

## A THEORETICAL MODEL OF TRANSITIONAL REGIME OF STEAM-WATER FLOW IN GEOTHERMAL WELLS

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### ABSTRACT

Transitional regime takes place in case of absence of conditions for bubble, slug and annular-mist regimes. Integral method of steam-water flow description is used (two-velocities). Theoretical basis for calculation of flow parameters of transitional regime is developed. Criteria of this regime existence are determined.

### INTRODUCTION

Often steam-water mixture flows in geothermal wells. In this case simulation of steam-water flow is used for exploration of geothermal reservoir by wellhead measurements and for determination of rational exploitation regime of reservoir. Importance of those problems causes continual interest to steam-water well simulators.

Advanced simulators take into consideration various regimes steam-water flow. Many regimes exist (Hewitt G.F. and Hall-Taylor N.S., 1974). Principal regimes are bubble, slug, transitional and annular mist flows. Other regimes may consider as particular case or combination of principal regimes. Theoretical models of principal regimes are absent. Practical calculations are realized by introduction of empirical formulas in model.

Transitional regime is considered most complicated for description. This regime has chaos flow structure. As a rule this regime is observed between slug flow and annular-mist flow. In the case of high velocities this regime may observe between bubble flow and annular-mist flow (slug flow is absent).

In present paper a theoretical model of transitional regime is proposed. The model is based on hypothesis about main role of local criticalness effect in this regime.

### GENERAL EQUATIONS

Any description of hydrodynamic process is based on conservation equations of mass, momentum and energy. Sort of equation depends on description method. There are integral and differential methods (Wallis G.B., 1972). Those methods have their merits and demerits. Also structure approach exists (conservation equations describe specific flow structure). For example equations are determined separately for liquid film and mist central flow in annular-mist regime.

Taking into consideration chaos of flow structure the structure approach is unacceptable for transitional regime. Application of hydrodynamic method creates essential trouble in model completion. Therefore the integral method is recommended to describe the transitional regime. It is remark, this method is usual for geothermal well simulators. Following advanced simulators, difference of phase velocities is recommended to take into consideration (two velocities: steam and water).

In this case the steam-water flow in vertical pipe is described by following conservation equations of mass, momentum and energy (axis z is pointed up-stream)

$$dG=0, \quad (1)$$

$$\rho'' \varphi v'' \frac{dv''}{dz} + \rho' (1 - \varphi) v' \frac{dv'}{dz} + (v'' - v') \frac{d(\rho'' \varphi v'')}{dz} = - \frac{dp}{dz} - \frac{4\tau}{D} - \rho g, \quad (2)$$

$$\frac{di}{dz} = \frac{\pi D q}{G} + \frac{Q}{G} \left( \frac{dp}{dz} + \frac{4\tau}{D} \right), \quad (3)$$

where G is a mass flow-rate of mixture (a total mass rate),  $\rho''$  and  $\rho'$  are steam and water densities,  $v''$

and  $v''$  are steam and water velocities,  $\phi$  is a volume steam fraction (void fraction),  $p$  is a pressure,  $\tau$  is a friction stress in pipe wall,  $q$  is a specific heat flux in pipe wall,  $Q$  is a volume flow-rate of mixture (a volumetric rate),  $D$  is a pipe diameter (bore),  $\rho$  and  $i$  are density and specific enthalpy of mixture

$$\rho = \rho''\phi + \rho'(1 - \phi) , \quad (4)$$

$$i = i''x + i'(1 - x) , \quad (5)$$

where  $i''$  and  $i'$  are specific enthalpies of steam and water,  $x$  is a mass output steam fraction (a quality).

### BINDING EQUATIONS

Relationships of general equation parameters are realized by well-known equations. Those equations supplement mathematical model.

$$G = G'' + G' , \quad (6)$$

where  $G''$  and  $G'$  are steam and water mass flow-rates.

$$Q = Q'' + Q' , \quad (7)$$

where  $Q''$  and  $Q'$  are steam and water volume flow-rates.

$$Q'' = \frac{\pi D^2 \phi v''}{4} , \quad (8)$$

$$Q' = \frac{\pi D^2 (1 - \phi) v'}{4} . \quad (9)$$

$$x = \frac{G''}{G} . \quad (10)$$

$$G'' = Q'' \rho'' . \quad (11)$$

$$G' = Q' \rho' . \quad (12)$$

Condition equations determine specific enthalpies and densities of steam and water. Geothermal steam and water does not are pure. However usually condition equations for pure steam and water may use (such as IAPWS-IF97).

General and binding equations does not are completion mathematical model. We must determine friction stress in pipe wall. Also we must determine volume steam fraction (or one in two of phase velocities). Usually empirical formulas are used for those purposes. Different formulas are used for different regimes.

Specific heat flux in pipe wall depends on heat flux in rock. This problem do not is considered in this paper.

### THEORETICAL PROPOSITIONS FOR TRANSITIONAL REGIME

Chaos of flow structure point at some phenomenon having destructive effect. Such effect has local criticalness. Saturated water in front part bubble (or slug) has velocity that is equal steam velocity. Local criticalness effect wrecks front part bubble (or slug) when steam velocity exceeds critical velocity of saturated water. Hence the transitional regime appears from bubble or slug flow when the steam velocity exceeds critical velocity of saturated water

$$v'' > v'_c , \quad (13)$$

where  $v'_c$  is critical velocity of saturated water (Shulyupin A.N., 1996)

$$v'_c = \left( \frac{dp'}{dp} + \frac{(\rho' - \rho'')\rho'}{(i'' - i')\rho''} \left( \frac{di'}{dp} - \frac{1}{\rho'} \right) \right)^{-0.5} . \quad (14)$$

Simple formula is recommended for rough engineering calculations in diapason  $10^6 < p < 10^7$  (Pa)

$$v'_c = 2.5 + 6.25p10^{-4} . \quad (15)$$

The transitional regime has two stages. Steam velocity is similar to critical velocity of saturated water in first stage. Attempt to increase the steam velocity raises local pressure bound in front part of babbler or slag. Such bounds prevent from increase of steam velocity. Also such bounds raise local cross-section flows of water. Namely those flows cause structure chaos.

Water velocity is similar to critical velocity of saturated water in second stage. Steam has greater velocity. Upper limit of second stage (and transitional regime in general) is condition of stable annular-mist flow. It is interesting to note that steam velocity is similar water velocity in change of stages.

### COMPLETION FORMULAS

The friction stress in pipe wall consists of steam and water friction stresses. If steam fraction of friction area amount to volume steam fraction then

$$\tau = \tau''\phi + \tau'(1 - \phi) , \quad (16)$$

where  $\tau''$  and  $\tau'$  are steam and water friction stresses

$$\tau'' = \frac{\lambda'' \rho'' v''^2}{8}, \quad (17)$$

$$\tau' = \frac{\lambda' \rho' v'^2}{8}, \quad (18)$$

where  $\lambda''$  and  $\lambda'$  are steam and water friction factors.

Altshul's formula is recommended for friction factors

$$\lambda'' = 0.11 \left( \frac{\delta}{D} + \frac{68}{\text{Re}''} \right)^{0.25}, \quad (19)$$

$$\lambda' = 0.11 \left( \frac{\delta}{D} + \frac{68}{\text{Re}'} \right)^{0.25}, \quad (20)$$

where  $\delta$  is equivalent height of roughness in pipe wall,  $\text{Re}''$  and  $\text{Re}'$  are steam and water Reynolds's numbers

$$\text{Re}'' = \frac{v'' D}{\nu''}, \quad (21)$$

$$\text{Re}' = \frac{v' D}{\nu'}, \quad (22)$$

where  $\nu''$  and  $\nu'$  are steam and water kinematics viscosity coefficients.

Taking into consideration the theoretical propositions we have for first stage

$$v' = v'_c, \quad (23)$$

for second stage

$$v'' = v''_c. \quad (24)$$

## CONCLUSION

The present theoretical model does not pretend to absolute precision. Especially it is to quantitative estimations. For example it is possible in front part of slug the water velocity is smaller than steam velocity or water is not saturated (owing to dynamic pressure of slug). Consequently it is possible the steam velocity rather exceeds critical velocity of saturated water in first stage. Also it is possible the water velocity rather exceeds critical velocity of saturated water in second stage.

The present model may consider as basis for precise models. In order to introduce improvements the experimental investigations are necessary.

## REFERENCES

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