

## A STEADY STATE FLOW WITH PHASE TRANSITION IN WATER OVER VAPOUR GEOTHERMAL RESERVOIRS

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### ABSTRACT

A steady state flow under action of gravity with water-vapour phase transition is considered. It is assumed that water layer lies over the layer of superheated vapour in a stratum that locates between two parallel high permeability domains saturated with water and vapour, respectively. When fluid flows from one high permeability domain to another one through the stratum, phase transition occurs at the water-vapour interface. Under the assumption of smallness of advective energy transfer as compared with the conductive one, the stationary distribution of the characteristics in the stratum with water-vapour phase transition is obtained. Consequently, the problem is reduced to the investigation of the boundary value problem at the unknown interface. The solution is shown to describe both vaporization and condensation processes in reservoirs. It is also found that three different solutions can exist for the same boundary conditions for the pressure and temperature functions. Investigation of normal stability of the interface shows that stable configuration corresponds to the unique solution whereas unstable configuration arises when steady state solution is not unique. It is found that stable water over steam configuration can exist when permeability value is about 1 millidarcy. This value is greater by the order than the critical value in Schubert-Straus example of the geothermal system. The fairly high value of the permeability makes it possible to explain the existence of a wide class of stable natural geothermal reservoirs where vapour layer underlies the water one.

### INTRODUCTION

Full-scale investigations of natural geothermal systems showed, that in great number of reservoirs the situation takes place, when a water layer of a considerable thickness locates over a layer of superheated vapor (White et al., 1971; Grant, 1983). The existence of such a configuration is explained from the thermodynamic point of view by a temperature gradient which presence in geothermal

systems is typical. In this case the thermodynamic conditions at large depths relate to the domain occupied by vapor, and at smaller depths - to the domain occupied by water. On the other hand, investigation of hydrodynamic stability shows, that the state, when the layer of heavy liquid lies over the layer of lighter one occurs to be unstable if the layers are immiscible (Chandrasechar, 1967). In order to explain the possibility of existence of stable geothermal systems containing the water layer overlying the vapor layer, some qualitative hypotheses about physical mechanisms of stability of such a configuration were proposed (Grant, 1983).

In Schubert, Straus (1980) the example of the geothermal system is given, where the water layer overlies the vapor one, and also the stability analysis of the phase transition interface, separating water and vapor domains, was fulfilled. In the unperturbed state the phases assumed to be motionless and the phase transition is absent. Numerical analysis of the dispersion equation implies that the configuration in question may be both stable and unstable. The critical value of the permeability coefficient separating the domain of stable and unstable states of the system was found to be given by  $k \sim 4 \cdot 10^{-17} \text{ m}^2$ . For values of permeability higher than the critical one the system loses its stability. It was noted that the found critical value of permeability is less by the order of magnitude than the value characteristic for natural geothermal systems, and thus, the stability of most systems cannot be explained.

In the present paper we propose a more complex example of a geothermal system where motion of phases and the phase transition in the **undisturbed** state are taken into account. A solution of the steady-state bounded problem with a water-vapor phase transition interface is obtained under the assumption of smallness of the advective energy transfer as compared with the conductive one. The solution describes slow motion of the water (vapor) towards the interface, phase transition and flow of water (vapor) out of the interface. In the first case the process of vaporization is realized, in the second case the process of condensation. The quantities to be determined are the location of the interface of phase

transition and also the values of temperature and pressure on this surface. These quantities are determined from the solving of the transcendental equation on the interface. The calculations show that at low values of permeability the transcendental equation has a unique solution, which determines the location of the interface of phase transitions. At comparatively high values of permeability of order  $k \sim 10^{-16} \text{ m}^2$  and higher, the solution of the transcendental equation in question may have not the unique root any more, but three ones. It means that for the same values of characteristics at the boundaries, there exist three different locations of the interface.

The topological method was used for the investigation of normal stability of the interface, based on the argument principle. This method makes it possible to detect complex roots of the dispersion equation, having the positive real part. The presence of these roots corresponds to the instability of the solution under consideration. It is essential, that we give no assumptions about the real nature of the roots of the dispersion equation. This assumption is contained implicitly in Schubert, Straus (1980). Our topological approach also allows to determine the number of the roots of the dispersion equation.

Investigation of stability of the interface shows that at values of permeability  $k \sim 10^{-16} \text{ m}^2$  and lower, i.e. when there exists the unique solution of the stationary problem, the interface is always stable. In the case when there exist three solutions of the stationary problem, the configuration, corresponding to the equilibrium without phase transition is always unstable. The second solution, relating to the water motion downwards and its further vaporization, is unstable with respect to short wavelength perturbations. Numerical investigation shows that the third solution, which describes the process of vapor motion upwards and its condensation at the interface, can be stable also at comparatively high values of permeability  $k \sim 6 \cdot 10^{-16} \text{ m}^2$ .

It seems to be important to note that the regimes obtained are realized within the range of parameters that appear to be characteristic for the geothermal systems. The stability mechanism of the considered class of geothermal systems has the clear physical meaning, implying the predominance of the conductive energy transfer over the advective one.

### FORMULATION OF THE PROBLEM

We consider a high-temperature geothermal reservoir, consisting of two high permeable layers, which are separated by a low permeable stratum. Let us assume that the thermodynamic conditions imply that the upper high permeable layer is filled in by water and the lower one - by vapor. Then in the low permeable layer there exists the phase transition

interface, separating domains occupied by water and vapor, respectively (Fig.1). In dependence on values of pressure in the high permeable layers, either the regime of vaporization, when the water moves downwards, or the regime of condensation, corresponding to the vapor motion upwards, takes place. In the rest state phase transitions are absent, and the pressure distribution in the high permeable layers coincides with the hydrostatic one.

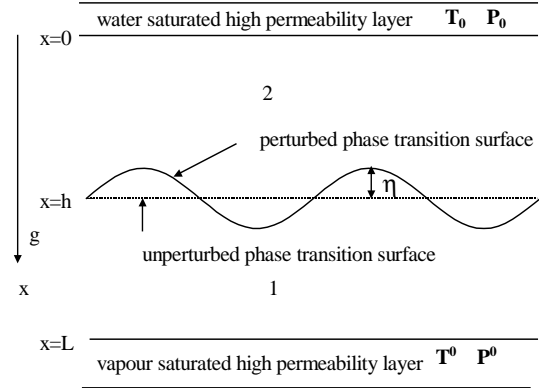


Figure 1

Processes of heat and mass transfer can be described by mass and energy conservation laws, the Darcy law for water and vapor with allowance for gravity, the equations of state and thermodynamic relations (O'Sullivan, 1985). Following Chandrasekhar (1967), Schubert, Straus (1980), we assume that the water and vapor are incompressible. Then the basic system of equations for both domains is written as

$$\text{div } \vec{v}_j = 0$$

$$\vec{v}_j = -\frac{k}{\mu_j} (\text{grad } P - \rho_j g \vec{e}_x) \quad j = w, v \quad (1)$$

$$(\rho C)_{1,2} \frac{\partial T}{\partial t} + \rho_j C_j \vec{v}_j \text{ grad } T = \text{div}(\lambda_{1,2} \text{ grad } T)$$

$$\lambda_{1,2} = m \lambda_j + (1-m) \lambda_s$$

$$(\rho C)_{1,2} = m \rho_j C_j + (1-m) \rho_s e_s$$

The relations on the interface are formulated as the conditions of thermodynamic equilibrium jump of the function of water saturation (Tsympkin, 1997). These relations have the form

$$\ln \frac{P_*}{P_a} = A + \frac{B}{T_*}, \quad A = 12.512, \quad B = -4611.73$$

$$T_+ = T_- = T_*, \quad P_+ = P_- = P_*, \quad P_a = 10^5 \quad (2)$$

$$m \left( 1 - \frac{\rho_v}{\rho_w} \right) V_n = \frac{k \rho_v}{\mu_v \rho_w} (\text{grad } P)_{n+} -$$

$$\frac{k}{\mu_w} (\text{grad } P)_{n-} + \frac{k}{\mu_w} \rho_w g \left( 1 - \frac{\mu_w \rho_v^2}{\mu_v \rho_w^2} \right) \\ m q \rho_w V_n = (\lambda \text{grad } T)_{n-} - (\lambda \text{grad } T)_{n+} - \\ - \frac{k q \rho_w}{\mu_w} ((\text{grad } P)_{n-} - \rho_w g)$$

Here,  $T$  is the temperature,  $P$  the pressure,  $v$  the filter velocity,  $k$  the permeability,  $\mu$  the viscosity,  $\rho$  the density,  $\lambda$  the thermal conductivity,  $m$  the porosity,  $C$  the heat capacity,  $g$  the gravity, the subscripts denote:  $w$  - water,  $v$  - vapor,  $s$  -porous medium skeleton.

We consider the flows, where the advective transfer of heat in (1) can be neglected. For the water domain the ratio of advective and conductive terms in the equation of energy is given by the dimensionless parameter

$$\frac{\rho_w C_w}{\mu_w \lambda_1} k (\delta P - \rho_w g l) \quad (3)$$

where permeability and pressure may vary strongly and  $l$  is the characteristic length scale. At the same time the other physical parameters vary slightly. Therefore, after substitution of the characteristic values of parameters in (3) the condition of smallness of advective transfer in the water domain can be written in the following form

$$\frac{\rho_w C_w}{\mu_w \lambda_1} k |\delta P - \rho_w g l| \approx 10^{10} N^{-1} k |\delta P - \rho_w g l| \ll 1$$

$$\text{or } k |\delta P - \rho_w g l| \ll 10^{-10} N$$

Analogously, one has for the vapor domain

$$\frac{\rho_v C_p}{\mu_v \lambda_2} k |\delta P - \rho_v g l| \approx 0.3 \cdot 10^8 N^{-1} k |\delta P - \rho_w g l| \ll 1$$

$$\text{or } k |\delta P - \rho_v g l| \ll 3 \cdot 10^{-9} N$$

It follows then, that the condition of smallness of the advective transfer in the vapor domain occurs to be weaker than that one in the water domain. We take the difference of values of pressure between high permeable layers as a characteristic variation of pressure, and the distance between these layers as a characteristic length scale. Neglecting the advective heat transfer in the equation of energy in (1) we get the equation of heat transfer for both domains.

## STATIONARY SOLUTION

In this section we consider one-dimensional problem about stationary phase cross-flow between high permeable layers. As concerns the real situation, the assumption of one-dimensionality is valid if characteristic horizontal length scales of pressure variation in high permeable layers are much greater

than the thickness  $L$  of the low permeable stratum. Let the upper water-saturated layer  $x < 0$ , have the pressure  $P_0$  and the temperature  $T_0$ , and the lower vapor saturated layer  $x > L$  have the pressure  $P^0$  and the temperature  $T^0$ . Let us note, that the thickness of the both high permeable layers is not essential for the problem under formulation. It is natural to assume that pressures and temperatures in the both high permeable layers are constant (constancy of temperature is guaranteed by heat flow from the surrounding rocks, and constancy of pressure - by the high permeability of the upper and lower layers). Then the phase transition surface, located in the low permeable stratum  $0 < x < L$ , occupies a certain equilibrium position  $x = h$  and the problem has a stationary solution. The domain  $0 < x < h$  is saturated by water, and the domain  $h < x < L$  - by vapor (Fig.1). Location of the interface  $x = h$ , and also the values of the temperature  $T_*$  and the pressure  $P_*$  on this interface are unknown and have to be found in the process of solving the stationary problem. Under the condition of smallness of the advective energy transfer the system of equations for the unknown pressure and temperature in the water and vapor domains are given by

$$P'(x) = 0, \quad T'(x) = 0$$

Conservation laws of mass and energy on the interface for the case under consideration read

$$\frac{\mu_w \rho_v}{\mu_v \rho_w} P'_+ - P'_- + \rho_w g \left( 1 - \frac{\mu_w \rho_v^2}{\mu_v \rho_w^2} \right) = 0$$

$$\lambda_+ T' - \lambda_- T_- + \frac{k q \rho_w}{\mu_w} ((\text{grad } P)_{n-} - \rho_w g) = 0$$

where prime denotes differentiation with respect to  $x$ . These conditions, along with the conditions of thermodynamic equilibrium on the interface, constitute the full system of relations on this interface.

The solutions in water and vapor domains have the form

$$P = P_0 + \frac{P_* - P_0}{h} x, \quad T = T_0 + \frac{T_* - T_0}{h} x$$

$$P = \frac{P^0 - P_*}{L - h} x + \frac{L P_* - h P^0}{L - h} \quad (4)$$

$$T = \frac{T^0 - T_*}{L - h} x + \frac{L T_* - h T^0}{L - h}$$

Substituting the solution (4) in the stationary relations on the interface we get the system of equations for the unknowns  $T_*$ ,  $P_*$ ,  $h$ :

$$\begin{aligned} \frac{\mu_w \rho_v}{\mu_v \rho_w} \frac{P^0 - P_*}{L - h} - \frac{P_* - P_0}{h} + \rho_w g \left( 1 - \frac{\mu_w \rho_v^2}{\mu_v \rho_w^2} \right) &= 0 \\ \lambda_+ \frac{T^0 - T_*}{L - h} - \lambda_- \frac{T_* - T_0}{h} + \frac{kq\rho_w}{\mu_w} \left[ \frac{P_* - P_0}{h} - \rho_w g \right] &= 0 \\ P_* = f(T_*) = P_a \exp \left( A + \frac{B}{T_*} \right) &\quad (5) \end{aligned}$$

From the first equation the pressure on the interface can be expressed as a function of  $h$ , and the temperature from the third equation - as a function of pressure. This makes it possible to reduce the system to the unique equation for the unknown location parameter  $h$ .

$$G(h) \equiv G(T_*(P_*(h))) = 0 \quad (6)$$

where  $G$  is the known function. Let us consider the characteristic regimes of stationary flows, or basic states, which are realized in geothermal systems at the following values of parameters:

$$\mu_w = 1.48 \cdot 10^{-4} \text{ Pa}\cdot\text{c}, \mu_v = 1.59 \cdot 10^{-5} \text{ Pa}\cdot\text{c}, \rho_w = 888.66 \text{ kg/m}^3, \rho_v = 4.82 \text{ kg/m}^3, q = 2 \cdot 10^6 \text{ J/kg}, L = 10, 20, 40 \text{ m}, C_s = 10^3 \text{ J/(kg}\cdot\text{K)}, \lambda_s = 2 \text{ W/(m}\cdot\text{K)}, g = 9.8 \text{ m/s}^2.$$

Since for the moderate values of porosity the heat capacity and the thermal conductivity of the rocks are determined by the corresponding parameters of the skeleton, we set for the sake of simplicity  $\lambda_1 = \lambda_2 = \lambda_s$ ,  $C_1 = C_2 = C_s$ .

Depending on the values of the temperature and pressure in the high permeable layers the different regimes of phase transitions can be realized, corresponding to different directions of phase motion. The flow downwards is accompanied by water vaporization and upwards - by vapor condensation.

At  $T_0 = 448 \text{ K}$ ,  $T^0 = 464 \text{ K}$ ,  $P_0 = P^0 = 10^6 \text{ Pa}$ ,  $L = 20 \text{ m}$ ,  $k = 10^{-16} \text{ m}^2$ , for example, we get the following values of the unknowns  $T_* = 456.18 \text{ K}$ ,  $P_* = 1.105 \cdot 10^6 \text{ Pa}$  and  $h = 13.27 \text{ m}$ .

The water moves downwards under the action of gravity, then finds itself in a domain with greater temperature and vaporizes on the phase transition surface. This leads to the pressure growth on the interface that becomes greater than the pressure on the boundaries of the low permeable stratum, but smaller than the hydrostatic pressure. The arising vapor moves from the phase transition surface and flows out from the vapour saturated domain of the low permeable stratum into the high permeable layer. If we increase the pressure in the high permeable

layer saturated with vapor up to value  $P^0 = 1.1 \cdot 10^6 \text{ Pa}$ , then the solving of the system of equations on the interface gives:  $T_* = 454.72 \text{ K}$ ,  $P_* = 1.07 \cdot 10^6 \text{ Pa}$  and  $h = 7.9 \text{ m}$ . The value of the pressure on the interface now decreases as compared to the previous case, though the pressure on the lower boundary of increases. This is due to vapor condensation driven by the motion of vapor upward from the lower high permeability layer and its penetration into the lower temperature zone. Water also moves upward, flowing out from low permeable rock into the water-saturated high permeable layer. The third possibility is also possible, relating to the absence of motion and phase transitions. The corresponding solution can be constructed by setting the location of phase transition interface *a priori*. Let the pressure on the upper water saturated layer be  $P_0 = 10^6 \text{ Pa}$ , and the phase interface locate in the middle of the low permeable, i.e.  $h = L/2$ . Then, one can uniquely find the pressure on the interface. The difference between this pressure and the pressure at each boundary exactly equals to the hydrostatic pressure. For  $L = 40 \text{ m}$ , for example,  $P_* = 1.174 \cdot 10^6 \text{ Pa}$ . Adding to this value of the pressure the hydrostatic pressure of vapor in vapor saturated domain of the low permeable stratum, we obtain the pressure in the lower boundary  $x = L$ :  $P^0 = 1.175 \cdot 10^6 \text{ Pa}$ . These values of pressure correspond to the dynamical rest state. In order to satisfy the thermodynamic conditions it is necessary to choose the temperature distribution with boundary values lying, respectively, in the water and vapor domains in the Clapeyron plane. Consequently, the temperature on the upper boundary  $x = 0$  must be smaller than the temperature of water boiling:  $T_0 < T_f(P_0) = 451.71 \text{ K}$ , and at the lower boundary  $x = L$  - greater than the temperature of vaporization:  $T^0 > T_f(P^0) = 458.97 \text{ K}$ .

On the interface the value of the  $T_*$  has to lie on the water-vapor phase equilibrium Clapeyron curve at values of pressure  $P_* = 1.174 \cdot 10^6 \text{ Pa}$ . This value of the temperature can be uniquely determined and it equals  $T_* = 458.93 \text{ K}$ . Arbitrariness exists only in a choice of the temperature gradient. For example, setting  $T_0 = 450 \text{ K}$  and using the absence of phase transitions (continuity of derivatives of temperature across the interface), and also the known value of the temperature on the interface, we get the following value of the temperature on the lower boundary  $x = L$ :  $T^0 = 467.86 \text{ K}$ . Note that this solution realizes at all values of the permeability coefficient.

The numerical experiments show that for sufficiently high values of the permeability the equation (6) has three different roots for the fixed boundary conditions.

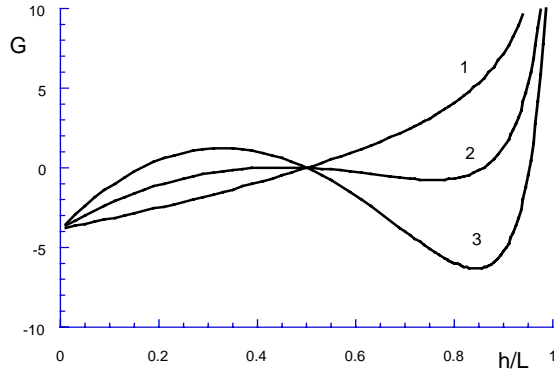


Figure 2

In Figure 2 the graphic of the transcendent function  $G(h)$  is given for three different values of permeability. In the first case (line 1), at  $k = 10^{-17} m^2$  the function is monotonous and it intersects the x-axis in a single point  $h = 20 m$ , relating to the rest state. For increasing permeability the curve deforms in a way, that the property of the monotonous behaviour ceases to exist and the curve intersects the x-axis in three different points (lines 2,3). It means that apart from the rest state there exit two roots of the equation (6) at the same boundary values. The smallest root corresponds to a regime with vapor condensation, the central one - to the rest state, and the largest one - to a regime with water vaporization. The non-uniqueness of the solution makes it necessary the attachment of the additional investigation in order to choose the physically correct solution. This investigation is usually related to the stability analysis of the solutions in question. This analysis is performed in next section.

The validity of the condition of smallness of advective energy transfer for the prescribed solutions must be also checked. Calculations show that in the first example the conductive heat flux is greater than the advective one by two orders of magnitude in both domains. In the second example the excess is equal to three orders in the water domain and two orders in the vapor domain. For the rest state solution there is no advective energy flux.

### STABILITY ANALYSIS

In this section we investigate normal stability of the stationary solution (4) with respect to small perturbations. The system of basic equations, linearized about the equilibrium state solution, under the condition of smallness of advective energy transfer have the form in both domains:

$$\Delta P = 0, \quad \frac{\partial T}{\partial t} = a_{1,2} \Delta T, \quad a_{1,2} = \frac{\lambda_{1,2}}{(\rho C)_{1,2}} \quad (7)$$

Hereafter, we assume for the sake of simplicity, that the heat capacities and heat conductivities in both zones are determined by the corresponding parameters of the porous medium skeleton  $a = a_1 = a_2$ . The boundary relations for perturbations take the form

$$P = 0, \quad T = 0, \quad x = 0, L$$

$$x = h:$$

$$P_- = P_+ + \Gamma_1 \eta, \quad \Gamma_1 = \frac{P_0}{h} + \frac{P^0 h - P_* L}{h(L-h)},$$

$$T_- = T_+ + \Gamma_2 \eta, \quad \Gamma_2 = \frac{T_0}{h} + \frac{T^0 h - T_* L}{h(L-h)} \quad (8)$$

$$P_- = \left( \frac{\partial f}{\partial T} \right)_{T=T_-} T_- + \left[ \left( \frac{\partial f}{\partial T} \right)_{T=T_*} \left( \frac{\partial T}{\partial x} \right)_- - \left( \frac{\partial P}{\partial x} \right)_- \right] \eta =$$

$$= -\Gamma T_- - \Gamma_0 \eta,$$

$$\Gamma = \frac{P_* B}{T_*^2}, \quad \Gamma_0 = \Gamma \frac{T_* - T_0}{h} + \frac{P_* - P_0}{h}$$

$$m(1-R) \frac{\partial \eta}{\partial t} = \frac{k}{\mu_v} R \left( \frac{\partial P}{\partial x} \right)_+ - \frac{k}{\mu_w} \left( \frac{\partial P}{\partial x} \right)_-, \quad R = \frac{\rho_v}{\rho_w},$$

$$mq \rho_w \frac{\partial \eta}{\partial t} = \lambda_- \left( \frac{\partial T}{\partial x} \right)_- - \lambda_+ \left( \frac{\partial T}{\partial x} \right)_+ - \frac{kq \rho_w}{\mu_w} \left( \frac{\partial P}{\partial x} \right)_-$$

where  $x = h + \eta(t, y)$  is the equation of the interface. We look for the expressions of unknown pressure and temperature in the form  $g(x) \exp(\sigma t + iky)$ . Then, from the condition of existence of a nontrivial solution of (7), (8), one gets the dispersion relation:

$$F(\sigma, \kappa) = \alpha \coth[\alpha(L-h)] \left[ \Gamma_3 \kappa \frac{k}{\mu_w} \coth(\kappa h) + \right.$$

$$\left. + \Gamma_4 R \kappa \frac{k}{\mu_v} \coth[\kappa(L-h)] - m(1-R)\sigma \right] + \alpha \coth(\kappa h)$$

$$\left[ \Gamma_5 R \kappa \frac{k}{\mu_v} \coth[\kappa(L-h)] - m(1-R)\sigma + \Gamma_0 \kappa \frac{k}{\mu_w} \coth(\kappa h) \right] +$$

$$+ \frac{\Gamma q \rho_w k R \kappa}{\lambda} \left[ \sigma \frac{m}{\mu_v} \coth[\kappa(L-h)] + \sigma \frac{m}{\mu_w} \coth(\kappa h) + \right.$$

$$\left. + \Gamma_1 \kappa \frac{k}{\mu_v \mu_w} \coth(\kappa h) \coth[\kappa(L-h)] \right] = 0,$$

$$\Gamma_3 = \Gamma \frac{T^0 - T_*}{L-h} + \frac{P_* - P_0}{h}, \quad \Gamma_4 = \Gamma \frac{T^0 - T_*}{L-h} + \frac{P^0 - P_*}{L-h}$$

$$\Gamma_5 = \Gamma \frac{T_* - T_0}{h} + \frac{P^0 - P_*}{L-h}, \quad \alpha = \sqrt{\kappa^2 + \frac{\sigma}{a}}$$

$F(\sigma, \kappa)$  is an even function of  $\kappa$   $F(\sigma, \kappa) = F(\sigma, -\kappa)$ , hence the dispersion curve  $\sigma = \sigma(\kappa)$  has to be symmetric about  $\sigma$  - axis (reversibility).

The form of the dispersion relation suggests the following asymptotic  $\sigma \rightarrow \sigma_0 |\kappa|$  at both infinities, where  $\sigma_0$  is a constant given by the expression

$$\sigma_0 = \left[ (\Gamma_4 + \Gamma_5) \frac{Rk}{\mu_v} + (\Gamma_0 + \Gamma_3) \frac{k}{\mu_w} + \frac{\Gamma \Gamma_1 q \rho_w R k^2}{\lambda \mu_v \mu_w} \right] \times \left[ 2m(1-R) - \frac{\Gamma q \rho_w R k^2 m}{\lambda} \left( \frac{1}{\mu_v} + \frac{1}{\mu_w} \right) \right]^{-1}$$

If for any fixed real  $\kappa$  there exist complex roots  $\sigma$  of the dispersion relation  $F(\sigma, \kappa) = 0$  with positive real part, then the corresponding perturbation will exponentially grow with time, and the basic equilibrium solution (4) will be unstable.

The transcendental function  $F(\sigma) = F(\sigma, \kappa)$  for fixed real  $\kappa$  (reversibility implies that one can consider the positive  $\kappa$  only) is analytic everywhere in the complex  $\sigma$ -plane except for negative real semi-axis, where it has the countable set of poles, bounded away from zero. Zeroes of  $F(\sigma)$  correspond to roots of dispersion relation for fixed  $\kappa$ .

The absence of zeroes with positive real part, which give exponential growth to small perturbations, implies stability of the stationary solution in question. For investigation of stability we use the well known argument principle of complex analysis. According to this principle the difference between the number of zeroes and poles of the function  $F(\sigma, \kappa)$  inside some contour  $C$  in the complex  $\sigma$  - plane is equal to the number of rotations of radius-vector in the  $F(\sigma)$  - complex plane in circuit of a contour  $C$ , being the image of the contour  $C$  for the mapping  $F(\sigma)$ . As a contour  $C$  in the  $\sigma$ -plane we choose the contour, consisting of the segment of the imaginary axis and a semi-circle, with the center in the origin, lying in the first and fourth quadrants. The function  $F(\sigma)$  has no singularities in the right hand half-plane. If  $F(\sigma)$  has zeroes at this half-plane, then by the suitable choice of the large enough radius, it is possible to put all zeroes inside the domain bounded by the contour  $C$ . The contour, being the union of the segment of the imaginary axis, symmetric about zero and the semi-circle, having the radius  $R$  and centered at the origin:

$$c_0 = \{\sigma = iy, |y| \leq R\} \cup c = \{\sigma = R \exp i\phi, -\pi/2 < \phi < \pi/2\}$$

where  $y$  is the real variable. The contour  $C$  transforms into the contour  $C_1 = F(c_0) \cup F(c)$  under the mapping  $\sigma \rightarrow F(\sigma)$ . The components of the contour  $C_1$  are given by the following equations in the complex  $F(\sigma)$ -plane:  $F(c_0) = r_1(y) \exp[i\phi_1(y)]$ ,  $F(c) = r_2(y) \exp[i\phi_2(y)]$  with certain functions  $r_i, \phi_i, i=1,2$ .

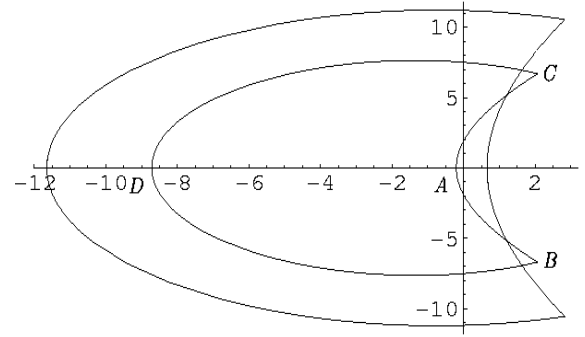


Figure 3.

The form of contours  $F(\sigma)$  presented in Figure 3 is typical for absolutely all the regimes considered in the present paper. The calculations show that the location of the point  $A$  depends on the decay (growth) rate. The smaller distance from  $A$  to origin the smaller the decay (or growth) rate of perturbations. The first equilibrium state is a stable one (inner contour). When we increase the value of permeability, the configuration becomes unstable. It can be seen that in the first case the roots with positive real part are absent, that implies the stability of the solution. In the second case the image of radius-vector of the points of the contour  $C$  makes one round about the origin in the  $F(\sigma)$ -plane, that means the existence of the root  $\sigma$  with the positive real part and corresponds to the instability regime of the basic solution. The dependence of the decay and growth rates (corresponding roots are real) on the wavenumber  $\kappa$  is illustrated in fig.4

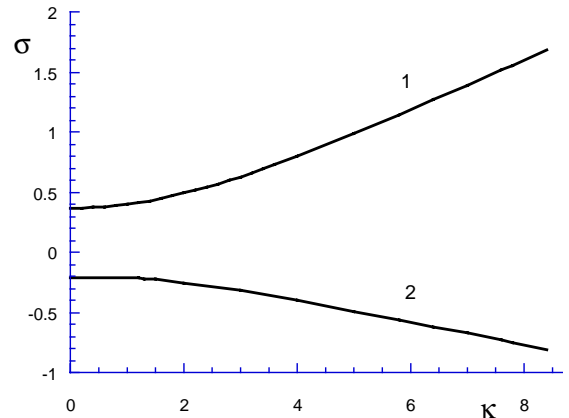


Figure 4

There exists the critical value of permeability  $k_{cr} \sim 8.9 \cdot 10^{-17} m^2$  for this solution separating stable and unstable equilibrium states. For the values of permeability lower than the critical one, the solution is stable and in becomes unstable, when the permeability exceeds the critical value  $k_{cr}$ . As it was noted in the preceding section there exist three different roots at  $k_{cr} = 10^{-16} m^2$  relating to the rest state solution, which was already considered, solution with water vaporization interface and also the solution with condensation interface. In Figure 5 the dependence of decay (growth) rates corresponding to the second and the third regimes on  $\kappa$  are shown.

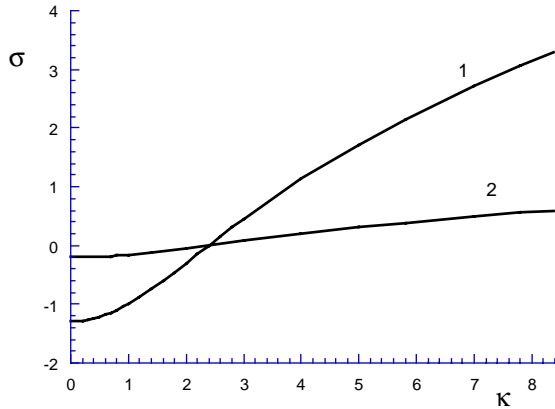


Figure 5

As follows from Figure 5, these solutions are stable at small and unstable at large enough wave numbers. We examined the stability of the steady basic state for a lot of the boundary values of the pressure and permeability meeting the criterion of smallness of the advective energy transfer as compared with the conductive one. It was found that for a stable equilibrium state the solution (4) is unique in all cases. Numerical experiments show, that besides the permeability, other physical parameters can influence the stability. For example, the unstable regimes, demonstrated in Figures 4 and 5, are subjected to qualitative changes when the pressure in the lowest high-permeability layers increases from the value  $P^0 = 1.175 \cdot 10^6 Pa$  to  $P^0 = 1.2 \cdot 10^6 Pa$ . Initially, there existed three different basic solutions of the undisturbed equations. The solutions, relating to the vapor condensation and water vaporization were stable at small  $\kappa$  and unstable at large enough  $\kappa$  (Figure 5). The solution, relating to the rest state, was unstable for all wave numbers of perturbation. Increase of the pressure at the low boundary leads to the uniqueness of the unperturbed basic solution. This solution corresponds to the condensation regime and relates to the unique root of the corresponding transcendental equation. The solution is stable. It is

interesting to note, that if the pressure in the low layer is decreased (for example,  $P^0 = 0.99 \cdot 10^6 Pa$ ), then also only one root remains, but now it corresponds to the stable regime of vaporization. Consequently, it is reasonable to make a conclusion, that phase motion in the basic stationary solution can stabilize the system. May be the absence of such a flow in the basic solution explains the fact that in the paper Schubert, Straus (1980) lower values of critical permeability were found, when the stable configuration is realized, because the rest state occurs, as a rule, to be the most unstable one. This conclusion is also in agreement with data of Eastwood, Spanos (1994), where the comparison of the decay at different wave numbers was performed for an unbounded and bounded geothermal systems and it was found that the presence of the boundary makes the system more stable. The numerical experiments were also performed in order to find the solutions stable for all wave numbers at higher values of permeability. For the parameters of the system, given in example 1 (see sec. 3), growth of the permeability leads to the appearance of three solutions of the undisturbed system, though the condensation regime is stable for an arbitrary wave number. In fig.6 the dependence of the decay rate on the wave number is shown for  $k = 4 \cdot 10^{-16} m^2$ .

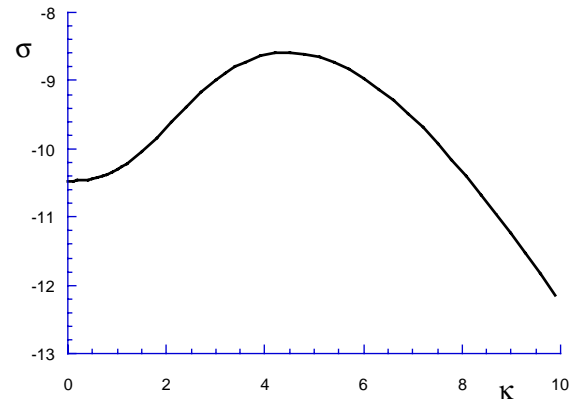


Figure 6

Verification of the smallness of the advective energy transfer in comparison with the conductive one shows that the conductive energy transfer more than by an order of magnitude exceeds the advective heat transfer. The analogous dependence of the decay rate on the wave number, testifying the stability of the solution, takes place also for  $k = 6 \cdot 10^{-16} m^2$ , though in this case the conductive energy transfer only three times greater than the advective one. Therefore, increasing the permeability, we get either the transfer to instability, or find ourselves without the frame of our approximation. Let us note, that in Schubert, Straus (1980), Eastwood, Spanos (1994) it is assumed that the roots are purely real, not complex. This

assumption, generally speaking, requires the substantiation. In our analysis we are able to detect also the complex roots  $\sigma$  of the dispersion relation with positive real part. The existence of such roots implies the instability of the physical system under investigation.

### **CONCLUSION AND DISCUSSION**

In the present paper we find the stationary solution of the problem of phase motion in a geothermal system where the water layer overlies the vapor layer under the assumption of the smallness of the advective energy transfer as compared with the conductive one. It is found that for low values of the permeability the basic solution is unique. If the value of permeability exceeds some critical value, there exist three different locations of the phase transition surface. It means that for the same boundary values three different regimes of phase motion can be realized.

The analysis of normal stability of basic solutions shows, that in the cases when the solution is unique, it is always stable. If the basic solution is not unique, there exist three different solutions and the question about the stability of each solution has to be treated separately. In this case the regime with the intermediate location of the interface occurs to be most unstable one, and the other two regimes can be either stable or unstable. In some cases the condition of smallness of the advective energy transfer is violated before the bifurcation of basic state takes place and therefore, the criterion of stability coincides with the condition of smallness of the advective heat transfer. This fact makes it possible to treat the physical mechanism of the stability in the sense that perturbations of the interface, formation and penetration of water fingers in the vapor domain are suppressed by the dominating conductive heat flux, which leads to the vaporization of the liquid phase. The calculations show that there exist stable basic states for the values of permeability  $k \sim 0.6 \cdot 10^{15} \text{ m}^2$  and higher. These values exceed more than by an order of magnitude the critical value, given in Schubert, Straus (1980). At higher values of the permeability the role of the advective heat transfer becomes considerable and the use of the solution of the unperturbed system is not rightful. In cases when the violation of our criterium precedes the transfer to instability, it seems to be reasonable to conjecture the growth of permeability will not lead to the transition of the system to instability at once, i.e. in reality the water layer can exist over the vapor layer even for higher values of the rock permeability.

### **ACKNOWLEDGEMENTS**

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