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QUANTIFYING CHANGES OF GEOPHYSICAL PROPERTIES IN SMALL REGIONS WITHIN EGS RESERVOIRS

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ABSTRACT

Double-difference elastic-waveform inversion is a promising tool for quantitatively monitoring reservoir changes in enhanced geothermal systems (EGS). The traditional Tikhonov regularization used in waveform inversion tends to smooth reconstruction results and gives inaccurate values of changes of geophysical properties in small regions within EGS reservoirs. We develop a new double-difference elastic-waveform inversion method with a modified total-variation (TV) regularization to improve the quantification of reservoir changes within small regions. The double-difference elastic-waveform inversion method inverts for reservoir changes utilizing differences of time-lapse seismic data and the compressional- (P-) and shear-wave (S-wave) velocity models obtained using waveform inversion of the baseline data. The modified TV regularization improves not only the reconstruction of the baseline models, but also reservoir changes, particularly in small regions within EGS reservoirs. We use synthetic time-lapse seismic data for a Brady's EGS model to verify our new method. Our numerical results clearly demonstrate that our double-difference elastic-waveform inversion with a modified total-variation regularization scheme can accurately reconstruct the changes in small regions within an EGS reservoir.

INTRODUCTION

Quantitative monitoring for enhanced geothermal systems can help optimize the geothermal production and the placement of new wells. Conventionally, reservoir changes are obtained from differences of independent inversions of time-lapse data. Full-waveform inversion is a quantitative method for estimating subsurface geophysical properties. It can be implemented in both the time domain (Tarantola, 1984; Mora, 1987) and the frequency domain (Pratt et al., 1998; Sirgue and Pratt, 2004). In recent years, many new full-waveform inversion schemes were developed, for example, those based on regularization (Hu et al., 2009; Burstedde and Ghattas, 2009; Ramirez and Lewis, 2010), *a priori* information (Ma et al., 2010), preconditioning (Guitton and Ayeni, 2010; Tang and Lee, 2010), and dimensionality reduction (Moghaddam and Herrmann, 2010). Images of the conventional approach for time-lapse seismic data usually contain significant image noise and artifacts, and the values of changes in geophysical properties are not accurate. Watanabe et al. (2004) proposed a differential waveform tomography method in the frequency domain for time-lapse crosswell seismic data, and clearly showed its improvement compared to the conventional method. Denli and Huang (2009) introduced a double-difference elastic-waveform tomography method in the time domain for time-lapse surface seismic reflection data. These methods jointly invert time-lapse seismic data for reservoir changes.

The reconstruction of the baseline models plays an important role in the double-difference waveform inversion. To further improve the accuracy and robustness of double-difference waveform inversion, we develop an elastic-waveform inversion method with a modified TV regularization scheme to reconstruct the baseline models. We apply the modified TV regularization scheme to the double-difference elastic-waveform inversion method, and test the new method using time-lapse synthetic seismic reflection data for a Brady's EGS reservoir model.

THEORY

Elastic-waveform Inversion

The forward modeling of elastic-wave propagation can be written as

$$\mathbf{p} = f(\mathbf{K}, \boldsymbol{\rho}, \mathbf{s}), \quad (1)$$

where the function of f is the propagation operator, $\rho(\mathbf{r})$ is the density, $K(\mathbf{r})$ is the bulk modulus, and $\mathbf{s}$ is the source term. Numerical techniques, such as finite difference and spectral element methods, can be used to solve for forward problem. Let $\mathbf{m}$ be the model parameters, Eq. (1) becomes

$$\mathbf{p} = f(\mathbf{m}). \quad (2)$$

To invert the forward modeling given by Eq. (2), we need to solve the minimization problem

$$E(\mathbf{m}) = \min_{\mathbf{m}} \left\{ \| \mathbf{d} - f(\mathbf{m}) \|_2^2 \right\}, \quad (3)$$

where $E(\mathbf{m})$ is the misfit function, $\|\cdot\|_2$ stands the L_2 norm, and $\mathbf{d}$ represents the recorded waveforms. The resulting model $\mathbf{m}$ minimizes square difference between observed and synthetic waveforms.

Double-difference Elastic-Waveform Inversion

Conventionally, two independent inversions in (3) are carried out to obtain the time-lapse changes in reservoir, that is

$$\delta \mathbf{m}_{\text{conv}} = f^{-1}(\mathbf{d}_{\text{time 2}}) - f^{-1}(\mathbf{d}_{\text{time 1}}), \quad (4)$$

where f^{-1} means the general inverse of waveform data, and $\mathbf{d}_{\text{time 1}}$ and $\mathbf{d}_{\text{time 2}}$ are data collected at two different times.

For double-difference elastic-waveform inversion, the data misfit in the cost function is replaced by

$$\delta d \equiv (\mathbf{d}_{\text{time 2}} - \mathbf{d}_{\text{time 1}}) - (\mathbf{d}_{\text{sim\_time 2}} - \mathbf{d}_{\text{sim\_time 1}}), \quad (5)$$

where the first term is the time-lapse difference in data, and the second term is the difference in synthetic time-lapse data. The method uses time-lapse seismic data to jointly invert for changes in reservoir geophysical properties.

Total-Variation Regularization

A general form of regularization can be usually written as,

$$E(\mathbf{m}) = \min_{\mathbf{m}} \left\{ \|\mathbf{p} - f(\mathbf{m})\|_2^2 + \lambda \mathrm{R}(\mathbf{m}) \right\}, \quad (6)$$

where $R(\mathbf{m})$ is the regularization term, whose form depends on the type of the regularization being invoked, Tikhonov regularization and TV regularization being the most common. Mathematically, the TV-norm for 2D model can be defined as,

$$\| \mathbf{m} \|_{\text{TV}} = \sum_{1 \le i,j \le n} \sqrt{ |(\nabla_x \mathbf{m})_{i,j}|^2 + |(\nabla_z \mathbf{m})_{i,j}|^2 }, \quad (7)$$

with the gradient terms $(\nabla_x \mathbf{m})_{i,j} = f_{i+1,j} - f_{i,j}$ and $(\nabla_z \mathbf{m})_{i,j} = f_{i,j+1} - f_{i,j}$. Because of the nonlinearity of the TV term in Eq. (7) and its non-differentiability at the origin, problems arise when optimizing Eq. (7). Therefore, various approaches have been used to approximate the original TV function so that it can be differentiated. One common method to approximate TV is to introduce a small constant $\grave{o}$ in the TV term,

$$\|\mathbf{m}\|_{\text{TV},\grave{o}} = \sum_{1 \le i,j \le n} \sqrt{ |(\nabla_x \mathbf{m})_{i,j}|^2 + |(\nabla_z \mathbf{m})_{i,j}|^2 + \grave{o} }, \quad (8)$$

so that $\| \mathbf{m} \|_{\text{TV},\grave{o}}$ becomes differentiable, and the gradient and Hessian can be calculated. However, as pointed out in (Huang et al., 2008; Vogel, 2002; Li and Santosa, 1996), TV solvers using such an approximation usually suffer a slow convergence rate or even fail to converge to the proper solution. Therefore, the traditional TV formulation and computational methods may not be appropriate for our elastic-waveform inversion. To accelerate the convergence rate and acquire a better reconstruction, a different TV-based regularization is needed.

Modified Total-Variation Regularization

In recent years, researchers have proposed various approximate form of TV-based regularization to yield a faster convergence. Following the work of (Huang et al., 2008; Vogel, 2002), we develop a modified TV regularization for (double-difference) elastic-waveform inversion. The cost function with the modified TV regularization is given as (Huang et al., 2008),

$$E(\mathbf{m}, \mathbf{u}) = \min_{\mathbf{m},\mathbf{u}} \{ \|\mathbf{p} - f(\mathbf{m})\|_2^2 + \lambda_1 \|\mathbf{m} - \mathbf{u}\|_2^2 + \lambda_2 \|\mathbf{u}\|_{\text{TV}} \}, \quad (9)$$

where λ_1 and λ_2 are both positive regularization parameters. There is an additional term, $\|\mathbf{m} - \mathbf{u}\|_2^2$, compared to Eq. (6). The modified TV regularization in Eq. (9) can be equivalently written as,

$$E(\mathbf{m}, \mathbf{u}) = \min_{\mathbf{u}} \{ \min_{\mathbf{m}} \{ \|\mathbf{p} - f(\mathbf{m})\|_2^2 + \lambda_1 \|\mathbf{m} - \mathbf{u}\|_2^2 \} + \lambda_2 \|\mathbf{u}\|_{\text{TV}} \}. \quad (10)$$

From the above formulation, the meaning of the two regularization parameters can be inferred. The parameter λ_1 controls the trade-off between the

regularization term and the data misfit, and λ_2 controls the amount of edge-preserving in the reconstruction.

An alternating-minimization algorithm can be therefore employed to solve the double minimization problem in Eq. (10). Beginning with a starting model $\mathbf{u}^{(0)}$, solving for Eq. (10) leads to the solutions of two minimization problems:

$$\begin{aligned} \mathbf{m}^{(k)} &= \underset{\mathbf{m}}{\operatorname{argmin}} \left\| \mathbf{p} - f(\mathbf{m}) \right\|_2^2 + \lambda_1 \left\| \mathbf{m} - \mathbf{u}^{(k-1)} \right\|_2^2, \\ \mathbf{u}^{(k)} &= \underset{\mathbf{u}}{\operatorname{argmin}} \left\| \mathbf{m}^{(k)} - \mathbf{u} \right\|_2^2 + \lambda_2 \left\| \mathbf{u} \right\|_{\mathrm{TV}}, \end{aligned} \tag{11}$$

for i=1, 2, ...

NUMERICAL RESULTS

We use synthetic elastic time-lapse surface seismic data for the models in Figs. 1-4 to demonstrate the improvement of the double-difference elastic-waveform inversion with a modified TV regularization scheme. The models are constructed using geologic features found at the Brady's EGS site. They contain several steep fault zones. There is a region in Figs. 1 and 2, and Figs. 3 and 4 with a decreased velocity caused by water/fluid injection for stimulation, as shown in Figs. 5 and 6. Twenty common-shot gathers of synthetic time-lapse seismic data with 500 receivers along the upper boundary of the models are used to jointly invert for the reservoir changes. The shot interval is 125 m and the receiver interval is 5 m. A Ricker wavelet with a center frequency of 25 Hz is used as the source function.

The implementation of the double-difference waveform inversion requires the reconstruction of the baseline models, which play an important role in obtaining accurate results of EGS reservoir changes. Figures 7 and 8 are used as the initial models for the inversion of the baseline models.

For comparison purposes, we provide the baseline reconstruction using the Tikhonov regularization, where Fig. 9 is the reconstruction of P-wave velocity model and Fig. 11 is the reconstruction of the S-wave velocity model. To better illustrate the reconstruction effectiveness on the model interfaces, we also give the vertical profiles along $x = 1200$ m in Figs. 10 and 12. Figures 13 and 15 illustrate the reconstruction of the baseline models of P-wave velocity and S-wave velocity using the modified TV regularization. Similarly, we also provide the vertical profiles along $x = 1200$ m in Figs. 14 and 16.
We observe that the model interfaces between different layers are much better preserved in Figs. 13 and 15 than in those in Figs. 9 and 11. The image artifacts and noise in Figs. 13 and 15 are also suppressed better than those in Figs. 9 and 11. The vertical profile gives a better visualization of the quantitative values. By looking at Figs. 10 and 12, and Figs. 13 and 15, the reconstruction using the modified TV regularization yields a result that closely matches the true image, while the Tikhonov reconstruction does not.

Once we obtain the reconstructions of the baseline models, we can use them for the double-difference elastic-waveform inversion. To ensure an accurate quantitative reconstruction, we use the spatially-variant regularization technique in Ref (Lin et al., 2011). The resulting reconstructions of P-wave velocity and S-wave velocity changes are illustrated in Figs. 17 and 19. Their vertical profiles are shown in Figs. 18 and 20 to better visualize the quantitative values. Both double-difference elastic-waveform inversions yield very good results in terms of both the location and quantitative values.

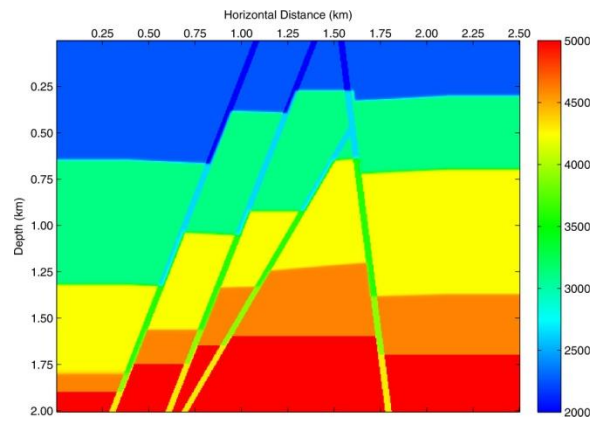

Figure 1: P-wave velocity of a baseline EGS model.

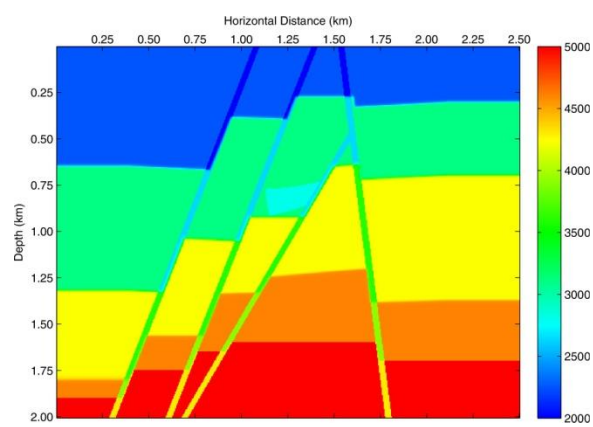

Figure 2: Time-lapse P-wave velocity model after stimulation.

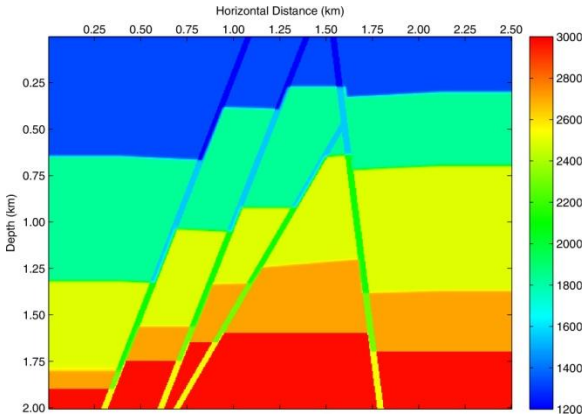

Figure 3: S-wave velocity of the baseline EGS model.

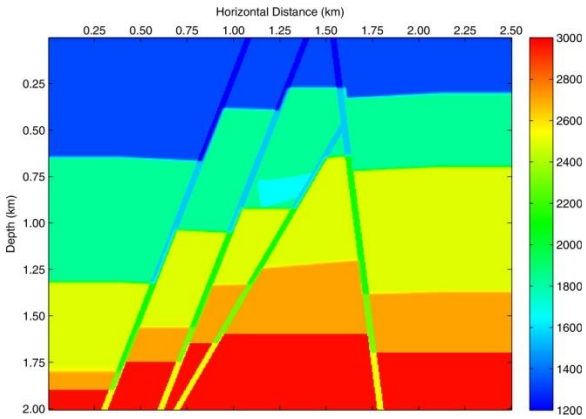

Figure 4: Time-lapse S-wave velocity after stimulation.

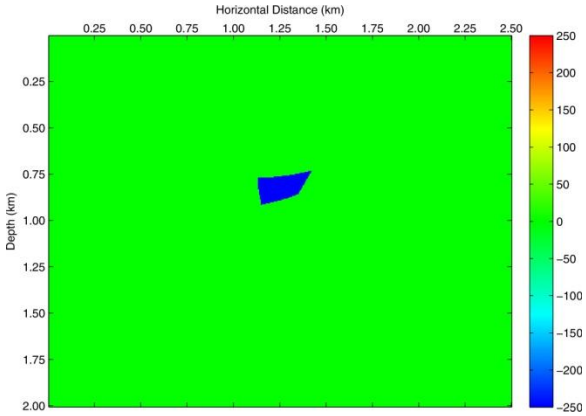

Figure 5: Time-lapse difference of P-wave velocity.

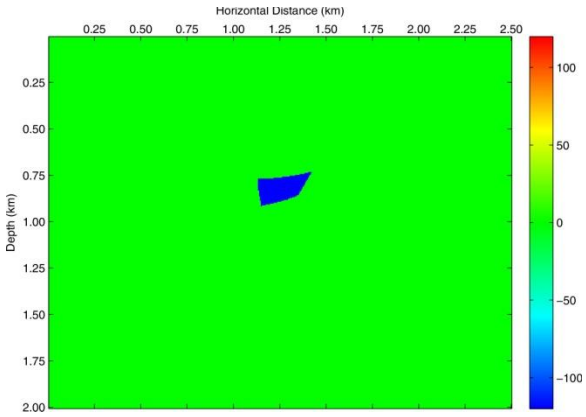

Figure 6: Time-lapse difference of S-wave velocity.

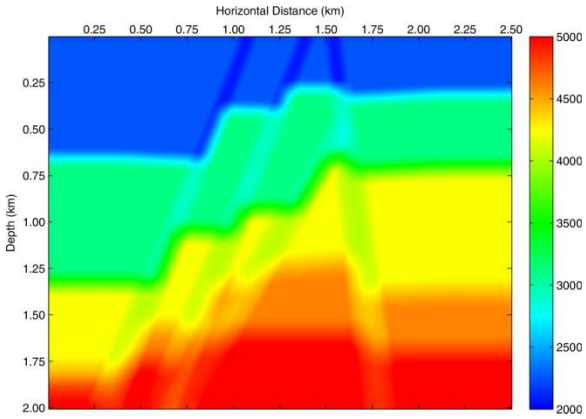

Figure 7: Initial guess of the baseline P-velocity model.

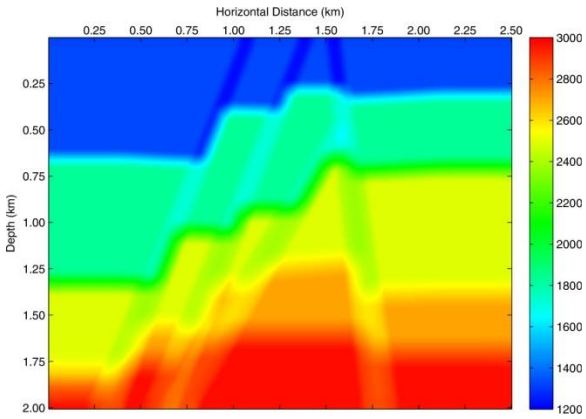

Figure 8: Initial guess of the baseline S-velocity model.

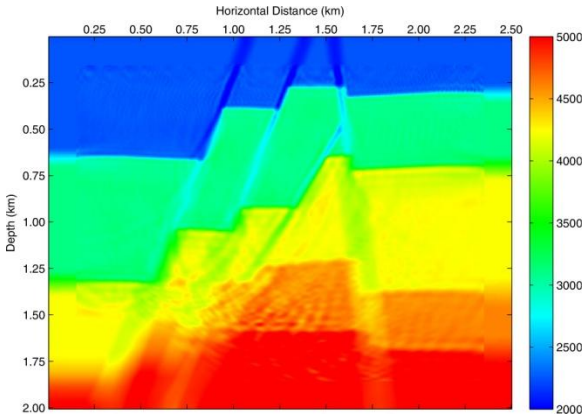

Figure 9: Tikhonov reconstruction result of P-wave velocity using the baseline seismic data.

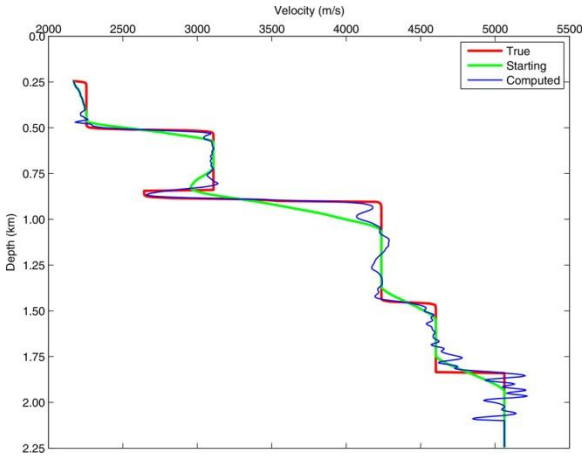

Figure 10: A vertical profile of the Tikhonov reconstruction of P-wave velocity using the baseline seismic data.

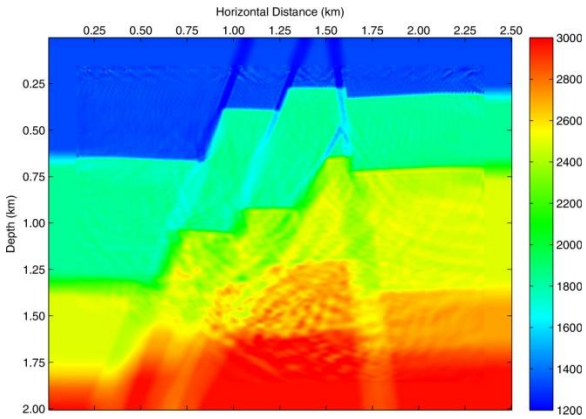

Figure 11: Tikhonov reconstruction of S-wave velocity using the baseline seismic data.

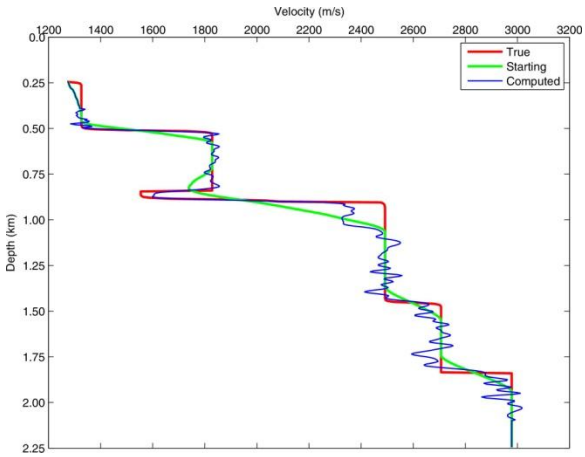

Figure 12: A vertical profile of the Tikhonov reconstruction of S-wave velocity using the baseline seismic data.

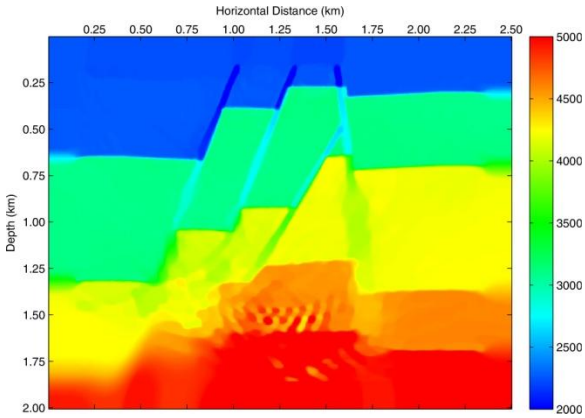

Figure 13: Modified TV reconstruction of P-wave velocity using the baseline seismic data.

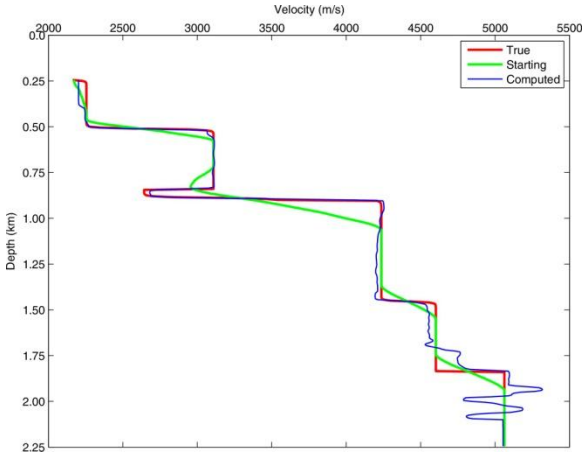

Figure 14: A vertical profile of the Modified TV reconstruction of P-wave velocity using the baseline seismic data.

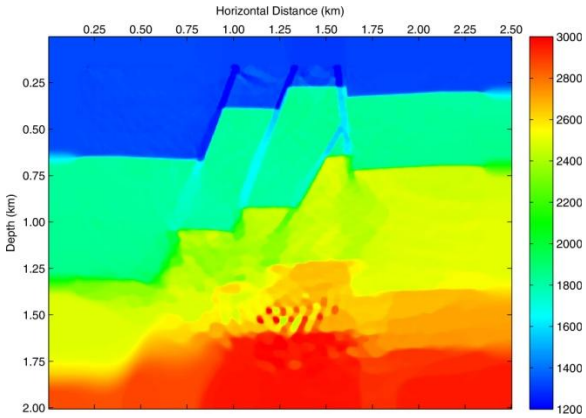

Figure 15: Modified TV reconstruction of S-wave velocity using the baseline seismic data.

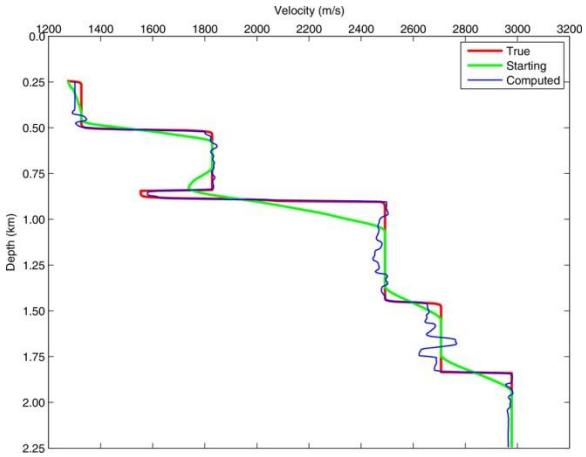

Figure 16: A vertical profile of the Modified TV reconstruction of S-wave velocity using the baseline seismic data.

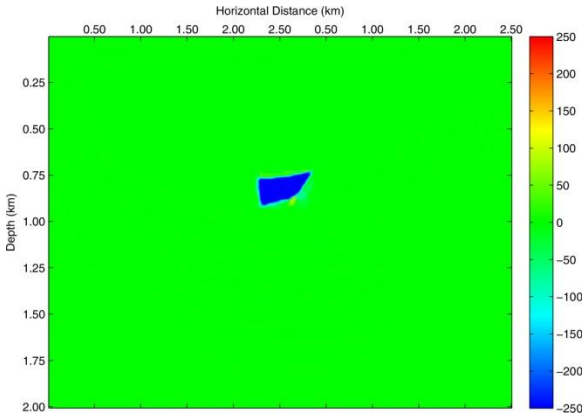

Figure 17: Reconstruction of the P-wave velocity change using double-difference waveform inversion.

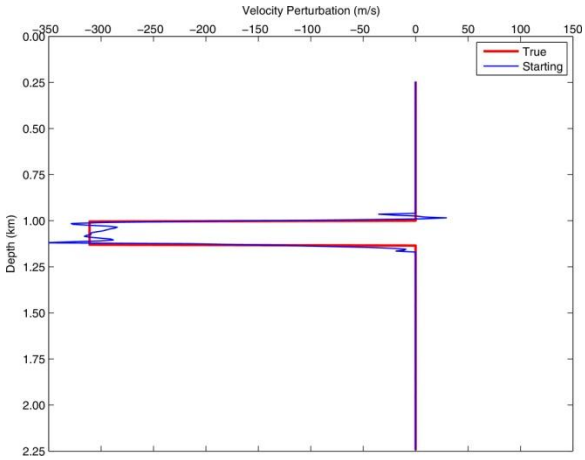

Figure 18: A vertical profile of the reconstructed P-wave velocity changes obtained using the double-difference waveform inversion.

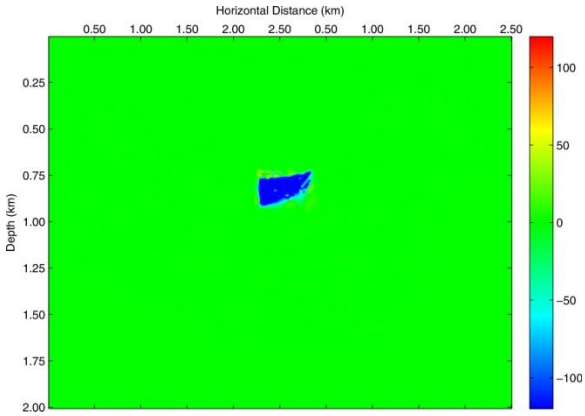

Figure 19: Reconstruction of the S-wave velocity change obtained using the double-difference waveform inversion.

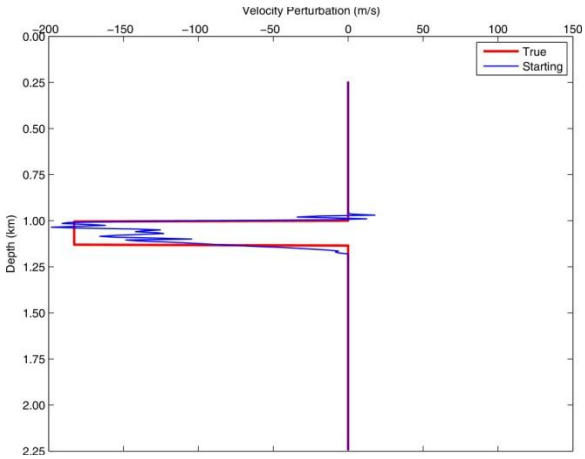

Figure 20: A vertical profile of the S-wave velocity changes obtained using the double-difference waveform inversion.

CONCLUSIONS

We have developed a modified TV regularization scheme for elastic-waveform inversion and double-difference elastic-waveform inversion. The resulting reconstructions not only very well preserve the model interfaces, but also significantly suppress the image noise. Our results of synthetic time-lapse seismic data for a Brady's EGS model demonstrate that our new method can accurately reconstruct values of velocity changes caused by water/fluid injection for geothermal stimulation. The double-difference elastic-waveform inversion with the modified total-variation regularization can quantify the spatial and temporal changes in reservoirs of enhanced geothermal systems using time-lapse seismic data.

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