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REPRESENTATION AND INDUCTIVE RESPONSE OF FRACTAL RESISTIVITY DISTRIBUTIONS

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ABSTRACT

A long standing approximation in EM logging for fractures has been to assume that a fracture is an extended thin sheet, perhaps a half-plane. This approximation can be useful and is theoretically and numerically convenient. However, examination of core, FMS logs, and outcrop reveals that fracture zones can be geometrically self-similar over dimensional scales of less than millimeters to kilometers. In such a case, a strict geophysical model would have to account for such a fractal structure to truly represent the resistivity structure of the earth. The purpose of this work is to suggest some methods for forward and inverse EM modeling of geophysical fractal distributions.

We model fractal distributions of conductivity with band-limited Weierstrass functions. The conductivity is then discretized over some averaging window to give thin isotropic or thicker anisotropic layers, whose response can be calculated by traditional means. This representation gives a facile means of scaling small alternating sequences of open fracture and matrix rock, as observed in a borehole, to fracture packets. Parallel and transverse resistivities as functions of averaging window are easily calculated from such a distribution assuming that dip of the bedding with respect to the borehole is known - as would be the case with the auxiliary use of a FMS log, for example.

INTRODUCTION

An extensive literature exists demonstrating a correspondence between electrical and fluid flow (see, e.g., the 1996 National Research Council Report). For this reason, inversion of EM data to conductivity or current flow patterns has been touted as a means of imaging fluid flow paths. This electric current flow - fluid flow equivalence is a powerful construct, which is critically dependent on the notion that both flows have the same geometry. Hence, it is critical in modeling EM data that the scattering geometries not be simplified to excess.

Fractal electrical conductivity distribution packets seem to be ubiquitous in geothermal reservoirs. Fracture spatial occurances, fracture surfaces, and fracture aperature widths - all can have fractal geometries. (Baran et al., 1992; Barton, 1989; Russ, 1994; Thompson, 1991; National Research Council, 1996; Bruhn et al., 1997). The electromagnetic expression of fractally distributed fluid filled voids is fractal. Unfortunately, there do not seem to be any publications in the geophysical literature concerning the EM response of fractal materials. The purpose of this work is to suggest some methods for forward and inverse EM modeling of geophysical fractal distributions.

REPRESENTATION OF FRACTAL CONDUCTIVITY

Forward and inverse modeling of the EM response of a fractal conductivity distribution requires a set of basis functions which can represent the petrophysical conductivity distribution and whose EM response is readily calculated. Suppose that we have a borehole which penetrates a fractured formation. Then the resistivity log can have the character of a fractal function. There are several representations of such functions which facilitate EM modeling.

Weierstrass function representation

Resistivity logs in fractal material would be "spiky", and any interpolating function should be capable of reproducting the spiky behavior. One classical function which is "spiky" is the Weierstrass function

$$F(x) = \Sigma\, b^{-\rho n} \exp(ib^{n} x)$$

For $0<\rho<1$ and $b>1$, this function is not differentiable, as discussed by Hardy (1916).

Mandebrot (1982) extended the Weierstrass function to define the Weierstrass- Mandebrot function as an infinite summation

$$F(x) = \Sigma \left[\{1 - \exp(ib^{n} x)\}\exp(i\phi_n) / b^{(2-D)n}\right],$$

where $1<D<2$, and ϕ_n is arbitrary.

The infinite number of terms limit the use of this function for computations. For these Jaggard and Kim (1987) and Jaggard (1990) define the bandlimited cosine Weierstrass function W(z) as:

$$W(z) = \eta\, S\, \Sigma\, T_n(z),$$

where η^2 is the variance of the function;

$$S = \mathrm{sqrt}\,[\,2 - 2b^{(2D-4)}\,] / \mathrm{sqrt}\,[\,b^{(2D-4)N_1} - b^{(2D-4)(N_2+1)}\,],$$

and

$$T_n(z) = b^{(D-2)} \cos(2\pi s b^{n} z + \theta_n),\ N_1 < n < N_2 .$$

In this formula, sb^{N_1} $(b > 1)$ is the fundamental spatial frequency; D is the fractal dimension of the function which ranges from 1 to 2; s is a scaling factor; θ_n are specified phases taken, for example, from a random distribution on $[0, 2\pi]$; and $N = (N_2 - N_1 + 1)$ is the number of tones in the representation.

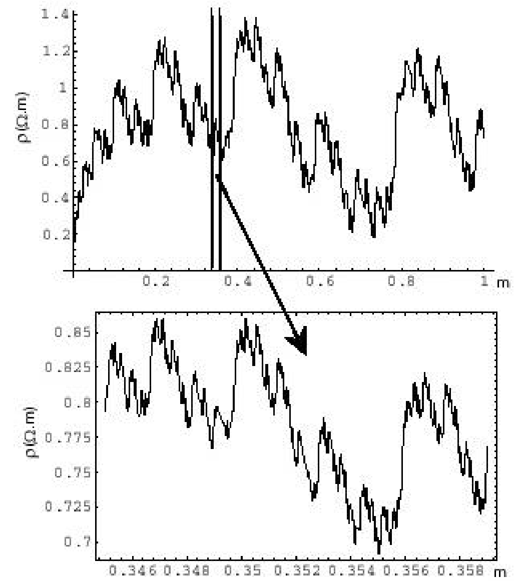

Figure. 1. Example of Weierstrass function having similar oscillating behavior on different scales.

The function W(z) has self-scaling behavior, as illustrated in Figure 1. For a limited number of tones, the function is fractal from an inner scale $(sb^{N_2})^{-1}$ to an outer scale of $(sb^{N_1})^{-1}$. As N tends to infinity, and $\forall n$, $\theta_n = 0$,

$W(\beta z) = \beta^{(2-D)} W(z)$, where $\beta = b^{-m}$, and m is an integer. Weierstrass functions are almost periodic (Corduneanu,1989).

An alternate representation, may be provided by the Rademacher functions, as discussed by Zygmund (1968), or wavelets (Grossman and Morlet, 1984), that are well known to be useful for describing nonperiodic, multiscale properties, and thus can provide an efficient represention of conductivity variations in boreholes.

The representation of resistivity data as parametric functions assumes the form of a superposition of a collection of sinusoids. This faciltates analytic manipulation of the expressions, including calculating the EM response of a fractal fracture packet. We assume that resistivity data can be represented by such in the sense that any field data set over a certain interval can be modelled by a parametric expression - an assumption which seems reasonable given the number of free parameters in the parametric representations.

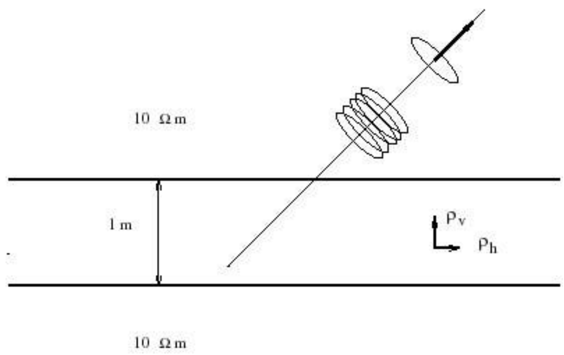

Figure 2. Induction log model for a dipping well. The resistivity in the 1 meter thick layer is fractal. Transmitter-receiver spacing is 0.5 m. The frequency of excitation is 10 kHz.

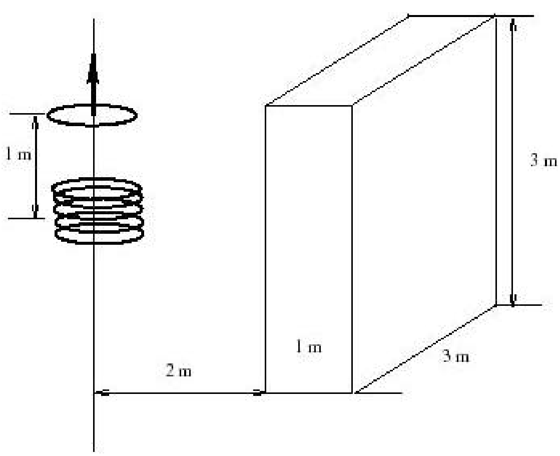

Figure 3. Induction log model for a fracture using fractal resistivity inside the block. The frequency of excitation is 10 kHz.

Fracture Packet Resistivity Composition

Suppose that a geological column is composed of thin beds, some of which could be fractures. Along a line perpendicular to the layering we assume that the isotropic point resistivities have the form of a Weierstrass function W(z). It is well known that a stack of thin resistive and conductive layers will give an anisotropic formation resistivity. Similarly, we can form an anisotropic tensor resistivity from W(z) by averaging W(z) over a window $[z_i, z_{i+1}]$ to give vertical and horizontal resistivities ρ_v and ρ_h . Thus

$$|z_{i+1} - z_i| \ \rho_v = \int_{z_i}^{z_{i+1}} W(z)\ dz$$

and

$$\rho_h = \frac{|z_{i+1} - z_i|}{\int_{z_i}^{z_{i+1}} \frac{dz}{W(z)}} .$$

The first expression can be evaluated explicitly.

FORWARD AND INVERSE SCATTERING FOR FRACTAL DISTRIBUTION

Analytic Dipole Response of Fracture Packets

The expressions for lateral and transverse resistivities of a fractal conductivity packet can be used directly in EM algorithms which permit anisotropic earth conductivities. For many petrophysical studies, whole-space formulae, such as the frequency domain expressions contained in Moran and Gianzero (1979) or the time domain expressions contained in the Appendix below, are suitable. For example, these should be very useful in assessing the tradeoff between sampling interval and fractal resolution (See, e.g. Dimri, 1998).

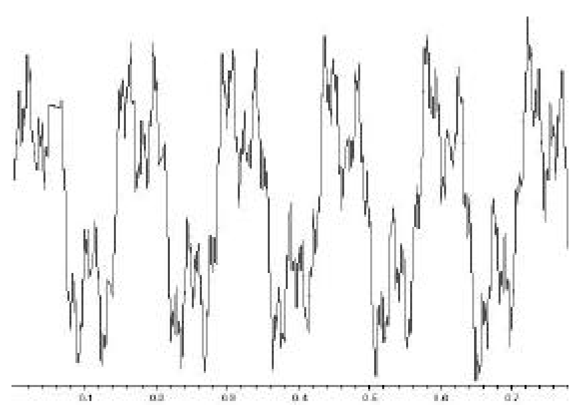

Figure 4. Example of fractal resistivity function generating anisotropic response.

For low-loss plane wave excitation, Maxwell's equations for cosine or periodic one-dimensional conductivity distributions reduce to Mathieu and Hill equations respectively (Brillouin, 1953). Since the Weierstrass function should be approximated to an arbitrary accuracy by a periodic function, it seems reasonable to expect that the theory of wave propagation on a periodic structure could be used to approximate the EM response of the almost periodic Weierstrass function.

Plane wave response by D+ Theory

For lossy geophysical earth, a more developed technique can be adapted directly. Since the Weierstrass functions or fractal harps can be approximated by a sequence of Dirac Delta Functions, an EM scattering response of such a sequence should approximate the response of the represented fractal function. Parker (1980) and Parker and Whaler (1981) develop the plane wave (MT) response of the conductivity distribution

$$\sigma(z) = \Sigma\, \tau_\iota\, \delta(z - z_\iota)\ ,\ z_o = 0,\ 0 < z_1 < z_2 \ldots < z_k < h,$$

in terms of a continued fraction. Such a conductivity distribution is called a D+ distribution. Thus, given a delta function approximation of a fractal function, a forward plane wave response is easily calculated. Parker (1980) and Parker and Whaler (1981) also discuss an inverse theory for D+ materials. An unconstrained inversion of finite amounts of data give non-unique D+ models. Each inverse D+ model can be interpolated by a set of Weierstrass functions, thus leading to a fractal interpretation.

Dipole Fractal Response by Wavelets

General references to the use of wavelets in EM modeling include Goswami et al. (1999), Sarkar et al. (1998), and Sarkar and Su (1998). The literature on the use of wavelets in integral equations modeling is particularly extensive. Algorithms using traditional basis functions can be adapted to a wavelet basis via similarity transformations, as is discussed by Chew et al.(1997), Deng and Ling (1998), Steinberg and Leviatan (1993), and Pan et al.(1999).

NUMERICAL SIMULATIONS

We demonstrate here results of numerical modeling of the response of the fractal conductivity to a low frequency induction tool. Two models were used to numerically simulate the response of a fractured zone to a magnetic dipole in a borehole. The models are shown in Figures 2 and 3. The first model is a finely fractured layer embedded in 10 ohm.m earth. The anisotropic resistivity of a fractured layer was derived by averaging the fractal isotropic resistivity distribution shown in Figure 4. The induction tool lowered along the dipping well generates an anisotropic response which is shown in Figure 5. The second model is a model of a fractured region 2 m distant from the borehole. The resistivity of the fracture was formed by averaging the fractal isotropic resistivity function shown in Figure 4. The inductive dipole in the borehole generates the magnetic field Hz, the imaginary component of Hz is shown in Figure 6.

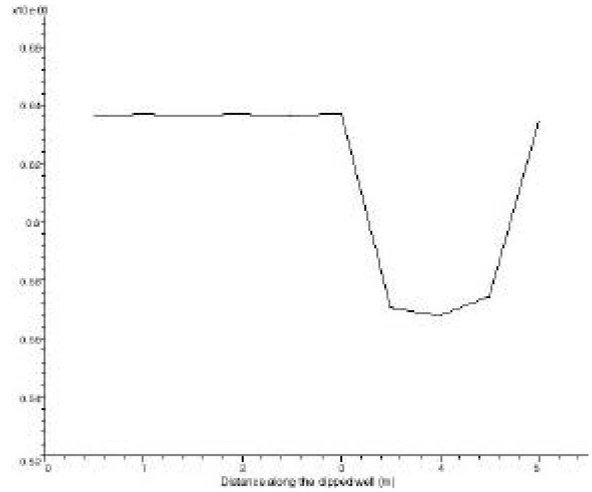

Figure 5. Magnetic field amplitude response (in A/m) of fractal anisotropic layer in model 1 to the induction dipole in the dipped well shown in Fig. 2.

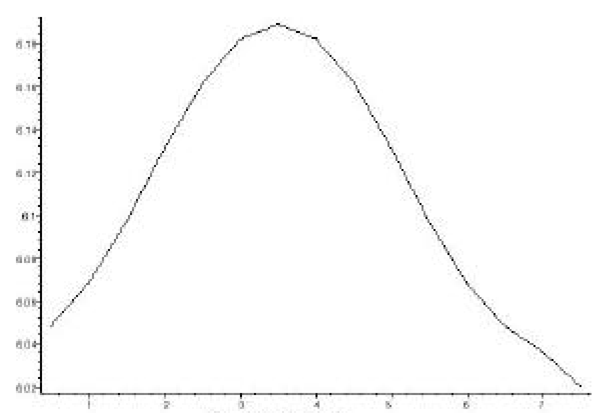

Figure 6. Response of the fractured zone shown in Fig.3 to a induction dipole moving along the borehole.

CONCLUSIONS

Methods for calculating the EM response of fractal materials can be adapted to geophysical parameters. We examined available techniques and used them to numerically model the EM response of fractured models for geothermal applications.

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APPENDIX:

TEMPORAL HERTZ POTENTIALS FOR A MAGNETIC DIPOLE IN A HOMOGENEOUS UNIAXIAL ANISOTROPIC FORMATION

$\lambda^2 = \sigma_h / \sigma_v$ $\quad r = \sqrt{(x^2 + y^2 + z^2)}$

$\Theta_h^2 = \mu\sigma_h / 4t$ $\quad s = \sqrt{(x^2 + y^2 + \lambda^2 z^2)}$

$\Theta_v^2 = \mu\sigma_v / 4t$ $\quad \rho = \sqrt{(x^2 + y^2)}$

$M_x = M,\ M_y = M_z = 0$

$\pi_x(t) = (M / 4\pi\lambda s)\ \mathrm{erfc}(s\Theta_v)$

$\pi_y(t) = 0$

$\pi_z(t) = (M / 4\pi)(x / \rho^2)\{(\lambda z / s)\ \mathrm{erfc}(s\Theta_v) - (z / r)\ \mathrm{erfc}(r\Theta_h)\}$

$\Phi(t) = (M / 4\pi)(x / \rho^2)\{\lambda[(2 / \sqrt{\pi})\Theta_v \exp(-s^2\Theta_v^2)] - [(2 / \sqrt{\pi})\Theta_h \exp(-r^2\Theta_h^2)] + (\rho^2 / r^2)((2 / \sqrt{\pi})\Theta_h \exp(-r^2\Theta_h^2) - (1/r)\ \mathrm{erfc}(r\Theta_h))\}$

$M_z = M \quad M_x = M_y = 0$

$\pi_x = 0$

$\pi_y = 0$

$\pi_z = (M / 4\pi r)\ \mathrm{erfc}(r\Theta_h)$

$\Phi(t) = (Mz / 4\pi r^2)\{(2 / v\pi)\Theta_h \exp(-r^2\Theta_h^2) - (1 / r)\ \mathrm{erfc}(r\Theta_h)\}$

$M_y = M \quad M_x = M_z = 0$

$\pi_x(t) = 0$

$\pi_y(t) = (M / 4\pi\lambda s)\ \mathrm{erfc}(s\Theta_v)$

$\pi_z(t) = (M / 4\pi)(y / \rho^2)\{(\lambda z / s)\ \mathrm{erfc}(s\Theta_v) - (z / r)\mathrm{erfc}(r\Theta_h)\}$

$\Phi(t) = (M / 4\pi)(y / \rho^2)\{\lambda[(2 / \sqrt{\pi})\Theta_v \exp(-s^2\Theta_v^2)] - [(2 / \sqrt{\pi})\Theta_h(\exp(-r^2\Theta_h^2)] + (\rho^2 / r^2)((2 / \sqrt{\pi})\Theta_h \exp(-r^2\Theta_h^2) - (1 / r)\ \mathrm{erfc}(r\Theta_h))\}$
