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THE CONSTRUCTION OF A TENSORIAL MODEL TO REPRESENT THE POROUS VOLCANIC ROCK DEFORMATION COUPLED TO THE FLOW OF FLUID

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ABSTRACT

In this paper we present the initial development of an operational Tensorial Model to represent the deformation of porous volcanic rocks. The fundamental equation of the model is the total flow of mass, solid plus fluid, in a porous rock which conduces to a natural generalization of Darcy=s Law for a deformable medium. The total stress tensor is the sum of the tensor acting on the solid rock plus the tensional force acting on the fluid. This total stress tensor must satisfy some classical equilibrium conditions. Assuming that the geothermal rock is only subjected to small deformations, a linear Hooke=s Law can be used to represent the relationship between strains and stresses. In the most general case an elastic matrix of 49 elements will be obtained in this way. All its elastic coefficients must be determined experimentally. Those coefficients could be constants or be some functions of *pressure* and *temperature*. Some practical assumptions can lead to simplify this general model. Simplified models could be adapted to well known fluid-energy flow simulators just by including into its fundamental equations the Terzaghi=s effect which states that in saturated rock the effective stresses acting in the pores will be decreased by the pore-water pressure. These results can be useful in the study of enhanced hydrothermal reservoirs, in hot dry rock systems and in the interpretation of microseismicity data.

Keywords: *Rock deformation, poroelasticity, tensorial model, subsidence, volcanic reservoirs, hot dry rock, low permeability, low porosity, fractures collapse, enhanced geothermal systems.*

INTRODUCTION

The Earth=s crust is formed almost completely by volcanic rocks (–95%; Farmer,1968); sedimentary rocks represents only about 5% and methamorphic rocks are a very small percentage of the total. Thus the greatest amount of geothermal reservoirs in the world are found in igneous or volcanic matrices. Volcanic rock is formed by clustered aggregates of minerals. Its geomechanical properties depend on both, the crystal structure and the mode in which the mineral particles are cemented. Igneous rocks originated by the solidification of fluid magma. The global mechanical behavior of volcanic rocks is affected by porosity, pressure, temperature, the presence of cracks, fissures and faults. But there is a general consensus that the most important factor is the total amount of interconnected pore space. It is well known that fluid extraction from reservoirs, can produce subsidence because of the reduction of both the internal pore pressure and the effective pores= diameter. Some volcanic hydrothermal systems contain portions of fractured rock where many fractures appear closed. This phenomenon could be caused by autosealing during water-rock chemical interactions at high temperature. Fractures in volcanic systems can also be closed because of fluid lost by natural means or through human activity. In both cases the deformation differences between dry rock and saturated rock play an important role.

In geothermal systems rock deformation properties are affected by the presence of fluid inside the pores. It is well known that the cohesive structure of volcanic rocks is weakened by the presence of liquid in both pores, fissures and microfractures. This is generally called the ***pore-fissure-water effect***, which affects more or less all rock geomechanical parameters. Rock cohesion is also affected by pressure, temperature and by the amount of fluid inside the solid matrix. The deformation of poro-elastic media produces several interesting phenomena. Dry rock deformation and saturated rock deformation are different; the former can be anelastic or plastic, while the latter could be totally elastic. The fluid extraction causes the reduction of both the internal pore-fracture pressure and the effective aperture of pores and fissures. In some type of volcanic systems the fluid could be scarce since the beginning of its formation, producing a natural closure of fissures. Compared with air, liquid water is almost incompressible and this property tends to reduce rock elasticity and stiffness. In geothermal reservoirs, the different values in parameters of dry rock and wet rock are determined by the amount of liquid water saturation, porosity, permeability, pressure and temperature.

Rocks forming volcanic systems are discontinuous at the small scale of pore spaces and also on local and regional scales they present fractures and faults or major discontinuities. If at the moment of its formation the reservoir was unable to store abundant water, such lack of fluid could cause the collapse and closing of many fissures, fractures and even faults, originating a global drop on permeability and permitting, at the same time, the coexistence of very strong pressure gradients between the matrix blocks and the residual open fractures. If the rock is accepted to be poroelastic, then we have to accept also that fractures aperture has some variability and the related permeability is also affected by fluid extraction and by liquid injection, thus the rock acts as a solid sponge.
In this paper we present the initial development of a general tensorial model to represent the volcanic rock deformation coupled to the fluid flow. It can include both, pore and fissure deformations. Andesites are some kind of transitional igneous rocks because they are composed of intermediate percentages of silica, between 52% and 66%. Rhyolite is a fine texture igneous acid rock which contains more than 66% of silica. Both types of rocks are massive, hard and strong. They are conforming almost the entirety of hydrothermal systems discovered in Mexico (Suarez, 1995).

EFFECTS OF FLUID ON GEOTHERMAL ROCK PROPERTIES

As already mentioned, in geothermal reservoirs the rocks are poroelastic and their geomechanical properties are affected by the presence of fluid inside the pores. The different values between dry and saturated rock parameters are determined by the amount of liquid saturation, porosity, permeability, pressure and temperature (Farmer, 1968). Compared with gases, liquid water has very low compressibility. This property tends to reduce rock elasticity and stiffness, but saturated rock density and the magnitude of wave propagation are increased, while strength is reduced. In other words, the presence of water in a rock always makes its fracture easier. Such effects are well known in porous materials since 1943. This is generally called the *pore-water* effect, which affects more or less the whole rock thermo-mechanical behavior. In the study of fractured rocks, we have to extend those concepts to include new effects such as the *fissure-water* effect and the *fissure elasticity*. On the basis of the classical poroelastic theory, we can calculate approximately how internal stresses and deformations are affected by the fluid in a fractured rock matrix. There are two extreme cases to consider: dry rock and totally saturated rock.

FUNDAMENTAL FLOW EQUATIONS

There are several different formulations and hypothesis found in the specialized literature to represent the porous rock deformation coupled to the flow of fluids (Biot,1941; Muskat,1937; Terzaghi,1943; Farmer,1968; Germain,1973; Bear & Bachmat,1984 & 1990; Bles & Feuga, 1986; Coussy,1991; Spanos,2002). A model of volcanic rock mechanics means a group of equations capable to predict the porous medium deformation under different internal and external forces. Because of the different objectives followed by different authors and in spite of many experimental data and several strong founded theoretical works, a unique, robust, well consolidated theory of porous mechanics apparently does not exist. Particularly if compared to the well established solid mechanics. In our opinion the most fruitful, unifying and essential concept in this matter is, perhaps, the viewpoint of assigning all the dynamics of saturated porous rocks to that of the solid skeleton. We start this work from this outlook which is based on the pioneering classical work of Biot (1941).

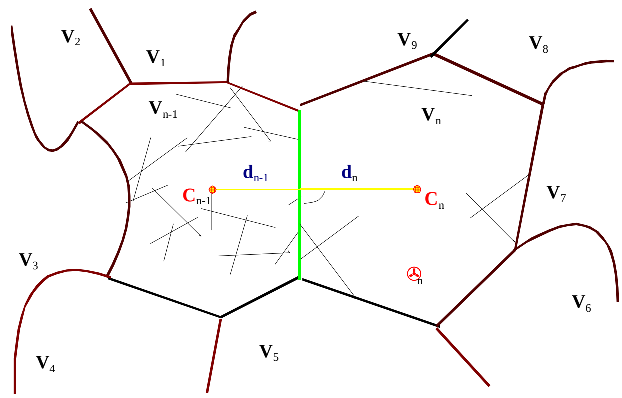

Fig. 1.- *Representation of a finite porous volume V_n subjected to deformation and connected to other volumes through its total surface S_n. ▶ $_n$ represents any physical property.*

There are different possible numerical schemes to represent the mechanical behavior of porous continuum. Because of practical and intuitive reasons, we choose the Volume-Surface Integrated Finite Diference technique (Fig. 1), as defined by Narashimhan (1982) and Pruess (1979). We develop this preliminary model for a single phase fluid. In the more general case the set of equations become more numerous and complicated, but the fundamental ideas can be understood considering only liquid and rock phases. If ρ_α, $\boldsymbol{F}_\alpha$ and q_α, represent density, flow and mass variation of phase α respectively (α =F, is the fluid, α = R, is the rock), the integral equation representing mass conservation is:

$$\frac{\partial}{\partial t}\int_{V_n}(\phi_\alpha\ \rho_\alpha\ )\,dV+\int_{V_n}\vec{\nabla}\bullet\vec{F}_\alpha\ dV=\int_{V_n}q_\alpha\ dV$$

Where $\varphi_F = V_F/V_n$ and $\varphi_R = V_R/V_n$, represent the volume fraction of phase α in V_n. The flow of mass of phase α is:

$$\vec{F}_\alpha=\rho_\alpha\ \phi_\alpha\ \vec{u}_\alpha$$

Notice that $\boldsymbol{u}_F$ is the effective microscopic velocity of the fluid particles, while Darcy=s velocity or specific discharge (*Dupuit=s* hypothesis) is $\varphi_F \boldsymbol{u}_F = \boldsymbol{v}_F = M\boldsymbol{r}_F/Mt$, where $\boldsymbol{r}_F$ is the average displacement of the fluid particles. For the poro-elastic solid medium $\boldsymbol{u}_R = \boldsymbol{v}_R = M\boldsymbol{r}_R/Mt$ and $\boldsymbol{r}_R$ is the displacement of the solid particles. $V_n(t)$ is a representative finite deformable volume in the spatial discretization. In the general case the total volume $V_n = V_R + V_F + V_O + V_U$, where subscripts mean R = solid rock, F = fluid, O = voids without fluid and U = unconnected pores respectively. In this study we will simplify the total volume assuming $V_0 = V_U = 0$. The Darcy=s Law for a fluid flowing in a deformable porous medium (Biot, 1941, 1955) in standard notation is:

$$\phi (\vec{u}_F - \vec{u}_R) = - \frac{K}{\mu_F} (\vec{\nabla} p_F - \rho_F g \nabla \vec{z})$$

$\boldsymbol{K} = k_{ij}\, \mathbf{e}_i \theta \mathbf{e}_j$ is the permeablity tensor, θ is the tensorial product of the vectors of a basis $\{\mathbf{e}_i\}$ of 3D space, μ is viscosity, p is pressure and g is the acceleration of gravity. Coupling eq. (3) to eq. (1) we get:

$$\int_{V_n} \vec{\nabla} \bullet \left[\frac{\rho_F\, K}{\mu_F} (\vec{\nabla} p_F - \rho_F \vec{g}) \right] dV = \frac{D}{Dt} \int_{V_n} \phi_F \rho_F\, dV + \int_{V_n} \phi_F \rho_F\, \vec{\nabla} \bullet \vec{v}_R\, dV$$

D/Dt is the total or material or convective derivative of the fluid mass referred to a deformable porous rock with velocity of deformation $\boldsymbol{v}_R$ in V_n and is equal to:

$$\frac{D\, m_F}{Dt} = \frac{\partial\, m_F}{\partial t} + \vec{v}_R \bullet \vec{\nabla}\, m_F$$

Introducing this derivative into equation (1) for the solid phase and assuming a constant mass for the rock:

$$\vec{\nabla} \bullet \vec{v}_R = \frac{-1}{\phi_R\, \rho_R} \frac{D}{Dt} (\phi_R\, \rho_R) = \frac{1}{V_n} \frac{D V_n}{Dt}$$

Of course, for a non-deformable medium, both sides of this equation are 0. If there is internal production of rock mass we need to add the term $q_R / \varphi_R \rho_R$. Finally expanding the right side of eq. (4), introducing eq. (6) and replacing the relations between volumes and porosity we get:

$$\int_{V_n} \vec{\nabla} \bullet \left[\frac{\rho_F\, K}{\mu_F} (\vec{\nabla} p_F - \rho_F \vec{g}) \right] dV = \int_{V_n} \phi_F\, \rho_F \left(\frac{1}{\rho_F} \frac{D \rho_F}{Dt} + \frac{1}{V_F} \frac{D V_F}{Dt} \right) dV$$

Equation (7) relates the fluid flow to both the derivative of fluid density and the rate of change of the volume of pores. This is a fundamental equation for the coupling of rock deformation and fluid flow.

TOTAL STRESS TENSOR OF THE POROUS ROCK

Let σ_{ij} and σ_F be the components of the stress tensors of porous rock and fluid respectively:

$$(\sigma_{ij}) + (\sigma_F) = \begin{pmatrix} \sigma_{11} + \sigma_F & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} + \sigma_F & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} + \sigma_F \end{pmatrix} = \Sigma_T$$

$\boldsymbol{\Sigma}_T$ represents the total stress acting on the faces occupied by the solid rock through σ_{ij} plus the tension force σ_F acting on the fluid. The total stress tensor must satisfy the following equilibrium relationship (Germain, 1973):

$$\vec{\Delta} \bullet \Sigma_T + \rho_T\ g\ \vec{\Delta}\ z = 0$$

Where ρ_T is the total density of the bulk porous rock:

$$\rho_T = \phi_F\ \rho_F + \phi_R\ \rho_R$$

Let $\boldsymbol{\Xi}^R = (\varepsilon^R_{i\,j})$ and $\boldsymbol{\Xi}^F = (\varepsilon^F_{i\,j})$ be the strain tensors of the solid rock and fluid respectively:

$$\varepsilon^{\alpha}_{ij} = \frac{1}{2} \left(\frac{\partial r_i}{\partial x_j} + \frac{\partial r_j}{\partial x_i} \right)^{\alpha} , \alpha = R , F$$

The stress-strain tensor relationship is: $\boldsymbol{\sigma_{ik} = \lambda_{ik}\, \varepsilon_{jk}}$, (k = 1,7; i, j=1,3). $\mathbf{tr}\,(\boldsymbol{\varepsilon_F}) = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$. All the elastic coeficients λ_{ik} must be determined experimentally. They could be constants or be some functions of *p* and *T*.

Substituting this tensor equation into eq. (9) we get 3 equations to solve the 3 unknowns of the rock displacement vector $\boldsymbol{r}_S = (r_1 , r_2 , r_3)^R$ Some practical assumptions can lead to simplify this general model (Suarez & Samaniego, 2003).

CONCLUSIONS

- # We presented the initial development of a general tensorial model to calculate the rock deformation of poroelastic rocks.
- # For practical reasons the general equations were formulated using an integral approach based on the Volume-Surface Integrated Finite Difference Method.
- # The development of a numerical code using this model is a current research work in progress.

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