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Development of a Fast Analytical Method of Heat Transfer from a Geothermal Well Into a Thin Shallow Aquifer

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ABSTRACT

Ground Source Heat Pumps (GHSP) utilize the earth’s shallow depths for storing and extracting heat during different seasons in the year. The installation and efficient operation of those geothermal energy systems requires an accurate prediction of their thermal performance. Numerical models are able to simulate the thermal performance accurately but lack the desired computational speed. In this study, a fast and simple analytical method is proposed for evaluating the performance of an injection-extraction well system. The proposed method is benchmarked against a more sophisticated numerical model and observed to be yielding results with a close match.

1. INTRODUCTION

Ground Source Heat Pumps (GSHP) are a centralized heating and cooling system that consists of a heat pump with a ground heat exchanger for closed-loop systems or directly utilize ground water from a geothermal borehole for open-loop systems (Sanner et al., 2003). GSHPs uses earth as a heat source during the winter and a heat sink during the summer. During the injection and production of geothermal energy through GSHPs, it is important to understand the thermal performance of the system and the corresponding heat losses to the cap rock and underburden. In order to evaluate the performance of a geothermal setup, the temperature profile along the aquifer that is used to store/extract heat should be accurately predicted for given operational conditions. In order to conduct a performance analysis in a fast and accurate fashion, an analytical solution that successfully predicts the temperature profile beneath the ground is required.

Lu (2019) developed an analytical solution for deep geothermal systems with a thick aquifer. In Lu (2019)’s analytical solution, a source term is used to evaluate the heat transfer between the aquifer and overburden layer. Through numerous simulations, Lu (2019) proposed an empirical expression to estimate the coefficient of heat transfer that is in the source term. This empirical expression gives good performance for a deep geothermal system with a thick aquifer. However, when the same empirical expression is used to evaluate the heat loss from an aquifer to the cap rock for a shallow geothermal system with a thin aquifer, calculated results did not reach the desired accuracy since the empirical expression could not describe the heat dissipation in a thin aquifer. In this paper, we introduce a semi-analytical solution routine for calculating the heat loss between aquifer and cap rock through the application of Vinsome and Westerveld’s (1980) method that was developed for thermal reservoir simulators in order to calculate the cap and base rock heat losses in a simple way. The semi-analytical solution matched well with the numerical simulations for temperature profile calculations along a thin aquifer within a shallow geothermal system.

2. ANALYTICAL SOLUTION

2.1 Problem Description

Properties related to geometry and site conditions are adopted from a case study in Portland, Oregon where a research team from the United States Geological Survey (USGS), AltaRock Energy, and Portland State University are currently conducting a feasibility study for a deep direct use geothermal application (e.g., Burns et al., 2018). As provided in Figure 1, the aquifer thickness is 20 m and the overburden thickness is 100 m, above which the ground surface temperature is held fixed. The radius of the model is set to 600 m for 40 years of simulation time. Further simplifications include neglecting gravity effects and assuming an impermeable overburden layer. Taking advantage of the symmetry of geometry provided in Figure 1, model geometry was further simplified into a two-dimensional radial axisymmetric model as illustrated in Figure 2.

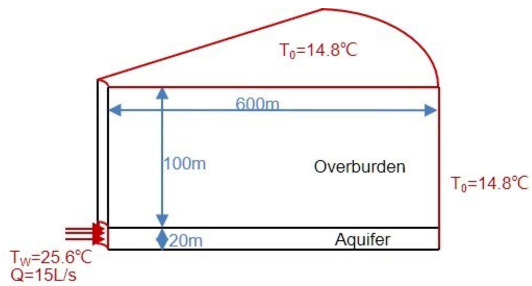

Figure 1: The three-dimensional view of the problem geometry

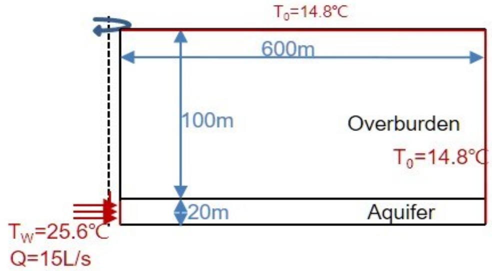

Figure 2: The simplified two-dimensional view of the problem geometry

2.2 Governing Equations and Parameters

The heat transfer in the aquifer is mainly governed by heat convection and conduction. In the overburden layer, heat conduction is the dominant factor. For the aquifer, simulations are performed for the convection-diffusion equation in radial coordinates. The governing differential equation for convection-diffusion is provided in Eq. (1).

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial \theta}{\partial r}\right) - \frac{Q}{2\pi r H}\frac{(\rho c)_f}{\rho c}\frac{\partial \theta}{\partial r} - \frac{\partial \theta}{\partial t} = \frac{w_0}{\rho c H} \tag{1}$$

where θ is the temperature within the aquifer, D is the thermal diffusivity, H is the thickness of the aquifer, ρ is the density of the aquifer, c is the specific heat capacity of the aquifer, Q is the fluid flow rate, r is radius, w_0 is the heat flux at the interface between the aquifer and overburden layer which is equal to h*(θ-T_c) in which h is the convective heat transfer coefficient and T_c is the ambient temperature of the caprock. Lu (2019) conducted several numerical simulations to get the empirical formula for the dimensionless heat transfer coefficient h^* as shown in Eq. (2) and Eq. (3).

$$h^* = 0.652\lambda^{*0.3926}\alpha^{0.2465-0.0051\lambda^*}, 1 \le \lambda^* \le 10, 0.6 \le \alpha \le 600 \tag{2}$$

$$h^* = 0.6722\alpha^{0.231}\lambda^{*0.45}, 0.1 \le \lambda^* < 1, 0.6 \le \alpha \le 600 \tag{3}$$

The relation between heat transfer coefficient h and dimensionless heat transfer coefficient h^* is given in Eq. (4), where α is the dimensionless injection rate representing the ratio between heat convection and heat conduction while $\lambda^*$is the dimensionless thermal conductivity representing the ratio of heat conductivity of the overburden layer to the aquifer.

$$h^* = \frac{H}{4\lambda}h \tag{4}$$

where H and λ are thickness and thermal conductivity of the aquifer, respectively.

A complete list of hydrothermal properties used during the simulation is provided in Table 1.

Table 1: Model parameters

Parameter	Value	Unit
Aquifer Thickness	20	m
Overburden Thickness	100	m
Aquifer Thermal Conductivity	1.45	W/mK
Aquifer Heat Capacity	$3.03*10^6$	J/m^3K
Overburden Thermal Conductivity	1.45	W/mK
Overburden Heat Capacity	$2.90*10^6$	J/m^3K
Fluid Heat Capacity	$4.00*10^6$	J/m^3K
Injection/Extraction Rate	15	L/s
Ambient Temperature	14.8	C
Injection Temperature	25.8	C

Lu (2019) obtained an analytical solution to Eq. (1) with $w_0 = h^*(\theta - T_c)$:

$$T^*(r,t) = \int_{\frac{r^{*2}}{t^*}}^{\infty} e^{-\left(x + h^* r^{*2} \frac{1}{x}\right)} x^{\alpha - 1} dx \tag{5}$$

In Eq. (5), T^* is the dimensionless temperature of water, α is the dimensionless injection rate, α^* is the dimensionless revised injection rate (revised after α for a doublet well system), $r^*$is the dimensionless distance between the injection well and the point of temperature evaluation and $t^*$is the dimensionless time. The detailed expressions of the latter terms are provided in Eq. (6).

$$\alpha = \frac{1}{4\pi n R} \frac{\rho c}{H\lambda} Q,\ r^* = \frac{1}{H} r,\ t^* = \frac{4\lambda}{\rho c H^2} t,\ \alpha' = \alpha + 0.0914\alpha^2 e^{-0.801\alpha^{-0.281L^*}},\ L^* = \frac{1}{H} L \tag{6}$$

where *n* is the porosity, *R* is the delay factor, ρ is the density of the aquifer, *c* is the specific heat capacity of the aquifer, *H* is the thickness of the aquifer, λ is the thermal conductivity of the aquifer, *r* is the radius, *t* is the time and L^* is the dimensionless distance between the injection well and extraction well.

The analytical solution derived by Lu (2019) works well in matching the numerical simulation results for a deep geothermal system with thick aquifer conditions. The same analytical solution routine has been implemented for the thin aquifer case with the geometry given in Figure 1. A significant discrepancy between the analytical solution and the numerical simulation results has been observed.

2.3 Explanation for Poor Performance of Analytical Solution on Shallow Geothermal Systems

The validity of the empirical expressions used for h* in the analytical solution for the shallow geothermal storage problem was examined by conducting 2D TOUGH + Qloss (Pruess et al., 2012) numerical simulations with different aquifer geometries (20-100 meters of thickness). In the TOUGH + Qloss model, heat transfer into an overburden layer (100 meters of thickness) was modeled semi-analytically (Vinsome and Westerveld, 1980). TOUGH + Qloss results were used as benchmark cases. The comparison between the TOUGH + Qloss solutions and analytical solutions showed that the temperature profile for thin aquifers (20 meters with an injection rate of $0.015 m^3/s$) was affected by large heat dissipation from the aquifer to the overburden, which in turn impacted the matching of the temperature profile between 2D TOUGH + Qloss solution and 1D analytical solution as shown in Figure 3.

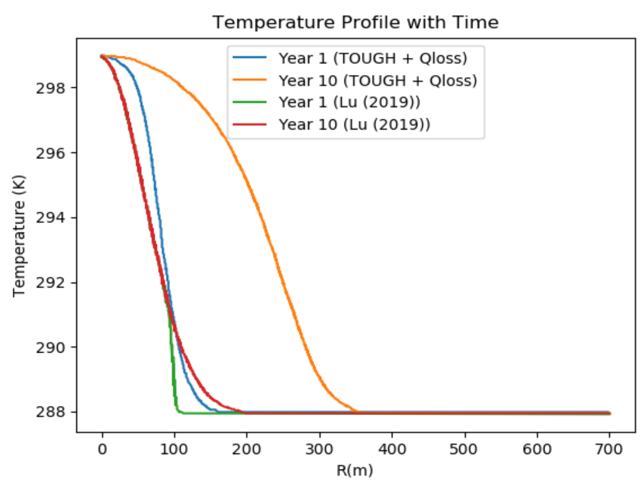

Figure 3: Comparison of numerical simulation results with Lu (2019)'s analytical solution for shallow geothermal system with a thin aquifer

The two solutions matched better when aquifer thickness was increased up to 100 meters and a smaller percentage of heat loss occurs through the overburden. The performance of the 1D analytical model breaks down when a large amount of heat is lost into the overburden layer. This becomes more apparent when the aquifer thickness is small.

Due to inadequate performance of the analytical solution for the shallow geothermal case, a semi-analytical solution for calculating the heat loss from aquifer to the cap rock has been developed.

3. SEMI-ANALYTICAL SOLUTION

To avoid error in the estimation of the heat dissipation between aquifer to the overburden leading to the deviation between the analytical solution and numerical solution, the semi-analytical solution derived by Vinsome and Westerveld (1980), which is used in TOUGH, is applied. The heat transfer between the aquifer and overburden is modeled as a source term using Vinsome & Westerveld (1980)'s method which was developed for a fast calculation of heat losses to the cap rock and underburden from an aquifer.

The proposed FE coupled semi-analytical method is based on the finite element method. The governing differential equation of convection-diffusion provided in Eq. 1 is used as a starting point for the development of the weak form of the equation. The corresponding finite element code is coded using Python. The Eq. (1) can be discretized into Eq. (7) by the backward Euler method.

$$[M]\{\dot{\theta}\} + [K]\{\theta\} = \{F\} \tag{7}$$

Where M is the mass matrix, θ is the matrix storing nodal unknown temperature values, K is the stiffness matrix and F is the forcing vector.

Applying the Backward Euler time integration scheme to the discrete form provided in Eq. (7) provides the following expression in order to compute the unknown values of temperature at the nodes:

$$\{\theta\}_{s+1} = ([M] + \Delta t[K])^{-1}\big[[M]\{\theta\}_s + \Delta t\{F\}_{s+1}\big] \tag{8}$$

In Eq. (8), subscript s represents the current time step and s+1 denotes the next time step while Δt is the time-step size used in the finite element simulation.

For examining the performance of the proposed semi-analytical solution, an aquifer that is overlain by a caprock configuration is considered. The problem is comprised of an aquifer of 20 meters thickness, where there is a caprock and a bedrock of 100 meters thickness lying above and below it respectively. The computational domain has been developed for a radial geometry with the radius extending to 700 meters. Due to the symmetry of the geometry, only the upper half of the model is needed. The initial temperature in the aquifer is 14.8 degrees Celsius, while the injection well is pumping hot water with a constant temperature of 25.8 degrees Celsius. The finite element model developed for this study simplifies the geometry to a one-dimensional problem. The radial domain has been discretized with linear elements of length of 1 meter, resulting in a total of 700 nodes where the temperature values have been calculated as shown in Figure 7.

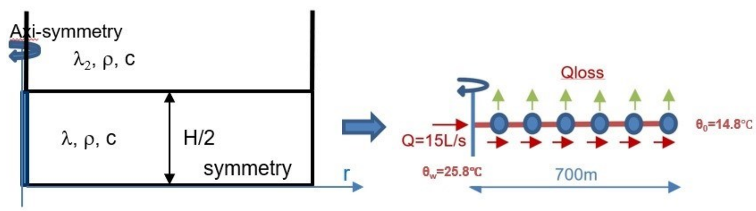

Figure 7: The simplified geometry of the one-dimensional finite element model

The Vinsome and Westerveld (1980) method predicts the temperature profile along the cap or base rock as

$$T(t,z) = (\theta + pz + qz^2)e^{-z/d} \tag{9}$$

The heat loss term obtained using the previous expressions is given in as Eq. (10) as in Vinsome and Westerveld (1980), which is used as the source term in Eq. 1:

$$Heat\ Loss\ Term = \frac{w_0}{\rho cH} = \lambda_2\left(\frac{\theta}{d} - p\right) \tag{10}$$

in which λ_2 represents the thermal conductivity of the caprock, θ is the temperature value at the node, d is the diffusion length and p is a fitting parameter.

The finite-element simulation of the problem has been run for five years. The reason for selecting this time interval is based on the interest of understanding the change in the thermal conditions within the aquifer in a relatively small but also meaningful time span. The temperature profiles along the domain for the first and fifth years are provided in Figure 8. According to the results that are obtained from finite element simulations, the proposed one-dimensional FE code and TOUGH + QLoss code yield similar temperature profiles with a slight difference in the shape of the fronts. The slight difference between the simple one-dimensional FE code and TOUGH + QLoss can be due to a couple of reasons. First of all, TOUGH is based on the finite difference method, while the developed simple code is rooted on the finite element method. Also, the TOUGH model uses multiple layers to represent the aquifer, whereas the simple code uses only one.

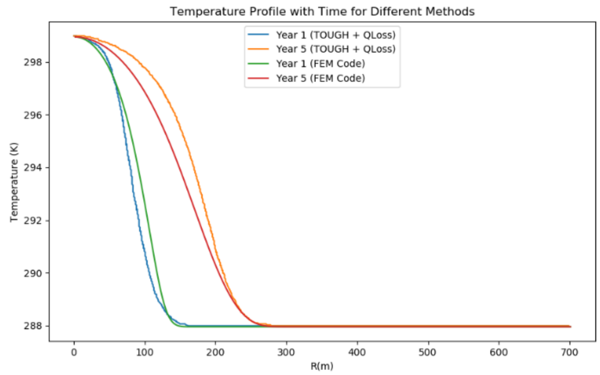

Figure 8: The temperature profiles for first five years for different approaches

4. CONCLUSION

For the fast prediction of the variation of aquifer temperature during heat injection and extraction, an analytical solution by Lu (2019) is derived. However, the analytical solution with a constant coefficient of the heat transfer characterizing the heat dissipation from the aquifer to the overburden was not able to provide accurate temperature profiles during the heat injection for the thin aquifer applicable to a shallow geothermal problem. A fast and simple one-dimensional finite element code for heat extraction and injection that is to be used in shallow geothermal applications was developed to overcome the drawback of the analytical solution. Heat loss at the interface of aquifer and caprock was modelled using Vinsome and Westerveld (1980)'s simplified method. The results generated from a one-dimensional FE code was compared with those of TOUGH + Qloss and a slight difference between those two was observed. It was concluded that the possible reason for that difference could be the nature of the solution scheme that has been adopted differently in those methods.

As for the next steps, advanced algorithms will be implemented into the one-dimensional finite element code in order to improve the accuracy of the calculations and speed up the computational time.

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