

Approximate Solutions to Fracture-Matrix Heat Transfer for Numerical and Analytical Modeling of Enhanced Geothermal Systems

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ABSTRACT

Dual-continuum models, such as dual-porosity and dual-permeability models, have often been employed to simulate heat transfer induced by fluid injection and production in geothermal systems, despite the known shortcomings of first-order approximations that may significantly underestimate fracture-matrix heat transfer, leading to earlier and larger thermal drawdown at production wells. For accurate modeling of enhanced geothermal systems (EGS), cooling-induced fracturing that occurs in EGS reservoirs demands high-resolution non-isothermal modeling. To avoid the effects of first-order approximations, we have developed unified-form approximate solutions to fracture-matrix heat transfer based on existing analytical solutions with infinite exponential series (Zhou et al., 2017). Our newly developed approximate solutions combine the early- and late-time solutions that are continuous at a dimensionless switchover time, t_{d0} . The early-time solutions are based on three-term polynomial functions in terms of square root of dimensionless time, $\sqrt{t_d}$, with the first coefficient dependent on dimensionless area-to-volume ratio. The last two coefficients solely depend on the aspect ratios for anisotropic rectangular blocks, while they are determined analytically for isotropic blocks of various shapes. For the late-time solutions, only the leading exponential term is needed for isotropic blocks, while a few additional exponential terms are needed for highly anisotropic rectangular blocks. The developed solutions are used to demonstrate the effects of anisotropy, multiple rates, and early- and late-time heat transfer in idealized geothermal systems. These solution can be easily extended to modeling reservoir-scale heat transfer with any shape and size of rock matrix blocks, with the aid of time-convolution.

1. INTRODUCTION

Numerical modeling of heat transfer between fluids in fractures and the rock matrix requires high-resolution modeling, in particular near the fracture-matrix interfaces, because thermal diffusion in the rock matrix is a slow process. The dual-porosity and dual-permeability models initially developed for hydraulic diffusion in fractured reservoirs (Barenblatt et al., 1960; Warren and Root, 1963) may produce large errors for heat-transfer modeling because the thermal diffusion coefficient is on the order of 10^{-6} m²/s (Zhou et al., 2007; see Figure 4 in Zhou et al., 2017). Different improvements have been made for conventional dual-porosity models, including the multiple interacting continuum (MINC) model by further dividing a block into nested sub-elements (Pruess and Narasimhan, 1985), and nonlinear fracture-matrix transfer equations using specific analytical solutions (Zimmerman et al., 1993). However, these improvements also have many limitations.

Analytical solutions have been developed to couple the advective and diffusive transport in fractures with diffusion in the rock matrix by the temperature and thermal-flux continuity at the fracture-matrix interfaces (e.g., Lauwerier, 1955; Jung and Pruess, 2012). In all these solutions, one set of parallel fractures is assumed to simplify the geometry of matrix blocks. Most recently, Zhou et al. (2017) focused on the diffusion process at the matrix block scale and the diffusive fracture-matrix transfer and developed unified-form approximate solutions for 1D, 2D, and 3D rectangular matrix blocks, as well as spherical and cylindrical blocks; as they showed, the simplification of anisotropic blocks to isotropic ones and the reduction in the block dimensions produce large simulation errors. These approximate solutions for block-scale diffusion process for any regular block shapes (spheres, cylinders, slabs, square columns, cubes, rectangular columns, and rectangular parallelepipeds) can be used as a building block for numerical and analytical modeling of heat transfer in fractured geothermal reservoirs and EGS systems.

2. ANALYTICAL AND APPROXIMATE SOLUTIONS OF FRACTURE-MATRIX HEAT TRANSFER

The heat transfer equation for 1D thermal diffusion in slab-like matrix blocks, 2D diffusion in rectangular-column blocks, and 3D diffusion in rectangular-parallelepiped blocks can be written

$$\frac{\partial T_r}{\partial t} = D \frac{\partial}{\partial x_i} \frac{\partial T_r}{\partial x_i} \quad (1a)$$

where $T_r(x_i, t)$ is the spatially-varying temperature change relative to the constant temperature change at fractures ΔT_f , t is the time, x_i is the coordinates ($= x, y, z$ in 3D), and D is the thermal diffusion coefficient in the porous matrix defined by

$$D = \lambda_m / [(1 - \phi_m)\rho_m c_m + \phi_m \rho_w c_w], \quad (1b)$$

where λ_m is the thermal conductivity of the matrix, ϕ_m is the matrix porosity (i.e., pore volume per unit volume of the matrix), ρ is the density, c is the specific heat, and the subscripts m and w denote for rock matrix grains and water.

Without loss of generality, it is assumed that the matrix block and fractures are saturated with water, the fractures have a constant temperature of T_f that is fixed with time as a boundary condition, and the fractures and matrix block have an initial uniform temperature of T_0 , i.e., $\Delta T_f = T_0 - T_f$. It is also assumed that the matrix porosity and thermal diffusion coefficient are constant in space and time. We can define a dimensionless temperature change for the matrix block:

$$T_d = (T_0 - T_{mt}) / (T_0 - T_f) = M_t / M_\infty \quad (2a)$$

where T_{mt} is the block-wise average temperature, M_t is the time-dependent thermal energy depletion, $M_\infty = ((1 - \phi_m)\rho_r c_r + \phi_m \rho_w c_w)(T_0 - T_f)V$, and V is the volume of the block. T_d can be written in terms of dimensionless time defined by

$$t_d = Dt / l^2 \quad (2b)$$

where l is the half-width of a slab or the minimum fracture half-spacing of a rectangular column or parallelepiped.

A slab-like, rectangular-column, and rectangular-parallelepiped block can be formed by one set of parallel fractures with fracture spacing of $2l$, two orthogonal sets of parallel fractures with fracture spacing of $2l_x$ and $2l_y$, and three orthogonal sets of parallel fractures with fracture spacing of $2l_x$, $2l_y$, and $2l_z$, respectively. Here we introduce $l = l_x \leq l_y$ or $l = l_x \leq l_y \leq l_z$, and the aspect ratios $R_{li} = l / l_i$, $i = y, z$, where x, y, z are the local coordinates and may not be the same as the global ones.

2.1 Analytical Solutions

For a rectangular matrix block bounded by one, two, and three orthogonal sets of parallel fractures in fractured geothermal reservoirs, the dimensionless temperature change can be calculated using existing analytical solutions (Carslaw and Jaeger, 1959; Crank, 1975; Lim and Aziz, 1995):

$$T_d = 1 - 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2 \pi^2} \exp(-(2n-1)^2 \pi^2 t_d / 4), \quad (3a)$$

$$T_d = 1 - \left(\frac{8}{\pi^2}\right)^2 \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \frac{1}{(2n_x-1)^2 (2n_y-1)^2} \exp\left[-\left((2n_x-1)^2 + (2n_y-1)^2 R_{ly}^2\right) \frac{\pi^2}{4} t_d\right], \quad (3b)$$

$$T_d = 1 - \left(\frac{8}{\pi^2}\right)^3 \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \sum_{n_z=1}^{\infty} \frac{1}{(2n_x-1)^2 (2n_y-1)^2 (2n_z-1)^2} \exp\left[-\left((2n_x-1)^2 + (2n_y-1)^2 R_{ly}^2 + (2n_z-1)^2 R_{lz}^2\right) \frac{\pi^2}{4} t_d\right]. \quad (3c)$$

However, for a very early time, these calculations are computationally expensive. For example, for $t_d = 10^{-7}$, we need $N_x \times N_y \times N_z = 1000 \times 1000 \times 1000$ calculations of the exponential function for a rectangular-parallelepiped block.

2.2 Approximate Solutions

Zhou et al. (2017) developed unified-form approximate solutions for any shape of matrix blocks (spheres, cylinders, slabs, rectangular columns, and rectangular parallelepipeds). The unified solutions take the following form of the early-time solution and late-time solution:

$$T_d = \begin{cases} a_1 \sqrt{t_d} + a_2 t_d + a_3 (t_d)^{3/2}, & t_d \leq t_{d0} \\ 1 - \sum_{j=1}^N b_{1j} \exp[-b_{2j} t_d], & t_d > t_{d0} \end{cases} \quad (4)$$

where a_1 , a_2 , and a_3 are the three coefficients of the three-term polynomial function for the early-time solution, b_{1j} , b_{2j} and N are the capacity ratios, heat-transfer rate coefficients, and the number of leading exponential terms in the exact solutions for the late-time solution, and t_{d0} is the dimensionless switchover time between the early- and late-time solutions. These solution coefficients are different for different shapes of matrix blocks. For a slab-like matrix block, we have $a_1 = 2/\sqrt{\pi}$, $a_2 = 0$, $a_3 = 0$; $b_1 = 8/\pi^2$, $b_2 = \pi^2/4$, $N = 1$ (also see Eq. 3a).

For a rectangular-column matrix block, we have the following solution coefficients:

$$a_1 = 2(1 + R_{ly})/\sqrt{\pi}, \quad (5a)$$

$$a_2 = -1.2735 R_{ly}, \quad (5b)$$

$$a_3 = 0, \quad (5c)$$

$$b_{1j} = \left(\frac{8}{\pi^2}\right)^2 / ((2n_{xj} - 1)^2 (2n_{yj} - 1)^2), \quad (5d)$$

$$b_{2j} = \frac{\pi^2}{4} c_j; \quad c_j = (2n_{xj} - 1)^2 + (2n_{yj} - 1)^2 R_{ly}^2 \quad (5e)$$

The solution coefficients in Eq. (4) for a rectangular parallelepiped are written:

$$a_1 = 2(1 + R_{ly} + R_{lz})/\sqrt{\pi} \quad (6a)$$

$$a_2 = -1.2735R_{ly} - 1.2645R_{lz} - 1.2791R_{ly}R_{lz} \quad (6b)$$

$$a_3 = 1.4232R_{ly}R_{lz} \quad (6c)$$

$$b_{1j} = \left(\frac{8}{\pi^2}\right)^3 / ((2n_{xj} - 1)^2(2n_{yj} - 1)^2(2n_{zj} - 1)^2) \quad (6d)$$

$$b_{2j} = \frac{\pi^2}{4} c_j; \quad c_j = ((2n_{xj} - 1)^2 + (2n_{yj} - 1)^2 R_{ly}^2 + (2n_{zj} - 1)^2 R_{lz}^2). \quad (6e)$$

For a rectangular column or parallelepiped, the number of exponential terms ($j = 1, N$) needed in Eq. (4) can be determined practically by

$$c_j \leq 11 \text{ with } b_{1j} \exp(-b_{2j} T_{d0}) > \text{tol}, \quad (7)$$

where *tol* is a cutoff, depending on the degree of block anisotropy. $\text{tol} = 5 \times 10^{-4}$ for $R_{ly} > 0.1$, while $\text{tol} = 1 \times 10^{-4}$ is needed for $R_{ly} \leq 0.1$ to control the relative error below 0.5% for $R_{ly} = 0$.

The details of the derivation of these solution coefficients can be found in Zhou et al. (2017). The dimensionless switchover time of 0.22 can be used for 1D, 2D, and 3D rectangular matrix blocks. Figure 1 shows the comparison between the analytical (using $N_x \times N_y = 1000 \times 1000$ or $N_x \times N_y \times N_z = 1000 \times 1000 \times 1000$) and the approximate solutions for a 2D rectangular-column block with aspect ratio of $R_{ly} = [0, 1]$ with an increment of 0.2, and for a 3D rectangular-parallelepiped block with an increment of 0.1 and $0 \leq R_{ly} \leq 1$ and $0 \leq R_{lz} \leq R_{ly}$.

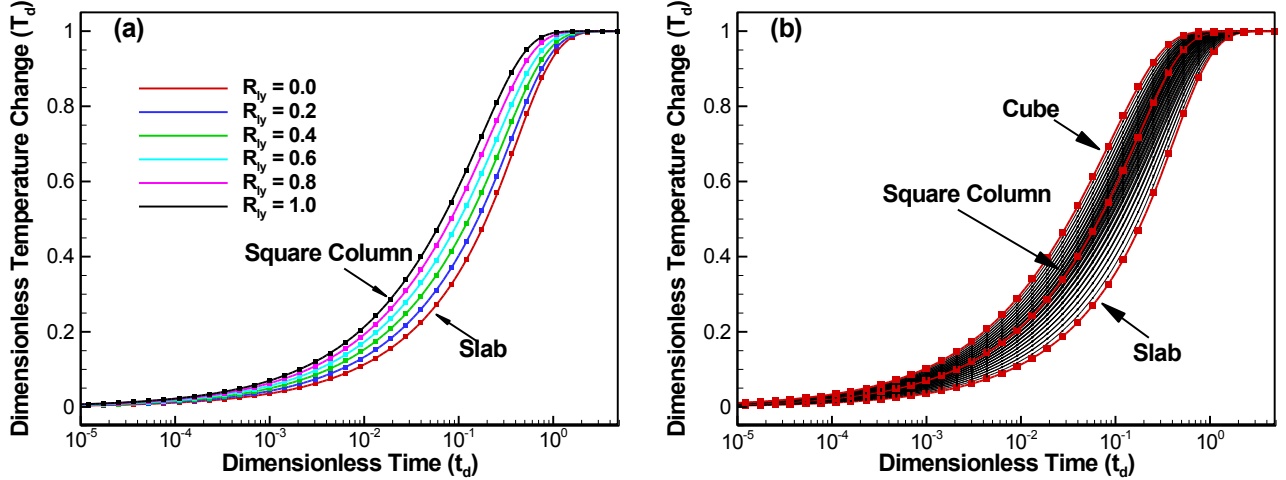


Figure 1: Comparison between analytical solutions (in lines) and approximate solutions (in symbols) for (a) a rectangular-column matrix block and (b) a rectangular-parallelepiped block with different values of aspect ratios.

3. CALCULATIONS OF FRACTURE-MATRIX HEAT TRANSFER

We are interested in calculation of the time-dependent dimensionless temperature change in matrix blocks induced by a constant temperature change in the surrounding fractures. The matrix is of a porosity $\phi_m = 0.10$, a thermal conductivity $\lambda_m = 2.1 \text{ W/m}^\circ\text{C}$, specific heat $c_m = 1000 \text{ J/kg}^\circ\text{C}$, and a density $\rho_m = 2650 \text{ kg/m}^3$, and the water has a density $\rho_w = 1000 \text{ kg/m}^3$ and specific heat $c_w = 4200 \text{ J/kg}^\circ\text{C}$ (Jung and Pruess, 2012). The resulting thermal diffusion coefficient is $0.75 \times 10^{-6} \text{ m}^2/\text{s}$.

We are interested in local thermal diffusion processes in a unit pore volume of fractured matrix that consists of blocks of single shape and size or a mixture of blocks of different sizes. Three cases of single-size blocks are considered, including (1) slabs, (2) cubes, or (3) rectangular parallelepipeds with $R_{ly} = 0.5$ and $R_{lz} = 0.2$. In the last case, four cubical blocks with fracture half-spacing of $l, 2l, 3l$, and $6l$ and their volumetric fractions of 0.42, 0.24, 0.17 and 0.17 respectively are considered. In each case, the characteristic minimum fracture half-spacing varies from 0.19 m to 58.1 m.

Figure 2a-c shows the contours of dimensionless temperature change as functions of minimum fracture half-spacing and the time in the three cases of single-size blocks. It can be seen that for a given l , cubical blocks are more effective than the rectangular-parallelepiped blocks that are in turn more effective than the slab-like blocks for heat depletion. The multi-block case shows a more gradual increase in T_d with time because of the effects of blocks of large half-spacing.

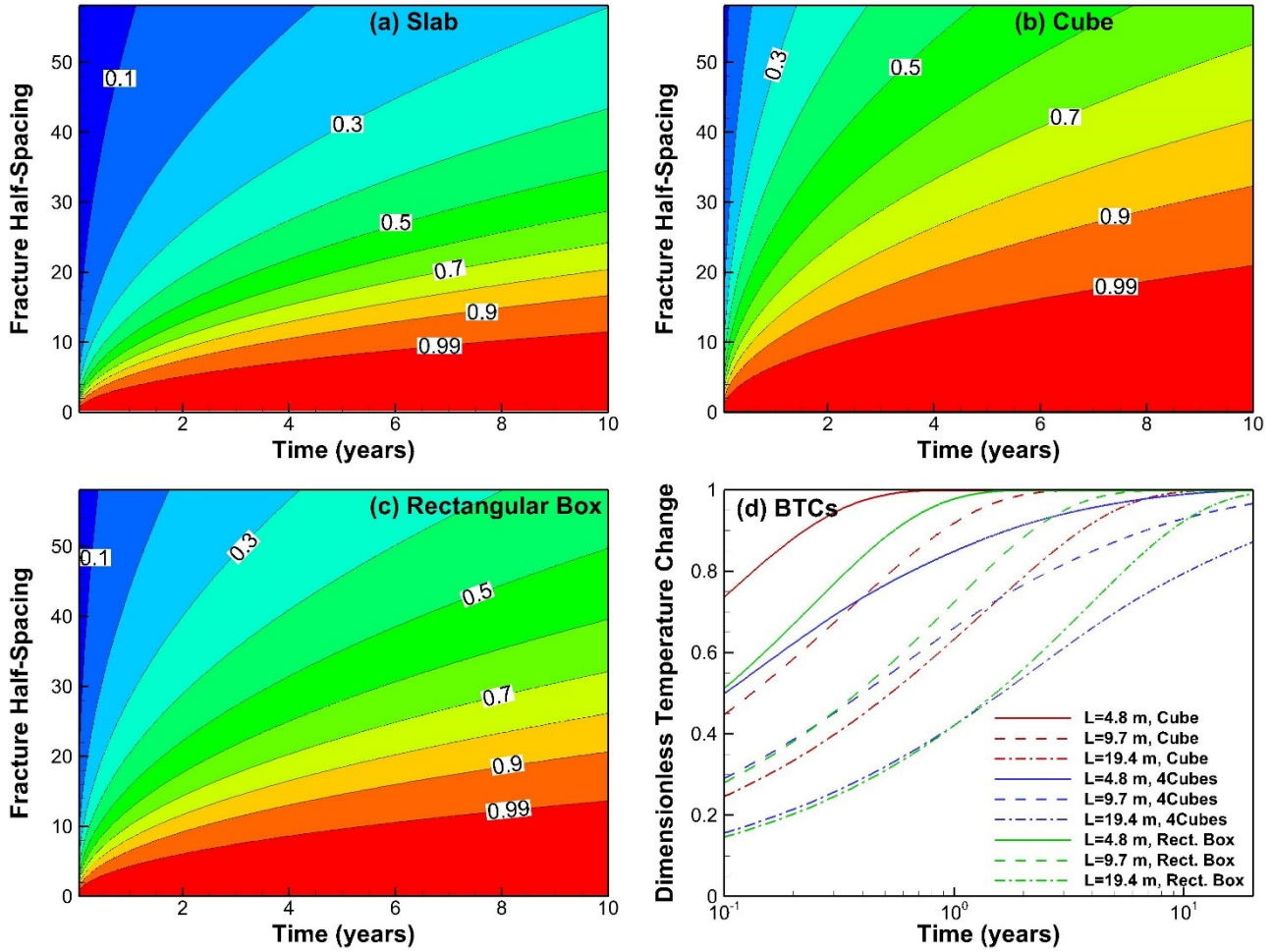


Figure 2: Contours of dimensionless temperature change as functions of time and fracture half-spacing for (a) slab-like blocks, (b) for cubical blocks, and (c) for rectangular-parallelepiped blocks with $R_{ly} = 0.5$ and $R_{lz} = 0.2$, and (d) time-dependent dimensionless temperature change for three cubes and three rectangular parallelepipeds, as well as three composites of 4 cubes, with different minimum half-spacing. All calculations were conducted using the developed approximate solutions (Zhou et al., 2017).

4. CONCLUSIONS

Dual-continuum models, such as dual-porosity and dual-permeability models, have often been employed to simulate heat transfer induced by fluid injection and production in geothermal systems, despite the known shortcomings of first-order approximations that may significantly underestimate fracture-matrix heat transfer, leading to false predictions of earlier and larger thermal drawdown at production wells. For accurate modeling of enhanced geothermal systems (EGS), cooling-induced fracturing that occurs in EGS reservoirs demands high-resolution non-isothermal modeling. To avoid the effects of first-order approximations, Zhou et al. (2017) developed unified-form approximate solutions to fracture-matrix heat transfer based on existing analytical solutions with infinite exponential series. The newly developed approximate solutions combine the early- and late-time solutions that are continuous at a dimensionless switchover time.

The developed solutions are used to demonstrate the effects of anisotropy, multiple rates, and early- and late-time mass transfer in idealized geothermal systems. These solution can be easily extended to modeling reservoir-scale heat transfer with any shape and size of rock matrix blocks, with the aid of time-convolution.

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