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# Preliminary study of dependence of the modification factor controlling generation of periodic wellbore flow

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## ABSTRACT

The authors previously revealed a universal condition controlling generation of periodic wellbore flow due to inflow of a lower-enthalpy fluid at a shallow feed zone, while producing from a deep feed zone. This paper reports a preliminary study of the dependence of a modification factor on dimensionless parameters involved in this universal condition, which remains unstudied. Revealing this dependence by deriving an empirical equation, we will be able to reduce computational loads dramatically to determine the modification factor. Based on the total derivative of the modification factor, two dimensionless parameters representing the productivity index of the deep reservoir and the shallow reservoir depth were selected for a further study. Investigating changes in the modification factor depending on these two dimensionless parameters, we have found nonlinear relationship with each dimensionless parameter, including steep and highly nonlinear changes depending on the dimensionless shallow reservoir depth. Vertical pressure gradient, which has been assumed to be constant, is also expected to play a key role in the dependence of the modification factor.

## 1. INTRODUCTION

Periodic wellbore flow occasionally observed in production wells (e.g., Grant et al., 1979; Iwata et al., 2002; Yanto et al., 2019) causes difficulties in maintaining steam production and the output of a geothermal power plant. One of the periodic flow generation mechanism is the periodic formation and flash of a single-phase interval with a liquid phase because of the inflow of a lower-enthalpy fluid at a shallow feed zone while producing from a deep feed zone (Itoi et al., 2013; Yamamura et al., 2017; Matsumoto et al., 2021). Assuming this mechanism, Matsumoto et al. (2021) numerically reproduced the periodic flow and revealed a universal condition for the periodic flow generation through a parametric study in terms of dimensionless parameters. The condition is described by the mean specific enthalpy of the shallow and deep reservoirs weighted by their productivity indices. A modification factor adjusting the weight is also required to successfully control the periodic flow generation. Referring to numerous simulation runs, the value of this modification factor must be determined by trial and error. This study preliminarily investigates the dependence of the modification factor on the dimensionless parameters, which remains unstudied. If we successfully obtain an empirical equation describing this dependence, we will be able to reduce computational loads dramatically and rapidly determine the generation of periodic flow. This is advantageous for developing effective mitigation measures of periodic flow and improving the output of a power plant.

## 2. MODIFICATION FACTOR APPEARING IN THE FLOW PATTERN BOUNDARY

Assuming a vertical wellbore flow model with a uniform inner diameter intersecting shallow and deep reservoirs, Matsumoto et al. (2021) proposed that periodic flow is generated when the mean specific enthalpy value falls below the lower limit of the specific enthalpy in the deep reservoir to sustain production without the shallow reservoir. This mean specific enthalpy $\bar{h}$ is defined using the specific enthalpies in both reservoirs weighted by their productivity indices as follows:

$$\bar{h} = \frac{\kappa \mathrm{PI_D} h_{\mathrm{reD}} + \mathrm{PI_S} h_{\mathrm{reS}}}{\kappa \mathrm{PI_D} + \mathrm{PI_S}}, \tag{1}$$

where $h_{\mathrm{re}}$ and PI denote the specific enthalpy and productivity index of a reservoir, respectively, while subscripts D and S indicate the deep and shallow reservoirs, respectively. The factor $\kappa$ modifies the weight and was found to be generally larger than unity, implying an excess impact of the deep reservoir beyond the weights defined by the productivity indices. Rearranging Eq. (1), we found that a variant of the specific enthalpy ratio $1 - h_{\mathrm{reS}}/h_{\mathrm{reD}}$ obeys a linear function of the inverse of the productivity index ratio $(\mathrm{PI_S}/\mathrm{PI_D})^{-1}$ between the shallow and deep reservoirs along the flow pattern boundary separating the constant and periodic flows as follows:

$$1 - \frac{h_{\mathrm{reS}}}{h_{\mathrm{reD}}} = \kappa \left(1 - \frac{\bar{h}}{h_{\mathrm{reD}}}\right) \left(\frac{\mathrm{PI_S}}{\mathrm{PI_D}}\right)^{-1} + 1 - \frac{\bar{h}}{h_{\mathrm{reD}}}, \tag{2}$$

where $\bar{h}$ is set to the lower limit of the specific enthalpy in the deep reservoir $h_{\mathrm{L}}$, while the value of $\kappa$ is selected by trial and error to match the curve drawn by Eq. (2) with the flow pattern distribution obtained by many numerical simulations. If we find an empirical equation describing the dependence of $\kappa$ on several conditions, we will be able to determine the periodic flow generation without numerical simulations, and dramatically reduce computational loads when performing coupled numerical simulations of wellbore and reservoir models. Because transient wellbore flow simulations require extremely small time step sizes such as 1.0 s, as described by

Matsumoto et al. (2021), effective treatments to reduce computational loads for transient wellbore flow simulations are essential to perform the coupled numerical simulations successfully.

## 3. EVALUATING DEPENDENCE ON EACH DIMENSIONLESS PARAMETER

Based on the discussions by Matsumoto et al. (2021), we can assume that the modification factor $\kappa$ depends on, at most, six dimensionless parameters as follows:

$$\frac{\partial}{\partial x}\kappa(\alpha_1,\beta_1,\beta_2,\gamma,z_S^*,P_S^*) = \frac{\partial\kappa}{\partial\alpha_1}\frac{\partial\alpha_1}{\partial x} + \frac{\partial\kappa}{\partial\beta_1}\frac{\partial\beta_1}{\partial x} + \frac{\partial\kappa}{\partial\beta_2}\frac{\partial\beta_2}{\partial x} + \frac{\partial\kappa}{\partial\gamma}\frac{\partial\gamma}{\partial x} + \frac{\partial\kappa}{\partial z_S^*}\frac{\partial z_S^*}{\partial x} + \frac{\partial\kappa}{\partial P_S^*}\frac{\partial P_S^*}{\partial x}, \tag{3}$$

where $\alpha_1$, $\beta_1$, $\beta_2$, $\gamma$, $z_S^*$, and $P_S^*$ are dimensionless parameters representing the productivity index of the deep reservoir, thermodynamic productivity of the deep reservoir, pressure loss due to friction while flowing in the wellbore, lower limit of the specific enthalpy in the deep reservoir to sustain production, shallow feed zone depth, and shallow reservoir pressure, respectively, while $x$ is an arbitrary dimensional parameter. Because the numerical code originally developed by Matsumoto et al. (2021) is based on the dimensional form, we obtain an equation obeying Eq. (3) by selecting an appropriate dimensional parameter $x$, which yields a linear equation with respect to the six partial derivatives of $\kappa$ appearing on the right-hand side of Eq. (3). The partial derivatives with respect to $x$ on both sides of Eq. (3) are approximated using finite differences.

Numerous simulation runs are required to obtain a change in $\kappa$ while modifying $x$ when computing the finite difference approximating the left-hand side of Eq. (3). In fact, determining a single value of $\kappa$ under a condition (i.e., determining each case listed in Table 1) requires 50 runs. This results in 100 runs in total to determine the values of $\kappa$ before and after modifying $x$. Referring to the conditions described by Matsumoto et al. (2021), which correspond to Cases 0–3 in Table 1, we can obtain three equations obeying Eq. (3) with different dimensional parameters $x$. These conditions consist of a reference case and three modified cases. The reference case assumes a vertical well with a uniform inner diameter of 0.2 m intersecting the shallow and deep reservoirs at depths of 1400 and 2000 m. The wellhead pressure is assumed to be constant at 0.7 MPa. The roughness of the wellbore inner surface is 0.0478 mm. The shallow reservoir temperature varies between 180 and 240°C, while the deep reservoir temperature is constant at 260°C. The pressure, permeability–thickness product, and radial distance of the external boundary of the shallow reservoir are 4.5 MPa, $4.0 \times 10^{-12}$ $m^3$, and 1000 m, respectively, while those of the deep reservoir are 10.0 MPa, $1.0 \times 10^{-12}$ $m^3$, and 1000 m, respectively. The modifications involved in the modified cases (Cases 1–3) are described in Table 1.

We performed numerous additional simulations (Cases 4–6) and obtained seven cases in total, as summarized in Table 1. Applying the reference case (Case 0) and one of the other modified cases (Cases 1–6) to Eq. (3) using finite differences, we derived a linear system with respect to the six partial derivatives of $\kappa$ at the condition of the reference case as follows:

$$\begin{pmatrix} 44.9 & 0 & 38.4 & 0 & 0 & 0 \\ 0 & 52.0 & 0 & -0.00522 & 0 & 0.0244 \\ 0 & 0 & 0 & 0 & 0 & 0.0538 \\ -0.0268 & 1.50 & 0 & -0.000646 & 0 & -0.0208 \\ -0.477 & 0 & 0 & 0.000049 & 0 & 0 \\ 0.175 & -0.0694 & 0.178 & 0 & -0.00175 & 0 \end{pmatrix} \begin{pmatrix} \partial\kappa/\partial\alpha_1 \\ \partial\kappa/\partial\beta_1 \\ \partial\kappa/\partial\beta_2 \\ \partial\kappa/\partial\gamma \\ \partial\kappa/\partial z_S^* \\ \partial\kappa/\partial P_{reS}^* \end{pmatrix} = \begin{pmatrix} 1.5 \\ -0.6 \\ -0.9 \\ 0.2 \\ -0.1 \\ -0.5 \end{pmatrix}. \tag{4}$$

The partial derivatives are obtained solving Eq. (4) directly using an inverse matrix, as shown in Table 2. The distributions of the flow pattern depending on $PI_S/PI_D$ and $h_{reS}/h_{reD}$ and the flow pattern boundary obeying Eq. (2) for Cases 4–6 are shown in Figure 1.

**Table 1: Conditions for each case.**

| Case | $\kappa$ | $\alpha_1$ | $\beta_1$ | $\beta_2$ | $\gamma$ | $z_S^*$ | $P_{reS}^*$ | Modification of the dimensional parameter $x$ |
|---|---|---|---|---|---|---|---|---|
| 0* | 2.3 | 46.5 | 27.8 | 71.1 | 0.0530 | 0.700 | 0.409 | Reference case |
| 1* | 3.8 | 91.5 | 27.8 | 110 | 0.0530 | 0.700 | 0.409 | Inner diameter from 0.20 to 0.14 m |
| 2* | 1.7 | 46.5 | 79.8 | 71.1 | 0.0478 | 0.700 | 0.433 | Wellhead pressure from 0.7 to 0.3 MPa |
| 3* | 1.4 | 46.5 | 27.8 | 71.1 | 0.0530 | 0.700 | 0.462 | Shallow reservoir pressure from 4.5 to 5.0 MPa |
| 4 | 2.5 | 46.6 | 29.3 | 71.1 | 0.0524 | 0.700 | 0.388 | Deep reservoir pressure from 10.0 to 10.5 MPa |
| 5 | 2.2 | 46.1 | 27.8 | 71.1 | 0.0530 | 0.700 | 0.409 | Deep reservoir radius from 1000 to 1100 m |
| 6 | 1.8 | 46.8 | 27.8 | 71.3 | 0.0531 | 0.698 | 0.409 | Deep reservoir depth from 2000 to 2005 m |

* Described by Matsumoto et al. (2021).

**Table 2: Values of the partial derivatives of the modification factor.**

| $\partial\kappa/\partial\alpha_1$ | $\partial\kappa/\partial\beta_1$ | $\partial\kappa/\partial\beta_2$ | $\partial\kappa/\partial\gamma$ | $\partial\kappa/\partial z_S^*$ | $\partial\kappa/\partial P_{reS}^*$ |
|---|---|---|---|---|---|
| 0.181 | 0.0244 | -0.172 | 280 | 286 | -16.7 |

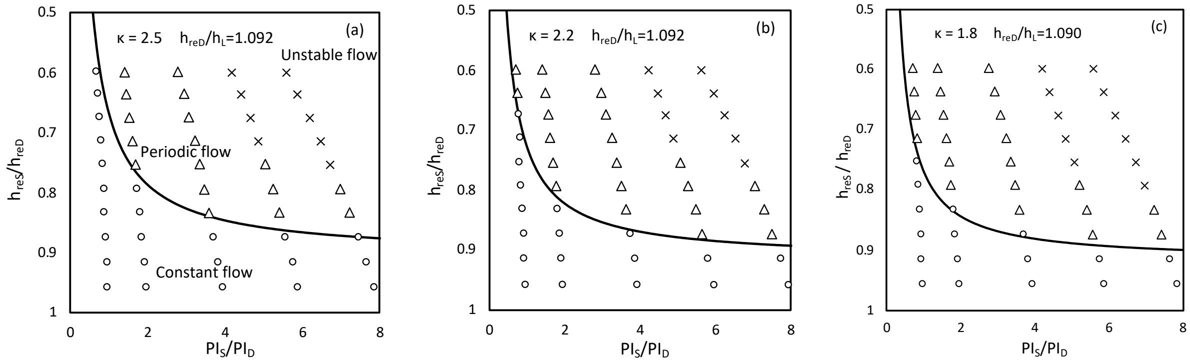


**Figure 1: Distributions of the flow pattern and the flow pattern boundary obeying Eq. (2) and assuming $\bar{h} = h_L$. Cases 4–6 correspond to (a–c), respectively.**

Accordingly, we obtained an approximate value of the partial derivative of the modification factor $\kappa$ with respect to each dimensionless parameter under the reference case condition (Case 0). Taking the total derivative of $\kappa$ with respect to the dimensionless parameters, we evaluate the dependence of $\kappa$ on each dimensionless parameter. The term $(\partial\kappa/\partial\alpha_1)d\alpha_1$ is unnegligible because $\alpha_1$, including a permeability–thickness product, generally varies by several orders of magnitude in real fields. The effect of the shallow reservoir depth $(\partial\kappa/\partial z_S^*)dz_S^*$ is also effective because of the relatively large value of $\partial\kappa/\partial z_S^*$ and $z_S^*$ ranging up to unity. In contrast, $\beta_1$, $\beta_2$, and $\gamma$ generally range less widely within an order of magnitude. Hence, we consider the dependence of $\kappa$ on $\alpha_1$ and $z_S^*$ in the following section. Assuming that the shallow reservoir pressure naturally changes with depth, we regard $P_{reS}^*$ as a linear function of $z_S^*$ by interpolating or extrapolating the relationship between the pressure and depth assumed in the reference case.

## 4. DEPENDENCE ON DEEP RESERVOIR PRODUCTIVITY INDEX AND SHALLOW RESERVOIR DEPTH

We investigated the dependence of the modification factor $\kappa$ on the two dimensionless parameters $\alpha_1$ and $z_S^*$ that we have selected in the previous section. The former $\alpha_1$ denotes the scaled productivity index of the deep reservoir, while the latter $z_S^*$ is the scaled shallow reservoir depth. The shallow reservoir pressure is assumed to change linearly with depth, as mentioned in the previous section. For a preliminary study to outline the dependence of $\kappa$, we selected ranges of $40 \leq \alpha_1 \leq 70$ and $0.5 \leq z_S^* \leq 0.9$. When assuming a wellbore with an inner diameter of 0.2 m intersecting a deep reservoir with a thickness of 1 m at a depth of 2000 m, the corresponding productivity index value ranged from $4.5 \times 10^{-6}$ kg s$^{-1}$ Pa$^{-1}$ (16 t h$^{-1}$ MPa$^{-1}$) to $7.9 \times 10^{-6}$ kg s$^{-1}$ Pa$^{-1}$ (28 t h$^{-1}$ MPa$^{-1}$). The shallow reservoir depth ranged from 1000 to 1800 m. Within these ranges, the values of $\kappa$ in 20 cases were obtained by performing numerous simulations.

**Table 3: Modification factor values depending on the two dimensionless parameters.**

| | | $z_S^*$ | | | | |
|---|---|---|---|---|---|---|
| | | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| $\alpha_1$ | 40 | 21 <* | 16.5 | 1.9 | 1.1 | 1.0 |
| | 50 | 21 <* | 20.5 | 2.0 | 1.1 | 1.0 |
| | 60 | 21 <* | 21 <* | 2.3 | 1.1 | 1.1 |
| | 70 | 21 <* | 21 <* | 2.4 | 1.2 | 1.1 |

* All simulation runs exhibit stable flow.

Table 3 summarizes the results of the investigation. The modification factor $\kappa$ exhibits a slight increase with an increase in the scaled productivity index of the deep reservoir $\alpha_1$, while $\kappa$ steeply decreases and approaches unity with an increase in the scaled shallow reservoir depth $z_S^*$. The dependence of $\kappa$ on $\alpha_1$ indicates slight nonlinearity with respect to the productivity indices involved in the definition of the mean specific enthalpy $\bar{h}$ described by Eq. (1). In contrast, we have found a high nonlinearity with respect to $z_S^*$. As Matsumoto et al.

(2021) discussed, the excess impact of the deep reservoir beyond the weights defined by the productivity indices, which is implied by $\kappa > 1$, is because of the asymmetry of sharing the wellbore depth interval between the fluids originating in the shallow and deep reservoirs. The fluid originating in the shallow reservoir totally shares the wellbore depth interval, while that in the deep reservoir partially shares the depth interval above the shallow feed zone depth. The investigation results indicate highly nonlinear increases in the excess impact of the deep reservoir while the shallow reservoir moves upward.

## 5. DISCUSSIONS

As described in the previous section, the modification factor $\kappa$ sensitively and nonlinearly depends on the scaled shallow reservoir depth $z_{\mathrm{S}}^*$. This implies that the inflow depth of a lower-enthalpy fluid into a wellbore is key in the generation of periodic flow. Inflow at a relatively shallow depth, such as that of ground water due to the failure of a casing pipe at a shallow depth, may not frequently generate periodic flow. It must be noted that the dependence of the shallow reservoir pressure is also essential. As seen in Table 2, $\kappa$ and $z_{\mathrm{S}}^*$ have a positive correlation ($\partial\kappa/\partial z_{\mathrm{S}}^* > 0$) contrarily to the results shown in Table 3, while $\kappa$ and $P_{\mathrm{resS}}^*$ have a negative correlation ($\partial\kappa/\partial P_{\mathrm{resS}}^* < 0$). This means that the negative correlation observed in the previous section results is due to the predominant effect of change in the shallow reservoir pressure. The pressure gradient in the vertical direction, which remains unstudied, probably plays a key role in the dependence of $\kappa$ on $z_{\mathrm{S}}^*$, and will be an important subject of future studies.

The dependence of the modification factor $\kappa$ on the scaled shallow reservoir depth $z_{\mathrm{S}}^*$, including the dependence through the scaled shallow reservoir pressure $P_{\mathrm{resS}}^*(z_{\mathrm{S}}^*)$ and assuming that the other dimensionless parameters are constant, can be written as follows:

$$\frac{d\kappa}{dz_{\mathrm{S}}^*} = \frac{\partial}{\partial z_{\mathrm{S}}^*}\kappa(z_{\mathrm{S}}^*, P_{\mathrm{resS}}^*) + \frac{\partial}{\partial P_{\mathrm{resS}}^*}\kappa(z_{\mathrm{S}}^*, P_{\mathrm{resS}}^*)\frac{dP_{\mathrm{resS}}^*}{dz_{\mathrm{S}}^*}, \quad (5)$$

where $dP_{\mathrm{resS}}^*/dz_{\mathrm{S}}^* = 1.97$ when assuming dimensionless depth and pressure values at the shallow reservoir, which are scaled using the deep reservoir values, as described for Case 0 in Table 1. Referring to the partial derivative values in Table 2, we found that the value of $d\kappa/dz_{\mathrm{S}}^*$ is positive and inconsistent with the correlation described in Table 3. Even though the authors have not yet resolved this inconsistency, one probable cause may be the lack of numerical accuracy when computing the partial derivatives involved in Eq. (3) using finite differences. As mentioned in Section 3, each case listed in Table 1 requires 50 simulation runs, which can be performed in parallel using multiple computers, and the execution time of a run varies up to a half-day. The subject of the future study, which will be addressed with high priority, is to improve the numerical code to perform further investigations rapidly, and resolve inconsistencies.

## 6. CONCLUSIONS

We preliminarily investigated the dependence of the modification factor $\kappa$ on the dimensionless parameters derived by Matsumoto et al. (2021). Based on the total derivative of $\kappa$ with respect to the dimensionless parameters, the scaled productivity index of the deep reservoir $\alpha_1$ and the scaled shallow reservoir depth $z_{\mathrm{S}}^*$ were selected for further investigation. The modification factor $\kappa$ slightly increases with an increase in $\alpha_1$, while $\kappa$ steeply and nonlinearly decreases with an increase in $z_{\mathrm{S}}^*$ and approaches unity. The vertical pressure gradient $dP_{\mathrm{resS}}^*/dz_{\mathrm{S}}^*$, which has been assumed to be constant, also plays a key role in forming the correlation between $\kappa$ and $z_{\mathrm{S}}^*$. Acceleration of simulation runs is required to perform further investigations efficiently and to resolve the inconsistency observed in the results of this study.

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