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MATRIX-FRACTURE TRANSFER FUNCTIONS FOR PARTIALLY AND COMPLETELY IMMERSED FRACTURES

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ABSTRACT

Modeling multiphase flow in fractured porous media relies on the accurate description of matrix to fracture transfer of water. The rate of mass transfer between the rock matrix and fractures is significant, and calculation of this rate, within dual-continuum models, depends on matrix-fracture transfer functions incorporating the shape factor. Typically, matrix-to-fracture transfer functions are obtained by assuming all fractures to be instantaneously immersed in water (instantly-filled), with a uniform fracture pressure distribution under pseudo-steady state conditions. The result is constant, time-independent, shape factors. Clearly, this is not necessarily true. Partially immersed fractures and other unsteady-state conditions do not lead to constant shape factors.

A new time-dependent matrix-fracture transfer shape factor formulation and transfer functions for both filling- and instantly-filled fracture transfer are derived based on dimensional analysis. The general shape factor is expressed as the area of the matrix block contacted by the wetting phase divided by the product of the bulk volume of the rock matrix times the distance between the saturation at the fracture and the matrix average saturation. These parameters are obtained by means of either the analytical model for imbibition proposed by Rangel-German and Kovscek (2002) or experimental data (Rangel-German, 2002, for example). One of the advantages of the new shape factor is that it is based on dimensional analysis. This avoids simplifications that may lead to expressions that do not represent the physics of matrix-fracture transfer. The new shape factor carries information of the transient behavior of the water saturation, S_w, and so it leads to more accurate description of the matrix-fracture transfer. Good agreement was found between the experimental data and analytical model, and the modified dual porosity formulation with the new time-dependent shape factor and transfer function.

INTRODUCTION

Fractured systems are usually modeled by means of a dual-porosity or dual-permeability formulation. One of the chief features of such formulations is the evaluation and modeling of the matrix-fracture mass transfer, that is, the mass rate at which a matrix feeds an adjacent fracture. Rock matrix blocks contribute the main portion of the reservoir pore volume, but they have much smaller permeabilities than do the fractures. Flow between the matrix block and the fracture is essential to the productivity of fractured formations. This rock matrix flow can be estimated using a shape factor in a transfer function.

Unfortunately, there is no clear physical understanding of how the matrix-to-fracture transfer occurs, and, therefore, the few mathematical expressions for matrix to fracture fluid transfer found in the literature may provide erroneous recovery information. There is a need for expressions that account for realistic boundary conditions (partially covered or totally immersed boundaries), as a function of parameters that can be obtained either in the laboratory or the field with high certainty. The following sections describe the transfer functions derived for a fractured system. Both filling- and instantly-filled fractures under unsteady state conditions with uniform and variable fracture conditions are considered.

Typically, matrix-to-fracture transfer functions are obtained by assuming all fractures to be instantaneously immersed in water (instantly-filled), with a uniform fracture pressure distribution under pseudo-steady state conditions. The boundary conditions represent a step change in the fracture pressure, p_f, at the initial time that is held constant for the rest of the process. The result is, constant, time independent, shape factors. This type of derivation is equivalent to assuming a linear pressure gradient between the center of a matrix block and the fractures surrounding the block (Ueda *et al.*, 1989). Clearly, this is not necessarily true.

This paper will first summarize the experimental results and the analytical model for capillary imbibition previously presented (Rangel-German and Kovscek, 2002). It will then describe the definition of the new general shape factor and transfer function and how the former results are used to evaluate such transfer functions. Finally, an example compares the results among experimental data, analytic model, and the modified dual porosity formulation.

MULTIDIMENSIONAL IMBIBITION IN FRACTURED POROUS MEDIA

Whereas most experimental studies have focused upon single factors important to imbibition, Rangel-German and Kovscek (2002) investigated the rate of fracture to matrix transfer and the pattern of wetting fluid imbibition as a function of the rate of water propagation in a fracture. Experimental equipment was constructed to allow detailed and accurate measurement of the extent and rate of imbibition in an idealized fracture and matrix block.

An apparatus was designed to insure minimal artifacts while imaging and the collection of maximum saturation distribution information. In summary, the apparatus consists of cubic rock samples, epoxied on all sides, potted in cylindrical PVC containers. Once the core was covered by epoxy in the PVC container, the bottom face of the cylinders was cut open to expose the rock. Fracture apertures were set by means of metallic shims (essentially feeler gauges of precise thickness) placed between the block and end-caps. Thus, a uniform fracture aperture is guaranteed along the entire core. Using this system, fluids can be injected and produced in any combination desired, as shown in Fig. 1.

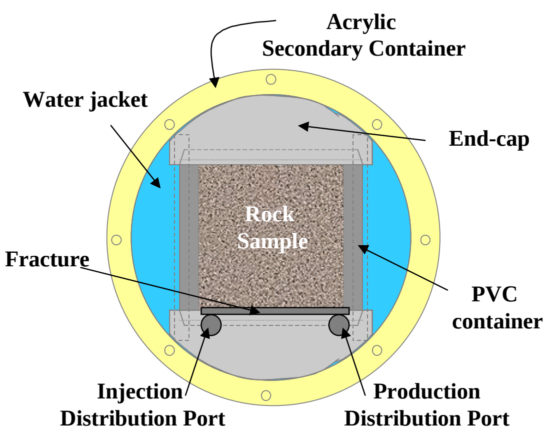

Figure. 1. The core holder: Frontal view.

Using this coreholder, a single X-ray CT exposure was used to image the entire length of the matrix block and the matrix/fracture interface. Hence, the spatial distribution of water was measured as a function of time. Two regimes were identified with respect to the amount of water within a fracture: instantly-filled and filling fracture regimes. For details on experimental design and exhaustive results, refer to Rangel-German (2002).

Completely immersed fractures: Instantly-filled fracture regime

The typical assumption while dealing with fractured media is to have all the fractures "instantly-filled" by water. In this instantly-filled fracture regime, little water imbibes before the fracture fills with water. Indeed, for some combinations of water injection flow rate and fracture aperture, fractures can be considered to be completely immersed. Fig. 2 shows nearly a one-dimensional water advance in one of the CT-scanning multiphase, single-matrix block experiments by Rangel-German and Kovscek (2002).

CT images were obtained throughout the duration of injection to monitor the progress of imbibition, as shown in Fig. 2. Dark shading indicates zero water saturation while white indicates fully water saturated. The mass of water imbibed is anticipated to scale linearly with the square root of time. The behavior of this regime is very similar to that observed during both counter current and cocurrent imbibition experiments reported previously in the literature (Akin *et al.*, 2000; Handy, 1960; Cil and Reis, 1996; Reis and Cil, 1999; Rangel-German and Kovscek, 2002; Li and Horne, 2000, Zhou *et al.*, 2002).

Partially immersed fractures: Filling-fracture regime

Rangel-German and Kovscek (2002) showed both experimentally and analytically that the "instantly-filled" fractured regime is not always the case during water injection. If a combination of a relatively small injection flow rate and wide fracture occurs, the fractures will first be on the "filling-fracture" regime. This regime shows a variable length plane source due to relatively slow water flow through fractures. Water advance in the fracture is controlled by the interaction between the matrix and the fracture as shown in Fig. 3. Similarly, CT images were obtained throughout the duration of injection to monitor the progress of imbibition. Again, dark shading indicates zero water saturation while white indicates fully water saturated. Because the advance of water (inside the matrix) in the horizontal and vertical directions each scale linearly with the square root of time, we anticipate that the mass of water imbibed scales

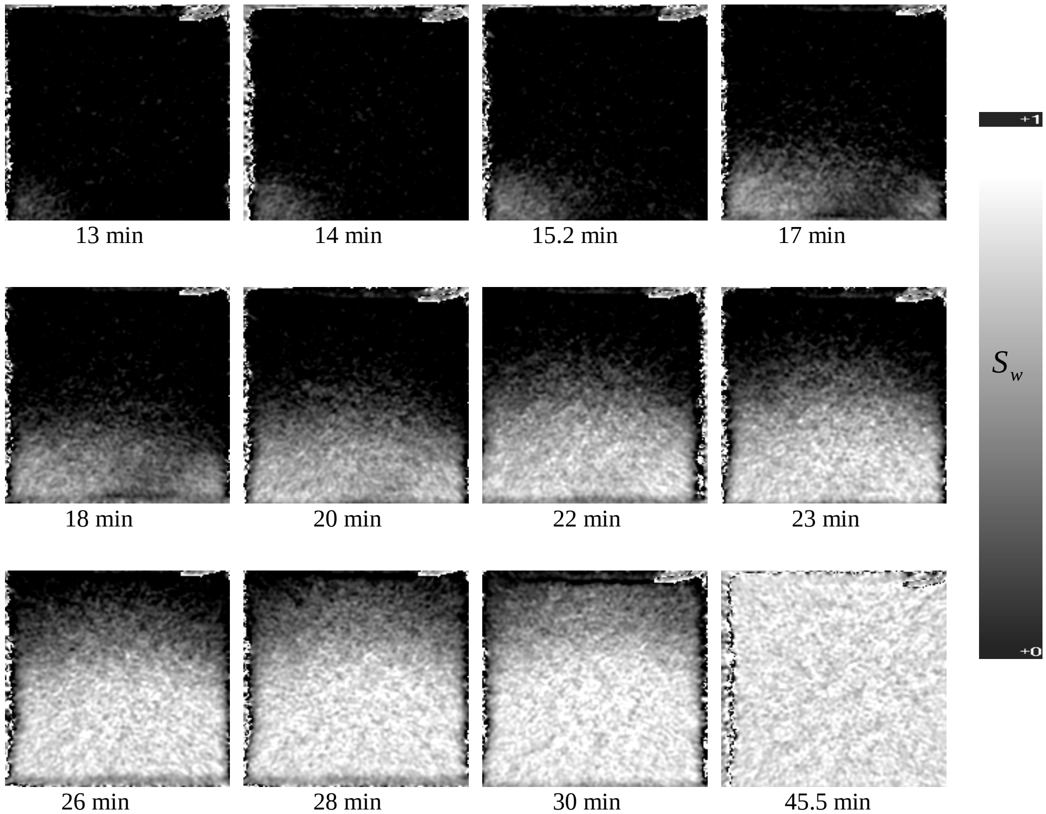

Figure. 2. CT images for "instantly-filled fracture" system for different times. Water injection at 1 cc/min in a fracture 0.025 mm thick. Injection is from lower left corner and production from lower right corner.

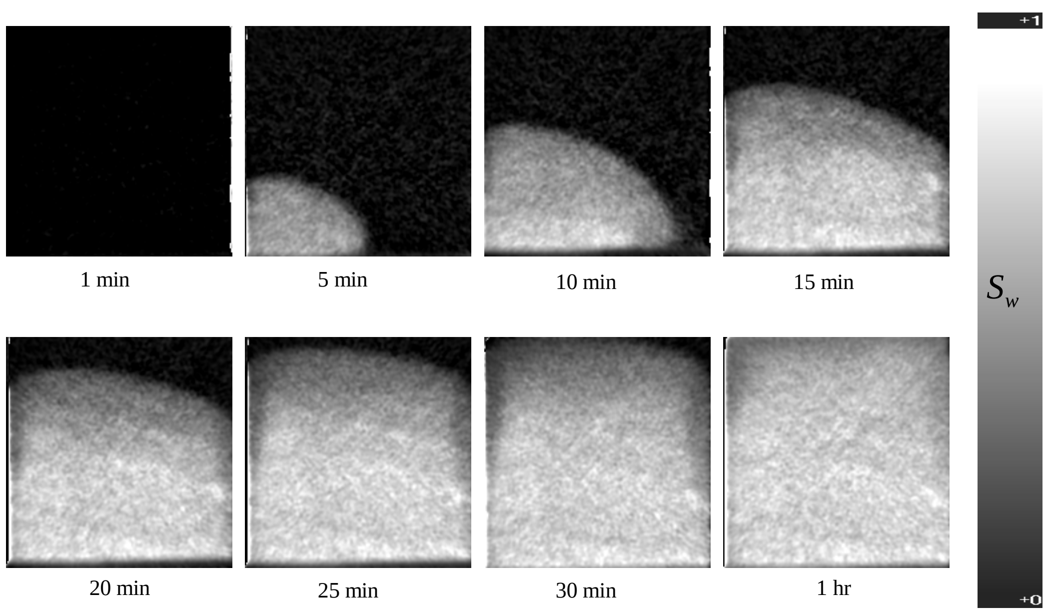

Figure. 3. CT images for "filling-fracture" system for different times. Water injection at 1 cc/min in a fracture 0.1 mm thick. Injection is from lower left corner and production from lower right corner.

linearly with time. That is, the mass imbibed in this two-dimensional geometry is proportional to the product of these two length scales. The "filling-fracture" regime presented by Rangel-German and Kovscek (2002) was also found to lead to a more efficient recovery of non-wetting fluid.

In summary, the average water saturation, $\overline{S_w}$, is linearly proportional to the mass of water imbibed and nonwetting fluid expelled. Due to the variable-extent of the imbibing area in the filling-fracture regime, a new analytical model for imbibition is necessary.

It is also clear that the 'completely immersed fractures' assumption is not necessarily correct. When fractures behave under unsteady state conditions such as the "instantly-filled fracture" regime with a long transient (of water saturation), or the "filling-fracture" regime, a more general transfer shape factor and matrix-fracture transfer is required.

ANALYTICAL MODEL FOR IMBIBITION: FILLING- AND FILLED-FRACTURES

This section summarizes a model for early-time behavior (Rangel-German, 2002). The "instantly-filled" regime is well described by the square-root-of-time model and so is not reanalyzed here. Thus, the analytical model for imbibition contains two parts: the filling-fracture and the filled-fracture regimes.

The model is based on the material balance principle and accounts for the growing matrix area in contact with the water in the fracture. A linear superposition of one-dimensional solutions is applied as illustrated in Fig. 4. Capillary-based diffusion equations with equivalent constant diffusivity values have been shown to model accurately imbibition problems (Beckner *et al.*, 1987, Chen *et al.*, 1995; Reis and Cil, 1999), so the solution to the one-dimensional saturation diffusion equation is used in the superposition. A dimensionless time was also developed so that scaling from laboratory to field data can also be performed.

Three assumptions were used: 1) the water saturation in the matrix is constant initially, 2) the water saturation in the matrix at an infinite distance from the fracture remains constant, and 3) the convection/diffusion equation with constant diffusivity, α_h, applies, where $\alpha_h = kk_{rw}/\mu_w \, (dP_c/dS_w)$.

Upon neglecting convection in the matrix, the one-dimensional solution of the diffusion equation subject to these boundary conditions is (Carslaw and Jaeger, 1959)

$$S_w(z,t) = erfc\left(\frac{z}{2\sqrt{\alpha_h t}}\right) \qquad (1)$$

A linear superposition of Eq. 1 in the (horizontal) x-direction approximates the S_w distribution obtained in the matrix within the "filling-fracture" regime, as shown in Fig. 4. The matrix is not allowed to begin filling until the time, τ, when a front within the fracture reaches a particular position, x. The implications of this assumption are discussed in Rangel-German (2002). Another necessary approximation is to treat matrix/fracture transfer analytically. Given the horizontal position of the front, as obtained analytically in the following section, and using Eq. 1 with an estimated value for hydraulic diffusivity, the vertical position of any iso-saturation, S_w, is calculated. The model is validated by the experimental results.

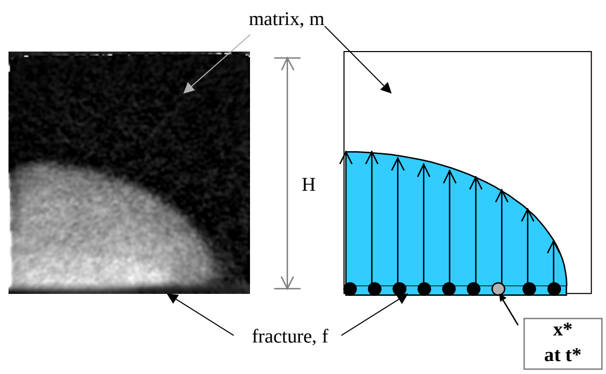

Figure. 4. Schematic of the linear superposition approximation for the "filling-fracture" regime. Left image taken from Fig. 3.

It is not simple to obtain the location of the water front in the fracture. Mass transfer between the fracture and the matrix slows the frontal advance within the fracture. In order to obtain the location of the front in the fracture, a material balance was used, with the assumptions that water and the rock are incompressible:

$$q_{w,inj} = q_{w,m} + q_{w,f} \qquad (2)$$

where $q_{w,inj}$ is the volumetric water injection rate, $q_{w,m}$ is the volumetric rate at which water enters the matrix, and $q_{w,f}$ is volumetric water advance rate in the fracture.

Equation 3 gives the water transfer rate from the fracture to the matrix, $q_{w,m}$,

$$q_{w,m} = 2n\int_0^t \left(\alpha_h \frac{\partial S_w(z,t-\tau)}{\partial z}\bigg|_{z=0} \frac{dA(t)}{d\tau} \right) d\tau \qquad (3)$$

where n is the number of fractures associated with the matrix block and A is the area of the matrix exposed to wetting fluid and τ is the time when a portion of the fracture surface is exposed to water. The factor 2 appears in Eq. 3 because a fracture is, generally, bounded by two matrix blocks. The water advance in the fracture is given as

$$q_{w,f} = n\phi_f w_f \frac{dA}{dt} \qquad (4)$$

where ϕ_f is the fracture porosity, w_f is the fracture width. The solution for the area of the fracture filled with water is

$$A(t) = \frac{q_{w,inj}\,\phi_f\, w_f}{4n\,\alpha_h\,\phi^2}\left\{ e^{t_D} erfc\left[\sqrt{t_D}\right] + 2\sqrt{\frac{t_D}{\pi}} - 1 \right\} \qquad t \le t_c \qquad (5)$$

where

$$t_D = 4\left(\frac{\phi}{\phi_f}\right)^2 \frac{\alpha_h}{w_f^2}\, t \qquad (6)$$

Let R_{ff} be the rate of imbibition during the filling-fracture regime, a constant for a given set of constant injection rate, fracture aperture, and fluid system. We can also write the material balance as

$$Q_m(t) = q_{w,inj}t - Q_{w,f} \qquad (7)$$

where Q_m is the volume of water imbibed to the matrix and $Q_{w,f}$ is the volume of water in the fracture. Then

$$R_{ff} = \frac{dQ_m(t)}{dt} = q_{w,inj} - q_{w,f} \qquad t \le t_c \qquad (8)$$

where $q_{w,f}(t)$ is given by Eq. 4 and dA(t)/dt (in Eq. 4) is the derivative of Eq. 5. We note that the “instantly-filled” regime is well described by Eq. 1 and needs no further analysis. The rate of imbibition during instantly-filled fracture regime, R_{if}, is a constant for a given set of constant injection rate and fluid system. Thus for both regimes and accounting for initial water saturation, S_{wi}, the equation for average water saturation in the matrix block is:

$$\overline{S_w} = tR_{ff}\left[1 - \mathrm{H}(t-t_c)\right] + \left[t_cR_{ff} + (t-t_c)^{0.5}R_{if}\right]\mathrm{H}(t-t_c) + \overline{S_{wi}} \qquad (9)$$

where H is the Heaviside function; and t_c is the characteristic time for the fracture. The characteristic time, t_c, is the boundary between imbibition regimes. The fracture fills instantly with respect to the matrix if t_D (= t/t_c) is large. This time can be found implicitly from Eq. 1 when A is set to the fracture characteristic area.

PROPOSED GENERAL TRANSFER FUNCTION FORMULATION

Due to the variable-extent of the imbibing area in the filling-fracture regime, it cannot be approached in as straight-forward a fashion as the instantly-filled fracture regime. To obtain a characteristic, dimensionless solution, a dimensional analysis was performed. It has two objectives: (i) to obtain a better understanding of the definition of shape factor within the dual porosity/dual permeability formulation, and (ii) provide a general shape factor for any kind of geometry, shape and regime that can be applied to current commercial numerical simulators without major corrections.

Equation 10 is the typical form of the transfer function in any dual porosity/dual permeability simulator. Simplified versions of the behavior of the shape factor, σ, are used to obtain computational efficiency.

$$q = \frac{\sigma k V}{\mu}\left(\overline{p_m} - p_f\right) \qquad (10)$$

Here, q is the flow rate in an element V of bulk reservoir volume, p_m is the volumetric average matrix pressure and p_f is the pressure in the fracture. The shape factor, σ, reflects the geometry of the matrix elements in pseudo-steady state, single-phase flow at all times.

Transfer functions and shape factors have not received sufficient attention. Enormous discrepancies also exist for values of shape factors proposed by different investigators. For blocks of size L, and for one-dimensional systems, values range from $4/L^2$ (Kazemi *et al.*, 1976) to $12/L^2$ (Warren and Root, 1963; de Swaan, 1990). Whereas for three-dimensional systems shape factors range from $12/L^2$ (Kazemi *et al.*, 1976) to $60/L^2$ (Warren and Root, 1963; de Swaan, 1990). A fuller discussion of shape factors and transfer functions is presented in Rangel-German (2002).

Exponential approximations or using only the first term of the summations in the analytical solutions helps keep the calculations efficient by giving a constant shape factor; however, these approaches do not necessarily capture the physics of the process. Apparently, this is a common case of trade-off between computational efficiency and accuracy of a solution. However, the definition of the transfer function in Eq. 10 is a Darcy's law type equation applied to a characteristic distance between the matrix block and the fracture. Dimensional analysis teaches that the proportionality constant, σ (shape factor), can be expressed as:

$$\sigma(t) = \frac{A(t)}{V\ l^*(t)} \tag{11}$$

where $A(t)$ is the area of the matrix block contacted by water, V is the bulk volume of the rock matrix block and l^* is the distance between the saturation at the fracture and the matrix average saturation, $\overline{S_w}$. Thus, the transfer function proposed in this work is

$$\tau = V\alpha_h\sigma\left(S_{w\max} - \overline{S_{wm}}\right) \tag{12}$$

with the shape factor, σ, is calculated using Eq. 11. This makes its implementation in any dual porosity simulator straightforward.

The analytical model summarized in the previous section also includes an equation for the water advance in a single fracture in the presence of capillary imbibition into adjacent matrix blocks (Eq. 5). This equation was obtained by coupling the matrix and fracture implicitly, and it is used here to derive or estimate the transfer functions needed in numerical reservoir simulation. The area of the matrix block contacted by water, A(t), is by Eq. 5, or its approximation (Rangel-German and Kovscek, 2002).

It is important to mention that Eq. 11 is a general expression as it is derived from the dimensional definition of shape factor; thus, $A(t)$ can be obtained from any given model of imbibition. However, Eq. 5 is used here because, to the best of our knowledge, it is the only proposed model for the filling-fracture regime. It is easy to implement, and its evaluation is fast because the equation is analytic and expressed in terms of the rock and fluid properties. Most importantly, it was validated with experimental data (Rangel-German and Kovscek, 2002; and Rangel-German, 2002).

The only term missing to compute the shape factor in Eq. 11 is the distance between the water saturation at the fracture and the matrix average water saturation, l^*. For a steady-state problem, it is clear that l^* is the half distance between the fractures (i.e. half-length of the matrix block). However, for unsteady state behavior such as early-time filled-fracture or during the filling-fracture regime, the behavior of l^* is not obvious. In this work, a simple analytical method to obtain l^* is proposed. The integral of the saturation distribution over the entire domain (n-D, if needed) is equal to the average saturation in the matrix times a characteristic distance to the centroid of the domain. The characteristic distance, l^*, for one set of orthogonal fractures is

$$l^*(t) = \frac{\int_0^{L_x} S_w(x,t)\,dx}{\overline{S_{wm}}(t)} \tag{13}$$

thus, l^* depends on the shape of the matrix block. A similar analogous $\sigma(t)$ is needed when dealing with pressure; thus, l^*_p is obtained in a manner similar to Eq. 13, but using the pressure solution and the average pressure in the matrix.

From experimental results obtained with CT scanning, the characteristic length, l*, is computable for every CT image. That is, every CT image (corresponding to a single time) of every experiment is translated into a bitmap file containing the water saturation value at each pixel. The integral of Eq. 13 is calculated numerically. Then, one computes the average water saturation in the matrix. Thus, Eq. 13 is evaluated for every single CT image. Fig. 5 illustrates a plot of the shape factor, $\sigma(t)$, calculated by means of Eq. 13 against dimensionless time, $t_D = \frac{\alpha_h t}{L_x^2}$, in a log-log plot, for four different experiments.

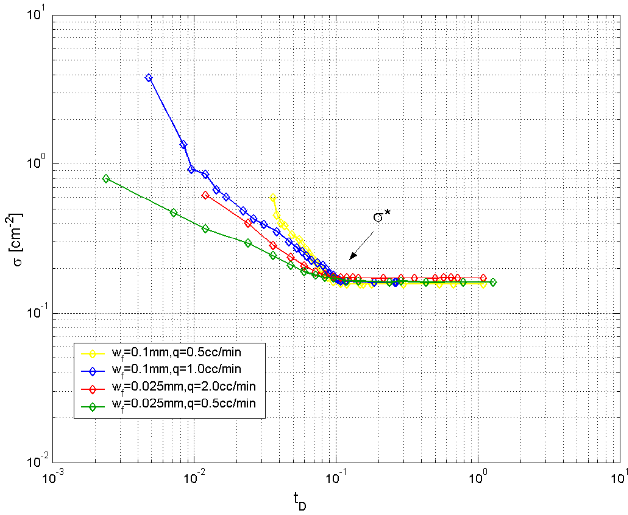

Figure. 5. Shape factor versus dimensionless time for both filling-fracture and instantly-filled fracture experiments (log-log coordinates).

All the experiments converge to the constant shape factor, σ^*, at the same characteristic dimensionless time, $t_D{}^*$. The experimental data indicate that the dimensionless time at which the new shape factor converges to a constant value is approximately, t^*_D = 0.1. In general, the new shape factor can be written as:

$$\sigma = \sigma^* \left(\frac{t_D}{t_D^*} \right)^{-m} \qquad \text{for} \quad t_D < t_D{}^* \qquad (14)$$

and

$$\sigma = \sigma^* \qquad \text{for} \quad t_D \geq t_D{}^* \qquad (15)$$

where t^*_D is 0.1 and m is a function of the flow rate and the fracture aperture. Note that this formulation converges to the classical constant σ (Kazemi *et al.*, 1976), and also has a time varying portion for t_D less than $t_D{}^*$. A numerical validation of the proposed shape factor is presented in the next section with comparisons made to the experimental data and analytical model, and the modified dual porosity formulation with the new time-dependent shape factor and transfer function.

VALIDATION

As explained above, it is possible to express the shape factor analytically. Fig. 6 shows the best fit obtained using Eq. 14 and 15 to match the experiment performed with a fracture aperture of 0.1 mm and flow rate of 1 cm^3/min (also in Fig. 3). This experiment has a large fracture and medium flow rate, and so lay well within the filling-fracture regime. After water filled the fracture, imbibition switched to filled-fracture behavior. The entire experiment (both filling and filled fracture) is matched using Eq. 14 and 15 in Fig. 5. The best match is obtained using a value for the slope, *m*, of 0.592.

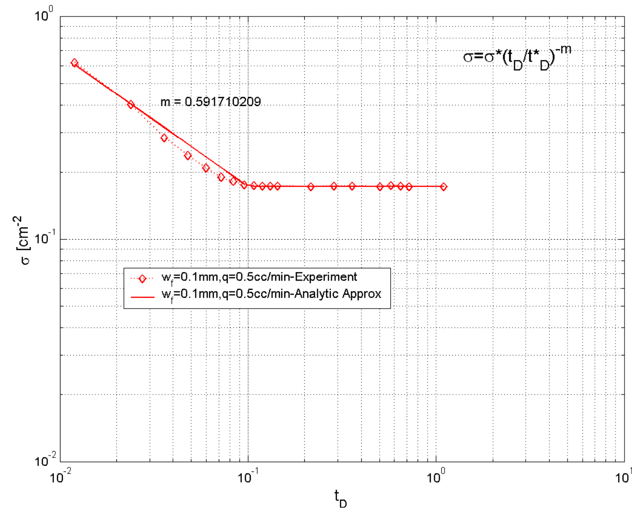

Figure. 6. Comparison of the shape factor versus dimensionless time between the experimental results with the analytical approximation (Eqs. 14 and 15).

Using Eq. 12, one can calculate the transfer to the matrix for a time step. For the first time step the matrix average saturation, $\overline{S_w}$, is equal to the initial water saturation (=0 for this experiment). Multiplying the transfer times Δt and dividing by the pore volume of the matrix, an increment in the matrix average water saturation is obtained:

$$\Delta \overline{S_{wm_i}} = \frac{\tau_w(t_i)}{\phi V} \Delta t \qquad (16)$$

and the new matrix average saturation is

$$\overline{S_{wm_{i+1}}} = \overline{S_{wm_i}} + \Delta \overline{S_{wm_i}} \qquad (17)$$

This computed value is input into Eq. 12 and the process is repeated for the next time steps. It is clear from Eq. 12 that the imbibition rate is large initially and, as water imbibes into the matrix, and the fracture is filled with water, the rate drops until converging to zero at late times. Fig. 7 compares the matrix average saturation versus time between the experimental

results and the $\overline{S_w}$ calculated with the new formulation. Agreement is excellent, as expected.
When there is no flow between matrix blocks, the matrix-fracture transfer of a grid block with no source terms is expressed, for constant porosity as

$$\tau_w = -V\,\phi \frac{d\,\overline{S_{wm}}}{dt} \qquad (18)$$

This equation indicates that the change in water saturation in the matrix is equal to the rate of water imbibed. Using the analytic equation for the average water saturation (Eq. 9) discussed in the previous section, the water transfer in a matrix block (exclusively for dual porosity formulation) is calculated. Fig. 7 also includes a curve of the matrix average saturation calculated using Eq. 9.

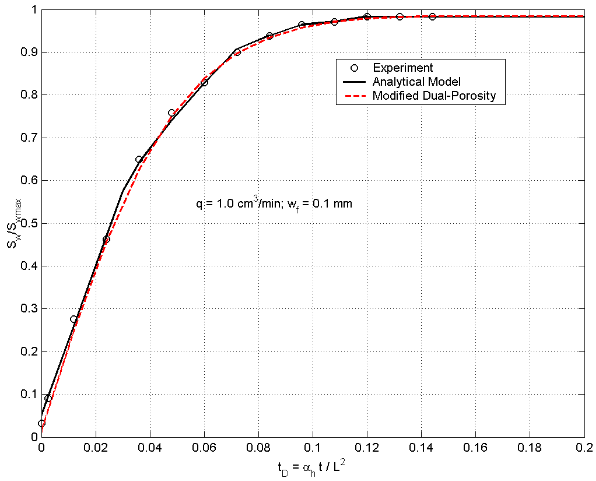

Figure. 7. Comparison of the average water saturation versus dimensionless time between the experimental results with the proposed transfer function.

SUMMARY AND CONCLUSIONS

The pseudo-steady state approach for describing matrix-fracture transfer leads to constant shape factors. However, these conditions do not necessarily describe the flow in fractured porous media. Thus, true behavior of the shape factor is clearly time dependent, and therefore the use of constant shape factor is incorrect. The current formulations for modeling flow in fractured porous media need to be reconsidered.

Based on our experimental and analytical results, more general transfer functions are derived. These functions provide a better understanding of the discrepancies in the values for shape factors found in the literature. A new transfer function is also presented in such a way that it can be easily included in current reservoir simulators without coding complications. This transfer function is general and can be applied to both instantly-filled and filling-fracture regimes. Although, it is not presented in this paper, the new shape factor can be applied for different block geometries as it was obtained from dimensional analysis based on the area open to imbibition, matrix block volume and a characteristic length that depends on the solution for either water saturation or pressure in the matrix.

One of the advantages of the new shape factor is that it is based on dimensional analysis. This avoids simplifications that may lead to expressions that do not represent the physics of matrix-fracture transfer. The experimental results helped guide us toward simple expressions for the shape factor, such as those shown in Eqs. 14 and 15. The laboratory experimental results have an advantage over a numerical experimental result, as they provide flow behavior that is closer to reservoir reality. CT scan measurements embody minimal errors and the water saturation maps are reliable data.

NOMENCLATURE

A = area
CT = CT number (Hounsfields)
H = Heaviside function
k = absolute permeability
K_r = relative permeability
n = number of fractures
P_c = capillary pressure
Q = rate (m^3/s)
R = rate of imbibition (m^3/s)
S = saturation
t = time (s)
W = fracture width (m)
x = horizontal distance(m)
X = location of wetting fluid front in fracture (m)
z = vertical distance (m)

α_h = hydraulic diffusivity
ϕ = porosity
μ = viscosity
σ = shape factor

Subscripts.
c = critical
D = dimensionless
f = fracture
ff = filling fracture
if = instantly filled
inj = injected
m = matrix
o = oil
w = water

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