PROCEEDINGS, Twenty-Eighth Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, January 27-29, 2003
SGP-TR-173

NEW LUMPED PARAMETER MODELS FOR SIMULATION OF LOW-TEMPERATURE GEOTHERMAL RESERVOIRS

Hulya Sarak, Mustafa Onur, and Abdurrahman Satman

Petroleum and Natural Gas Engineering Department
Istanbul Technical University
Maslak, 80626, Istanbul, Turkey
e-mails: hulya@itu.edu.tr, onur@itu.edu.tr, mdsatman@itu.edu.tr

ABSTRACT

Axelsson (1989) described a method of lumped modeling to simulate data from several low-temperature geothermal reservoirs in Iceland. His lumped model is based on a general capacitor/conductor network. In his formulation, the basic system of equations was derived in matrix form and, thus, the solution is presented in implicit form.

The lumped parameter models presented in this paper are similar in concept to Axelsson's model. As in Axelsson's work, our solutions are valid for the low-temperature liquid reservoirs only and assume that variations in temperature within the system can be neglected. However, our model equations are given in terms of the well-known material balance equations, and the solutions are in the form of explicit analytical expressions.

In this paper, the analytical solutions are presented for 1 reservoir-1 aquifer, 1 reservoir-2 aquifers, and 1 upper reservoir-1 lower reservoir-1 aquifer systems. The reservoir simulates the innermost (production) part of the geothermal system, and the aquifers simulate the outer parts of the system. The outer aquifer can either be closed or can be connected to a constant-pressure source, which supplies the recharge to the geothermal system. The equations are presented in terms of pressure change, or the water level change of the reservoir. The rate of water influx (or recharge) between the aquifer and reservoir or between the aquifers is expressed by using Schilthuis's steady-state equation with water influx constant. The systems with more than 1 aquifer are unsteady-state hydraulic analogues of water influx into a reservoir. The Duhamel's principle is applied to obtain the solutions for the variable mass flow rate.

INTRODUCTION

Most of the low-temperature geothermal fields are produced by pumping from boreholes, while in some cases the production is by spontaneous discharge. The production causes the pressure in the geothermal systems to decline, which is reflected in the lowering of the water level in boreholes. The rate of pressure decline is determined by the rate of production, the size and properties of the geothermal system, and the recharge characteristics of the system. The recharge water invades the system in response to the lowered pressure or water level. Depending on the type of the system, temperature decline may or may not be observed.

Simple analytical models as well as complex numerical models can be used to simulate geothermal systems. A simpler approach is known as lumped-parameter or zero-dimensional simulation. In this case average properties are assigned to the parts of the geothermal system. The changes in pressure and/or water level are monitored and production/reinjection rates are recorded. If the temperature change is negligible or not observed, then the main factors describing the behavior of the system are the production/injection flow rate, pressure and/or water level. These data are important for all evaluation of changes in the reservoir and are important for simulations. Information on the nature, properties and size of a geothermal reservoir are obtained by careful monitoring of its production and reservoir history.

Several lumped-parameter models have been reported in the literature (Whiting and Ramey, 1969; Brigham and Ramey, 1981; Grant, 1977, Brigham and Neri, 1980; Grant, 1983; Castanier, Sanyal, and Brigham, 1980; Olsen, 1984; Gudmundsson and Olsen, 1987; Axelsson, 1989; Axelsson and Dong, 1998; Alkan and Satman, 1990). Axelsson and Gunnlaugsson (2000) discuss the usefulness of lumped parameter models of interpreting the monitored production data of low-temperature geothermal fields.

In this paper, an effective method of lumped parameter modeling is discussed. The lumped parameter models presented in this paper are similar in concept to Axelsson's (1989) model. As in

Axelsson's work, our solutions are valid for low-temperature liquid reservoirs only and assume that variations in temperature within the system are neglected. However, our model equations are given in terms of the well-known material balance equations, and the solutions are in the form of explicit analytical expressions. A companion paper by Sarak, Onur, and Satman (2003) presents field applications of the models.

MODELS OF LOW TEMPERATURE GEOTHERMAL SYSTEMS

For reservoirs with liquid only, the production path is for all practical purposes isothermal. The heat balance thus can be omitted. The reservoir is treated as one lump with average properties. In a confined system, the production is due to expansion of the compressed liquid. When this system is produced, the water level decreases in the same manner as emptying a tank.

During the drawdown history of a geothermal reservoir, recharge will maintain pressure in the reservoir, by replacing the produced liquids. A term of influx mass has to be added to the mass balance equations. Assuming no mass loss, the mass balance becomes

$$W_c = W_i - W_p + W_a \tag{1}$$

where the current mass, W_c, equals that initially in the reservoir, W_i, minus what has been produced, W_p, plus any water influx, W_a.

The initial fluid in place in a liquid-dominated reservoir may be compressed water. In this case, when the reservoir is produced, the water expands because of its compressibility. This is called a confined reservoir. For a reservoir of volume V_r, the liquid mass in place is given by

$$W_c = V_r \phi_r \rho_r \tag{2}$$

where ϕ_r is reservoir porosity, and ρ_r is liquid density. When this relationship and Equation 1 are differentiated with respect to time and the definition of isothermal compressibility is used, the following equation in terms of mass flow rate, w, results,

$$w_a - w_p = V_r \phi_r \rho_r c_t \frac{dp}{dt} \tag{3}$$

where c_t is the total (liquid+formation) compressibility for the reservoir system.

We use the steady-state Schilthuis (1936) water-influx method assuming that the recharge is proportional to the pressure difference between the tank and the recharge source. The pressure at the outside boundary of the system is constant, and the rate of influx is expressed as

$$w_a = \alpha_a (p_i - p) \tag{4}$$

where α_a is the "productivity index" of the aquifer, p_i is the pressure of the recharge source and p is the pressure of the reservoir. In cases where we model a closed system, we set $w_a = 0$ in Equation 3. Here and throughout, we work with pressures. However, the equations and solutions given in this paper can be represented in terms of water level $h(t)$ by using the relation $p(t) = \rho g h(t)$. Throughout, we use the SI system units.

We studied several variations of geothermal systems using the tank model approach and obtained explicit analytical solutions describing the reservoir pressure behavior. The systems studied in detail are 1 reservoir with recharge source, 1 reservoir-1 aquifer, 1 reservoir-2 aquifers, 1 reservoir-3 aquifers, 1 upper reservoir-1 lower reservoir-1 aquifer, and 1 upper reservoir-1 lower reservoir-2 aquifers. Here we discuss some of them and present results.

1 Reservoir With Recharge Source

Consider a geothermal system sketched in Figure 1 consisting of a reservoir and a recharge source. The system (reservoir and recharge source) is in equilibrium at $t = 0$. Reservoir is produced at a mass rate of w_p and the recharge source at a constant pressure of p_i supplies water.

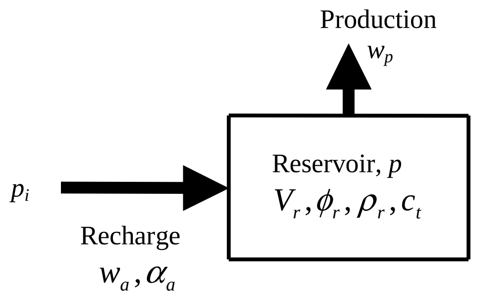

Figure 1. Schematic of a single tank model with recharge source.

Using Equation 4 in Equation 3 and rearranging the resulting equation gives

$$w_p = \alpha_a (p_i - p) - \kappa_r \frac{dp}{dt} \tag{5}$$

where

$$\kappa_r = V_r \phi_r \rho_r c_{tr} \tag{6}$$

Because p_i is constant, we can recast Equation 7 in terms of $\Delta p = p_i - p$ as

$$\frac{d\Delta p}{dt} + \frac{\alpha_a}{\kappa_r} \Delta p = \frac{w_p}{\kappa_r} \tag{7}$$

Equation 7 is a first order ordinary differential equation and its solution is given by

$$\Delta p = \frac{w_p}{\alpha_a} \left[1 - \exp(-\frac{\alpha_a t}{\kappa_r}) \right] \tag{8}$$

Equation 8 gives the pressure behavior of a geothermal system as a function of production time under the conditions of a constant production rate and a constant aquifer outer boundary pressure.

For sufficiently small times, t, the exponential term in Equation 8 can be approximated as $\exp(-\alpha_a t / \kappa_r) \approx 1 - \alpha_a t / \kappa_r$. Thus, Equation 8 becomes

$$p(t) = p_i - \frac{w_p t}{\kappa_r} \tag{9}$$

which clearly indicates that reservoir pressure will decline linearly with time, and recharge to reservoir will be negligible over these small times.

For large time values under constant production rate, Equation 8 reduces to

$$p(t) = p_i - \frac{w_p}{\alpha_a} = const. \quad for \quad t >> \kappa_r / \alpha_a \tag{10}$$

It is worth noting that for all practical purposes, Equation 10 becomes valid for $t \geq 5\kappa_r / \alpha_a$. So Equation 10 indicates that for all t such that $t \geq 5\kappa_r / \alpha_a$, the reservoir pressure stabilizes at a value determined by a balance with the recharge. The pressure decline becomes independent of reservoir storage coefficient, κ_r, but dependent on the aquifer productivity, α_a.

The pressure equation for the variable production rate case will be discussed later in the paper.

1 Reservoir-1 Aquifer Systems

The second lumped parameter model considered in this work consists of two tanks. A schematic of the 2-tank lumped model is shown in Figure 2. The first tank represents the reservoir, the inner or central part of the geothermal system, where the production occurs. The second tank simulates the outer part of the system (aquifer) recharging the reservoir. The aquifer may be treated as a closed one or with a constant pressure source. Both tanks are interconnected. If the second tank is connected to a constant pressure source, then this source supplies recharge with a rate of $w_{a1} (= \alpha_{a1} (p_i - p_1))$ to the geothermal system.

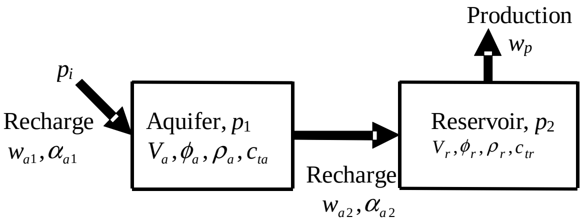

Figure 2. Schematic of a 2-tank model.

Hot water is pumped out of the first tank (reservoir), which causes the pressure and water level to decline. This in turn causes the decline of pressure and water level in the second tank (aquifer). Thus the total geothermal system is affected by the production.

When using this method of lumped parameter modeling, the data fitted (simulated) are the pressure or the water level data for an observation well in the reservoir, while the input for the model is the production/injection history of the field.

The mass balance written for the 2-tank system shown in Figure 2 yields the following differential equations describing the flow:

For aquifer:

$$\alpha_{a1} (p_i - p_1) - \alpha_{a2} (p_1 - p_2) = \kappa_a \frac{dp_1}{dt} \tag{11}$$

where $\kappa_a = V_a \phi_a \rho_a c_{ta}$

For reservoir:

$$w_p - \alpha_{a2} (p_1 - p_2) = -\kappa_r \frac{dp_2}{dt} \tag{12}$$

Initial condition:

$$p_1(t=0) = p_2(t=0) = p_i \tag{13}$$

The solutions describing the aquifer and reservoir pressure behaviors can be obtained from Equations 11-13 by the use of Laplace transformation. Details of the solution procedure can be found in Sarak (2003). Here, we only record the solution describing the reservoir pressure behavior, $\Delta p_2 = p_i - p_2$,

$$\Delta p_2 = \frac{w_p}{\kappa_r}\Big(\frac{d}{\mu_1\mu_2} + \frac{\mu_1 - d}{\mu_1(\mu_2-\mu_1)}e^{-\mu_1 t} + \frac{\mu_2 - d}{\mu_2(\mu_1-\mu_2)}e^{-\mu_2 t}\Big) \tag{14}$$

where $d = \frac{\alpha_{a1}+\alpha_{a2}}{\kappa_a}$, and μ_1 *and* μ_2 are the roots of

$$\left(s + \frac{\alpha_{a1}+\alpha_{a2}}{\kappa_a}\right)\left(s + \frac{\alpha_{a2}}{\kappa_r}\right) - \frac{\alpha_{a2}^2}{\kappa_a\kappa_r} = (s+\mu_1)(s+\mu_2) = 0 \tag{15}$$

The analytical solution of the 2-tank problem, Equation 14, yields the magnitude of the reservoir pressure decline and as well as the time at which the steady-state pressure drop occurs (Karaalioglu et al. 2002). The steady-state pressure drop is given by

$$(\Delta p_2)_{ss} = \left(\frac{\alpha_{a1}+\alpha_{a2}}{\alpha_{a1}\alpha_{a2}}\right)w_p = \left(\frac{1}{\alpha_{a1}} + \frac{1}{\alpha_{a2}}\right)w_p \tag{16}$$

and the time at which the steady-state pressure drop occurs (denoted by t_{ss}, steady-state or stabilization time) is found from

$$t_{ss} \cong \frac{10(\kappa_a\kappa_r)}{-\kappa_a\alpha_{a2} - \kappa_r(\alpha_{a1}+\alpha_{a2}) + SQT} \tag{17}$$

where SQT is given by

$$SQT = \sqrt{[\kappa_a\alpha_{a1} + \kappa_r(\alpha_{a1}+\alpha_{a2})]^2 - 4\alpha_{a1}\alpha_{a2}\kappa_a\kappa_r}$$

It is interesting to note from Equation 16 that the steady-state reservoir pressure drop, $(\Delta p_2)_{ss}$, is a function of the harmonic average of reservoir and aquifer productivities and the production rate (α_{a1}, α_{a2}, w_p), whereas the stabilization time, t_{ss}, is dependent on aquifer and reservoir properties and independent of the production rate. These results are similar to the ones obtained from the 1 reservoir tank with recharge source case.

We have also derived the solutions for the two-tank model for the case where the aquifer is closed. For this case, we set $\alpha_{a1} = 0$ in Equation 11 and then solve the resulting system of equations for p_1 and p_2 by using Laplace transformation (Sarak, 2003). Here, we only record the pressure change solution for the reservoir, i.e, $\Delta p_2(t)$, for the case of constant production rate (i.e., w_p is constant). This solution is given by

$$\Delta p_2(t) = \left(\frac{w_p}{\kappa_a+\kappa_r}\right)t - \left(\frac{\alpha_{a2}w_p}{\kappa_a\kappa_r\mu} - \frac{w_p}{\kappa_r}\right)\left(\frac{1-e^{-\mu t}}{\mu}\right) \tag{18}$$

where μ is given by

$$\mu = \frac{\alpha_{a2}(\kappa_a+\kappa_r)}{\kappa_a\kappa_r} \tag{19}$$

It can be shown that for sufficiently small values of time such that $1-e^{-\mu t} \approx \mu t$, Equation 18 reduces to

$$\Delta p_2(t) = \frac{w_p}{\kappa_r}t \tag{20}$$

which indicates that the pressure drop in the reservoir increases linearly with time with a slope equal to w_p/κ_r.

Similarly for sufficiently large times such that $1-e^{-\mu t} \approx 1$ in Equation 18, then it can be shown that Equation 18 becomes

$$\Delta p_2(t) = \frac{w_p}{(\kappa_a+\kappa_r)}t + \frac{w_p\kappa_a^2}{\alpha_{a2}(\kappa_a+\kappa_r)^2} \tag{21}$$

which indicates that the pressure drop in the reservoir increases linearly with time with a slope equal to $w_p/(\kappa_a+\kappa_r)$.

A Comparison of Behavior of the Open and Closed **2-***Tank Systems*

We already discussed the behavior of an open tank system with constant pressure outer boundary condition. If no recharge is allowed for the reservoir-aquifer system then the system is described as a closed system.

Figure 3 shows early time and late time reservoir pressure drawdown for the open and closed 2-tank lumped models for constant production rate. As long as the production rate is constant the early time pressure response of a closed system is the same as of an open system. The pressure drop increases linearly with the production time and is given by Eq. 20. This corresponds to pseudo-steady state flow behavior of the reservoir itself.

After some transition time the pressure (or water level) declines steadily with time. The pressure drop during this long-term production is given by Eq. 21, which corresponds to pseudo-steady state behavior of the total system of reservoir and aquifer.

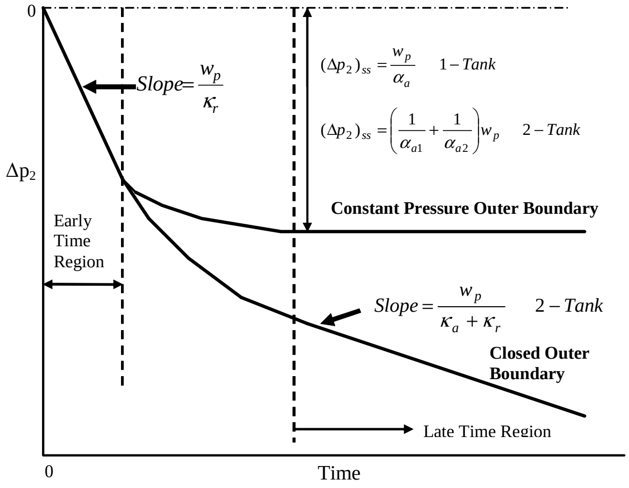

Figure 3. A comparison of early time and late time reservoir pressure drawdown in 2 tank closed and open lumped models for constant production rate.

1 Upper Reservoir-1 Lower Reservoir Case

Here, we consider a lumped model, where the reservoir consists of two parts; 1 upper (shallower) reservoir and 1 lower (deeper) reservoir. Both are supplied by the same recharge source. However, due to depth differences between the upper and lower reservoirs, the initial pressures for both reservoirs at equilibrium condition may vary. The pressure of the recharge source stays constant at p_i. A schematic of the model discussed is presented in Figure 4.

Let p_{1i} be the initial pressure of the upper reservoir and p_{2i} be the initial pressure of the lower reservoir. Then the basic equations are the mass flow equations:

for upper reservoir:

$$\alpha_{a1}(p_i - p_1) + \alpha_{12}(p_2 - p_1) - w_{p1} = \kappa_{r1}\frac{dp_1}{dt} \qquad (22)$$

for lower reservoir:

$$\alpha_{a2}(p_i - p_2) - \alpha_{12}(p_2 - p_1) - w_{p2} = \kappa_{r2}\frac{dp_2}{dt} \qquad (23)$$

and the initial conditions

$$p_1(t=0) = p_{1i}, \quad p_2(t=0) = p_{2i} \qquad (24)$$

where w_{p1} and w_{p2} are the mass flow rates from the upper and lower reservoirs, respectively.

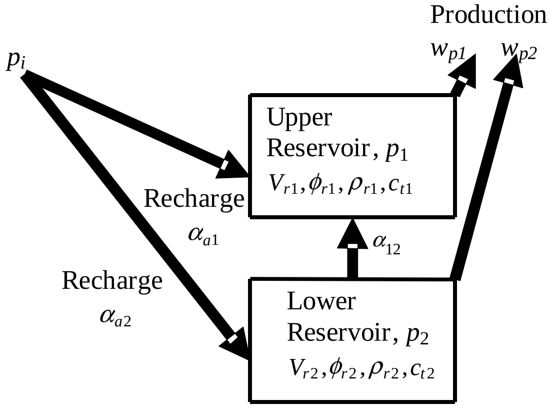

Figure 4. Schematic of a lumped model with 1 upper reservoir and 1 lower reservoir, both supplied by the same recharge source.

Assuming w_{p1} and w_{p2} are constant, the solution of system of equations given by Equations 22-23 with the initial condition of Equation 24 can be obtained by use of the Laplace transformation (Sarak, 2003). The solutions in terms of pressure change for upper and lower reservoirs (Δp_1 and Δp_2) are given by, respectively,

$$\begin{aligned}\Delta p_1(t) &= \left(\frac{w_{p1}}{\kappa_{r1}} - \beta_1 \Delta p_{1i} + \frac{\alpha_{12}}{\kappa_{r1}}\Delta p_{2i}\right) \\ &\times \left[\frac{\mu_1 - \beta_2}{\mu_1(\mu_1 - \mu_2)}\left(1 - e^{-\mu_1 t}\right) + \frac{\mu_2 - \beta_2}{\mu_2(\mu_2 - \mu_1)}\left(1 - e^{-\mu_2 t}\right)\right] \\ &+ \left(\frac{\alpha_{12} w_{p2}}{\kappa_{r1}\kappa_{r2}} + \frac{\alpha_{12}^2}{\kappa_{r1}\kappa_{r2}}\Delta p_{1i} - \frac{\alpha_{12}\beta_2}{\kappa_{r1}}\Delta p_{2i}\right) \\ &\times \left[\frac{1}{\mu_1(\mu_2 - \mu_1)}\left(1 - e^{-\mu_1 t}\right) + \frac{1}{\mu_2(\mu_1 - \mu_2)}\left(1 - e^{-\mu_2 t}\right)\right]\end{aligned} \qquad (25)$$

and

$$\Delta p_2(t) = \left(\frac{w_{p2}}{\kappa_{r2}} + \frac{\alpha_{12}}{\kappa_{r2}} \Delta p_{1i} - \beta_2 \Delta p_{2i} \right)$$
$$\times \left[\frac{\mu_1 - \beta_1}{\mu_1(\mu_1 - \mu_2)} \left(1 - e^{-\mu_1 t}\right) + \frac{\mu_2 - \beta_1}{\mu_2(\mu_2 - \mu_1)} \left(1 - e^{-\mu_2 t}\right) \right]$$
$$+ \left(\frac{\alpha_{12} w_{p1}}{\kappa_{r1}\kappa_{r2}} - \frac{\alpha_{12}\beta_1}{\kappa_{r2}} \Delta p_{1i} + \frac{\alpha_{12}^2}{\kappa_{r1}\kappa_{r2}} \Delta p_{2i} \right)$$
$$\times \left[\frac{1}{\mu_1(\mu_2 - \mu_1)} \left(1 - e^{-\mu_1 t}\right) + \frac{1}{\mu_2(\mu_1 - \mu_2)} \left(1 - e^{-\mu_2 t}\right) \right] \quad (26)$$

where

$$\Delta p_j(t) = p_{rji} - p_j(t) \quad (27)$$

$$\Delta p_{ji} = p_i - p_{rji} \quad (28)$$

$$\beta_j = \frac{\alpha_{aj} + \alpha_{12}}{\kappa_{rj}} \quad (29)$$

and

$$\kappa_{rj} = V_{rj} \phi_{rj} \rho_{rj} c_{trj} \quad (30)$$

for j=1,2. In Equations 25 and 26, μ_1 and μ_2 are the roots of

$$\left(s + \frac{\alpha_{a1} + \alpha_{12}}{\kappa_{r1}} \right) \left(s + \frac{\alpha_{a2} + \alpha_{12}}{\kappa_{r2}} \right) - \frac{\alpha_{12}^2}{\kappa_{r1}\kappa_{r2}} .$$

$$= (s + \mu_1)(s + \mu_2) = 0 \quad (31)$$

Other Cases Considered

We have also considered lumped parameter models containing more than two tanks. For example, Figure 5 shows a system containing 1 reservoir and 2 aquifers. The system is represented by three tanks. The outer aquifer is connected to a recharge source at a constant pressure of p_i. Thus the system is called open three-tank model.

When the number of tanks is increased the analytical solution of the problem becomes complicated. The analytical solution for this system has been obtained (Sarak, 2003), however, it is not included here due to the length of the equations.

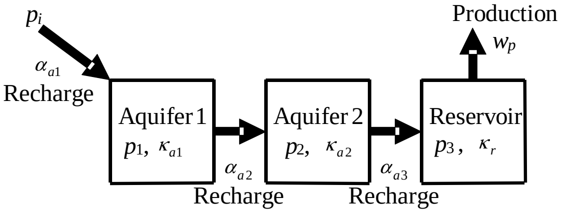

Figure 5. Schematic of an open three-tank (1 reservoir-2 aquifers) model.

Modeling Variable Mass Flow Rate

The Duhamel's principle is applied to obtain the solutions for the variable mass flow rate. Applying the Duhamel's principle, the pressure drop in the reservoir is given by

$$\Delta p(t) = \int_0^t w_p(\tau) \Delta p_u^{'}(t - \tau) d\tau \quad (32)$$

or in terms of pressure,

$$p(t) = p_i - \int_0^t w_p(\tau) \Delta p_u^{'}(t - \tau) d\tau \quad (33)$$

where Δp_u is the pressure drop that would be obtained with the unit constant mass rate of w_p and $\Delta p_u^{'}$ is the time rate of change (or simply time derivative) of Δp_u. For example, Equation 8 is the general solution for the constant mass withdrawal w_p, then if we set w_p = 1 kg/s in Equation 8, we can obtain the unit rate pressure change, $\Delta p_u(t)$, and its time derivative, $\Delta p_u'(t)$, respectively, as

$$\Delta p_u(t) = \frac{1}{\alpha_a} \left[1 - \exp\left(-\frac{\alpha_a t}{\kappa_r} \right) \right] \quad (34)$$

and

$$\Delta p_u'(t) = \frac{1}{\kappa_r} \exp\left(-\frac{\alpha_a t}{\kappa_r} \right) \quad (35)$$

For the other lumped models discussed in this paper, one can similarly derive the unit-rate pressure change and its time derivative and apply Equation 32 or 33 to generate the pressure response for a given variable mass flow rate history. In cases where we have step changes of mass flow rate history, we consider a partition of the time interval (0,*t*) as $0 = t_0 < t_1 < t_2 < \cdots < t_n < t_{n+1} = t$ and write Equation 32.

$$\Delta p(t) = p_i - p(t) = \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} w_p(\tau)\Delta p_u'(t-\tau)d\tau \quad (36)$$

Following Thompson and Reynolds (1986) and Kuchuk and Ayestaran (1985), we approximate Equation 36 as:

$$\Delta p(t) = \sum_{j=0}^{n} \Delta w_p(t_{j+1})\,\Delta p_u(t-t_j) \quad (37)$$

where $\Delta w_p(t_{j+1}) = w_p(t_{j+1}) - w_p(t_j)$ represents the mass flow rate steps. Note that in deriving Equation 37, we used $t_0 = 0$, $w_p(0)=0$ and $\Delta p_u(0) = 0$.

DISCUSSION AND CONCLUSIONS

The models presented in this paper are based on material balance only. We recognize that in geothermal reservoirs, the heat content of the rock and fluid constitute the main resource, so an energy balance should be included to describe the system more accurately. However, unless excessively disturbed, the cooling effect in the low-temperature geothermal systems may be neglected. For example, the production histories of the low-temperature geothermal reservoirs in Turkey do not indicate significant changes in the temperature of the water produced. Generally speaking, pressure (or water-level) decline is the parameter affected from production operations.

Our main objective was to study the effect of recharge on the pressure behavior of the low-temperature geothermal systems. We developed the explicit analytical solutions for several types of geothermal systems. Solutions yield the importance of the reservoir and aquifer parameters on simulation of the geothermal system's pressure (or water-level) behavior.

The lumped parameter models presented in this paper are appropriate in cases where data on reservoir and aquifer conditions are scarce but where the pressure (or water level) response of a reservoir has been recorded for some time.

REFERENCES

Alkan, H., Satman, A. (1990), "A New Lumped Parameter Model For Geothermal Reservoirs in the Presence of Carbon Dioxide," *Geothermics*, Vol. 19, No. 5, 469-479.

Axelsson, G. (1989), "Simulation of Pressure Response Data From Geothermal Reservoirs by Lumped Parameter Models," *14th Workshop on Geothermal Reservoir Engineering*, Stanford University, USA, 257-263.

Axelsson, G., Dong, Z. (1998), "The Tanggu Geothermal Reservoir (Tianjin, China)," *Geothermics*, Vol. 27, No. 3, 271-294.

Axelsson, G., Gunnlaugsson, E. (2000), Long_term Monitoring of High-and Low-Enthalpy Fields Under Exploitation, International Geothermal Association-International Institute for Geothermal Research, Auckland, New Zealand.

Brigham, W.E., Neri, G. (1980), "A Depletion Model for the Gabbro Zone (Northern Part of Larderello Field)," *Proceedings Second DOE-ENEL Workshop for Cooperative Research in Geothermal Energy*, Oct. 20-22, LBL-11555.

Brigham, W.E., Ramey, H.J.Jr. (1981), "Material and Energy Balance in Geothermal Reservoirs," *Reservoir Engineering Assessment of Geothermal Systems*, Ramey, H.J.Jr. (editor), Petroleum Engineering Department, Stanford University.

Castanier, L.M., Sanyal, S.K., Brigham, W.E. (1980), "A Practical Analytical Model for Geothermal Reservoir Simulation," *SPE Cal. Regional Meeting*, Los Angeles, Ca, April 9-11, SPE 8887.

Grant, M.A. (1977), "Approximate Calculations Based on a Simple One Phase Model of a Geothermal Reservoir," *New Zealand Journal of Science*, Vol. 20, 19.

Grant, M.A. (1983), "Geothermal Reservoir Modeling," *Geothermics*, Vol. 12, No. 4, 251.

Gudmundsson, J.S., Olsen, G. (1987), "Water-Influx Modeling of the Svartsengi Geothermal Field, Iceland," *SPE Reservoir Engineering* (Febr.), 77-84.

Karaalioglu, H., Onur, M., Satman, A. (2002), "Effects of Natural Recharge on Reservoir Behavior," *UTES'2002(IV. National Clean Energy Symposium)*, Proceedings, 629-637, Istanbul, Turkey, 16-18 October.

Kuchuk, F.J., Ayestaran, L. (1985), “Analysis of Simultaneously Measured Pressure and Sandface Flow Rate in Transient Testing,” *Journal of Petroleum Technology*, (February), 323-334.

Olsen, G. (1984), Depletion Modeling of Liquid Dominated Geothermal Reservoirs, *Technical Report SGP-TR-80*, Stanford Geothermal Program, Stanford University, USA.

Sarak, H. (2003), Effect of Recharge on Geothermal Reservoir Behavior, PhD Thesis (in progress), Petroleum and Natural Gas Engineering Department, Istanbul Technical University, Turkey.

Sarak, H, Onur, M., Satman, A. (2003), “Applications of Lumped Parameter Models for Simulation of Low-Temperature Geothermal Reservoirs,” *28th Stanford Workshop on Geothermal Reservoir Engineering*, Stanford University, USA, 27-29 Jan.

Schilthuis, R.J. (1936), “Active Oil and Reservoir Energy,” *Trans. AIME*, 118, 33-52.

Thompson, L.G., Reynolds, A.C. (1986), “Analysis of Variable Rate Well-Test Pressure Data Using Duhamel’s Principle,” *SPE Formation Evaluation*, (October), 453-469.

Whiting, R.L., Ramey, H.J.Jr. (1969), “Application of Material and Energy Balances to Geothermal Steam Production,” *Journal of Petroleum Technology* (July), 893-900.