

Three-dimensional Elastic-waveform Inversion with Compressive Sensing for Imaging Geothermal Fields Using Sparse VSP Data

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ABSTRACT

Three-dimensional elastic-waveform inversion of multi-component seismic data is one of the most powerful tools for obtaining high-resolution subsurface velocity models. These compressional- and shear-wave velocity models are crucial for accurate subsurface migration imaging, microseismic location and focal mechanism inversion, and geothermal energy exploration and characterization of enhanced geothermal systems. Obtaining a high-accuracy inverted velocity model usually requires seismic data acquired with dense source/receiver arrays. However, seismic sources and/or geophones are often sparsely distributed. To improve velocity inversion with sparse seismic data, we incorporate a compressive sensing technique into elastic-waveform inversion. In our new inversion method, we use an alternating-minimization algorithm to solve the optimization problem. We apply our new inversion method to synthetic 3D vertical seismic profiling (VSP) data for a 3D geophysical model built using geologic features and well log data at the Raft River geothermal field. The inversion results obtained using our new method and synthetic 3D VSP data with a sparse source array are similar to those produced with the conventional elastic-waveform inversion of 3D VSP data with a dense source array.

1. INTRODUCTION

High-quality three-dimensional elastic (compressional- and shear-wave) velocity models are important for accurate seismic migration, microseismic location and focal mechanism inversion in geothermal fields, and geothermal reservoir characterization. In complex geologic environments, such as geothermal reservoirs, one of the most advanced tools for velocity model building is elastic-waveform inversion. Elastic-waveform inversion updates both compressional- and shear-wave velocity models using forward modeling of elastic seismic wavefields and back propagation of residual seismic wavefields (Tarantola, 1984; Mora, 1987; Virieux and Operto, 2009).

To obtain high-resolution velocity models, elastic-waveform inversion usually requires dense seismic source and receiver arrays satisfying the spatial sampling criteria. However, seismic sources and/or geophones are often sparse in reality because of the economic and logistic limitation. How to reconstruct the model with sparse seismic data is thus of great interest. The compressive sensing technique has been shown to have the potential to recover the signal information using fewer measurements than those required by the sampling criteria (Candes et al., 2006; Donoho, 2006). This technique employs the sparsity of the inversion in a transformed domain, such as gradient domain (Chartrand, 2012; Sidky et al., 2013) and wavelet domain (Candes et al., 2006).

To improve velocity reconstructions with sparse seismic data, we have recently developed an elastic-waveform inversion method with a compressive sensing technique (Lin and Huang, 2015). We used a gradient-domain compressive sensing technique and an L_p norm minimization scheme in our elastic-waveform inversion method, and employed an alternating direction method of minimization to solve the minimization problem (Lin and Huang, 2015).

In this study, we demonstrate the capability of our new three-dimensional elastic-waveform inversion method with compressive sensing using synthetic 3D VSP data from a geophysical model of the Raft River geothermal field. Our numerical example shows that our new inversion method can preserve the accuracy of velocity inversion using only a fraction of 3D VSP data acquired with a dense source array.

2. METHOD

We briefly describe our recently developed elastic-waveform inversion method with a compressive sensing technique (Lin and Huang, 2015). Elastic-wave propagation is a forward problem that relates elastic-wavefields \mathbf{w} to model parameters \mathbf{m} as

$$\mathbf{w} = f(\mathbf{m}), \tag{1}$$

where f is the propagation operator. Model parameters \mathbf{m} include a source term and elastic properties of media through which seismic waves propagate. Given a source term and an initial velocity model, we solve a three-dimensional elastic-wave propagation problem using a high-order finite-difference elastic-wave scheme with a perfectly matched layer absorbing boundary condition.

Elastic-waveform inversion progressively fits synthetic elastic waveforms with recorded elastic waveforms \mathbf{d} to obtain elastic properties \mathbf{m} . We pose this inversion as the minimization of the following cost (or objective) function:

$$E(\mathbf{m}) = \min_{\mathbf{m}} \{\|\mathbf{d} - f(\mathbf{m})\|_2^2\}, \quad (2)$$

where $E(\mathbf{m})$ is the misfit function, and $\|\cdot\|_2$ is the L_2 norm. The model parameters \mathbf{m} are solved by minimizing square difference between recorded and synthetic elastic waveforms.

To improve inversion for sparse seismic data, we incorporate a compressive sensing technique in the gradient domain (Chartrand, 2012; Sidky et al., 2013) into our elastic-waveform inversion. The cost function is now given by

$$E(\mathbf{m}) = \min_{\mathbf{m}} \{\|\mathbf{d} - f(\mathbf{m})\|_2^2 + \lambda \|\nabla \mathbf{m}\|_p^p\}, \quad (3)$$

where $\|\nabla \mathbf{m}\|_p^p$ is the compressive sensing term, and λ is a regularization parameter.

For a 3D model, the compressive sensing term has the form of

$$\|\nabla \mathbf{m}\|_p^p = \sum_{i,j,k} (|(\nabla_x \mathbf{m})_{i,j,k}| + |(\nabla_y \mathbf{m})_{i,j,k}| + |(\nabla_z \mathbf{m})_{i,j,k}|)^p, \quad (4)$$

where $(\nabla_x \mathbf{m})_{i,j,k} = \mathbf{m}_{i+1,j,k} - \mathbf{m}_{i,j,k}$, $(\nabla_y \mathbf{m})_{i,j,k} = \mathbf{m}_{i,j+1,k} - \mathbf{m}_{i,j,k}$, and $(\nabla_z \mathbf{m})_{i,j,k} = \mathbf{m}_{i,j,k+1} - \mathbf{m}_{i,j,k}$.

According to Chartrand (2012), we select $p = 1/2$, and employ an alternating direction method of minimization to solve Eq. (3).

3. MODELS AND INVERSION RESULTS

We demonstrate the capability of our new three-dimensional elastic-waveform inversion method with compressive sensing for velocity inversion using a 3D geophysical model of the Raft River geothermal field in Idaho (see a 2D slice in Fig. 1). This 3D geophysical model contains several horizontal layers and a vertical Narrows Zone (a low-velocity vertical structure) below the depth of approximately 1.6 km (Fig. 1). This low-velocity Narrows zone is north-east trending, and represents a basement shear (Mabey et al., 1978; Ayling and Moore, 2013). VSP receivers are placed within a well extending to the depth of approximately 1.75 km and crossing part of the low-velocity Narrows Zone.

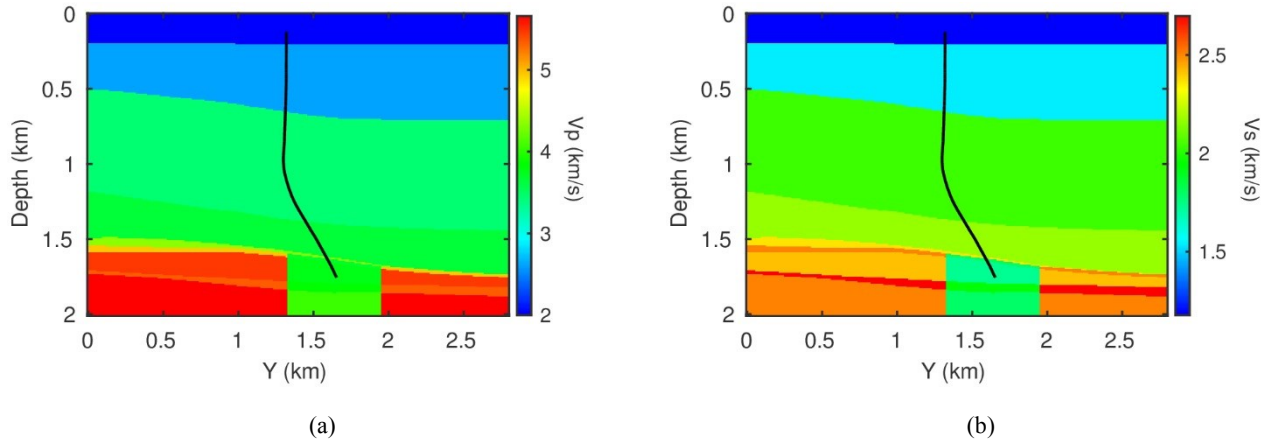


Figure 1: One cross section of true velocity models built for the Raft River geothermal field: (a) compressional-wave velocity, (b) shear-wave velocity. Black line indicates the location of the VSP well.

We conduct elastic-waveform inversion using synthetic 3D VSP data from the 3D velocity models of the Raft River geothermal field. In our synthetic VSP study, geophones are placed within a well with a vertical spacing of 8 m from the depth of 0.3 km to 1.75 km. A total of 740 sources are evenly distributed at the top surface of the 3D model with a horizontal spacing of 38 m. The source is a vertical force, and the source time function is a Ricker wavelet with a center frequency of 15 Hz.

We first generate synthetic 3D VSP elastic waveforms for all sources and receivers using the true velocity models shown in Fig. 1. Then we conduct elastic-waveform inversion using synthetic 3D VSP elastic waveforms and some starting velocity models (Fig. 2). The smoothed starting velocity models are created by averaging the slownesses of true velocity models over one wavelength for the center frequency of the source wavelet. The differences between true and starting models are most significant near the interfaces (Fig. 3).

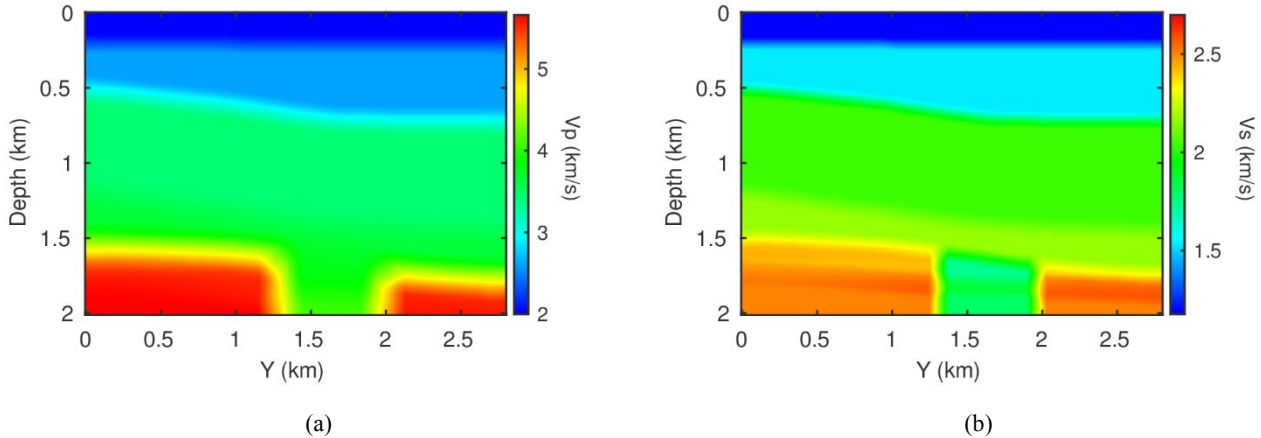


Figure 2: One cross section of smoothed velocity models used as the starting models for elastic-waveform inversion: (a) compressional-wave velocity, (b) shear-wave velocity.

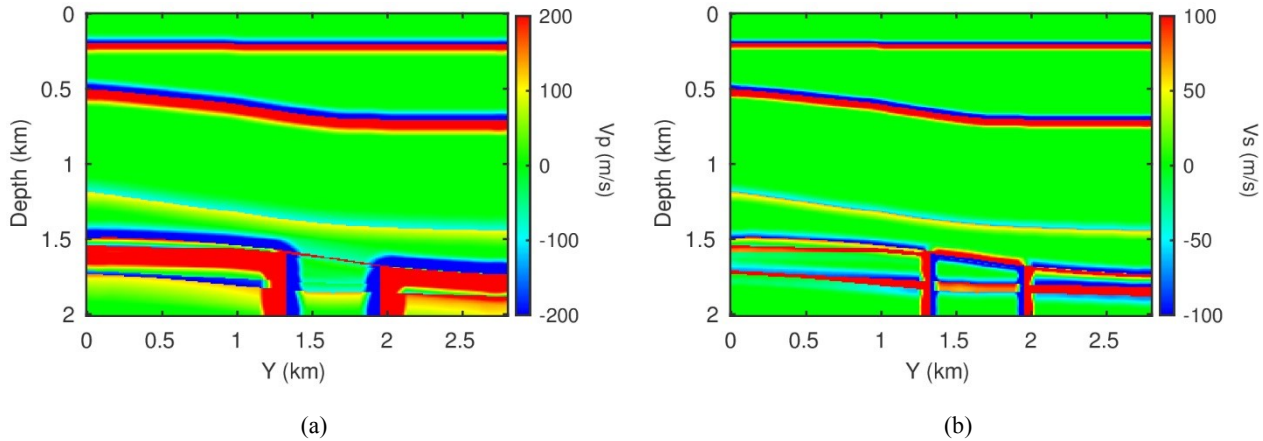


Figure 3: One cross section of velocity difference between true and starting models: (a) compressional-wave velocity difference, (b) shear-wave velocity difference.

We first conduct elastic-waveform inversion using all synthetic 3D VSP data from 740 sources, and then perform elastic-waveform inversion with compressive sensing using only 1/4 of the synthetic 3D VSP data, that is, the sparse data from 185 sources. The velocity updates from these two inversions are shown in Fig. 4 and Fig. 5 respectively. Comparing with Fig. 3, both inversions recover part of the velocity differences between true and starting models. Even with only the VSP data from 1/4 of the sources (a sparse source array), elastic-waveform inversion with compressive sensing is able to reconstruct velocity models similar to those obtained using elastic-waveform inversion with the VSP data from all the sources in a dense source array.

4. CONCLUSIONS

We have studied the capability of our recently developed elastic-waveform inversion method with a compressive sensing technique for 3D VSP inversion. The method incorporates an L_p norm for regularization, and is able to reconstruct a velocity model using sparse seismic data. We apply our method with compressive sensing to only a fraction of three-dimensional synthetic VSP data for the Raft River geothermal field, and compare the results with those obtained using conventional elastic-waveform inversion for a full data set from a dense source array. We demonstrate that our new method has the capability of preserving the accuracy of elastic velocity inversion with 3D VSP data acquired with a sparse source array.

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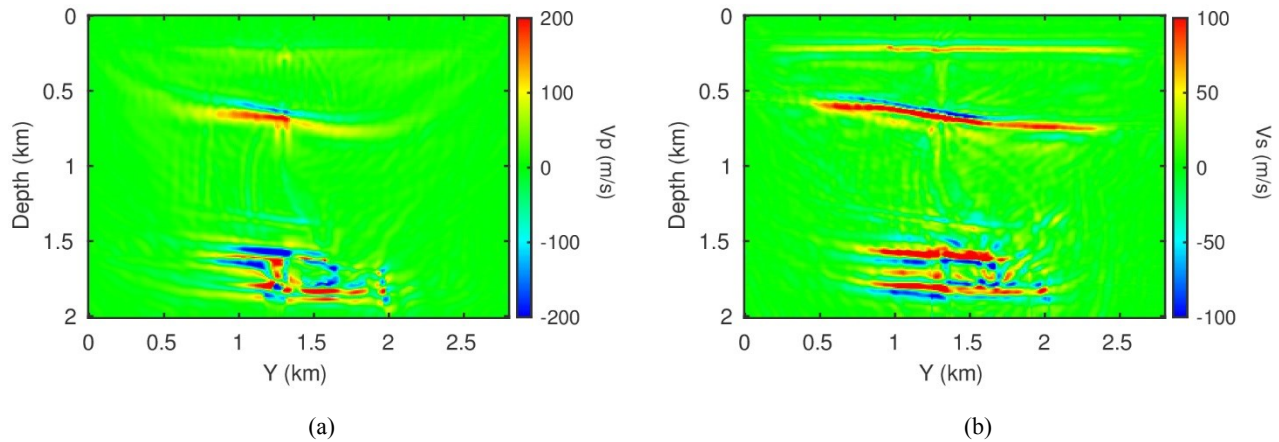


Figure 4: One cross section of velocity updates using conventional elastic-waveform inversion with full data set: (a) compressional-wave velocity, (b) shear-wave velocity.

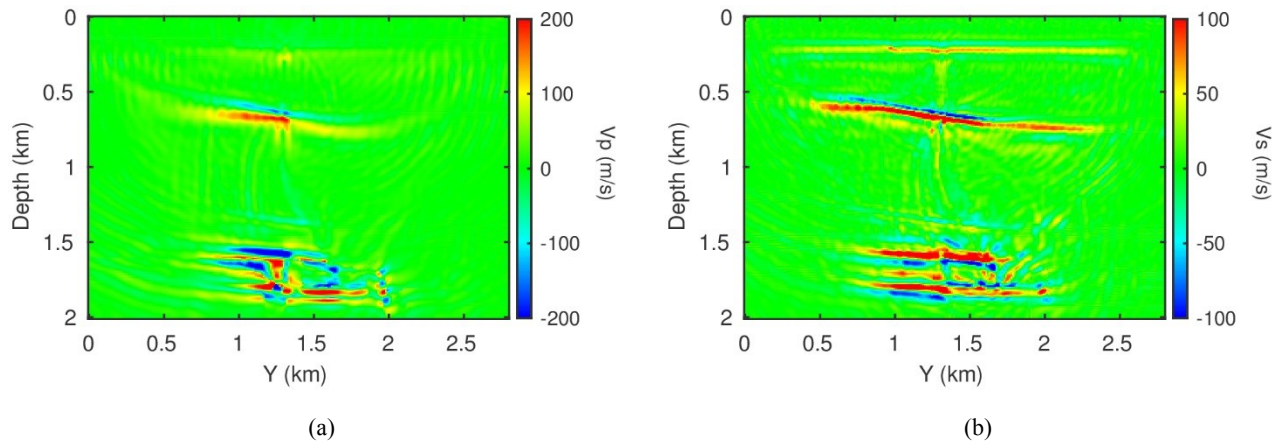


Figure 5: One cross section of velocity updates using elastic-waveform inversion with compressive sensing for 1/4 of full data set: (a) compressional-wave velocity, (b) shear-wave velocity.

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