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APPLICATIONS OF LUMPED PARAMETER MODELS FOR SIMULATION OF LOW-TEMPERATURE GEOTHERMAL RESERVOIRS

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ABSTRACT

A companion paper titled “New Lumped Parameter Models for Simulation of Low-Temperature Geothermal Reservoirs” presents practical analytical models for simulation of low-temperature geothermal reservoirs. This paper deals with the applications of the new lumped-parameter models to field cases. The models are used to match the long-term observed water level or pressure response to a given production history. For history matching purposes, we use an optimization algorithm based on the Levenberg-Marquardt method to minimize an objective function based on weighted least-squares for estimating relevant aquifer/reservoir parameters. In addition, we constrain the parameters during nonlinear minimization process to keep them physically meaningful and compute statistics (e.g, standard 95% confidence intervals) to assess uncertainty in the estimated parameters. Three field examples taken from the literature are considered to show the use of the models and optimization algorithm. The observed and simulated water level changes obtained from the models are discussed. Results show a very good agreement between the observed field data and simulated data from the lumped models given in this work, in spite of long data sets.

INTRODUCTION

The modeling approach used in this paper is described as lumped-parameter. A companion paper (Sarak et al. 2003) presents new lumped parameter models for simulation of low-temperature geothermal reservoirs. In lumped-parameter modeling average properties are assigned to the two components of the geothermal system, the reservoir and the aquifer. The changes of these properties are monitored and predicted.

Sarak et al. (2003) report and discuss the lumped parameter models in the literature and present the new ones (Figure 1). In this paper, our objective is to apply these new lumped parameter models within the context of automatic history matching to liquid-dominated fields and to show their validities. The models were evaluated using production data from three fields as examples.

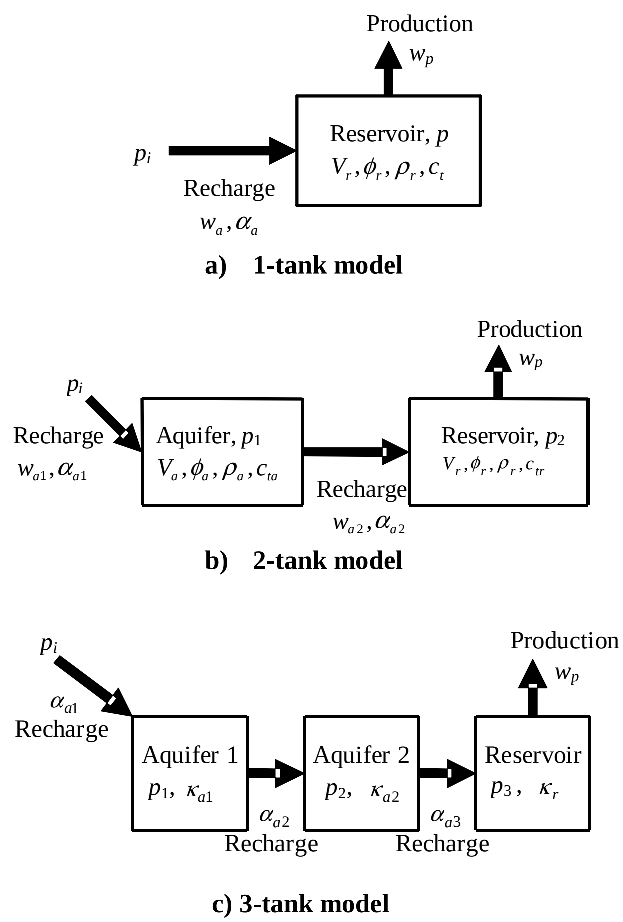

Figure 1. Schematics of lumped models considered in simulations.

OPTIMIZATION PROCEDURE

As is well known, the production causes the pressure in the geothermal systems to decline, which is reflected in the lowering of the water level in boreholes. The rate of pressure decline is determined by the rate of production, the size and properties of the geothermal system, and the recharge characteristics of the system. The recharge water invades the system in response to the lowered pressure or water level.

After a geothermal reservoir has been produced for a period of time, a lumped parameter model can be matched to observed pressure (or water level) data with the available production/reinjection rate history to obtain optimum parameters of a particular lumped model. As more data become available, more information can be obtained about the reservoir and the system. With time there are data available which may be used to improve the understanding the behavior of the reservoir. Therefore, in modeling, data must be collected as the reservoir is produced. The model is limited to the data used, so all the pressure (or water level) responses must be included for honoring all the data available. In matching observed production data, in general, more and more production data are desired. This is quite important in reducing the uncertainty in performance predictions as well as in further development of the system under consideration.

Fitting model parameters to the observed data requires accurate and fast approaches. The method of least squares fitting is a convenient one to apply. For example, Axelsson (1989) used the standard (unweighted) least squares fitting whereas Olsen (1984) used the standard least squares fitting and also graphical techniques for history matching. As is well known, the traditional (unweighted) least squares estimation is often unsatisfactory when some observations are less reliable than others and/or various measurements having disparate orders of magnitude are simultaneously used in estimation. In the former case, we want to make sure that our parameter estimates will be more influenced by the more reliable observations than by the less reliable ones. In the latter case, we wish to make sure that any information contained in the data with small magnitudes is not lost because of summing together squares of numbers of such disparate orders of magnitude. Therefore, in this work, we consider weighted least-squares fitting so that the above mentioned disadvantages associated with the standard least squares fitting can be overcome. The details of our optimization algorithm are given in the following subsection.

Parameter Estimation

The inverse problem of estimating unknown parameters from various lumped models derived in Sarak et al. (2003) can be formulated as a nonlinear optimization problem. We perform nonlinear parameter estimation by minimizing a weighted least-squares (LS) objective function (J) for which the weights (inverse of the variances of measurement errors assumed to be normal and independent) are assumed to be known. In general, we minimize a weighted least-squares objective function:

$$J(\vec{\chi}) = \sum_{j=1}^{M} \sum_{i=1}^{n} w_{j,i} \left[f_j(t_i, \vec{\chi}) - y_j(t_i) \right]^2 \qquad (1)$$

where M represents the total number of model function f and, $(t_i,\ y_j(t_i))$, i=1,…,n is a set of n observations of the model function f_j, j=1,…,M. $\vec{\chi}$ is an l-dimensional column vector whose elements are unknown parameters for a chosen lumped model. In Equation 1, the positive weights $w_{j,i}$ are the inverse of variance of measurement errors corresponding to measured value y_j at time t_i. In our applications, y_j could represent pressure (or water-level) data measured as a function of time from wells in reservoirs or aquifers.

One can construct weighted LS objective functions based on Equation 1, depending on the pressure data available and the lumped model chosen for regression, and consider matching of a single pressure data set (M=1 in Equation 1) as well as simultaneous matching of different pressure data sets (M>1 in Equation 1) to optimize $\vec{\chi}$. Suppose we consider a two-tank lumped model (Figure 1(b)), where the system is assumed to be consisted of one aquifer and one reservoir and assume that we have a set of n measured pressure data from a well in the reservoir and a set of n measured pressure data from a well in the aquifer. Then, $M = 2$ in Equation 1, and one can choose y_1 and f_1 to represent the measured and model pressure data for the reservoir, whereas y_2 and f_2 to represent measured and model pressure data for the aquifer, respectively. In this case, the positive weights $w_{1,i}$, i=1,…,n, will represent the inverse of variance of the measurement error for the ith measured pressure for the reservoir, whereas, $w_{2,i}$, i=1,…,n, will represent the inverse of variance of the measurement error for the ith measured pressure for the aquifer. It means that for this case, we construct a weighted-LS objective function based on two different sets of pressure data and simultaneously match both sets to optimize $\vec{\chi}$, where, for this lumped model, in general, $\vec{\chi}$ can be represented as

$$\vec{\chi} = \left[\kappa_r, \kappa_a, \alpha_{a1}, \alpha_{a2}, p_i \right]^T \qquad (2)$$

where T denotes the transpose. See Sarak et al. (2003) for the definition of parameters in Equation 2.

In our applications, we minimize the objective function given by Equation 1 by using the Levenberg-Marquardt method with a restricted step procedure as described by Fletcher (1987) and constrain the unknown parameters in nonlinear regression by using the so-called imaging method of Carvalho et al. (1996). In addition, we compute 95% confidence intervals and correlation coefficients by using the standard definitions (Dogru et al. 1977). As is well known, computing and inspecting such statistics in regression analysis is very useful for identifying which parameters can be reliably determined from available data.

As is well known, in nonlinear regression, parameter estimation from lumped models starts with a set of initial guess for the parameters, and then the parameters are updated by the method discussed above until a successful match of data with the model response can be obtained. We use the standard terminating criteria given by Gill et al. (1993). At termination, for each data set matched, we also compute the standard deviation of errors as well as the root mean square errors (RMS). Here, we use the standard definition of RMS given by

$$RMS_j = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[f_j(t_i,\vec{\chi}_o) - y_j(t_i)\right]^2}\ , \qquad (3)$$

where $\vec{\chi}_o$ represents the optimized parameter vector.

Before closing this section, we should note that choosing good initial guesses and constraints for parameters plays an important role in nonlinear regression analysis because nonlinear regression algorithms could often become trapped at unacceptable local minima. Particularly, this would be valid in cases where the models with a large number of unknown parameters are chosen for the data to be matched and/or the observed data contain large measurement errors. As shown in the next section, the analytical equations and the asymptotic expressions given in our companion paper (Sarak et al. 2003) are useful to obtain a good set of initial guesses for the parameters prior to performing nonlinear regression analysis.

FIELD APPLICATIONS

For the field applications discussed below, all observed data obtained from literature were given in terms of water levels. As we prefer formulating our models in terms of pressure, all the observed water level data first converted to pressure equivalence and then used in regression algorithm. Thus, all parameter estimates are given in pressure units. However, all graphical results are presented in terms of water levels to be consistent with the published field data.

Svartsengi Field

The Svartsengi field in Iceland is a liquid-dominated reservoir with fluids of nearly constant temperature at 235 ^oC.

Fluid production from the reservoir started in 1976. The composition of the fluids produced is about two-thirds seawater and one-third rainwater. Fluid extraction and reservoir drawdown in Svartsengi were monitored. The drawdown was measured as water level in monitoring wells. The water level was measured in wells 4, 5 and 6.

The resistivity measurements indicated a reservoir surface area of 5 km^2 at 200 m depth, and 7 km^2 at 600 m below sea level.

Olsen (1984) and Gudmundsson and Olsen (1987) studied the production data of the Svartsengi field. Their objective was to study the use of water influx methods in geothermal reservoir evaluation. They found that the steady state Schilthuis (1936) method gave a reasonable match and the Hurst (1958) simplified unsteady-state method gave the best match of the models they tried.

Production response data of the Svartsengi geothermal reservoir consist of a seven-year continuous record from nearby observation wells in the field. These data are presented in Figure 2.

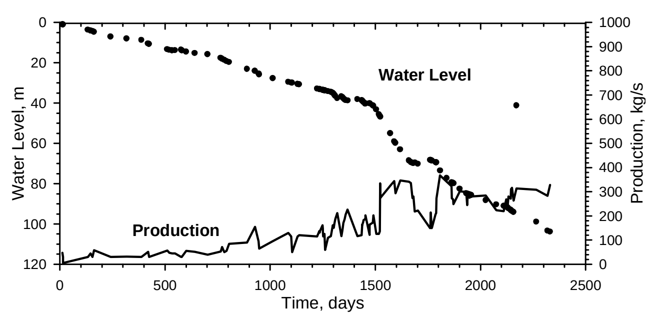

Figure 2. Production response data of the Svartsengi geothermal reservoir (Olsen, 1984).

Figure 3 shows the Schilthuis steady-state match and the Hurst (simplified) unsteady-state match obtained by Olsen (1984). The Schilthuis match is better for the early part of the data than for the later data. Olsen obtained the following values from the Schilthuis match: κ_r =5.34x10^8 kg/bar and α_a = 30.44 kg/(bar-s). Among the models that he tried to match the data, Olsen found that the Hurst simplified model assuming an infinite radial aquifer best matched to the data.

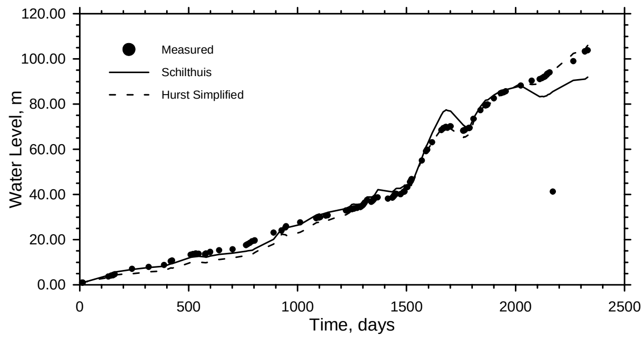

Figure 3. Schilthuis steady-state and Hurst (simplified) unsteady-state matches (Olsen, 1984).

As seen from Fig. 2, the production rate data can be considered almost constant in the time interval from 150 to 1000 days. In addition, we note that the water level data in this time interval increase linearly with time. Based on the asymptotic expression of Eq. 9 given in Sarak et al. (2003), we performed straight-line analysis of water level data in this time interval and found $\kappa_r = 2x10^9$ kg/bar. We also note that water-level data do not show any stabilized value. So, we cannot obtain an estimate of α_a from the asymptotic expression of Eq. 10 in Sarak et al. (2003), Then, using $\kappa_r = 2x10^9$ kg/bar and $\alpha_a = 30.44$ kg/(bar-s) as given by Olsen (1984) as initial guesses, we performed nonlinear regression of the Svartsengi production data using our 1-tank model. Figure 4 shows the match between the observed and simulated water level. The match obtained by Olsen using the Schilthuis model is also shown for comparison purposes. Notice that our match based on the 1-tank model fits the measured water level data better than the Olsen's match based on the Schilthuis model.

We then used our 1- and 2-tank models. Table 1 summarizes the parameters of the best fitting lumped models. Here and throughout, the percentages given in parentheses represent the 95% confidence interval in terms of percentages (Horne 1995). As seen from Table 1, the confidence percantages computed for the parameters of the 2-tank open model are quite high, (particularly see confidence intervals for κ_a and α_{a2}) compared to those of the 1-tank and 2-tank closed models. This gives an indication that the 2-tank open model is inappropriate for the data. However, 1-tank and 2-tank models appear to be appropriate for the data as the fits between the observed and simulated data are quite satisfactory (see Figure 5 and RMS values in Table 1) for both models. The 69% confidence interval for κ_a in the 2-tank closed model gives an indication that this parameter is not well determined compared to the other parameters of the model from the data available. Because results of our 1-tank and 2-tank closed model simulations did not exhibit any significant differences, further information and detailed analysis are required to identify the most appropriate model for the system.

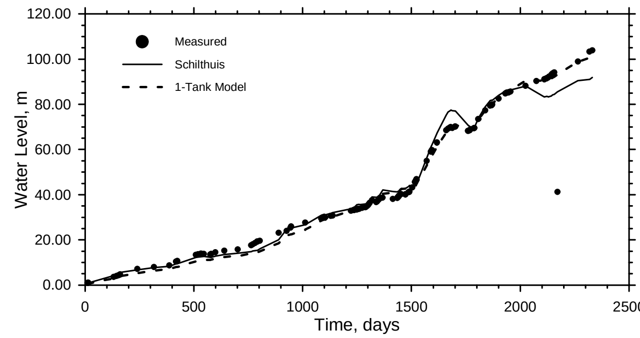

Figure 4. Comparison of observed and calculated water level changes in the Svartsengi field.

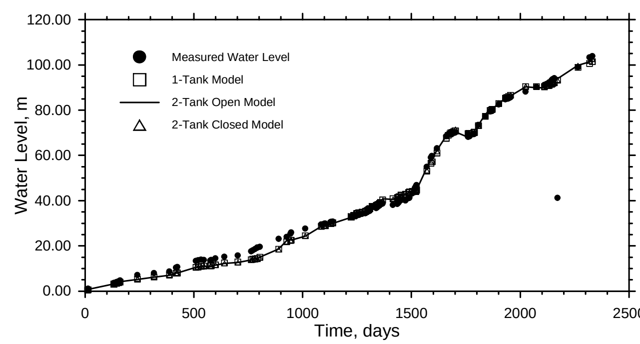

Figure 5. Simulation results of 1- and 2-tank models.

<u>Laugarnes Field</u>

The Laugarnes field in SW-Iceland is a considerably large field. The major feed zones are between depths of 700 and 1300 m and the water temperature is between 115 and 135 ^{o}C. A continuous water level record was available from one well. The Laugarnes field is discussed in Axelsson (1989), and Axelsson and Gunnlaugsson (2000). Axelsson (1989) used this data to simulate the pressure response of the field and to estimate its production capacity.

Table 1. Parameters of the best fitting lumped parameters

	Olsen Schilthuis	This Study 1-Tank	This Study 2-Tank Open	This Study 2-Tank Closed
κ_a, kg/bar	--	--	$6.9x10^8$ (63%)	$1.0x10^{10}$ (69%)
κ_r, kg/bar	$5.34x10^8$	$1.02x10^9$ (3%)	$6.1x10^8$ (38%)	$9.1x10^8$ (10%)
α_{a1}, kg/(bar-s)	--	--	28.3 (27%)	--
α_{a2}, kg/(bar-s)	30.44	24.6 (6%)	144 (150%)	30.2 (16%)
RMS, bar		0.504	0.507	0.504

Prior to exploitation the hydrostatic pressure at the surface in the geothermal field was 6-7 bars corresponding to a free water level 60-70 m above the land surface. Exploitation caused pressure drop in the field and water level fell. Figure 6 shows the water level changes and production history of the Laugarnes system.

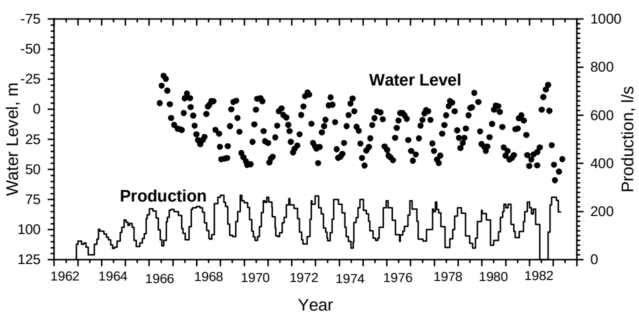

Figure 6. The water level changes and production history of the Laugarnes field (Axelsson, 1989; Axelsson, Gunnlaugsson, 2000).

Axelsson (1989) used a closed three capacitor lumped model (a three-tank with closed outer boundary model) for simulation. The simulations were carried out automatically by a computer. He treated the modeling as an inverse problem. He obtained quite a satisfactory match between the observed and calculated data (Figure 7). Results of our 3-tank closed model assuming the values of parameters given by Axelsson are also plotted for comparison. Axelsson's match and our match look almost identical.

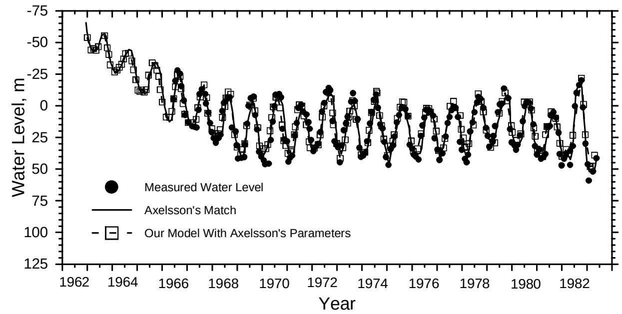

Figure 7. Comparison of observed and calculated water level changes in the Laugarnes field.

Next, using the parameter estimates of Axelsson (1989) as initial guesses for the parameters, we performed nonlinear regression analysis based on our model to estimate the parameters and obtained the best fit with the parameters given in Table 2. The RMS value for the fit was 0.581 bar. We note that the percantage confidence for the parameter κ_{a2} is quite high, indicating that this parameter is not well determined compared to the other parameters from the data. The Axelsson's parameters are also given for comparison purposes. Axelsson did not computed the confidence intervals for the parameters.

Figure 8 shows the observed and simulated water level changes in the Laugarnes field. There is a very good agreement between the observed and simulated data, in spite of long data sets, reflecting the flexibility of this method of lumped modeling and optimization algorithm.

Table 2. Parameters of the best fitting lumped parameters-(3-Tank with closed outer boundary model)

	Axelsson	This Study
κ_{a1}, kg/bar	$3.64x10^{10}$	$3.03x10^{10}$ (14%)
κ_{a2}, kg/bar	$2.09x10^9$	$2.47x10^9$ (107%)
κ_r, kg/bar	$7.73x10^7$	$7.94x10^7$ (20%)
α_{a2}, kg/(bar-s)	61.8	74.7 (21%)
α_{a3}, kg/(bar-s)	36.8	33.5 (9%)

Figure 9 presents the simulation results obtained from our 1-tank, 2-tank open, and 3-tank closed models. The 1-tank model does not give an acceptable match. Although the 3-tank closed lumped model gave the best match with the lowest RMS of the models tried, however, the 2-tank open model also gave a reasonable match as well as lower confidence intervals for the parameters (not shown here). This indicates that a 3-tank closed or 2-tank open model can be used to represent the system.

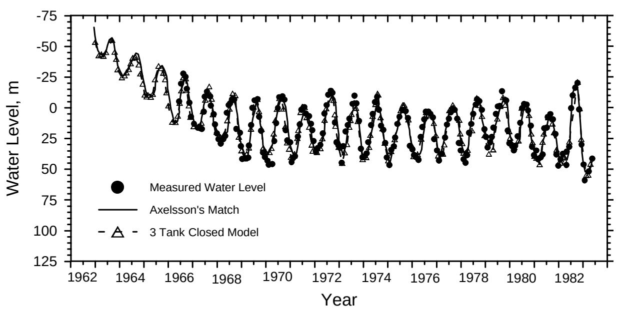

Figure 8. Comparison of Axelsson's match with our match.

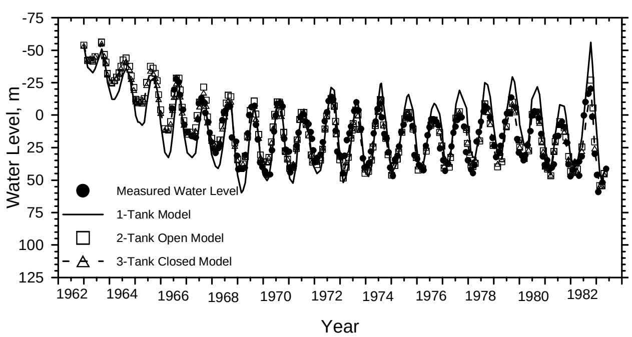

Figure 9. Simulation results of 1-, 2-, and 3-tank models.

Glerardalur Field

The water level and the production rate data in the Glerardalur low-temperature geothermal field in N-Iceland are presented in Figure 10 (Axelsson, 1989). This field has been utilized since 1982. The reservoir temperature at Glerardalur is about 61 ^{o}C. The main feed zone is at 450 m depth. Most of the wells drilled are shallow (100-300 m) exploration wells.

One problem involved in simulating the Glerardalur field was the absence of the initial reservoir pressure or the initial water level data. Lumped parameter modeling requires the initial water level to be known a priori. Hence, the simulations were carried out to determine the reservoir and aquifer parameters and as well as the initial water level. Our optimization study of the field data yielded an initial water level of -38 m.

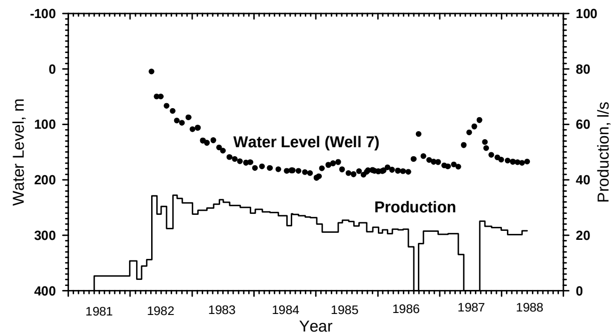

Figure 10. The water level data and data on the production in the Glerardalur field (Axelsson, 1989)

The observed water level behavior shown in Fig. 10 for the Glerardalur field resembles the behavior of a system with constant pressure outer boundary. For a constant production rate, the reservoir pressure of a constant pressure outer boundary system declines sharply at early times and then reaches to a constant value at late times. Equations 9, 10 and 16 given in Sarak et al. (2003) describe the early time and late time behaviors of the 1- and 2-tank systems with constant pressure source. The early time decline of the water level reflects the reservoir properties and its slope is equal to w_p / κ_r as given by Equation 9. In fact this slope relationship is valid for all systems since it is not a function of the aquifer properties and the outer boundary conditions. The late time steady-state reservoir pressure drop, however, is a function of the harmonic average of reservoir and aquifer productivities (α_{a1} and α_{a2}) and the production rate (w_p). Figure 10 exhibits a relatively constant production rate and a stabilized steady-state pressure drop. An equilibrium between production and recharge is eventually reached during long-term production, causing the reservoir pressure (or water level) drawdown to stabilize. Such a behavior is valid for systems with constant pressure outer boundary. Therefore we considered the 2-tank model with constant pressure outer boundary besides the 3-tank closed model as suggested by Axelsson (1989) for simulating the response of the Glerardalur field. Performing graphical analysis on the data by using the asymptotic equations given in our companion paper (Sarak et al. 2003), we estimated $\kappa_r = 2x10^7$ kg/bar and $\overline{\alpha}_a = 1.13$ kg/(bar-s). Here, $\overline{\alpha}_a$ represents the harmonic average. Based on these values, we chose our initial guesses of the parameters and performed nonlinear regression. Results of our simulation study are given in Table 3.

Table 3. Parameters of the best fitting lumped parameters

	Axelsson 3-Tank Closed	This Study 3-Tank Closed	This Study 2-Tank Open
κ_{a1} , kg/bar	$6.08x10^8$	$1.4x10^9$ (109%)	-
κ_{a2} ,kg/bar	$6.66x10^7$	$6.9x10^7$ (23%)	$8.07x10^7$ (12%)
κ_r , kg/bar	$5.9x10^6$	$7.6x10^6$ (36%)	$8.17x10^6$ (29%)
α_{a2} , kg/(bar-s)	1.89	1.5 (11%)	1.42 (5%)
α_{a3} , kg/(bar-s)	3.37	3.75 (21%)	3.41 (15%)
RMS, bar		0.6	0.61

Figure 11 shows the comparison of the observed data, calculated water level changes obtained by Axelsson and calculated by our 3-tank closed model using Axelsson's parameters of the best fitting models. Note that there are some differences between Axelsson's results and our results at early time data points. The reason for such difference could be due to the accuracy of the production data used in our model or due to the difference in initial water level values used in models. Axelsson used the measured values whereas we used the production data readings obtained from Figure 5 of Axelsson (1989).

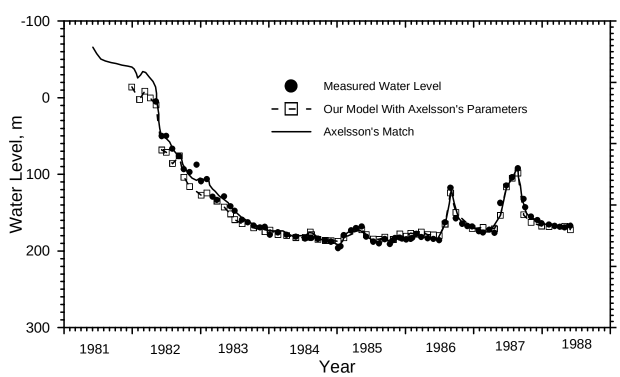

Figure 11. Comparison of observed and calculated water level changes (3-tank closed lumped model case).

As a next step, our model was used to determine the 3-tank closed lumped model parameters. Simulation results of our best fit yielded the parameters given in the third column of Table 3 with a RMS value of 0.6 bar. The results of the simulation, that is the comparison between observed and calculated water level, are presented in Figure 12.

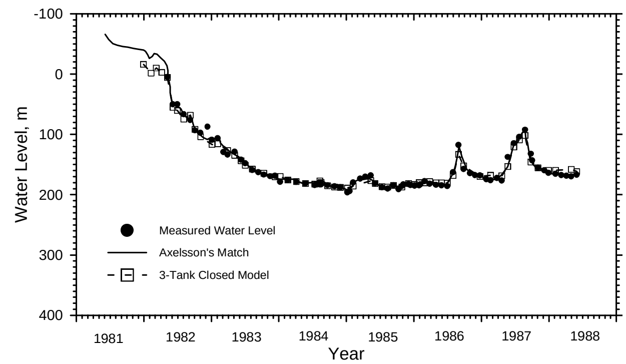

Figure 12. Comparison of observed and calculated water level changes.

Furthermore, we applied 1-tank and 2-tank open lumped models. Simulation results of those models as well as results of the 3-tank closed model are given in Figure 13 for comparison purposes. As is clear from Fig. 13, the water level computed from 1-tank model does not fit well the observed data. However, the 2-tank open and 3-tank closed models yield almost identical matches. This indicates that a 2-tank model with constant pressure outer boundary could also represent the Glerardalur field response.

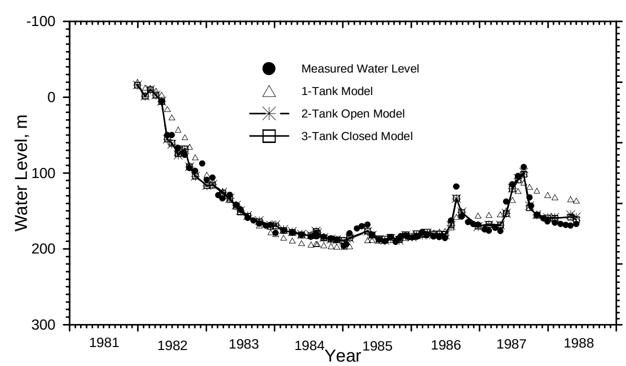

Figure 13. Simulation results of 1-, 2-, and 3-tank models.

Although Axelsson (1989) used a 3-tank closed model to match the data, a comparison of confidence intervals given in Table 3 for the parameters of both models indicates that a 2-tank open model is more appropriate for the data. Note from Table 3 that the confidence intervals of the estimated parameters for the 2-tank open model are narrower than the confidence intervals of the estimated parameters for the 3-tank closed model. However, the geological and geophysical conditions in the area should be considered in choosing the most appropriate model. This is not the scope of this study and no further analysis was conducted.

DISCUSSION

The pressure measured in the observation well is not necessarily representative of the average pressure in the reservoir. There may be interference from the producing wells around the observation well, causing the pressure to appear lower. To get the true average pressure, the reservoir should be shut in and allowing the pressure to stabilize. This is rarely accomplished since the reservoir is continually producing. To include the effects from each well, a superposition of the effects from all the wells would be necessary. However, in all the field cases discussed in this paper, the measured pressure is assumed to be representative for the reservoir.

Because the lumped models are based on many simplifying assumptions, their reliability is sometimes doubted. The predictions and simulations are made based on the data available. Amount (frequency) and duration (time length) of the data definitely affect the reliability of the simulations. The difference between the calculated and measured pressure or water level changes, if occurs, does not demonstrate unreliability of lumped modeling, but the uncertainty in such predictions. The amount and quality of the data and the type of model (1-, 2-, 3-tanks, with closed outer boundary or constant pressure boundary) chosen for modeling are the important parameters involved in uncertainty.

Geologic and geophysical data, whenever available, should be considered in preliminary estimates of reservoir and aquifer properties. Such data are particularly useful for early evaluation of $\kappa = V\phi\rho c$ for confined systems or $\kappa = A\phi / g$ for unconfined systems. Confined reservoir is the one produced by expansion. The water expands because of its compressibility. Unconfined reservoir is produced because of a fall in liquid level.

We should emphasize that the formulation of the lumped model is not affected by whether the system is assumed to be confined or unconfined. The κ value obtained from the simulation is analyzed to estimate the bulk volume *V* for the confined case or the lateral area *A* for the unconfined case. Checking the values of *V* or *A* with the those estimated from other sources can indicate the dominant producing mechanism of the system, whether it is confined or unconfined (Olsen, 1984; Gudmundsson and Olsen, 1987).

Effect of Injection

In order to maintain pressure in a reservoir, reinjection may be considered. The injection fluid will be colder and will cool down the reservoir. When the volume injected is known, an estimate of heat depletion in the reservoir can be made. In lumped parameter modeling the injected fluid must be included in the mass balance:

$$W_c = W_i - W_p - W_l + W_e + W_{in} \tag{5}$$

where W_i=initial mass, W_p=mass produced, W_l=mass loss, W_e=water influx(recharge), W_{in}=mass injected. The natural mass loss due to natural discharge or evaporation, W_l, has been assumed negligible in lumped parameter models. This may not be a good approximation. If the rate of mass loss is constant, this error is most pronounced for early time, since that is when the rate was low.

Assuming that the injection of fluid will not change the compressibility or total density of the system considerably, the injection and production terms can be lumped in a net production term:

$$W_{p,net} = W_p - W_{in} \tag{6}$$

which in terms of mass rate becomes:

$$w_{p,net} = w_p - w_{in} \tag{7}$$

Using this, the drawdown for a variety of production schedules can be predicted.

It should be noted that no transient effects in the reservoir and changes in temperature, density and compressibility as a result of injecting cold water have been considered.

CONCLUSIONS

The main conclusions of our work discussed in this paper are:

1. Lumped-parameter models used in this paper adequately match pressure (or water level) drawdown-production data.
2. Although up to three or four tanks can be considered in modeling the aquifer-reservoir system, however, the two tank models, one reservoir and one aquifer, seem to represent the low-temperature geothermal reservoirs sufficiently. Additionally, reducing the

number of the tanks in the model helps to decrease the nonuniqueness problem of the parameter estimation from the optimization procedure.

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