

NEW ANALYTICAL MODELS FOR MULTI-DIMENSIONAL THERMAL CONDUCTION IN FRACTURED GEOTHERMAL RESERVOIRS

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ABSTRACT

Accurate predictions of the performance of fractured geothermal reservoirs require good models for thermal crossflow between the matrix blocks and the fluid in the fractures. This paper reviews problems with classical analytical conduction models and then presents two new analytical models that address these problems.

INTRODUCTION

The ability to accurately predict the future performance of geothermal reservoirs is a critical element in the economic success of geothermal energy projects. A critical element in predicting such performance is the ability to predict heat and fluid flow through geothermal reservoirs. A variety of performance models for nonfractured reservoirs are available (Bodvarsson, et al., 1986). Most of these models are of limited accuracy because they do not account for the effect of natural fractures that are commonly found in geothermal reservoirs (Brook, et al., 1978).

One of the most important elements in predicting reservoir behavior in fractured reservoirs is an accurate model for the thermal crossflow between the matrix and fluid in the fractures. This crossflow can be handled through fine grid simulation, course grid simulation, or analytical models. This paper reviews some limitations to classical analytical models derived from the heat diffusion equation and presents new models that address these limitations.

CLASSICAL CONDUCTION MODELS

The classical conduction models are obtained by solving the constant-thermal-conductivity heat-diffusion equation for the temperature profile. The initial and boundary conditions assumed are a uniform initial temperature everywhere in the matrix block, T_i , with a simultaneous step change in the surface temperature on all six faces of a parallelepiped matrix block, T_s , at the starting time. The heat conduction rate into the matrix block can be determined from the average temperature in the block through the following energy balance:

$$q = \frac{dQ}{dt} = \frac{d}{dt} [\rho c_p V (\bar{T} - T_i)] \quad (1)$$

where \bar{T} is the average temperature in the matrix block and ρ , c_p , and V are the matrix block density, specific heat, and volume, respectively. Two classical models will be considered: infinite series and infinite medium models.

Infinite Series Models

One model for the temperature profile can be obtained by solving the heat diffusion equation using the method of separation of variables. For the geometry of simultaneous conduction into all six sides of a parallelepiped matrix block having a dimensions L_x , L_y , L_z the temperature profile is presented in Carslaw and Jaeger (1959). The resulting dimensionless conduction rate from Eq. 1 is given by

$$\frac{qt_c}{Q_\infty} = \frac{8L_x^2 D_L}{9\pi^4} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{1}{(2l+1)^2 (2m+1)^2 (2n+1)^2} \left[\frac{(2l+1)^2}{L_x^2} + \frac{(2m+1)^2}{L_y^2} + \frac{(2n+1)^2}{L_z^2} \right] \right] \left[* \exp \left\{ - \left[\frac{(2l+1)^2}{L_x^2} + \frac{(2m+1)^2}{L_y^2} + \frac{(2n+1)^2}{L_z^2} \right] \frac{\pi^2 L_x^2 D_L t}{576 t_c} \right\} \right] \quad (2)$$

where t is the time, t_c is a characteristic time for thermal conduction that is defined below, Q_∞ is the cumulative energy conducted after an infinite time, and D_L is a geometry factor that is defined below. This characteristic time can be considered the time when the thermal front reaches the center of the matrix block and the temperature fronts from opposing faces begin to interfere with each other.

The dimensionless cumulative energy conducted can be expressed as

$$\frac{Q}{Q_\infty} = \frac{512}{\pi^6} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{1}{(2l+1)^2 (2m+1)^2 (2n+1)^2} \right] \left[* \left[1 - \exp \left\{ - \left[\frac{(2l+1)^2}{L_x^2} + \frac{(2m+1)^2}{L_y^2} + \frac{(2n+1)^2}{L_z^2} \right] \frac{\pi^2 L_x^2 D_L t}{576 t_c} \right\} \right] \right] \quad (3)$$

Infinite Medium Models

For the case of conduction into a three-dimensional semi-infinite solid, i.e., into the corner of a rectangular parallelepiped, another model for the temperature profile can be obtained by solving the heat diffusion equation using similarity methods (Carslaw and Jaeger, 1959). The resulting dimensionless conduction rate for this geometry is

$$\frac{qt_c}{Q_\infty} = \frac{1}{12} \left(1 + \frac{L_x}{L_y} + \frac{L_x}{L_z} \right) \sqrt{\frac{D_L}{\pi}} \sqrt{\frac{t_c}{t}} - \frac{1}{36} \left(\frac{L_x}{L_y} + \frac{L_x}{L_z} + \frac{L_x^2}{L_y L_z} \right) \frac{D_L}{\pi} + \frac{1}{144} \frac{L_x^2}{L_y L_z} \sqrt{\left(\frac{D_L}{\pi} \right)^3} \sqrt{\frac{t}{t_c}} \quad (4)$$

and the dimensionless cumulative energy conducted into the matrix block is

$$\frac{Q}{Q_\infty} = \frac{1}{6} \left(1 + \frac{L_x}{L_y} + \frac{L_x}{L_z} \right) \sqrt{\frac{D_L}{\pi}} \sqrt{\frac{t}{t_c}} - \frac{1}{36} \left(\frac{L_x}{L_y} + \frac{L_x}{L_z} + \frac{L_x^2}{L_y L_z} \right) \frac{D_L t}{\pi t_c} + \frac{1}{216} \frac{L_x^2}{L_y L_z} \sqrt{\left(\frac{D_L}{\pi} \right)^3} \sqrt{\left(\frac{t}{t_c} \right)^3} \quad (5)$$

Discussion of Classical Conduction Models

The dimensionless conduction rate and cumulative energy conducted for these classical models are shown in Figs. 1 and 2, respectively for a cubic matrix block. The dimensionless time is the actual time divided by the characteristic time. For the infinite series model, solutions are provided for different numbers of terms in the series. It can be seen that even with tens of thousands of terms in the series, an accurate model for the cumulative energy conducted cannot be obtained with the infinite series model. The infinite medium model is considered valid before a dimensionless time of about one, but invalid thereafter.

These figures show that the classical conduction models obtained from a direct solution of the heat diffusion equation are not practical for reservoir thermal modeling. The infinite series model requires an impractical number of terms and the infinite medium model does not allow for finite-sized matrix blocks.

NEW MODELS

Two new models are presented in this paper. The first is based on an assumed temperature profile during conduction and the second is based on an empirical fit of an exponential model to the first new model.

Linear-Temperature Profile Model

In this model, the temperature profile is assumed to vary linearly with position in the matrix block. Conduction is divided into two periods. During the early-time period, conduction fronts advance simultaneously into all six faces of the parallelepiped matrix block without being affected by the

conduction fronts from the opposing sides. Interference from adjacent sides is included. This early-time period behavior lasts until the first set of conduction fronts reach the centerline of the matrix block. During the late-time period, the first set of conduction fronts have merged within the matrix block and interact with each other.

During the early-time period, the temperature profile is assumed to vary linearly from the surface temperature at the edge of the matrix block to the initial temperature at the leading edge of the conduction front within the matrix block. During the late-time period, the temperature is assumed to vary linearly from the same surface temperature to the temperature at the center of the matrix block. During the early-time period, conduction occurs through the advance of the conduction front. During the late-time period, conduction occurs through a changing temperature at the center of the matrix block. The linear-temperature-profile model is analogous to the linear-saturation-profile model for capillary imbibition in petroleum formations that was developed by Cil (1992), Reis and Cil (1993), and Reis and Cil (1999).

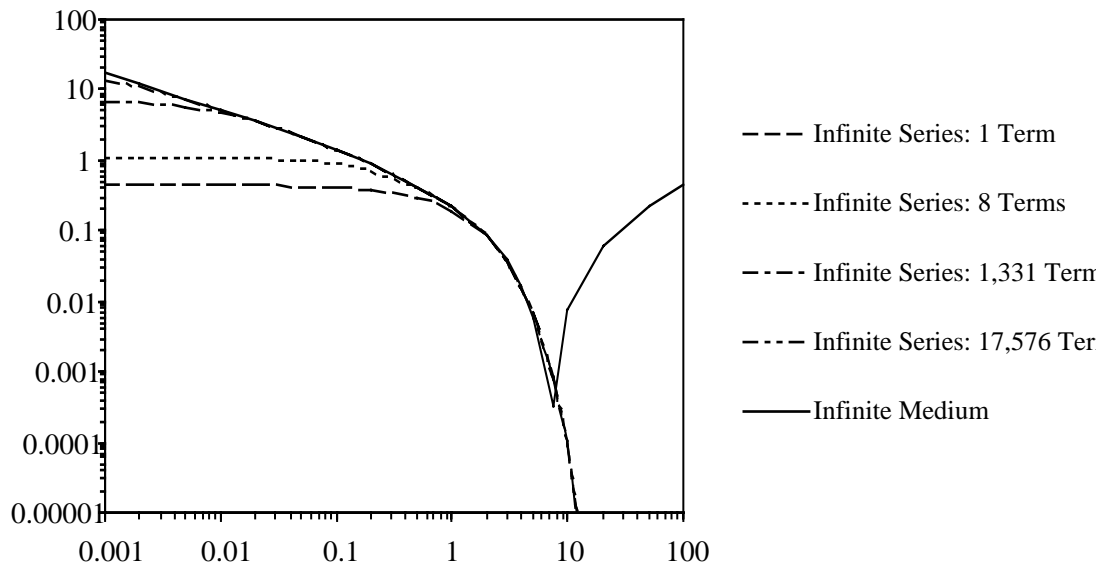


Figure 1: Classical Conduction Rate Models

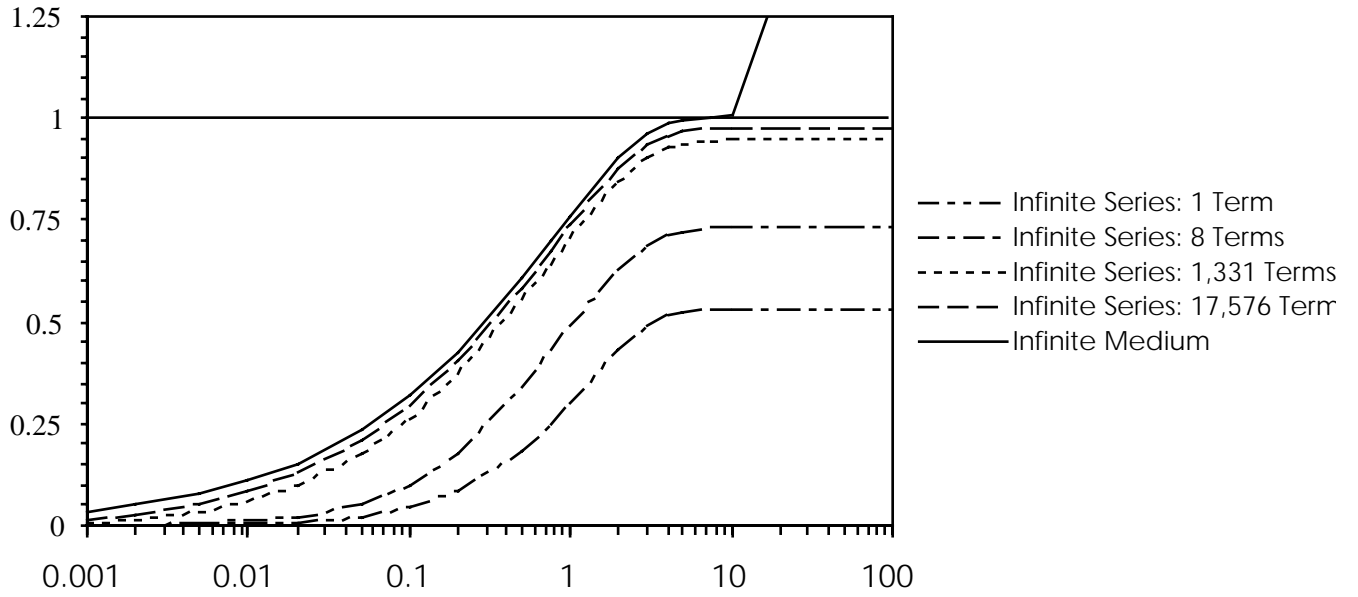


Figure 2: Classical Cumulative Energy Conducted Models

Early-Time Period

In this model, two equations are written relating the heat conduction rate to the location of the conduction front in the matrix block. The first equation is a direct expression of the conduction rate:

$$q = -KA_o \frac{(T_i - T_s)}{L'}, \quad (6)$$

where K is the thermal conductivity, A_o is the total surface area of the matrix block, and L' is the distance between the thermal front and the edge of the matrix block. The second equation is developed by substituting the assumed linear temperature profile into Eq. 1. After some algebra, this equation becomes

$$q = \rho c_p (T_s - T_i) \left[\frac{A_o}{2} - \frac{8}{3} (L_x + L_y + L_z) L' + 6L'^2 \right] \frac{dL'}{dt}. \quad (7)$$

Combining Eqs. 6 and 7 and solving for the resulting differential equation yields the following expression for the location of the thermal front,

$$t = \frac{1}{4\alpha} L'^2 - \frac{8}{9\alpha A_o} (L_x + L_y + L_z) L'^3 + \frac{3}{2\alpha A_o} L'^4, \quad (8)$$

where α is the thermal diffusivity. The time for the thermal front to reach the centerline of the matrix block can be obtained from Eq. 8 by noting that at this time, L' is one half of the smallest matrix block dimension. If the dimensions are ordered such that $L_x < L_y < L_z$, this characteristic time can be written as

$$t_c = \frac{L_x^2 D_L}{576\alpha}, \quad (9)$$

where a dimensionless geometry factor has been defined as

$$D_L = \frac{(4L_x L_y + 4L_x L_z + 36L_y L_z - 5L_x^2)}{(L_x L_y + L_x L_z + L_y L_z)}. \quad (10)$$

The dimensionless conduction rate for this model during the early-time period can be written as

$$\frac{qt_c}{Q_\infty} = \frac{1}{576} \frac{D_L A_o}{L_y L_z} \left(\frac{L_x}{L'} \right), \quad (11)$$

and the dimensionless cumulative energy conducted can be written as

$$\frac{Q}{Q_\infty} = \frac{1}{2} \frac{A_o}{L_y L_z} \left(\frac{L'}{L_x} \right) - \frac{4}{3} \frac{(L_x + L_y + L_z) L_x}{L_y L_z} \left(\frac{L'}{L_x} \right)^2 + 2 \frac{L_x^2}{L_y L_z} \left(\frac{L'}{L_x} \right)^3. \quad (12)$$

For a one-dimensional matrix block, i.e., a slab, the dimensionless cumulative energy conducted at the characteristic time is one-half, while for a cubic matrix block the dimensionless cumulative energy conducted is three-quarters. All other matrix block dimensions will fall between those two extremes.

Late-Time Period

For the late-time period, a similar approach was taken. The resulting expression for the dimensionless conduction rate is

$$\frac{qt_c}{Q_\infty} = \frac{1}{288} \frac{A_o D_L}{L_y L_z} \exp \left\{ \gamma \left(1 - \frac{t}{t_c} \right) \right\}, \quad (13)$$

where

$$\gamma = \frac{1}{12} \frac{(4L_x L_y + 4L_x L_z + 36L_y L_z - 5L_x^2)}{(6L_y L_z + L_x^2 - 2L_x L_y - 2L_x L_z)}. \quad (14)$$

The corresponding dimensionless cumulative energy conducted is

$$\frac{Q}{Q_\infty} = 1 - \left[\frac{1}{2} + \frac{1}{12} \frac{L_x^2}{L_y L_z} - \frac{1}{6} \frac{L_x}{L_z} - \frac{1}{6} \frac{L_x}{L_y} \right] \exp \left\{ \gamma \left(1 - \frac{t}{t_c} \right) \right\}. \quad (15)$$

Exponential Model

The exponential model is an empirical fit of a single-term exponential function to the cumulative energy conducted. The exponential model assumes that the dimensionless cumulative energy conducted after an infinite time is unity and that the dimensionless cumulative energy conducted at the characteristic time, i.e., at the time the first conduction front reaches the center of the matrix block, matches that predicted by the linear-temperature-profile model, i.e., Eq. 12 evaluated at $L'=L_x/2$.

The cumulative energy conducted for the exponential model is expressed as

$$\frac{Q}{Q_\infty} = 1 - \exp \left\{ -\delta \frac{t}{t_c} \right\}, \quad (16)$$

where δ is the match constant for multi-dimensional conduction and is given by

$$\delta = \ln \left[\frac{12L_y L_z}{L_x^2 + 6L_y L_z - 2L_x L_y - 2L_x L_z} \right]. \quad (17)$$

The dimensionless conduction rate for the exponential model is the time derivative of Eq. 16:

$$\frac{qt_c}{Q_\infty} = \delta \exp \left\{ -\delta \frac{t}{t_c} \right\}. \quad (18)$$

DISCUSSION OF MODELS

The dimensionless conduction rate and cumulative energy conducted for the new models are shown in Figs. 3 and 4, respectively, for a cubic matrix block. For comparison, the infinite series model with 17,576 terms and the infinite medium model are also shown.

For the conduction rate of Fig. 3, the early-time linear-temperature-profile model provides a good match to the infinite medium model throughout the early-time period. As expected, this model and the infinite medium model, however, provide a poor match during the late-time period. The late-time linear-temperature-profile and exponential models provide a poor fit to the conduction rate during the early-time period.

For the cumulative energy conducted of Fig. 4, both the exponential and late-time linear temperature profile model are offset during the late-time period from the infinite series model by about the amount that the infinite series model is in error because too few terms were used in the series. Thus it is concluded that both of these models provide a good model to the late-time period. These models, however, do not appear to provide a good model for the early-time period. Although these late-time models do not accurately predict the early-time behavior, the logarithmic scale of these figures magnifies their error. A more realistic assessment of their error can be seen in Fig. 5 where the cumulative energy conducted is shown on a linear time scale. On this scale, the errors are more difficult to identify.

SUMMARY

Two new models were presented in this paper for multi-dimensional thermal conduction.

The linear-temperature-profile model divides conduction into two periods and uses a separate algorithm for each period. As long as the transition time from one period to the next is calculated through the characteristic time and the appropriate algorithm is used for each respective period, the model provides a good model for conduction over the entire conduction time. The primary limitation to this model from the need to monitor when to switch equations during conduction.

The exponential model is a somewhat less accurate empirical approximation to conduction. It has the advantage of applying over the entire conduction time without the need to calculate a characteristic time and switch equations. The mathematical simplicity of this model makes it easy to implement in geothermal reservoir modeling. Because reservoirs virtually never contain parallelepiped matrix blocks with known dimensions, the use of more complex, more accurate conduction models may not be justified.

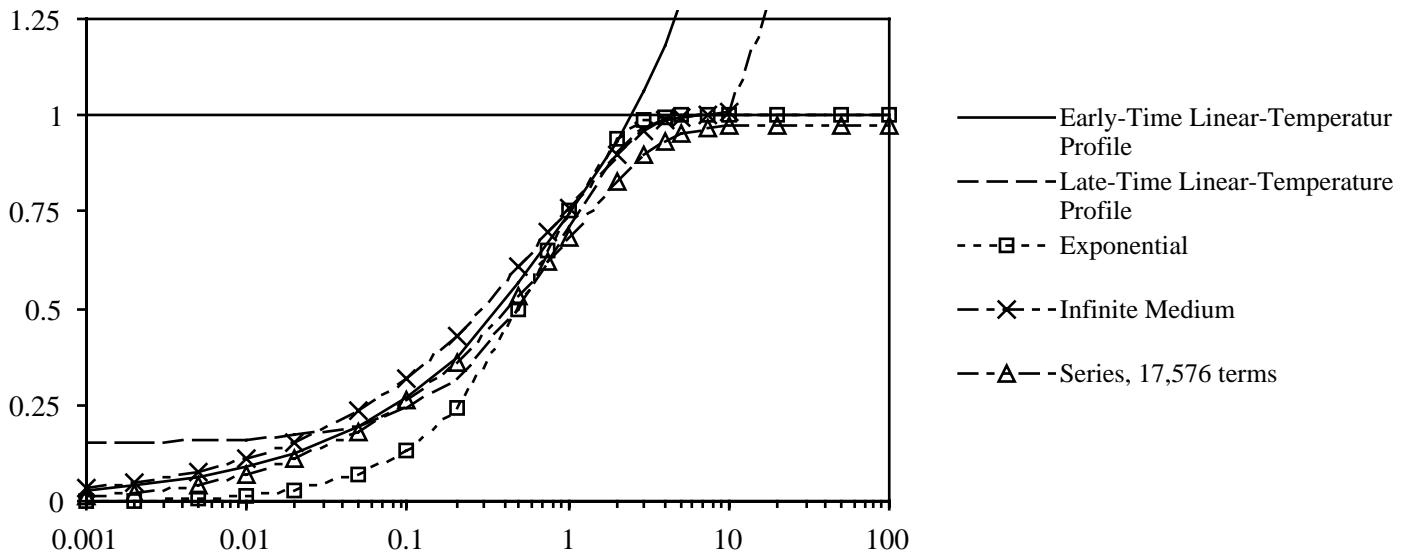
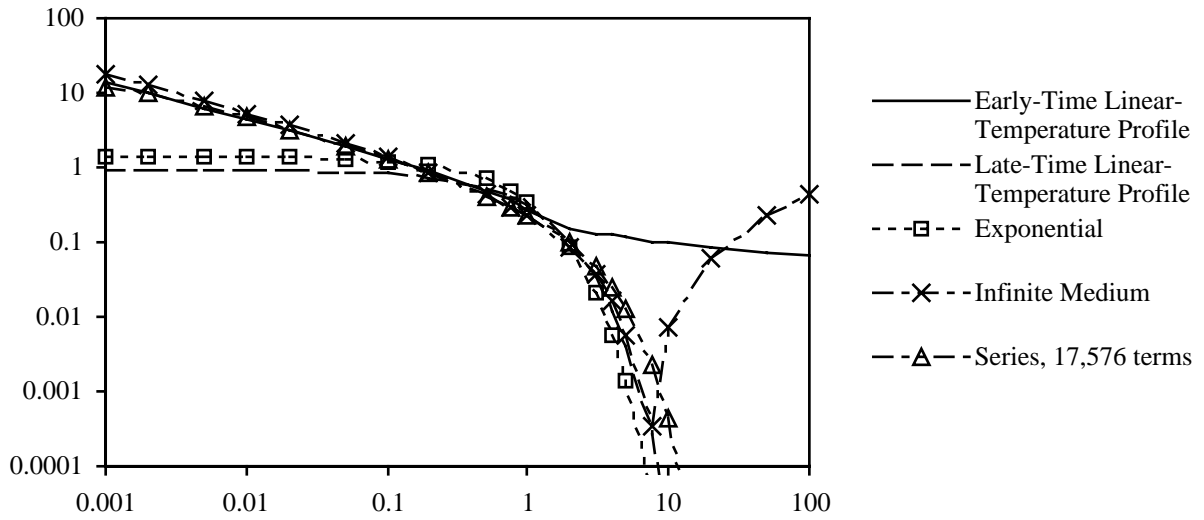


Figure 3: Comparison of New Model Conduction Rates

Figure 4: Comparison of New Cumulative Energy Conducted Models

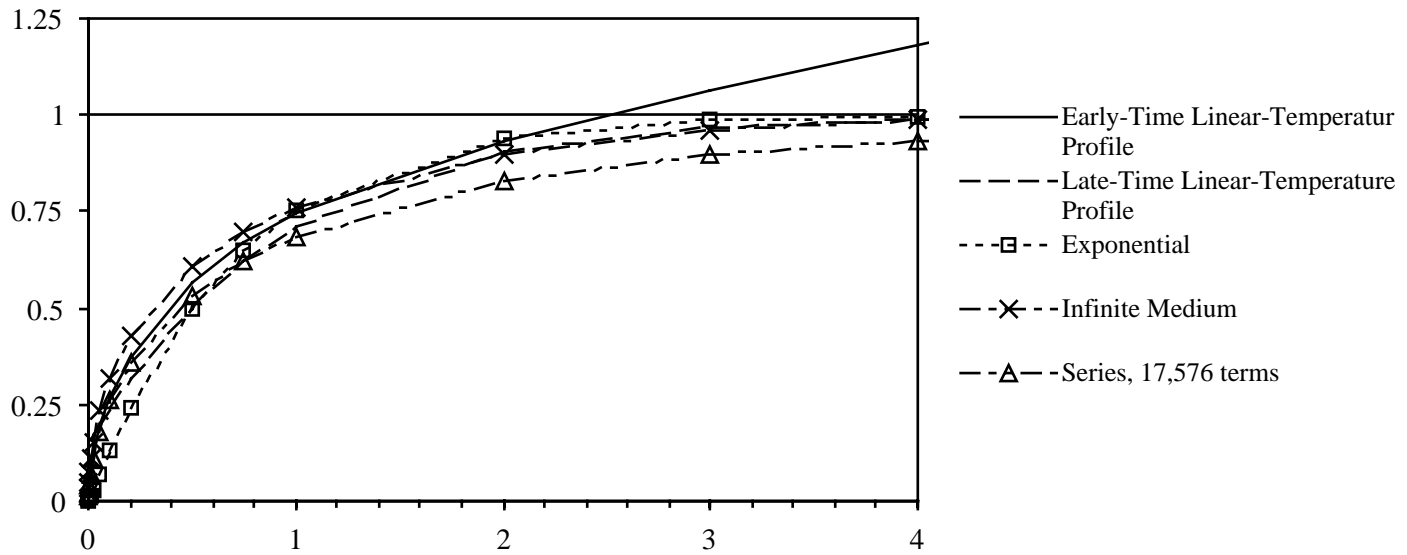


Figure 5: Comparison of new cumulative energy conducted models, linear scale

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