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THE STABILITY OF A HEAT PIPE TO LIQUID INFLOW/OUTFLOW

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Abstract

A model of a counterflowing liquid-vapour heat pipe is developed. The model identifies the existence of two steady flow regimes for low heat fluxes; both a liquid dominated high saturation and a vapour dominated, low saturation regime. In contrast, for sufficiently large heat fluxes, the simple heat pipe process is unable to transport sufficient heat and the steady heat pipe solution breaks down. We then examine the interaction of such a heat pipe with a permeable aquifer which may supply or remove fluid from the heat pipe. We find that even with a very slow exchange of fluid, the external aquifer destabilises the liquid dominated mode. In contrast, in some situations, for higher pressures, the vapour dominated mode is stable. The analysis has some interesting implications for the role played by heat pipes in transporting heat through geothermal systems

Introduction

High temperature geothermal systems are often characterized by two-phase zones of liquid and vapour in which the enthalpy flux associated with the relative motion of the two fluids is a key heat transfer mechanism. Heat pipes have attracted considerable attention, and the basic system is quite well understood (Bau and Torrance, 1980; McGuiness, 1996); the main area of empiricism in such models is the parameterisation of the relative permeability. This is an approximation to the real physical processes, and different

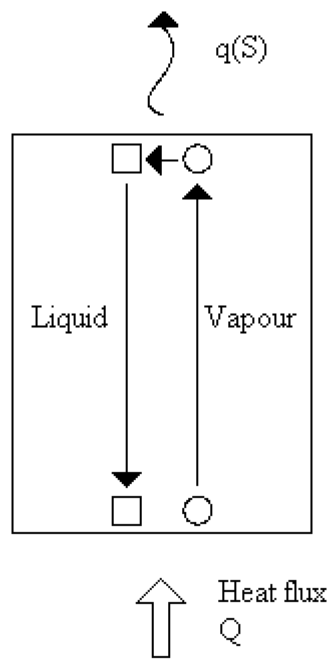

Figure 1 – simple heat pipe process

parameterisations can lead to quantitatively different, although qualitatively similar predictions (figure 2). Mindful of such limitations in the modelling approach, an important prediction of these models is the existence of two modes of convection – the liquid and the vapour dominated systems (e.g. McGuiness, 1996). Here we explore one aspect of the selection process which determines the stability of the two modes.

We examine the effect of a weak hydraulic connection between the fluid in a heat pipe and fluid in a neighbouring aquifer.

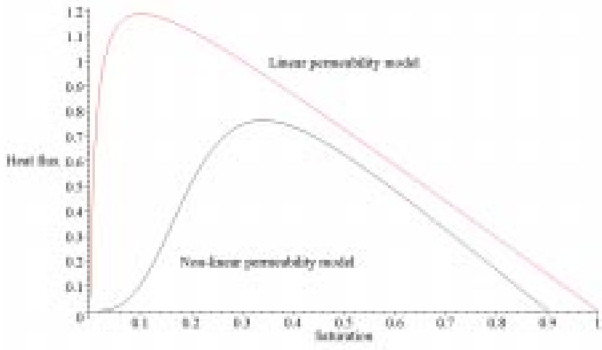

Figure 2 – heat flux as a function of saturation at a specific pressure, calculated using linear and cubic relations for relative permeability

We first examine the steady state flow regimes as a function of pressure and heat flux. We then examine the stability of the flow regime to an exchange of fluid with the neighbouring aquifer.

The Heat Pipe Model

In modelling the counterflow of liquid and vapour through a reservoir rock of low permeability, we adopt the Darcy flow equation for liquid and vapour,

$$u_l = -\frac{kf_l}{\mu_l}\left(\frac{dp}{dz} + \rho_l g\right),$$

$$u_v = -\frac{kf_v}{\mu_v}\left(\frac{dp}{dz} + \rho_v g\right).$$

Here, the relative permeability is modelled with a simple linear dependence on saturation, and the heat transport is then found in terms of the differential enthalpy flux (e.g. Bau and Torrance, 1982). The model is coupled with the Clausius Clapeyron relation to constrain the temperature and hence liquid and vapour properties as a function of pressure. The steam tables provide the data on the variation of the fluid properties, and we have parameterised these for convenience in the model development. Figure 3 illustrates the possible steady-flow conditions in a heat pipe, with each closed contour corresponding to a particular heat flux, and the axes illustrating the variation of the saturation (liquid void fraction) with pressure. For each heat flux, there is a range of pressures and hence depths for which two heat pipe configurations are possible; outside this range, no solutions exist. Essentially, at high pressure the difference in density between the liquid and vapour is insufficient to drive a substantial counterflow, and this limits the heat transfer through the heat pipe mechanism. With small pressures, the vapour density is so low that the liquid saturation tends to zero; this increases the resistance to liquid flow, and in turn this lowers the maximum mass flux and heat transfer through the system. As the heat flux increases, the difference in the saturation level of the two solutions decreases, and eventually the solutions coincide and cease to exist.

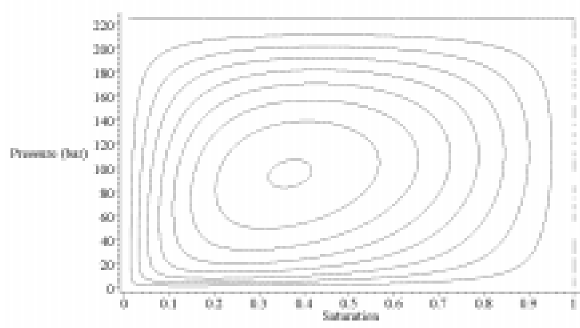

Figure 3: Contours shown are for heat fluxes q = 0, 2, 4, 6, 8, 10, 12, 14, 16, increasing inward.

This is because the counterflow is driven by the density difference between the phases; this flux, and the associated heat flux therefore have an upper bound.

Influence of Inflow/Outflow on Heat Pipe Stability

We now extend the model to allow for inflow/outflow of fluid from a neighbouring aquifer. We assume that flow will only arise if the pressure in the heat pipe evolves away from the local pressure in the aquifer. By allowing a small deviation in pressure, we can then examine whether the ensuing inflow/outflow restores the pressure to equilibrium, or drives the system awy from equilibrium and hence instability.

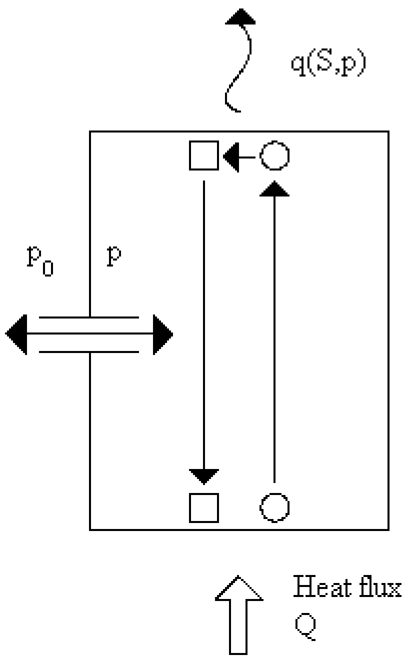

Figure 4 : Schematic of model system

The model system we examine includes the time-dependent mass conservation relation

$$V\frac{d}{dt}\left[\rho_l S + \rho_v(1-S)\right] = X\left(p_0 - p\right)$$

where S denotes the saturation, and X is the rate constant for the flux between the aquifer and the heat pipe. V denotes the volume of the system.

In equilibrium, the pressure matches the external pressure, $p = p_o$. The conservation of heat becomes modified to the form

$$V\frac{d}{dt}\left[\rho_l h_l S + \rho_v h_v(1-S)\right] =$$
$$\cdot\cdot Q - q(S,p) + X\left(p_0 - p\right)h_l\left(T_0\right)$$

where Q is the source heat flux at the base of the system. q(S,p) is the heat flux at the top of the domain associated with the two-phase steady heat-pipe, for a given saturation and pressure. The final term on the right hand side denotes heat transfer in/out of the system owing to the mass transfer with the aquifer.

In equilibrium, there is zero mass transfer, and the heat transfer reduces to the simpler steady form (as solved in the steady-state heat pipe earlier in the work, figure 3)

Q=q(S,p)

Analysis of the stability of small pertubations away from these equilibrium solutions (figure 4) leads to a matrix problem

$$\begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \end{pmatrix}\begin{pmatrix} \hat{S}_t \\ \hat{p}_t \end{pmatrix} = \begin{pmatrix} 0 & b_1 \\ a_2 & b_2 \end{pmatrix}\begin{pmatrix} \hat{S} \\ \hat{p} \end{pmatrix}$$

Substitution of a small perturbation

$(S,p)=(S_{o,}\ p_o)\exp(bt)$

leads to prediction of the growth rate of the small disturbances to the system (in terms of parameterised relations for the clausius-clapyron relation, and the viscosity, density and specific heat as a function of temperature). Figure 5 illustrates the transition from stable to unstable heat pipe solutions for a particular set of parameters. The closed curves represent the contours of constant heat flux. Two solid lines cut across these contours, a vertical tending line, A, and a near horizontal line B. The liquid dominated solutions are located to the right of line A. These solutions are unstable. The vapour dominated solutions to the left of line A are stable at higher pressures above the curve B, while they are unstable at lower pressures below the curve B.

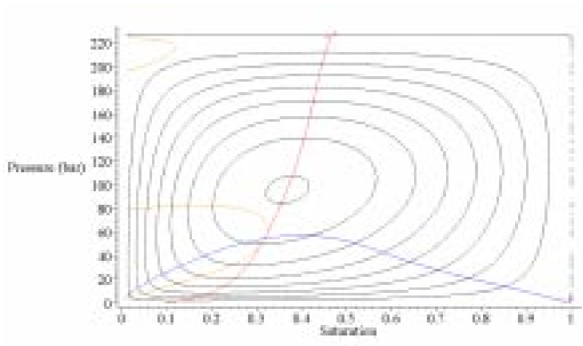

Figure 5 near vertical line A and horizontal line B demarcate the zones of stable and unstable heat pipe solutions.

The detailed shape of the boundary curves A and B dividing the stable and unstable solutions depends on the value of X, corresponding to the rate constant between the aquifer and the heat pipe, although figure 5 provides a qualitative guide to the nature of the transition.

The model suggests that for deeper systems at higher pressure, such heat pipe systems should tend to be vapour dominated in regions connected to neighbouring aquifers. For shallower systems, there may not be any steady state heat pipe solutions, but instead the system may become time dependent, with periods of liquid inflow and outflow. If in a deep system, the reservoir is flooded with water, then as the heat flux becomes re-established, the liquid will be driven from the system and the vapour dominated solution will be re-established. Further analysis shows that in some conditions, this adjustment may be monotonic, while in other cases it may be oscillatory.

References

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