

CALCULATION OF STEAM-WATER RELATIVE PERMEABILITY USING CAPILLARY PRESSURE DATA

Kewen Li and Roland N. Horne

Stanford Geothermal Program, Stanford University
Stanford, CA 94305-2220

e-mail: kewenli@stanford.edu horne@stanford.edu

ABSTRACT

Various capillary pressure approaches such as the Purcell, Burdine, Corey, and Brooks-Corey methods were used to calculate steam-water relative permeabilities using the measured data of steam-water capillary pressure in both drainage and imbibition processes. The calculated results were compared to the experimental data of steam-water relative permeability measured in Berea sandstone. The steam-water relative permeability and capillary pressure were measured simultaneously. The differences between the Purcell model and the measured values were almost negligible for water phase relative permeability in both drainage and imbibition but not for the steam phase. The insignificance of the effect of tortuosity on the wetting phase was revealed. A physical model was developed to explain the insignificance of the tortuosity. Steam phase relative permeabilities calculated by other models were very close to the experimental values for drainage, but very different for imbibition, as expected. The same calculation was made for the nitrogen-water flow to confirm the observation in the steam-water flow. The results in this study showed that it would be possible and useful to calculate steam-water relative permeability using the capillary pressure method, especially for the drainage case.

INTRODUCTION

Steam-water relative permeability plays an important role in controlling reservoir performance for water injection into two-phase geothermal reservoirs. However it is difficult to measure steam-water relative permeability because of the phase transformation and the significant mass transfer between the two phases as pressure changes. At the same time, Li and Horne (2001a) found significant differences between steam-water and air-water capillary pressures, and Horne *et al.* (2000) found significant differences between steam-water and air-water relative permeabilities. According to these studies, steam-water flow properties may not be

simply replaced by air (or nitrogen)-water flow properties. It would be helpful for reservoir engineers to be able to calculate steam-water relative permeability once steam-water capillary pressure is available.

There are a lot of papers (Gates and Leits, 1950; Rapoport and Leas, 1951; Fatt and Dykstra, 1951; Wyllie and Gardner, 1958; Land, 1968 and 1971; Huang *et al.*, 1997) related to the capillary pressure method for the calculation of oil-gas relative permeabilities. Honarpour *et al.* (1986) reviewed the literature in this field. The published literature and experimental data for relative permeability and capillary pressure were not sufficient to conclude which method should be the standard one.

Unlike for oil-gas flow properties, there are few studies for the calculation of steam-water relative permeabilities by the capillary pressure technique. Historically, the capillary pressure techniques were developed for drainage situations and were useful to obtain gas-liquid (oil or water) relative permeability when fluid flow tests were not practical. As stated previously, it is difficult to measure steam-water relative permeability. In this paper, we calculated the steam-water relative permeability by different capillary pressure techniques and compared to the measured data in the same core sample. The steam-water capillary pressure data from Li and Horne (2000) were used to compute steam-water relative permeability. The steam-water capillary pressure was measured at a temperature of about 120°C in Berea sandstone. The experimental data of steam-water relative permeability used to compare with the calculated data were measured by Mahiya (1999) and Horne *et al.* (2000) in the same core sample and at the same temperature.

METHOD

We chose four representative models developed by various authors (Purcell, 1949; Burdine, 1953; Corey, 1954; Brooks and Corey, 1966) to calculate steam-water relative permeabilities using the capillary

pressure techniques. The mathematical expressions of the four models are briefly described in this section.

Purcell Model. Purcell (1949) developed an equation to compute rock permeability by using capillary pressure data. This equation can be extended readily to the calculation of multiphase relative permeability. In two-phase flow, the relative permeability of the wetting phase can be calculated as follows:

$$k_{rw} = \frac{\int_0^{S_w} dS_w / (P_c)^2}{\int_0^1 dS_w / (P_c)^2} \quad (1)$$

where k_{rw} and S_w are the relative permeability and saturation of the wetting phase; P_c is the capillary pressure as a function of S_w .

Similarly, the relative permeability of the nonwetting phase can be calculated as follows:

$$k_{rnw} = \frac{\int_{S_w}^1 dS_w / (P_c)^2}{\int_0^1 dS_w / (P_c)^2} \quad (2)$$

where k_{rnw} is the relative permeability of the nonwetting phase. It can be seen from Eqs. 1 and 2 that the sum of the wetting and nonwetting phase relative permeabilities at a specific saturation is equal to one. This may not be true in most porous media. In the next section, the relative permeabilities calculated using this method are compared to the experimental data. The comparison shows that Eq. 1 is close to experimental values of the wetting phase relative permeability but Eq. 2 for nonwetting phase is far from the experimental results.

Burdine Model. Burdine (1953) developed equations similar to Purcell's method by introducing a tortuosity factor as a function of wetting phase saturation in the calculation of relative permeability by the capillary pressure method. The relative permeability of the wetting phase can be computed as follows:

$$k_{rw} = (\lambda_{rw})^2 \frac{\int_0^{S_w} dS_w / (P_c)^2}{\int_0^1 dS_w / (P_c)^2} \quad (3)$$

where λ_{rw} is the tortuosity ratio of the wetting phase. According to Burdine (1953), λ_{rw} could be calculated as follows:

$$\lambda_{rw} = \frac{\tau_w(1.0)}{\tau_w(S_w)} = \frac{S_w - S_m}{1 - S_m} \quad (4)$$

where S_m is the minimum wetting phase saturation from the capillary pressure curve; $\tau_w(1.0)$ and $\tau_w(S_w)$ are the tortuosities of the wetting phase when the

wetting phase saturation is equal to 100% and S_w , respectively.

In the same way, relative permeabilities of the nonwetting phase can be calculated by introducing a nonwetting phase tortuosity ratio. The equation can be expressed as follows (Burdine, 1953):

$$k_{rnw} = (\lambda_{rnw})^2 \frac{\int_{S_w}^1 dS_w / (P_c)^2}{\int_0^1 dS_w / (P_c)^2} \quad (5)$$

where λ_{rnw} is the tortuosity ratio of the nonwetting phase, which can be calculated as follows (Burdine, 1953):

$$\lambda_{rnw} = \frac{\tau_{nw}(1.0)}{\tau_{nw}(S_w)} = \frac{1 - S_w - S_e}{1 - S_m - S_e} \quad (6)$$

here S_e is the equilibrium saturation of the nonwetting phase; τ_{nw} is the tortuosity of the nonwetting phase.

Honarpour *et al.* (1986) pointed out that the expression for the wetting phase relative permeability (Eq. 3) fits the experimental data much better than the expression for the nonwetting phase (Eq. 5).

Corey Model. According to the Purcell and Burdine models, an analytical expression for the wetting and nonwetting phase relative permeabilities may be obtained if capillary pressure curves can be represented by a simple mathematical function. Corey (1954) found that oil-gas capillary pressure curves could be expressed approximately using the following linear relation:

$$1 / P_c^2 = C S_w^* \quad (7)$$

where C is a constant and S_w^* is the normalized wetting phase saturation which could be expressed as follows for the drainage case:

$$S_w^* = \frac{S_w - S_{wr}}{1 - S_{wr}} \quad (8a)$$

where S_{wr} is the residual saturation of the wetting phase or water phase in steam-water flow. In Corey's case, S_{wr} is the residual oil saturation.

Although the Corey model was not originally developed for imbibition case, in this study it was used to calculate the imbibition steam-water relative permeabilities by defining the normalized wetting phase saturation as follows:

$$S_w^* = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{nwr}} \quad (8b)$$

where S_{nwr} is the residual saturation of the nonwetting phase, representing the residual steam saturation in this study.

Substituting Eq. 7 into Eqs. 3 and 5 with the assumption that $S_e=0$ and $S_m=S_{wr}$, Corey (1954) obtained the following equations to calculate the wetting (oil) and nonwetting (gas) phase relative permeabilities for drainage cases:

$$k_{rw} = (S_w^*)^4 \quad (9)$$

$$k_{rnw} = (1 - S_w^*)^2 [1 - (S_w^*)^2] \quad (10)$$

A constraint to the use of Corey's Model (Eqs. 9 and 10) is that the capillary pressure curve should be represented by Eq. 7.

Brooks-Corey Model. Because of the limitation of Corey's model, Brooks and Corey (1966) modified the representation of capillary pressure function to a more general form as follows:

$$P_c = p_e (S_w^*)^{-1/\lambda} \quad (11)$$

where p_e is the entry capillary pressure and λ is the pore size distribution index.

Substituting Eq. 11 into the Burdine model (Eqs. 3 and 5) with the assumption that $S_e=0$, Brooks and Corey (1966) derived equations to calculate the wetting and nonwetting phase relative permeabilities as follows:

$$k_{rw} = (S_w^*)^{\frac{2+3\lambda}{\lambda}} \quad (12a)$$

$$k_{rnw} = (1 - S_w^*)^2 [1 - (S_w^*)^{\frac{2+\lambda}{\lambda}}] \quad (12b)$$

When λ is equal to 2, the Brooks-Corey model reduces to the Corey model.

RESULTS

The data of both drainage and imbibition steam-water capillary pressure from Li and Horne (2000) were used to calculate the corresponding steam-water relative permeability. Note that the capillary pressure data were represented using Eq. 11 in all the calculations by the Purcell model. The calculated results were compared to the experimental data of steam-water relative permeability (Mahiya, 1999; Horne *et al.*, 2000). During the process of the fluid

flooding tests, the water saturation in the core sample was first decreased from 100% to the residual water saturation, about 28%, representing a drainage process. The water saturation was then increased, representing an imbibition. The calculations and the comparisons are presented in this section.

Fig. 1 shows the experimental data of the steam-water relative permeability and capillary pressure in drainage (Mahiya, 1999; Horne *et al.*, 2000; Li and Horne, 2000). All these data were measured at a temperature of about 120°C in the same Berea core sample. The permeability and porosity of this core were 1400 md and 24.8%; the length and diameter were 43.2 cm and 5.04 cm, respectively. Because the relative permeability and the capillary pressure were measured simultaneously, the two curves had the same residual water saturations. This feature is important and will be discussed in more detail later. Note that the steam relative permeability data shown in Fig. 1 have been calibrated under the consideration of gas slip effect (Klinkenberg Effect) in two-phase flow by Li and Horne (2001b).

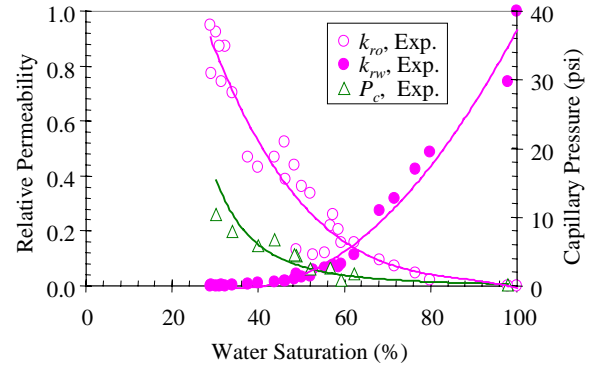


Figure 1: Experimental data of drainage steam-water relative permeability and capillary pressure.

The drainage steam-water relative permeabilities were calculated using the experimental data of the drainage steam-water capillary pressure shown in Fig. 1 and plotted versus the normalized water saturation that is defined in Eq. 8a. The calculated results and the comparison to the corresponding experimental data are shown in Fig. 2. The water relative permeabilities calculated using the Purcell model are the best fit to the experimental data. This implies that it may not be necessary to adjust the calculation of the wetting phase relative permeabilities by introducing the concept of the tortuosity factor in such a case. The water phase relative permeabilities calculated by all the other models are less than the experimental values. It can be seen from Fig. 2 that the steam phase (nonwetting phase) relative permeabilities calculated by all the models but the Purcell model are almost the same and

consistent with the experimental data for the drainage case. The steam phase relative permeabilities calculated by the Purcell model are not shown in Fig. 2 and all the figures following in this section because the curve is concave downwards, which is unexpected and far from the experimental values.

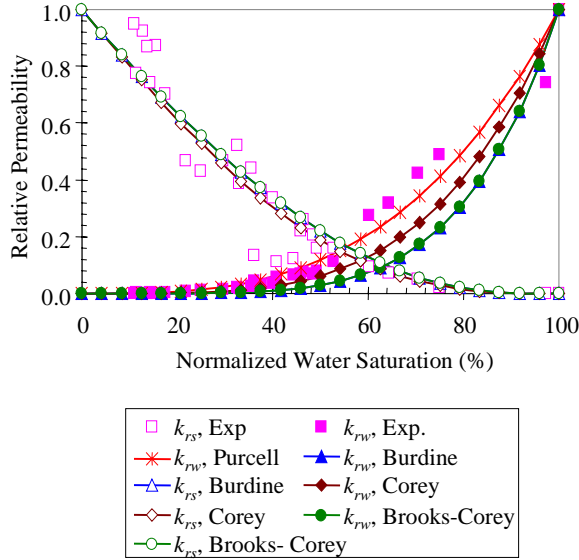


Figure 2: Calculated steam-water relative permeability and the comparison to the experimental data in the drainage case.

The experimental data of the imbibition steam-water relative permeability and the imbibition capillary pressure are shown in Fig. 3. These data were also measured simultaneously in the same Berea core sample (Mahiya, 1999; Horne *et al.*, 2000; Li and Horne, 2000) at a temperature of about 120°C. The steam relative permeability data shown in Fig. 3 have also been calibrated under the consideration of gas slip effect in two-phase flow (Li and Horne, 2001b).

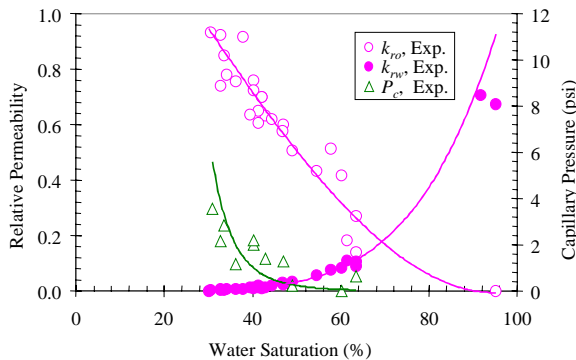


Figure 3: Experimental data of imbibition steam-water relative permeability and capillary pressure.

The imbibition steam-water relative permeabilities were then calculated using the measured data of the

imbibition steam-water capillary pressure shown in Fig. 3 and also plotted versus the normalized water saturation. Fig. 4 shows the calculated results and the comparison to the experimental values. The water relative permeabilities from the Purcell model are still the best fit to the experimental data. The results from the Corey model are a good fit too. The water phase relative permeabilities calculated by the Burdine and the Brooks-Corey models are less than the experimental values. Actually the results calculated using the two models are the same if the capillary pressure data in the Burdine model are represented using Eq. 11. The steam phase relative permeabilities calculated by all the models except the Purcell model are not significantly different from each other but are much less than the experimental data for the imbibition case.

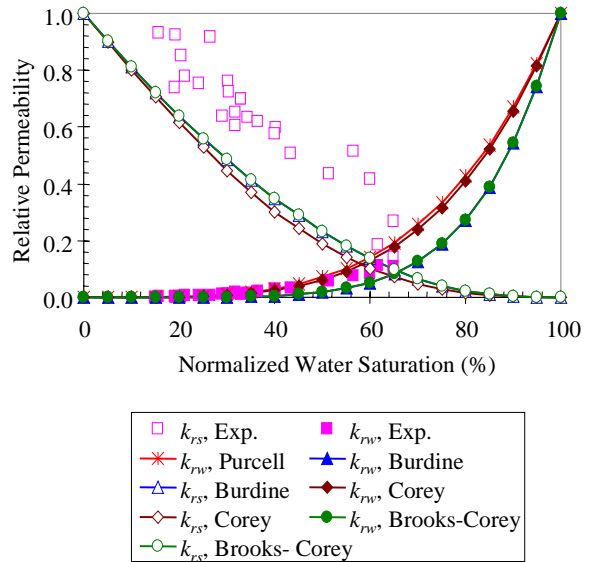


Figure 4: Calculated steam-water relative permeability and the comparison to the experimental data in the imbibition case.

In the following section, we will discuss the calculated results and the comparison in nitrogen-water systems to further confirm the phenomena that we observed. Li and Horne (2001b) measured the nitrogen-water relative permeabilities in a fired Berea core sample similar to that used in the measurement of steam-water relative permeabilities by Mahiya (1999). In this study, we drilled a plug from another part of the same fired Berea sandstone that was used by Li and Horne (2001b). The length and diameter of the plug sample were 5.029 cm and 2.559 cm respectively; the porosity was 24.37%. The drainage nitrogen-water capillary pressure of the plug was measured by using the semipermeable porous-plate method. The measured data of the drainage nitrogen-water capillary pressure along with the relative permeabilities from Li and Horne (2001b) are plotted in Fig. 5. Although the nitrogen-water capillary

pressure and relative permeability curves were not measured simultaneously, the residual water saturations were almost the same for both.

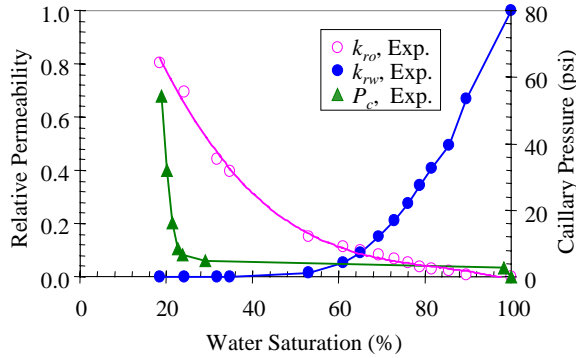


Figure 5: Experimental data of drainage nitrogen-water relative permeability and capillary pressure.

The results calculated using the capillary pressure models for the nitrogen-water flow (drainage) and the comparison to the experimental data are shown in Fig. 6. The experimental data of water relative permeability are located between the Purcell model and the Corey model. The two models provide a good approximation to the experimental data in this case. The features of gas phase relative permeability curve calculated by these models are similar to those of steam-water flow (see Fig. 4) except that the calculated results are greater than the measured data.

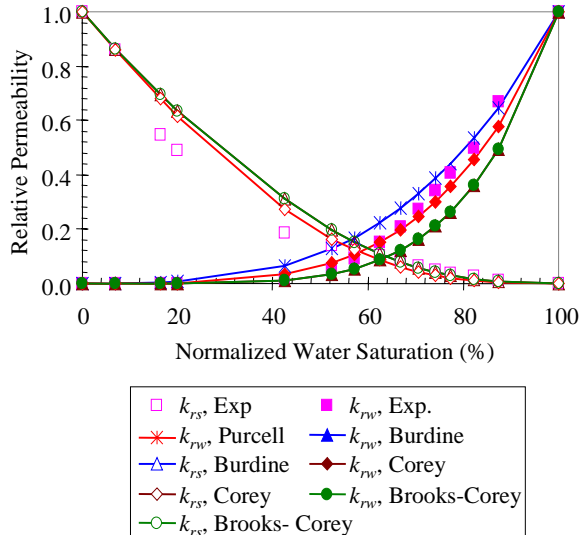


Figure 6: Calculated nitrogen-water relative permeability and the comparison to the experimental data in drainage.

We made the same calculation and comparison using the data of oil-water relative permeability and capillary pressure measured by Kleppe and Morse

(1974). We also observed that the best fit to the wetting phase relative permeability was from the Purcell model. However, we did not observe the same phenomenon for the data from Gates and Leitz (1950). In summarizing all the calculations that we have made, the Purcell model was the best fit to the wetting phase relative permeability if the measured capillary pressure curve had the same residual wetting saturation as the relative permeability curve.

DISCUSSION

The techniques of using capillary pressure to calculate relative permeability were developed in the late forties and were not widely utilized. Burdine (1953) pointed out that the calculated relative permeabilities are more consistent and probably contain less maximum error than the measured data because the error in measurement is unknown. This may be true in some cases. However, the differences between different capillary pressure models are obvious, especially for the wetting phase. Therefore, one of the questions is which model is most appropriate for practical use. The calculations in this study showed that the Purcell model was the best fit to the wetting phase relative permeability. This seems surprising because the concept of the tortuosity factor as a function of wetting phase saturation is not necessarily introduced for the calculation of the wetting phase relative permeability in such a case. A physical model was developed to demonstrate the insignificant effect of the tortuosity factor on the wetting phase, as shown in Fig. 7. L is the direct distance between the ends of a single capillary tube and L_a is the length of the tortuous capillary tube.

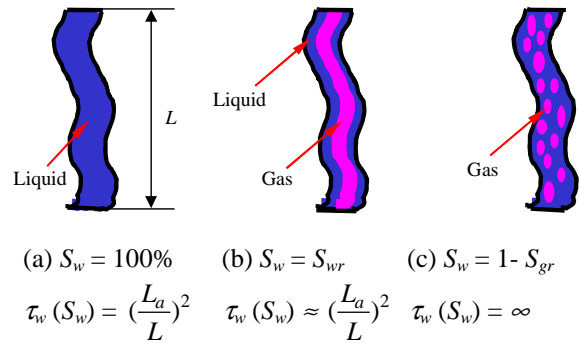


Figure 7: Tortuosity in a single capillary tube.

Burdine (1953) obtained an empirical expression of the effective tortuosity factor as a function of wetting phase saturation (see Eq. 4). λ_{rw} is actually the ratio of the tortuosity at 100% wetting phase saturation to the tortuosity at a wetting phase saturation of S_w . According to Eq. 4, the tortuosity of wetting phase is infinite at the minimum wetting phase saturation that is equal to residual water saturation S_{wr} here. This may not be true for the wetting phase because the

wetting phase may exist on the rock surface in the form of continuous film, as shown in Fig. 7b. In this case, τ_w ($S_m = S_{wr}$) may be close to τ_w (1.0) (see Fig. 7a), which demonstrates that there is little effect of the wetting phase saturation on the tortuosity of the wetting phase. Similarly, based on Eq. 6, the tortuosity of the nonwetting phase is infinite when the wetting phase saturation is equal to $1-S_e$. This may be true because the nonwetting phase may exist in the form of discontinuous droplets (see Fig. 7c). In this case, S_e is equal to S_{gr} .

It can be seen from the analysis here that the tortuosity of wetting and nonwetting phases would behave differently as a function of wetting phase saturation. This may be why it is necessary to introduce the tortuosity for the nonwetting phase but not for the wetting phase

CONCLUSIONS

Based on the present study, the following conclusions may be drawn:

1. In steam-water flow, the calculated results indicate that the Purcell model may be the best fit to the experimental data of the water phase relative permeability for both drainage and imbibition processes but is not a good fit for the steam phase.
2. It may not be necessary to introduce the tortuosity factor in calculating the wetting phase relative permeability with the capillary pressure technique.
3. The Corey model could also provide good approximation to the measured data of the wetting phase relative permeability in some cases.
4. Except for the Purcell model, the results of the steam phase relative permeability calculated using the models for the drainage case were almost the same and very close to the experimental values. However, those for the imbibition case were smaller than the measured data.
5. Because of the difficulty of measuring steam-water relative permeability, the capillary pressure technique would be valuable.

ACKNOWLEDGEMENT

This research was conducted with financial support to the Stanford Geothermal Program from the Geothermal and Wind division of the US Department of Energy under grant DE-FG07-99ID13763, the contribution of which is gratefully acknowledged.

NOMENCLATURE

C = constant
 k_{mw} = relative permeability of nonwetting phase

k_{rw} = relative permeability of wetting phase
 L = direct distance between the ends of a single capillary tube
 L_a = the length of the tortuous capillary tube
 P_c = capillary pressure
 p_e = entry capillary pressure
 S_e = equilibrium saturation of wetting phase
 S_m = minimum wetting phase saturation
 S_w = wetting phase saturation
 S_w^* = normalized wetting phase saturation
 S_{nwr} = residual saturation of nonwetting phase
 S_{wr} = residual wetting phase saturation
 λ = pore size distribution index
 λ_{rw} = tortuosity ratio of wetting phase
 λ_{nwr} = tortuosity ratio of nonwetting phase
 τ_w = tortuosity of wetting phase

REFERENCES

- Brooks, R. H. and Corey, A. T.: "Properties of Porous Media Affecting Fluid Flow", *J. Irrig. Drain. Div.*, (1966), **6**, 61.
- Burdine, N. T.: "Relative Permeability Calculations from Pore Size Distribution Data", *Trans. AIME*, (1953), **198**, 71.
- Corey, A. T.: "The Interrelation between Gas and Oil Relative Permeabilities", *Prod. Mon.*, (1954), **19**, 38.
- Fatt, I. and Dykstra, H.: "Relative Permeability Studies", *Trans. AIME*, (1951), **192**, 249.
- Gates, J. I. and Leitz, W. J.: "Relative Permeabilities of California Cores by the Capillary Pressure Method", paper presented at the API meeting, Los Angeles, California, May 11, 1950, 286.
- Honarpour, M. M., Koederitz, L., and Harvey, A. H.: *Relative Permeability of Petroleum Reservoirs*, CRC press, Boca Raton, Florida, USA, 1986, ISBN 0-8493-5739-X, 19.
- Horne, R.N., Satik, C., Mahiya, G., Li, K., Ambusso, W., Tovar, R., Wang, C., and Nassori, H.: "Steam-Water Relative Permeability," proceedings of the World Geothermal Congress, Kyushu-Tohoku, Japan, May 28-June 10, 2000; *GRC Trans.* (2000), **24**.
- Huang, D. D., Honarpour, M. M., Al-Hussainy, R.: "An Improved Model for Relative Permeability and Capillary Pressure Incorporating Wettability", SCA 9718, proceedings of International Symposium of the Society of Core Analysts, Calgary, Canada, September 7-10, 1997.
- Kleppe, J. and Morse, R. A.: "Oil Production from Fractured Reservoirs by Water Displacement," SPE 5084, presented at the 1974 SPE Annual Technical

Conference and Exhibition, Houston, TX, USA, October 6-9, 1974.

Land, C. S.: "Calculation of Imbibition Relative Permeability for Two- and Three-Phase Flow from Rock Properties", *SPEJ*, (June 1968), 149.

Land, C. S.: "Comparison of Calculated with Experimental Imbibition Relative Permeability", *Trans. AIME*, (1971), **251**, 419.

Li, K. and Horne, R.N.: "An Experimental and Analytical Study of Steam/Water Capillary Pressure," SPE 63224, presented at the 2000 SPE Annual Technical Conference and Exhibition, Dallas, TX, USA, October 1-4, 2000.

Li, K. and Horne, R.N. (2001a): "An Experimental Method of Measuring Steam-Water and Air-Water Capillary Pressures," paper 2001-84, presented at the Petroleum Society's Canadian International Petroleum Conference 2001, Calgary, Alberta, Canada, June 12-14, 2001.

Li, K. and Horne, R.N. (2001b): "Gas Slippage in Two-Phase Flow and the Effect of Temperature," SPE 68778, presented at the 2001 SPE Western Region Meeting, Bakersfield, CA, USA, March 26-30, 2001.

Mahiya, G.F.: *Experimental Measurement of Steam-Water Relative Permeability*, MS report, Stanford University, Stanford, Calif., 1999.

Purcell, W.R.: "Capillary Pressures-Their Measurement Using Mercury and the Calculation of Permeability", *Trans. AIME*, (1949), **186**, 39.

Rapoport, L. A. and Leas, W. J.: "Relative Permeability to Liquid in Liquid - Gas System", *Trans. AIME*, (1951), **192**, 83.

Wyllie, M. R. and Gardner, G. H. F.: "The Generalized Kozeny-Carmen Equation, Its Application to Problems of Multi-Phase Flow in Porous Media", *World Oil*, (1958), **146**, 121.