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FRACTURE SURFACE CHARACTERIZATION THROUGH X-RAY TOMOGRAPHY

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ABSTRACT

The surface of fractures and pore volumes in rocks affect many of the physico-chemical processes important to geothermal reservoirs and provides a record of the evolution of reservoir rock. Its geometric characterization is important. Computer tomographic imaging is developed into a technique for characterizing multiscale and fractal surfaces of pore-matrix interface of porous rock as well as inner fracture surfaces embedded in core samples. Horizontal slice images of surfaces and through the core images of vug structures are demonstrated by synthetic test cases.

INTRODUCTION

Heterogeneities in geothermal reservoir are encountered on every scale from kilometers to microscopic scales. One of the main difficulties in modeling and monitoring geothermal reservoirs and geothermal fractures is that the physical processes on the macroscopic level depend on interaction at microscopic scales. Small scale heterogeneities of the fractured rock have impact on physical and chemical processes in geothermal systems.

One of the important multiscale features of a geothermal fractured medium is the roughness of the fracture surface. The roughness of the fracture surfaces influences various processes in fractured rock. For example, mass transport in geothermal reservoirs is dependent on the geometry of the fracture surfaces dividing pore fluid and rock matrix (Brown, 1987; Brown et al., 1995; Glover et al., 1997). The modeling in (Glover et al., 1999) shows that rough surfaces increase the tortuosity of fluid flow, as well as influence the transport and mechanical properties of rocks which are extremely sensitive to the characteristics of the fractures. Hydrothermal alteration occurs at interfaces between the hydrothermal fluids and rock matrix. Water adsorbed on the interface might be significant in the fluid balance of a reservoir (Nielson et al., 1993). Likewise, heat transfer occurs across fracture surfaces. The nature and magnitude of electrical surface conduction (Ruffet et al, 1995; Revil and Glover, 1998) are important for understanding remote sensing of geothermal reservoirs using electro-magnetics. Electrical properties of fractured rock are influenced by the roughness of the fracture wall surface: through the electrical double layer at the fluid-matrix or at the pore-matrix interface (Revil and Darot, 1996; Revil et al., 1996). In (Ruffet et al., 1991) the internal surface area is characterized from electrical measurements of the complex resistivity, and the fractal dimension of the internal surface area is estimated from experimental data. For all these processes, and others besides, it is important to characterize the geometry of the fractures and fracture wall surfaces.

Traditional description of surface geometries have relied on direct appraisal of the interface, using such techniques as direct optical, X-ray backscattering (Hofgreve and Kunz, 1987), or SEM measurement (Berryman, 1987; Nielson et al., 1993; Russ, 1994). Fracture surfaces can have fractal characteristics and direct observation can be used to estimate fractal dimensions for these surfaces (Feder, 1988; Russ, 1994; Turcotte, 1997).

The present paper deals with the largely untackled problem of estimation of the geometry of fractures that are not opened for direct observation. It is a difficult problem because the surfaces are shielded by intervening rock. In our previous work, we used measurements of the effective complex permittivity (Cherkaeva and Tripp, 1996; Cherkaeva and Golden, 1998) and complex conductivity of a geophysical medium (Tripp et al., 1998) to derive estimates of the volume of one component in the mixture, such as pore or brine volume. This gives a volumetric characterization of fractured rock. The present work is concerned with problem of surface characterization. We use computer a tomographic imaging method for reconstruction of the rough surface and discuss possible characterizations of it. We consider a correlation functions approach to estimation of the surface specific area. It can be shown that the value of the derivative of the angularly averaged correlation function gives a specific surface area. This approach was previously investigated in (Debye et al., 1957; Berryman, 1987) for reconstruction of the surface area from SEM photographic images. We apply a similar method here for computer tomographic projection data that are given by the Radon transform. In conjunction with the Radon transform, the method works more efficiently, since the correlation functions of the medium can be calculated immediately from CT projection data. For rougher surfaces, we discuss the use of a fractal dimension for the characterization of roughness of the surface. The fractal characterization of the surface as well as the surface area can be estimated from the reconstructed images of the core samples as well as immediately from the raw tomographic data.

TOMOGRAPHIC RECONSTRUCTION

In X-ray tomography, a core sample is illuminated with X-rays emitted by sources encircling the sample. The density of the rock sample is the X-ray attenuation coefficient and the X-rays propagate in straight ray paths. The data gathered is the integral attenuation along the straight ray paths from source to detector. The attenuation path integrals are measured for a multitude of the sources, hence the projections are measured along numerous line paths through the object. The intent is that the interior density distribution of the core will be well sampled by the path rays from the applied sources. The projection data at the angle θ are given by the Radon transform of the density function $f(t, s)$:

$$P_\theta(t) = \int_{-\infty}^{\infty} f(t,s)\, ds$$

where s is the coordinate along the ray, t is the one in the orthogonal direction making the angle θ with the x direction of a fixed (x, y) Cartesian coordinate system.

Using the registered projection data $P_\theta(t)$, computer tomography (Kak and Slaney, 1988) reconstructs the spatially varying density of the rock without damaging the sample. There are numerous algorithms for effecting this information extraction (Deans, 1983). The reconstruction can be viewed as backprojecting filtered projections. Mathematical details of the reconstruction process are found in (Deans, 1983; Natterer, 1986). The efficiency of fracture surface characterization is based on a particular property of the Radon transform that permits evaluating the spectra of the density immediately from the measured data.

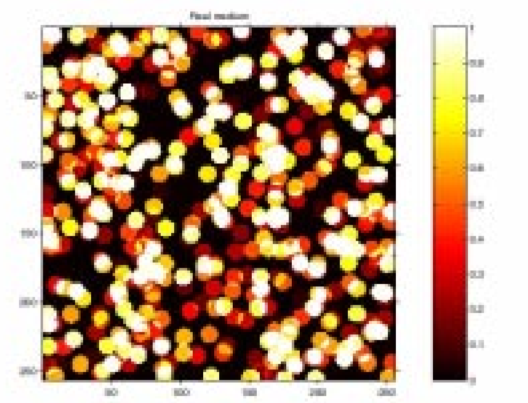

Figure 1. True density function for a section of a spherically pocked surface.

This property is formulated as Fourier Slice theorem which states that the Fourier image of the Radon transform $P_\theta(t)$ of the density function $f(x,y)$:

$$S_\theta(w) = \int_{-\infty}^{\infty} P_\theta(t)\, e^{2\pi i\, wt}\, dt$$

coincides with the Fourier transform of the density function $f(x, y)$

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{2\pi\, i(ux+vy)}\, dx\, dy$$

taken in a rotated coordinate system:

$$S_\theta(w) = F(w\cos\theta,\, w\sin\theta)$$

Using the Fourier Slice theorem, the object density function can be recovered from projections.
Indeed, with $t = x \cos\theta + y \sin\theta$, the object density function $f(x, y)$ can be reconstructed:

$$f(x,y) = \int_0^{\pi}\int_0^{\infty} S_\theta(w)\, |w|\, e^{2\pi\, i\, wt}\, dw\, d\theta$$

However, the inverse problem of the reconstruction of the density is an ill-posed problem. It manifests itself in amplification of the noise in the data caused by multiplication of $S_\theta(w)$ by fast growing term $|w|$ in the last integral. Hence special efforts are needed to ensure stability of the solution and high resolution which is necessary for reconstruction of such complicated objects as a fracture wall surface. We show below several simulated reconstruction examples. However, we will discuss the computational problems arising in high resolution CT imaging somewhere else, focusing here on deriving surface characterization from known spectra or correlation functions of the density. The above mentioned property of the Radon transform allows us to derive them immediately from the raw tomographic data.

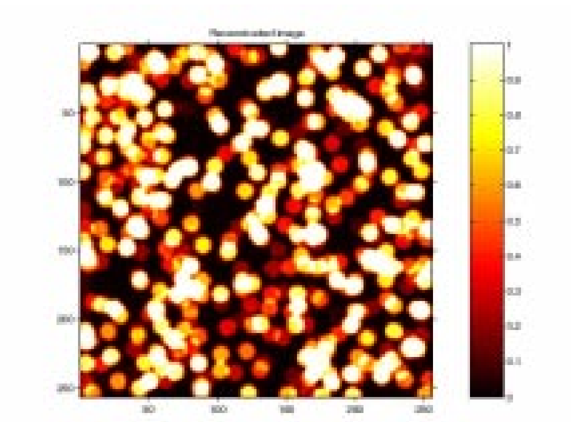

Figure 2. The reconstructed density function for the model shown in Figure 1.

TOMOGRAPHIC CORRELATION FUNCTION AND ROUGHNESS OF FRACTURE SURFACE

The present paper develops a new method of characterization of the fracture surface from tomographic data. The aforementioned property of the Radon transform permits computation of the two-point correlation function of the characteristic function of the medium immediately from the tomographic projections. Indeed, since the Fourier transform of the tomographic data $S_\theta(w)$ gives the Fourier image of the density $f(x, y)$ in a rotated coordinate system, the correlation functions can be computed using $S_\theta(w)$.

Two important properties of the fracture surface can be easily evaluated from the correlation functions. One is the specific surface area of a porous medium. Using an approach discussed in (Debye et al., 1957; Berryman, 1985; 1987), the specific surface area as well as the porosity of the medium can be estimated from the two-point correlation function as follows.

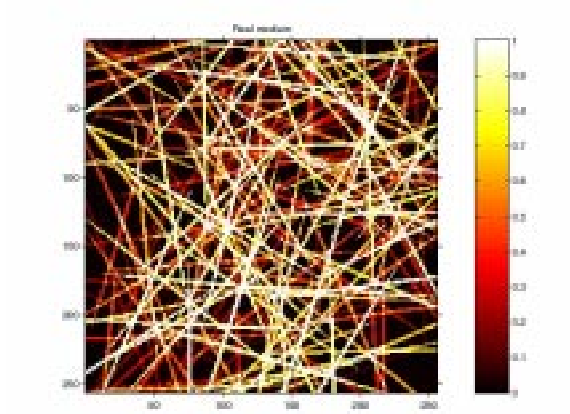

Figure 3. True random density function for a section of a surface.

For a porous medium, let the characteristic function $f(x)=1$ in void regions, $f(x)=0$ in the rock matrix. The two-point correlation function (with < > being spatial averaging over the x coordinate) is $F_2(r_1, r_2) = \langle f(x+r_1)\ f(x+r_2)\rangle$. A statistically homogeneous medium is translationally invariant, and the correlation function $F_2(r_1, r_2)$ depends only on a difference in coordinates, $F_2(r_1, r_2) = F_2(r_1 - r_2)$. The correlation functions contain a great deal of information about the structure of the medium. The porosity of the medium p is given by the value of the correlation function at zero: $p = F_2(0)$. In (Berryman, 1987) it is shown that the derivative $A(r)$ of the angular average of F_2 is related to the specific surface area s of the medium: $A(0) = -s/4$.

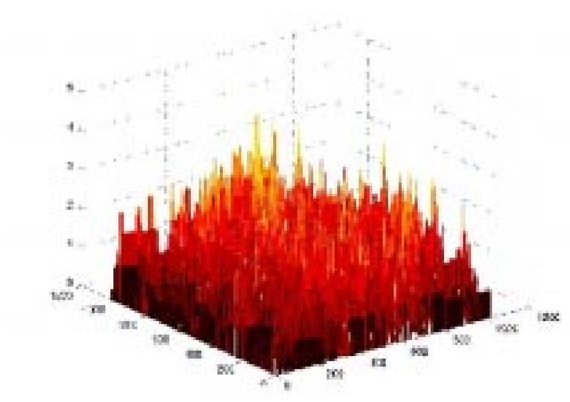

Figure 4. The true random density model.

This method of estimating the specific surface area was applied to analysis of digital SEM images and used in (Berryman and Blair, 1986) for evaluation of the fluid permeability from Kozeny-Carman relation. In combination with Radon transform, the method gives a possibility to calculate specific surface area from the tomographic data.

The second important property easily derived from the correlation function is a fractal dimension that characterizes roughness of the fracture surface. Geometrically complicated surfaces of fracture walls and porous media are often described using fractals and fractal surfaces (Russ, 1994). An example of such a random function is shown on Figure 4. Fractals are complex geometric shapes with fine structure at arbitrary small scales. Tiny parts of a fractal are approximately or statistically similar to the whole. Features of a part of a fractal are reminiscent of the whole fractal. One of the most important properties of such objects is that the power spectral density is proportional to a power of the spatial wave-frequency. Different definitions of fractal dimension have been introduced to describe particular features of a complicated object (e.g., Feder, 1988; Turcotte, 1992).

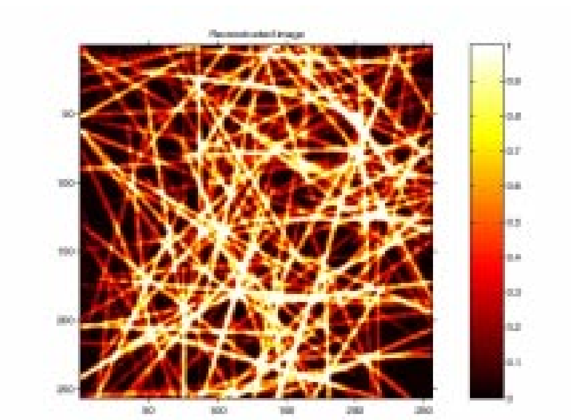

Figure 5. The reconstructed random density model.

The fractal dimension derived from Fourier analysis for a fractal profile is given by the slope of a log-log plot of the magnitude of the power spectral density versus the frequency. Similarly, for a two-dimensional density function or a surface, Fourier fractal characterization is derived from the corresponding two-dimensional or three-dimensional spectra.

SAMPLE RECONSTRUCTIONS

First, we consider two models with the source-detector plane parallel to a fracture surface which bisects the core. As we consider only two-dimensional reconstructions in the present paper, these models can also be viewed as two-dimensional slices of a porous volume.

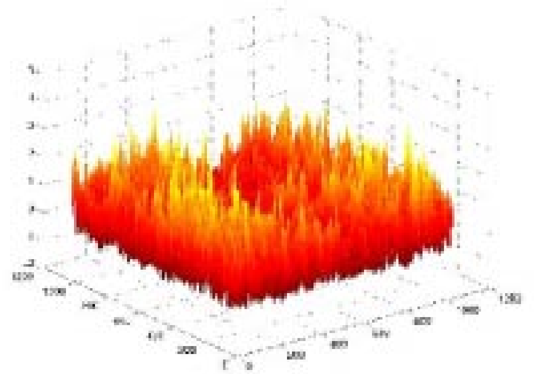

Figure 6. The correlation function for the random density model.

In the first model, the surface is "pocked" by randomly placed spherical voids. This might correspond to a situation where the rock contains some spherically shaped inclusion such as gas bubble pore volume. For the sake of the simulation, regular shaped inclusions provide a good initial test of the algorithm. Figures 1 and 2 illustrate the true and the reconstructed spherically pocked surface.

A more realistic representation for geometrically complicated geological surfaces and media can be generated using fractals and fractal surfaces (Russ, 1994). An example of such random function is shown on Figure 3. Figure 4 illustrates the corresponding density model which gives a Bird's Eye View of the complicated random surface. Figure 5 illustrates the reconstructed ray-path image. The reconstruction is good with minor artifacts, the resolution being dependent on pixel size and discretization. Physically, this random density model corresponds to a slice of filled with voids porous medium rather than three-dimensional surface dividing void from the rock matrix. Hence, we consider in this example the reconstruction of the inner surface of a fractured porous rock. In order to deal with real three dimensional surfaces we need to move to three dimensional "multi-slice" reconstruction algorithms where the presented "one-slice" reconstruction gives only one projection on a specific plane. However, even in a two-dimensional example, the complicated character of the inner surface in such a porous rock demonstrates the ability of the method to deal with the problem.

Figure 6 shows the two-point correlation function corresponding to this random density model. It is obvious from the picture that the surface is correlated on every spatial scale. The corresponding energy spectrum is shown on Figure 7.

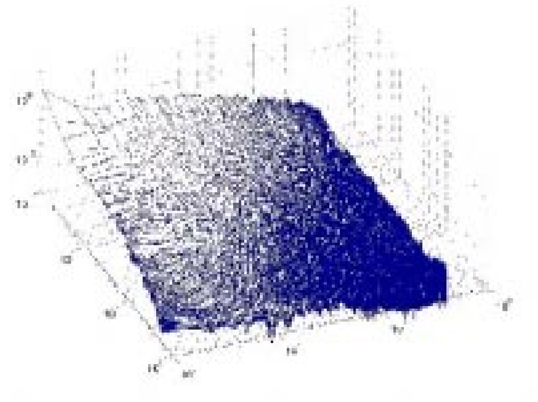

Figure 7. The spectrum of the random density model on a log-log scale.

The slope of the log power spectrum plotted with respect to the log spatial frequency in any direction gives an estimate of the fractal dimension of the surface in that direction. We have not discussed here a possible anisotropy of the surface, however, it is clear that in such a case, the anisotropy can be easily accessed from this estimate. Again, the correlation function, the energy spectrum, and fractal dimension can be derived immediately from the measured tomographic projections and do not require the reconstruction step.

The previous examples have assumed that a fracture has completely bisected the core sample. However, in many cases, surfaces of interest are completely shielded from the core exterior by intervening rock. To assess the resolution of reconstruction in such a case, we next consider a synthetic reconstruction of the gross surface features with a vug.

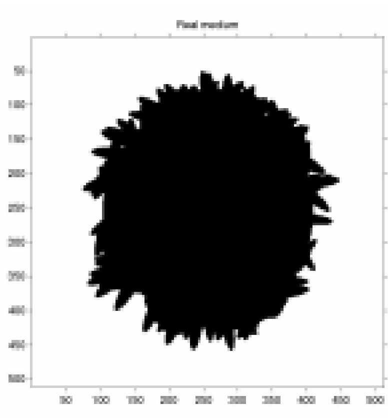

Figure 8. The true surface of a vug in a simulated core sample.

Figure 8 shows the two-dimensional slice of a synthetic core sample containing a void inside a complicated inner surface (true model). Figure 9 illustrates the reconstructed surface.

The sample reconstructions demonstrate good resolution achievable in through the core surface imaging. There are some possibilities of increasing information gained from fracture imaging, first of all, three-dimensional reconstruction which is a subject of our present study.

Figure 9. The reconstructed surface of a vug in a two-dimensional projection.

CONCLUSIONS

The paper demonstrates a through the core fracture surface imaging technique. The reconstructed two-dimensional images of synthetic core samples show good resolution and applicability of the technique to real materials. The fractal characterization of the fracture wall surface as well as the specific surface area can be derived immediately from the raw tomographic data.

ACKNOWLEDGEMENTS

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