – volume fraction of the i-th phase in the composite.

Conditional averaging refers to the operator

$$<\cdot>_i = <\cdot>, x \in \Omega_i, \quad (5)$$

and for any random field $y(x,z)$

$$< y >_i = < z_i y > / \mu_i, \quad (6)$$

i.e. for conditional averaging y, it is sufficient to unconditionally average $z_i y$ and the result is renormalized.

Now taking into account (4) – (6), we introduce the average pressure and filtration rate in phases

$$U_i = < u >_i, V_i = < v >_i \quad (7)$$

and let's move on to conditional averaging of differential operators, given that the conditional averaging operator, unlike unconditional averaging, generally speaking, does not commute with differentiation by spatial variables. It is easy to see that

$$< div\ v >_i = div\ V_i + {}^{Q_i}/_{\mu_i}, \quad (8)$$

where $Q_i = -< v\nabla z_i >, \sum_i Q_i = 0$.

Similarly

$$< \nabla u >_i = \nabla u_i + \psi_i / \mu_i, \quad (9)$$

where $\psi_i = -< u\nabla z_i >, \sum_i \psi_i = 0$.

Using (8) and (9) and the conditionally averaging system (1), we get

$$div\ V_i + \alpha_i\ {}^{c}D_t^{\alpha}U_i + \mu_i^{-1}Q_i = 1, \quad (10)$$

$$V_i = \sigma_i(\nabla u_i + \mu_i^{-1}\psi_i). \quad (11)$$

In addition, from (8) and (9) follow equalities

$$\sum_i \mu_i \nabla u_i = \nabla u, \sum_i \mu_i V_i = V, \sum \mu_i \sigma_i^{-1} V_i = -\nabla U. \tag{12}$$

Multiplyingequation (10) by μ_i and summing for all i, we get taking into account (12) and will find

$$div\ V + \sum_i \alpha_i \mu_i\ {}^c D_t^\alpha U_i = f. \tag{13}$$

The system (10), (11), although it is not closed, since correlations are not calculated Q_i and ψ_i, can be interpreted as exact transfer equations in the phase-continuum. In this case, since the generalized function Δz_i is different from zero only at the boundaries of different phases $\partial\Omega_i$, the correlation Q_i is the specific average flow from the i-th continuum-phase to the rest, and the correlation $\psi_i$is the average specific force of pressure flowed from other phases to the surface limiting the i-th phase.

To close the conditionally averaged system (10), (11) it is natural to take advantage of the results of the unconditional (global) averaging of the system (1). As is known, the averaged system is a consequence of the decomposition of the averaged operator into a series by powers of the parameter $\varepsilon = \delta / l_\Omega$. When small enough ε for fields U and V there is an averaged system that does not contain ε:

$$div\ V + \alpha^*\ {}^c D_t^\alpha U = f, V = -\sigma^* \Delta U; \tag{14}$$

$$U(x,0) = u_0(x) \quad U/\partial\Omega = \varphi(x,t), \tag{15}$$

where $\alpha^* =< \alpha \geq const$, and $\sigma^* = const$ – an effective conductivity tensor, which can be determined from an experiment or from the solution of some auxiliary stationary problem [1].

Thus, in terms of average fields U and V at known fields α^* , the σ^* in system (14) gives in the considered approximation a closed description of the process of non-stationary filtration in an inhomogeneous medium. The main difficulty of its implementation lies in the definition of the tensor σ^*, and the efficiency is determined by the degree of heterogeneity and fine scale of the medium, and also depends on the type of process under consideration, i.e. the smoothness of the functions $u_0(x)$ and $\varphi(x,t)$.

Comparing (13) with the globally averaged equation (14), write down the equation for U_i

$$\sum_{i=1}^m \alpha_i \mu_i\ {}^c D_t^\alpha = \alpha^*\ {}^c D_t^\alpha U, \sum_{i=1}^m \alpha_i \mu_i \tag{16}$$

to which we add an obvious relation

$$\sum_{i=1}^m \mu_i\ {}^c D_t^\alpha U_i = C\ D_t^\alpha U. \tag{17}$$

If $m = 2$ and $\alpha_1 \neq \alpha_2$, then the system (16), (17) has a single solution

$${}^c D_t^\alpha U_1 = {}^c D_t^\alpha U_2 = {}^c D_t^\alpha U. \tag{18}$$

Under $m > 2$ condition (18) is also the solution of the system, since it is natural to accept $U_i(x,0) = u_0(x)$, then from (18) follows $U_i(x,t) = U(x,t)$.

It should be noted that an approximate scheme of closing system of equations for filtering immiscible liquids in crack-porous media, in which the equality of pressures in cracks and blocks is assumed, and the total flow of liquids is determined, is given in (Svidler M.I. (1985)).

Let $m = 2$. Put in (11) and (12) $U_1 = U_2 = U$, find that the vectors ψ_i have the form

$$\begin{aligned} \psi_1 &= (\sigma_2 - \sigma_1)^{-1}(< \sigma > -\sigma^*)\nabla U \\ \psi_2 &= (\sigma_1 - \sigma_2)^{-1}(< \sigma > -\sigma^*)\nabla U, \end{aligned} \tag{19}$$

where is the tensor $< \sigma >= \mu_1\sigma_1 + \mu_2\sigma_2$.

From (19) it follows that $\psi_i = 0$ only in a layered system, provided that ∇U it is directed along the layers. Using the calculated ψ_1 and ψ_2 in (9) we will find the average values of the pressure gradient in the phases

$$< \nabla u >_2 = \mu_1^{-1}(\sigma_2 - \sigma_1)^{-1}(\sigma_2 - \sigma^*)\nabla U,$$

$$< \nabla u >_2 = \mu_2^{-1}(\sigma_1 - \sigma_2)^{-1}(\sigma_1 - \sigma^*)\nabla u.$$

Substituting (19) into (10), (11) we get the system

$$div\ V_i + \alpha_i\ {}^cD_t^\alpha U + \mu_i^{-1}Q_i = f_1, V_i = -\sigma_i f_i^* \nabla U,$$
$$f_i^* = \mu_1^{-1}(\sigma_2 - \sigma_1)^{-1}(\sigma_2 - \sigma^*), f_2^* = \mu_2^{-1}(\sigma_1 - \sigma_2)^{-1}(\sigma_1 - \sigma^*). \quad (20)$$

Tensors f_i, which are naturally called relative phase conductions, satisfy the obvious relation

$$\sum_i \mu_i f_i^* = I \qquad \sum_i \mu_i \sigma_i f_i^* = \sigma^*.$$

Since the system (20) is joint with the globally averaged system (14), it is possible to determine the flows between the phases from these systems,

$$Q_i = \mu_i[div\ [(\sigma_i f_i^* - \sigma^*)\nabla U] - (\alpha_i - <\alpha>)\ {}^cD_t^\alpha U]. \quad (21)$$

Suppose in the real component i-i phase is distributed as a set of inclusions. By isolating a Ω characteristic volume in spaceω_Δ, we calculate the specific flux through the surface of the inclusions of the i-th phase

$$q_i = |\omega_\Delta|^{-1} \int_{S_{ij}} v\ ds_{ij}, \quad (22)$$

where s_{ij}-surface - boundary - separating j-e inclusion of the i-th phase. In this case, a certain part of the inclusions will be dissected by the boundary $\partial\omega_\Delta$ and, therefore, in addition to the flow from the inclusions of i-th phase of the volume to other phases, in (22) there will also be a phase flow through the surface $\partial\omega_\Delta$. Moving to (22) to the integration by volume using (1) and taking into account for the scale Δ, we get the complete flow from i-th phase

$$q_i = \mu_i < div\ v >_i = \mu_i(f - \alpha_i\ {}^cD_t^\alpha). \quad (23)$$

Comparing (8) and (23), we find the relationship between the flows q_i and Q_i:

$$Q_i = q_i + \mu_i div(\sigma_i f_i^* \nabla U).$$

where Q_i , q_i have different physical meanings. If q_i is a complete flow on the i-th phase, then it Q_i is a characteristic of the mass transfer between different phases. Suppose, for example, stationary filtration in a medium without sources is considered. Obviously, then $q_i = 0$. On the other hand, if σ^* it is anisotropic, and $\Delta u \neq Q\ z$, then when $\sigma_i \neq 0$ the flow between phases $Q_i \neq 0$.

Consider a two-phase composite with isotropic phases at $\sigma_1 \gg \sigma_2$. Then (20) will take the form

$$\begin{cases} div\ V_1 + \alpha_1\ {}^cD_t^\alpha U + \mu_1^{-1}Q_1 = f, \alpha\ {}^cD_t^\alpha U - \mu_2^{-1}Q_1 = f \\ V_1 = -\mu_1^{-2}\sigma^*\nabla U, V_2 = 0. \end{cases} \quad (24)$$

In this case, $Q_i = q_i$, and the exclusion from the system (24) of the flow Q_2, as it should be, leads to the system (14).

Thus, the conditional averaging of the equations of the filtration process in inhomogeneous systems shows that the description in terms U and V is sufficient to determine the macroscopic phase characteristics of $< u >_i, < \Delta u >_i, \psi_i, < v >_i, Q_i, q_i$ flow slow, the more accurate the globally averaged equations used in the closure of phase systems.

4. FRACTIONAL STOCHASTIC DIFFERENTIAL EQUATIONS WITH THE LEVY PROCESS

This section is devoted to the study of one class of fractional differential equations of the diffusion type with the Levy process. An attempt is made to construct an analysis of the theory of the white noise for the Levy process.

4.1 Levy's processes

The Lewy process $h(t)$ denoted through the stationary and independent increments similar to Brownian motion $B(t)$, but unlike $B(t)$ carried from $\eta(t)$ allows discontinuous trajectories. This process allows us to explore more realistic models. For example, (Di-Ninno G. (2004)) found that classes of models based on Levy discontinuous processes more accurately describe data on stock prices compared to the classic Samuelson-Black-Scholes model derived from Brownian motion. The work (Osin A.V. (2007)) introduces the fractal Levy motion in the form of a fractional Riemann-Liouville integral

$$L_{\alpha,H}(t) = \frac{1}{\Gamma(H + 1/2)} \int_0^t (t - \tau)^{H-1/2} dL_\alpha(\tau),$$

here is the usual symmetry $L_\alpha(t)$-stable Levy motion, $0 < \alpha \leq 2, 0 < H \leq 1$. Such movements occur in the analysis of complex geophysical objects, for example, in the analysis of geothermal or oil-bearing reservoirs (Dyshin O.A. and Maharramov F.F. (2018)).

4.2 Levi-Hida-Kondratev spaces

This subsection defines the Levi analogs of the Hida-Kondratev basic functions space and the dual Hida-Kondratev distribution space.

The space of the main stochastic functions of Levi-Hida-Kondratev denoted by $(S)_\rho = (S)_\rho^L$ consists of functions

$$φ = \sum_{\alpha\in J} c_\alpha K_\alpha(\omega) \in L^2(\mu^{(L)}), c_\alpha \in \mathbb{R}, J \in \mathbb{R}_+$$

such that

$$\|\varphi\|^2_{\rho,k} = \sum_{\alpha\in J} c_\alpha^2 (\alpha!)^{1+\rho}(2\mathbb{N})^{k\alpha} < \infty.$$

The space of stochastic Levi-Hida-Kondratev distributions denoted by means $(S)_\rho$ — $(S)_\rho^L$ consists of formal decompositions

$$F = \sum_{\alpha\in J} b_\alpha K_\alpha(\omega)$$

such that

$$\|F\|_{-\rho,-q} = \sum_{\alpha\in J} b_\alpha^2 (\alpha!)^{1+\rho}(2\mathbb{N})^{q\alpha} < \infty, q \in \mathbb{N}.$$

Space $(S)_0$ — $(S)_0^{(L)}$ is called the Space of Basic Stochastic Levi-Hida Functions, and $(S)^*$ — $(S)_{-0}^{(L)}$ the space of stochastic Levi-Hida distributions.

Everywhere here $K_\alpha(\omega) = I_{|\alpha|}(\delta^{\widehat{\otimes}\alpha})(\omega), \mu(L)$ - Levy's random measure. The Levy White Noise Process $\dot{\eta}(t)$ defined as a decomposition

$$\dot{\eta}(t) = M^{1/2} \sum_{i=1}^{\infty} \xi_i(t) K_{\varepsilon^{(k(i,1))}}(\omega).$$

4.3 Vic's multiplication, the Levi-Hermite transform, and the Skorokhod integral

Let

$$F = \sum_{\alpha\in J} a_\alpha K_\alpha \in (S)_{-1}^{(L)}, G = \sum_{\beta\in J} b_\beta K_\beta \in (S)_{-1}^{(L)}.$$

Then Vic's multiplication $F \diamond G$ is specified by decomposition

$$F \diamond G = \sum_{\alpha,\beta\in J} a_\alpha b_\beta K_{\alpha+\beta} = \sum_{\gamma\in j}\left(\sum_{\alpha+\beta=\gamma} a_\alpha b_\beta\right) K_\gamma.$$

In (Holde H. at all (2010)) proves that space $(S)_\rho$ и $(S)_{-\rho}$ $0 \le \rho \le 1$ are closed relationships with respect to Vic`s multiplication.

Let $F = \sum_{\alpha\in J} a_\alpha K_\alpha \in (S)_{-1}^{(L)}$. The Levi-Hermite Transformation $\mathcal{HF}$ is a function from space $(\mathbb{C}^{\mathbb{N}})_c$ of all finite sequences of complex numbers in $\mathbb{C}$ defined by equality

$$\mathcal{HF}(\xi_1, \xi_2, \dots) = \sum_{\alpha\in I} a_\alpha \xi^\alpha \in \mathbb{C},$$

where is $\xi = (\xi_1, \xi_2, \dots) \in \mathbb{C}$ *and* $\xi^\alpha = \xi_1^{\alpha_1}, \xi_2^{\alpha_2}, \dots, \xi_m^{\alpha_m}$ *if* $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_m) \in I \subset \mathbb{R}_+$.

Let $Y(x), x \in \mathbb{R}^n$ is a stochastic process, and

$$E(Y^2(x)) < \infty \ for\ all\ x \in \mathbb{R}^n.$$

Then for everyone $x \in \mathbb{R}^n$ process $Y(x)$ allows decomposition

$$Y(x) = \sum_{n=0}^{\infty} I_n(f_n(\cdot, x)),$$

where $f_n(\cdot,x) \in L^2((\lambda \times \lambda)^n)$ with parameter x.

Let

$$\sum_{n=0}^{\infty}(n+1)!\,\|\tilde{f}_n\|^2_{L^2((\lambda\times n)^{n+1})} < \infty,$$

where $\tilde{f}(x^{(1)},z_1,\dots,x^{(n)},z_n,x,z)$ is symmetrization $zf_n(x^{(1)},z_1,\dots,x^{(n)},z_n,x)$ regarding $n+1$ variables

$$y_1 = (x^{(1)},z_1),\dots,y_n = (x^{(n)},z_n),y_{n+1} = (x^{(n+1)},z_{n+1}).$$

Then the Skorokhod integral from Y regarding η defined as follows

$$\int_{\mathbb{R}^n} Y(x)\delta\eta(x) = \sum_{n=0}^{\infty} I_{n+1}(\tilde{f}).$$

4.5 Levy's white noise differential equation

Consider the fractional stochastic equation

$$\begin{cases} {}^cD_t^{\gamma}U(t,x) = \frac{1}{2}\Delta U(t,x) + U(t,x)\diamond\dot{\eta}(t,x),(t,x)\in[0,T]\times\mathbb{R}^n \\ U(0,x) = f(x), x\in\mathbb{R}^n, 0<\gamma<1, \end{cases} \quad (25)$$

where f - deterministic function.

Take the Hermite transform and get a deterministic fractional equation by $u(x,t,\zeta)$ with parameter $\zeta \in (C^{\mathbb{N}})_c$

$$\begin{cases} {}^cD_t^{\gamma}u(t,x,\zeta) = \frac{1}{2}\Delta U(t,x,\zeta) + U(t,x,\zeta)\mathcal{H}\dot{\eta}(t,x),(t,x,\zeta) \\ U(0,x,\zeta) = f(x), 0<\gamma<1, \end{cases} \quad (26)$$

Problem (26) can be solved using the Feynman–Katz formula. In fact, let $\hat{B}(t)$ auxiliary Brownian motion on probabilistic space $(\hat{\Omega},\hat{\mathcal{F}},\{\mathcal{F}_t\}_t \geq 0,\hat{P})$ independent of $B(t)$.

Then the solution of the problem (26) can be written as

$$u(t,x,\zeta) = \hat{E}^x\left[f\left(\hat{B}(t)\right)\exp\left[\frac{1}{\Gamma(1-\alpha)}\int_0^t (t-s)^{-\alpha}\mathcal{H}\dot{\eta}(s,\hat{B}(s),\zeta)ds\right]\right]$$

where through $\hat{E}^x$ denoted by the mathematical expectation relative to $\hat{P}$, when $\hat{B}(t) = x$. Taking the inverse hermite transformation, we come to the following result:

$$\mathrm{U(t,x)} = \hat{\mathrm{E}}^{\mathrm{x}}\left[\mathrm{f}\left(\hat{\mathrm{B}}(\mathrm{t})\right)\exp\diamond\left[\frac{1}{\Gamma(1-\alpha)}\int_0^{\mathrm{t}}(\mathrm{t}-\mathrm{s})^{-\alpha}\times\dot{\eta}\left(\mathrm{s},\hat{\mathrm{B}}(\mathrm{s})\right)\mathrm{ds}\right]\right],$$

where through $\exp\diamond[\cdot]$ denoted the Vic exponent, usually defined by

$$\exp\diamond[\mathrm{F}] = \sum\frac{1}{\mathrm{n}}!\,\mathrm{F}^{\diamond\mathrm{n}},\mathrm{F}\in(\mathrm{S})_{-\mathrm{s}}$$

and

$$\mathrm{F}^{\diamond\mathrm{n}} = \mathrm{F}\diamond\mathrm{F}\diamond\dots\diamond\mathrm{F}$$

$\mathrm{F}\diamond\mathrm{F}\diamond\dots\diamond\mathrm{F}$ n time.

Note that fractional stochastic partial differential equations have been studied by other methods in the works (Ilolov M. at all (2021), Ilolov at all (2022)).

CONCLUSION

Modeling porous media (e.g., the Geelondi geothermal system) using stochastic fractals has significant potential. To build adequate models and create full-scale "digital twins" of porous media, cores, rocks, a range of methods for assessing the results of modeling and checking adequacy is necessary. Such tasks and methods require further development.

This article presents methods of averaging and perturbation associated with the evaluation of stochastic-fractal structures by the most important characteristics.

The analysis of the process of filtration transport of homogeneous liquid in a heterogeneous composite medium of several phases homogeneous in terms of physical characteristics is given. The task of describing the process is reduced to the definition of globally and conditionally averaged fields or equations that connect them, to elucidate the mechanism of mass transfer between phases.

The conditions for the existence and uniqueness of the solution of the Cauchy problem for a fractional stochastic differential equation with the Levy process in the right part are established. This solution can be the implementation of the temperature or pressure of the filtration flow in porous fractal media under uncertainty conditions.

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