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THREE-DIMENSIONAL MODELING OF INJECTION INDUCED THERMAL STRESSES WITH AN EXAMPLE FROM COSO

A. Ghassemi and S. Tarasovs

Department of Geology & Geological Engineering
University of North Dakota
Grand Forks, ND 58202

ABSTRACT

Poro-mechanical, thermal, and chemical processes can play a significant role when developing enhanced geothermal systems. These processes occur on various time scales and the significance of their interaction varies with the problem of interest. Of particular importance is the thermo-mechanical coupling during injection operations (time scale of months/years). In fact, the phenomena of the variation of injectivity with injection water temperature and reservoir seismicity can be attributed to thermal stresses. In this paper a three-dimensional integral equation formulation is presented for calculating thermally induced stresses associated with cooling of a fracture in a geothermal reservoir. The procedure is then implemented in a computer program and is used to treat the problem of injection into an infinite fracture. The thermally induced stresses are calculated using actual field data for an injection experiment. The resulting calculations are found to be consistent with those based on a semi-analytical solution as well as field observations.

INTRODUCTION

Thermally-induced stresses significantly contribute to seismicity in petroleum and geothermal fields (Sherburn, 2002; Stark, 1990). The variation of injectivity with injection water temperature (Petty, 2002) and reservoir seismicity in geothermal fields have been attributed to thermally-induced stresses. Stark (1990) has found that half the earthquakes in The Geysers field seem to be associated with cold water injection. The mechanism by which seismicity occurs is well understood namely, shear slip on natural fractures resulting from a reduction in effective stress acting across the fracture. The magnitude of the thermal stresses associated with advective cooling has been estimated analytically (Mossop, 2001) using an axisymmetric model of injection into a planar reservoir and a 1D heat flow in the rock mass. It has been shown that one- and two-dimensional heat flow models underestimate heat transfer to the fluid from the crack (Ghassemi *et al.* 2003). Thus, rock cooling and the associated thermal stresses should be studied using three-dimensional heat transfer and stress models. This requires coupling a 3D heat flow model to a 3D elasticity model. A reason for ignoring the three-dimensional nature of heat conduction in the reservoir is the difficulty in treating the infinite geothermal reservoir geometry by numerical discretization. However, it has been demonstrated (Ghassemi *et al.* 2003) that by using 3D Green's function for heat conduction and the integral equation formulation the need for discretizing the 3D reservoir is completely eliminated. In this paper we present a 3D integral equation formulation for calculating thermally induced stresses associated with cooling of a planar fracture in an infinite reservoir. A brief presentation of the fluid flow/heat transfer model is also provided for the sake of completeness. Additional details regarding the heat transfer modeling can be found in (Ghassemi *et al.* 2003).

FLUID FLOW & HEAT TRANSFER

A schematic view of heat extraction from a fracture or a fracture zone in rock is illustrated in Figure 1. With only a few exceptions such as the finite element solution by Kolditz (1995), Kolditz and Clauser (1998), and Kohl, *et al.* 1995, and the boundary element model by Cheng, *et al.* 2001, the heat conduction in the reservoir is typically modeled as one-dimensional heat flow perpendicular to the fracture surface (Bodvarsson, 1982; Heur, 1991; Kruger *et al.* 1991) The primary reason for such simplification is the inefficiency in modeling an unbounded three-dimensional domain by numerical discretization. The numerical difficulty can be overcome by utilizing the integral equation formulation and the three-dimensional Green's function of heat conduction.

In this work, the fracture is assumed to be flat, of finite size, and with arbitrary shape. The geothermal reservoir, on the other hand, is of infinite extent. Other physical assumptions are similar to these postulated in Cheng, *et al.* 2001. Specifically, it is assumed that the geothermal reservoir is impermeable to water, has constant heat conduction properties, and is non-deformable. The heat storage and dispersion effects in the fracture fluid flow are negligible. and production rate of hot water is equal to the injection rate in the fracture. It is further postulated that the fracture width is small such that the flow in the fracture is laminar and governed by the lubrication flow equation.

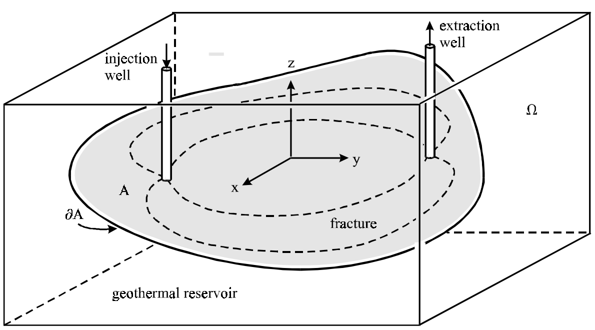

Figure 1: Heat extraction from a planar fracture.

In dealing with the governing equations of the problem, it is desirable to have a normalized solution field that becomes dimensionless with a value between zero and one. Therefore, a normalized temperature deficit is introduced:

$$T_d = \frac{T_o - T}{T_o - T_{inj}} \quad (1)$$

The heat transport occurs both in the geothermal reservoir and the fracture. For the geothermal reservoir, the heat conduction is governed by the three-dimensional diffusion equation:

$$K_r \nabla_3^2 \, T_d(x,y,z,t) = \rho_r c_r \frac{\partial T_d(x,y,z,t)}{\partial t} \quad (2)$$

where ρ_r is the rock density, c_r is the specific heat of rock, ∇_3^2 is the Laplacian operator in three dimensions, and Ω represents the infinite geothermal reservoir (Figure 1).

For heat transport in the fracture the governing equation is (Ghassemi *et al.* 2003):

$$\mathbf{q}(x,y) \cdot \nabla_2 \, T_d(x,y,0,t) = 2K_r \left. \frac{\partial T_d(x,y,z,t)}{\rho_w c_w \partial z} \right|_{z=0^+} \quad (3)$$

The governing equations are subject to initial and boundary conditions. Prior to the heat extraction operation, the temperature of the rock and the fracture fluid is assumed to be at a constant, $T(x,y,z,0)=T_o$ and at the injection point $(x_i, y_i, 0)$, the temperature equals that of the injected water: $T(x_i, y_i, 0, t)=T_{inj}$. The extraction temperature $T(x_e, y_e, 0, t)$ is unknown. The initial and boundary conditions can be expressed in terms of T_d:

$$T_d(x,y,z,0) = 0 \quad (4)$$

$$T_d(x_i, y_i, 0, t) = 1 \quad (5)$$

where ρ_w is the water density, c_w is the specific heat of water, K_r is the rock thermal conductivity, and T_{inj} is the injection water temperature. In the above, the term on the left hand side of (3) represents the heat advection by fracture fluid flow, the first term on the right hand side gives the heat supply through the fracture walls (two sides) by conduction, and the last two terms correspond to a heat sink and a heat source respectively caused by the extraction and injection of water. It should be noted that in (3) a single notation $T_d(x,y,z,t)$ is used to denote the temperature of the rock and the fracture fluid, because temperature is continuous across the two media. The water temperature T_w is equal to the rock temperature on the fracture plane $z = 0$, i.e., $T_w(x,y,t) = T(x,y,0,t)$, $x, y \in A$.

To facilitate the treatment of the time variable, we apply Laplace transform to the above equations and obtain:

$$K_r \nabla_3^2 \, \widetilde{T}_d(x,y,z,s) = s \rho_r c_r \widetilde{T}_d(x,y,z,s) \quad (6)$$

$$\mathbf{q}(x,y) \cdot \nabla_2 \, \widetilde{T}_d(x,y,0,s) = \frac{2K_r}{\rho_w c_w} \left. \frac{\partial \widetilde{T}_d(x,y,z,s)}{\partial z} \right|_{z=0^+} \quad (7)$$

$$\widetilde{T}_d(x_i, y_i, 0, s) = \frac{1}{s} \tag{8}$$

where the wiggle overbar denotes the Laplace transform, and s is the transform parameter. Note that the initial condition (4) has been absorbed into (6) thus, equations (6)–(8) form a complete solution system.

INTEGRAL EQUATION

The system of equations (6)–(8) is defined in three spatial dimensions. It has been demonstrated that by utilizing Green's function of three-dimensional diffusion equation, the solution system can be reduced to a two-dimensional integral equation (Cheng *et al.* 2001). The numerical discretization is performed on the fracture surface only, significantly reducing the computational cost. For the temperature on the fracture surface ($z = 0$) we have:

$$\widetilde{T}_d = \gamma \int_A \left[q(x', y') \cdot \nabla_2 \widetilde{T}_d(x', y', 0, s) \right] \varpi' dx'dy' \tag{9}$$

where $\gamma = \frac{-\rho_w c_w}{4\pi K_r}$ and:

$$\varpi' = \frac{1}{r} \exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\, r \right)$$

$$r = \sqrt{(x-x')^2 + (y-y')^2} \tag{10}$$

Equation (9) is entirely defined on the two-dimensional plane A. Together with the boundary condition (8), it forms a complete solution system for fluid temperature in the fracture. Equation (9) contains the temperature gradient as an unknown, which requires a finite difference approximation in numerical solution. The scheme for solving the system represented by (9) is shown in Ghassemi *et al.* 2003.

THERMOELASTICITY

When rocks are heated/cooled, the bulk solid as well as the pore fluid tend to undergo expansion/contraction. A volumetric expansion can result in significant pressurization of the pore fluid depending on the degree of containment and the thermal and hydraulic properties of the fluid as well as the solid. The net effect is a coupling of thermal and poromechanical processes. The theory of poroelasticity couples pore pressure and solid stress fields in deformable fluid saturated porous rocks. In the isothermal poroelastic theory, the time dependent fluid flow is incorporated by combining the fluid mass conservation with Darcy's law; and the basic constitutive equations relate the total stress to both the effective stress given by deformation of the rock matrix and the fluid pore pressure. Thermally induced pore pressure and stresses can also play an important role in geothermal reservoir development, for example in reservoir stimulation by hydraulic fracturing and borehole stability. In order to consider the influence of a temperature gradient on both pore pressure and stresses, it is necessary to use a non-isothermal poroelastic theory, or poro-thermo-elasticity (Ghassemi and Zhang, 2004). However, in the current three-dimensional analysis, a thermoelastic approach is used that combines the theory of heat conduction with elastic constitutive equations. Thus, only the coupling between the temperature and stress fields is included .

Let us consider an isotropic and homogeneous elastic body, subjected to permanent strain ε^0_{ij}. The total strain ε_{ij} consists of two parts the initial strain ε^0_{ij} and the elastic strain ε'_{ij}.

$$\varepsilon_{ij} = \varepsilon'_{ij} + \varepsilon^0_{ij} \tag{11}$$

The elastic strain ε'_{ij} is a linear function of the stress:

$$\varepsilon'_{ij} = 2\mu' \sigma_{ij} + \lambda' \delta_{ij} \sigma_{kk} \tag{12}$$

where $2\mu' = \frac{1}{2G}$, $\lambda' = \frac{1}{2G(3\lambda+2G)}$ and G is shear modulus and, $\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$ is a Lame's constant.

In the case of thermoelasticity, the permanent strain ε^0_{ij} can be written in the form:

$$\varepsilon^0_{ij} = \alpha_t \delta_{ij} \Delta T \tag{13}$$

where ΔT is the change of temperature and α_t is the coefficient of thermal expansion ($\frac{\beta_s}{3}$). Substituting equation 11 into 12 and solving the latter equations for the stresses yields:

$$\sigma_{ij} = 2G(\varepsilon_{ij} - \varepsilon^0_{ij}) + \lambda \delta_{ij} (\varepsilon_{kk} - \varepsilon^0_{ij}) \tag{14}$$

or:

$$\sigma_{ij} = 2G\varepsilon_{ij} + (\lambda\varepsilon_{kk} - \gamma T)\delta_{ij} \tag{15}$$

where $\gamma = (3\lambda + 2G)\alpha_t$.

The equilibrium equation in terms of displacements (in the absence of acceleration and body forces) can be written as:

$$Gu_{i,jj} + (\lambda + G)u_{j,ji} = \gamma T_{,i} \tag{16}$$

Introducing Goodier thermoelastic displacement potential, Φ, such that:

$$u_i = \Phi_{,i} \tag{17}$$

and substituting the latter in the equilibrium equations yields:

$$\Phi_{,kk} = mT, \quad m = \gamma/(\lambda + 2\mu) \tag{18}$$

The solution of this equation yields the function Φ, which can then be used to calculate the strain and stresses by means of the following relations:

$$\varepsilon_{ij} = \Phi_{,ij}, \quad \sigma_{ij} = 2\mu(\Phi_{,ij} - \delta_{ij}\Phi_{,kk}) \tag{19}$$

The function Φ can be represented by the Poisson integral:

$$\Phi(x) = -\frac{m}{4\pi}\int_V \frac{T(\xi)dV(\xi)}{R(x,\xi)} \tag{20}$$

However, calculation of the displacement potential using the latter volume integral requires significant computational time, therefore, a different approach is used. Similarly to the approach in the heat transfer part of the problem, the thermoelastic stresses are treated using the superposition of stresses induced by a distribution of instantaneous heat sources to generate the thermoelastic displacement potential.

Consider an instantaneous heat source acting at the origin of the coordinate system. The solution of the heat conduction equation for this case is known as (Carslaw and Jaeger, 1959):

$$T = \frac{Q}{\pi\vartheta^{3/2}\rho c}\exp\left(-\frac{R^2}{\vartheta}\right) \tag{21}$$

where $\vartheta = 4\varkappa t$, $\varkappa = \frac{K_r}{\rho c}$, Q is source strength, K_r is thermal conductivity of rock, ρ is the rock density, C_r is the specific heat capacity of rock, t is time, and R is the distance from the source to the point of observation. According to Nowacki (1973), the thermoelastic displacement potential Φ, in Laplace space, for an instantaneous point heat source of unit strength is given by:

$$\tilde{\Phi}^{*} = -\frac{m}{4\pi sR\rho c}\left(1 - \exp^{-\beta R}\right) \tag{22}$$

with $\beta = \sqrt{s/\varkappa}$. Or, after inverse Laplace transform:

$$\Phi^{*}(x,t) = -\frac{m}{4\pi R\rho c}\,\mathrm{erf}\left(\frac{R}{\sqrt{\vartheta}}\right) \tag{23}$$

Then, the displacement potential for a system of distributed non-stationary heat sources at time t can be calculated via a multi-dimensional integral:

$$\Phi(x,t) = \int_0^t\int_V Q(x,t)\Phi^{*}(x-\xi, t-\tau)\; d\tau\; d\xi \tag{24}$$

Performing the Laplace transform by using $L\int_0^t f_1(t-\tau)f_2(\tau)d\tau = L\left[f_1(t)\right]\cdot L\left[f_2(t)\right]$; yields:

$$\tilde{\Phi}(x,s) = \int_V \widetilde{Q}(x,s)\tilde{\Phi}^{*}(x-\xi,s)\; d\xi \tag{25}$$

where $\widetilde{Q}$ is a heat source intensity function in Laplace space. After inverse Laplace transform for displacement potential, the stresses can be calculated from the relation:

$$\sigma_{ij} = 2\mu(\Phi_{,ij} - \delta_{ij}\Phi_{,kk}) \tag{26}$$

HEAT SOURCE ON FRACTURE

For the problem of a rock cooled by fluid flow in a fracture, the heat source can be represented by the heat flux through the fracture boundaries:

$$\widetilde{Q} = 2K_r\frac{\partial\widetilde{T}}{\partial z} \tag{27}$$

Or, using the relation:

$$\nabla_2 \cdot \left[\mathbf{q}(x,y)\widetilde{T}(x,y,0,s)\right] = \frac{2K_r}{\rho_w c_w} \left. \frac{\partial \widetilde{T}(x,y,z,s)}{\partial z} \right|_{z=0^+} \quad (28)$$

it can be written as:

$$\begin{aligned} \widetilde{Q} &= \rho_w c_w \nabla_2 \cdot \left[q(x,y)\widetilde{T}(x,y,0,s)\right] = \\ & \rho_w c_w \left[q_x(x,y)\widetilde{T}_{,x} + q_y(x,y)\widetilde{T}_{,y}\right] \quad (29) \end{aligned}$$

Because the temperature distribution inside the fracture is already known in Laplace space, the heat source can be directly calculated in Laplace space as well, and the volume integral over all the heat sources reduces to a two-dimensional integral over the fracture surface.

The Green function for the displacement potential (Eq. 22) has a weak singularity ($1/R$), the fluid flow also has a $1/R$ singularity at the injection point. But, the heat flux through the fracture surface ($q \cdot \nabla T$) is not singular (Eq. 31) for the one-dimensional case). Therefore, overall integral Eq. (25) is only weakly singular. This type of integral, $\frac{f(x,y)}{R}$, can be computed with quite good accuracy using simple 9-point Gaussian procedure. Thus, similarly to the integrals for heat transfer, the thermoelastic stress integral is solved numerically using a 9-point integration procedure that has proven to be sufficiently accurate. The same mesh pattern is used in the thermoelastic portion of the calculation with different square element sizes for different solution times (3.25, 3.5 and 4.25 m). This is because the distance to the boundary of the cooled zone is different for each time and it is divided into the same number of element in each run (80x80 elements). A computer program has been developed based on the above procedures. In order to test the validity of the numerical algorithm and the analytical methods, an example is presented next.

EXAMPLES

We first treat the problem of injection into an infinite planar fracture with the injection well located at the origin (Figure 2). The initial rock temperature is a constant $T = T_o$, and $t = 0^+$ water is injected at a temperature of T_{inj} at a rate Q. The data for this problem are shown in Table 1 and approximates the conditions in **Coso** during stimulation of **Well 86-13**.

Table 1. Input Parameters

E	elastic modulus	3.75E04	MPa
ν	Poisson's ratio	0.25	
ρ_r	rock density	2650	kg/m^3
ρ_w	water density	1000	kg/m^3
C_r	rock heat cap.	790.0	J/(kg·C)
C_w	water heat cap.	4200	J/(kg·C)
c^*	thermal diff.	5.10E−6	m^2/ sec
β_s	rock exp coef.	2.40E−5	m/^{o}C
Q	injection rate	25.0	ℓ/ sec
T_R	rock Temp.	350	F
T_W	injection Temp.	86	F
μ	fluid viscosity	3.55E−4	Pa· sec
h	fracture width	10^{-3}	m

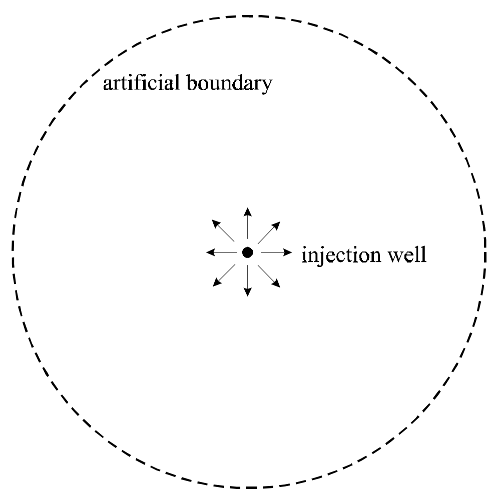

Figure 2: An infinite fracture with an injection well.

This problem can also be solved analytically if one assumes that the heat flow in the rock is one-dimensional (normal to the fracture plane). The temperature distribution can be written in closed-form (Mossop, 2001) as:

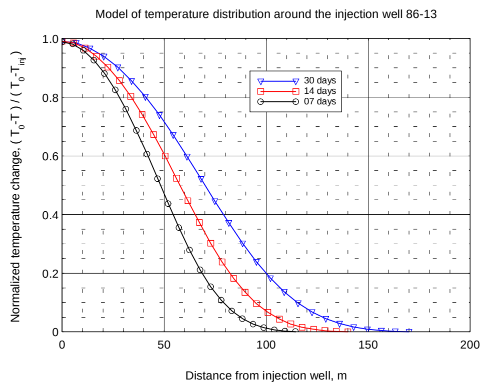

Figure 3: Temperature distribution around the injection point.

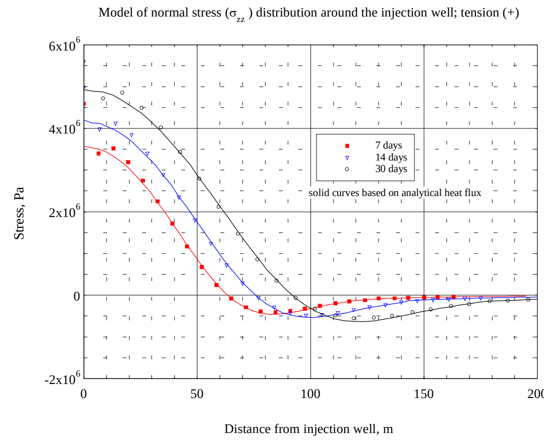

Figure 4: Normal stress distribution on a fracture surface around an injection well.

$$T = T_R - \Delta T \,\mathrm{erf}\, c\left(\frac{ar^2 + bz}{\sqrt{t - Cr^2}}\right) \qquad (30)$$

where $a=\frac{\pi K}{m_r c_w \sqrt{\kappa}}$; $b=\frac{1}{\sqrt{\kappa}}$; $C=\frac{\pi h \rho_w}{m_r}$; and $\Delta T=(T_R\text{-}T_W)$; K is a thermal conductivity of rock, m_r is the injection rate (kg/s); c_w is specific heat capacity of water; $\kappa=K/(c_r\rho_r)$ is the thermal diffusivity of rock; h is fracture thickness; ρ_w is the density of water; c_r is the specific heat capacity of rock; and ρ_r is the density of rock. This solution for T can then used in a hypersingular volume integral to solve for the thermoelastic stresses (Mossop, 2001). In contrast, we solve surface integrals to calculate the temperature distribution and the stresses. However, for the sake of comparison, we also calculate the thermal stresses by using the closed form solution for T (8) to obtain an analytical expression for the fluxes for use in numerical solution for stresses:

$$\widetilde{Q} \approx -2K\Delta T \frac{2b \exp\left[-2\sqrt{s}(ar^2 + bz)\right]}{\sqrt{s}} \qquad (31)$$

The temperature distribution for the simulation of the Well **83-16** is shown in Figure 3. As can be seen, the extension of the cooled zone increases with time to exceed 100 m from the injection well. The cooling induced stresses are illustrated in Figures 4-6. The solid curves show the solution based on the analytical expression for 1D heat flux. The 3D numerical results are shown using data markers. We note that there is good agreement between the two approaches. This is to be expected, as in the current problem the difference between the 3D and 1D heat flow models are small and appear only at very large times (see previous section). Several features are worth noting about the stress distribution. First, the normal (axial) stress on the fracture surface is smaller than that predicted by uniform cooling. Also, the normal stress is tensile to some distance from the injection well, its maximum occurs near the injection point and gradually approaches zero. Then, the normal stress switches sign (becomes compressive). This is because as the cooled rock shrinks, it tends to pull on the exterior rock material (strain compatibility) inducing a compressive stress in it.

Along the x-axis σ_{yy} and σ_{xx} represent the tangential and radial stresses, respectively. Both are an order of magnitude larger that the axial stress. The tangential stress distribution is different from the radial stress and similarly to the normal stress becomes slightly compressible, whereas the radial stress remains tensile throughout the domain and approaches zero. This behavior can be explained by the requirement that the radial stress be in equilibrium across the cooled zone boundary and

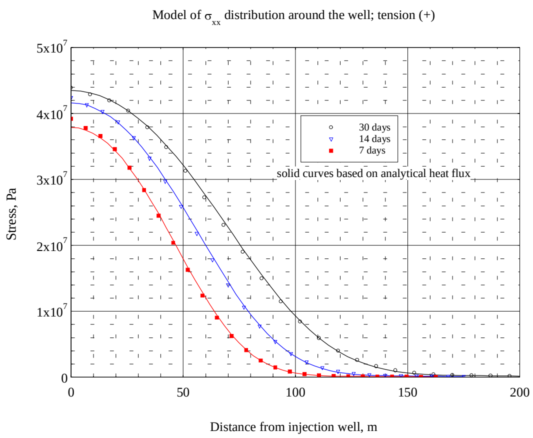

Figure 5: Radial stress distribution around an injection well for various injection times.

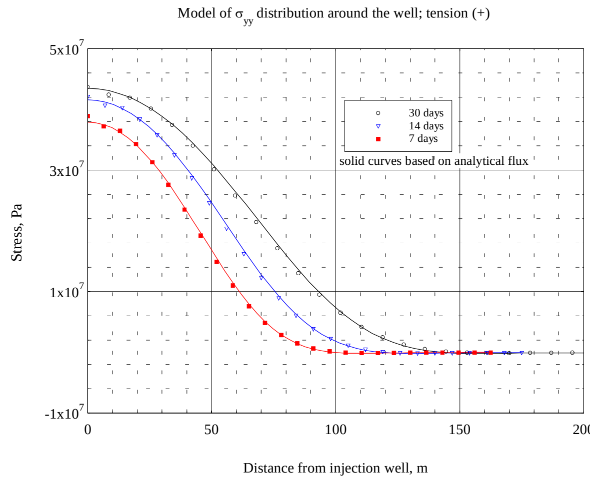

Figure 6: Tangential stress distribution around an injection well for various injection times.

the strains be continuous across it. Note that all induced stresses increase with the injection time, as expected. It is also interesting to note that the thermoelastic stresses around the injection well can produce the actual pressure drop (due to cold water injection) observed in the Well **83-16**.

The main source of error in stress calculations around the well is the heat flux term in the integral. Because the numerical temperature distribution has errors, the gradient of temperature also will have errors. The temperature gradient is, however, multiplied by large values of fluid flow near the well that result in magnification of this error to significant levels. This does not occur in other parts of the fracture since the error in computation of ∇T does not pose a problem as flow values are small. To partially reduce the error, a simple Gaussian smoothing procedure with a 3x3 filter size (Gonzales, 1992) is used for the temperature that involves recalculation of nodal values of T using the 9 nearest points with certain weights.

CONCLUSIONS

An integral equation formulation for 3D heat extraction from a planar fracture in an infinite hot rock reservoir has been used to calculate the temperature and heat flux distributions within the crack. The results are then used to determine the 3D thermal stresses resulting from cold water injection. This is in contrast to most existing work, analytical and numerical, that treats the heat conduction in the geothermal reservoir as one-dimensional and perpendicular to the fracture. The analytical procedures and computational schemes have been tested by considering the problem of injection into an infinite fracture and comparing the results with a semi-analytical solution. Then, using actual field data, the thermally induced stresses were calculated for an injection experiment. The results are consistent with the pressure drop and increase injectivity observed in the field. It has been found that the normal (axial) stress on the fracture surface is smaller than that predicted by uniform cooling of the crack surface. Also, the normal stress is tensile to some distance from the injection well, its maximum occurs near the injection point and gradually approaches zero away from it. The normal stress becomes compressive at some distance away from the well; this is because as the cooled rock shrinks, it tends to pull on the exterior rock material (strain compatibility) inducing a compressive stress in it. The model predictions are found to be consistent with those based on a semi-analytical. Although the present integral equation solution has been applied to a single fracture, the same concept can be applied to a reservoirs with a number of fractures. In addition, the stresses can be used in a rock mechanics study to analyze the details of reservoir/fracture response to water injection and cooling. Work in these directions are under development and will be reported in the future.

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